Flavour in S_3 multi-Higgs models

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FLASY 2014 - Sussex University

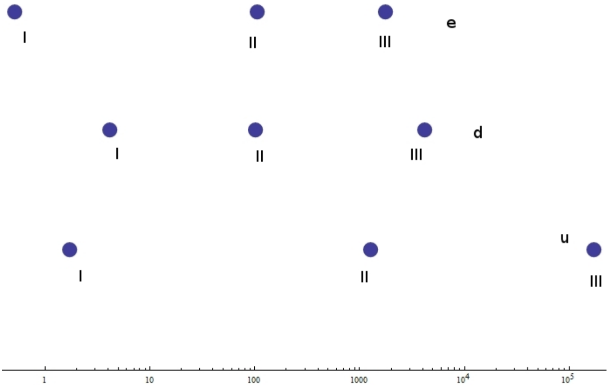
June 17, 2014

Contents

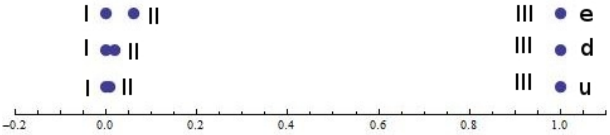
How do we choose a flavour symmetry?

- Find the smallest possible flavour symmetry suggested by the data
- Explore how generally ("universally") it can be applied
- Follow it to the end
- Compare with the data

Logarithmic plot of fermion masses



Plot of fermion mass ratios

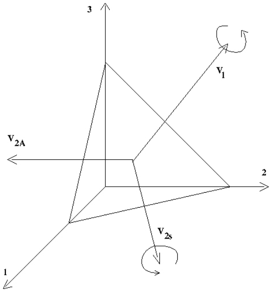


Fundamental fermions normalized by the heaviest of each type

suggests $\mathbf{2} \oplus \mathbf{1}$ structure

Also, prior to electroweak symmetry breaking all three families are interchangeable

The S_3 symmetry group: permutations of 3 objects.



Assignment between fermion fields and irreps:

$$\Phi \to F \quad = \quad F(\Phi_1, \Phi_2, \Phi_3)$$

F is a S_3 reducible representation $\mathbf{1_S} \oplus \mathbf{2}$

$$F_s \quad = \quad \frac{1}{\sqrt{3}}\Big(\Phi_1 + \Phi_2 + \Phi_3\Big); \qquad F_D = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2) \\ \\ \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3) \end{pmatrix}$$

Some references to work with an S_3 symmetry

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- E. Barradas et al, 2014
- P. Das et al, 2014
- There are many more, I apologize for those not included.

Facts

Some aspects of the flavour problem:

- Quark masses vastly different

$$m_u : m_c : m_t \approx 10^{-6} : 10^{-3} : 1, \quad m_d : m_s : m_b \approx 10^{-4} : 10^{-2} : 1,$$
$$m_e : m_\mu : m_\tau \approx 10^{-5} : 10^{-2} : 1.$$

- Quark weak mixing angles:
  - $\theta_{12} \approx 13.0^o$
  - $\theta_{23} \approx 2.4^o$
  - $\theta_{13} \approx 0.2^o$
- Lepton masses not known (only difference of squared masses), but extremely small
- Lepton weak mixing angles, best fit of recent experimental data
  - $\Theta_{12} \approx 33.9^o$
  - $\Theta_{23} \approx 46.1^o$
  - $\Theta_{13} \approx 9.2^o (IH\ 9.45^o)$
    $\Rightarrow \Theta_{13} \neq 0$
- CP-violation occurs in the weak sector

- In the SM with S_3, to give masses to the particles the flavor symmetry has to be broken

A. Mondragón, E. Rodríguez-Jáuregui, 1999, 2000

- The breaking of the symmetry can be parametrized with Z, satisfies cubic equation
- Possible to classify texture zeroes in equivalence classes, simplify analysis A. Mondragón and F. González, 2011
- If S_3 it's not explicitly broken then... $\Rightarrow$

  we need to introduce additionally two Higgs weak-doublets more to the SM to preserve the permutational symmetry (at least before eW breaking)

S_3 symmetry with 3 Higgs doublets in the Lagrangian

- To build an S_3 invariant Lagrangian:
  assign the first two families to the doublet irrep **2**
  the third one either to the singlet symmetric $\mathbf{1}_S$
  or the singlet antisymmetric $\mathbf{1}_A$.
- Add three right handed neutrinos to implement the seesaw mechanism
- All sectors follow this assignment: quarks, leptons (left and right) and Higgs
- Different assignments lead to different models
  Also important in symmetries left in the Higgs potential after eW breaking

González-Felipe, Ivanov, Nishi, Serodio and Silva, 2014

After electroweak symmetry breaking, the Higgs $SU(2)_L$ doublets acquire real vacuum expectation values (vev's),

$$w_1 \equiv \langle 0|H_1|0\rangle, \quad w_2 \equiv \langle 0|H_2|0\rangle,$$

$$v_S \equiv \langle 0|H_S|0\rangle, \quad \text{and} \quad v_A \equiv \langle 0|H_A|0\rangle,$$

J. Kubo, A. Mondragón, M. M., E. Rodríguez-Jáuregui, 2003

We get for every Dirac fermion the generic mass matrix:

$$\mathcal{M}_f = \begin{pmatrix} \mu_1 + \mu_2 & \mu_4 & \mu_5 \\ \mu_4 & \mu_1 - \mu_2 & \mu_6 \\ \mu_7 & \mu_8 & \mu_3 \end{pmatrix}$$

Furthermore, the concept of flavour is extended to the Higgs sector

Quarks

Numerical study of quarks and three Higgses showed compatibility with data

Kubo, Mondragón, M, Rodríguez-Jáuregui, 2003

FCNC's in quark sector are suppressed

Teshima, 2012

Quark lepton complementarity studied

Barranco, A. Mondragón, González Canales, 2009

Data on quarks has improved considerably $\Rightarrow$ important to go back to data and check compatibility with symmetry

Possible to classify different S_3 models in equivalence classes and obtain known textures

Comparison with recent data gives very good agreement between theoretical and experimental V_{CKM}

González-Canales, A& M Mondragón, Saldaña-Salazar, Velasco-Sevilla, 2013

- Depending on whether the singlet Higgs in the symmetric or anti-symmetric singlet irrep:
- $H_S \Rightarrow$ Viable models (quarks) only when left and right parts of the third family share the same assignement
- $H_A \Rightarrow$ Viable models (quarks) only when left and right part of the third family are in different irreps
- They lead to two zero textures (Fritzsch type) or NNI form
- Both known to give good phenomenology
- Case with four Higgs (H_A and H_S) reduces to the case with three Higgses, but Higgs potential may differ

2 zeroes mass matrices

We can bring the quark mass matrices from a symmetric basis to a hierarchical basis via a rotation and a shift

$$\mathcal{M}^f_{S_3} \longrightarrow \mathcal{M}^f_{Hier} \equiv \mathcal{R}(\theta)_{12} \mathcal{M}^f_{S_3} \mathcal{R}(\theta)^T_{12} = \begin{pmatrix} \mu^f_0 & a^f & 0 \\ a^{f*} & b^f & c^f \\ 0 & c^{f*} & d^f \end{pmatrix}$$
$$= \mu^f_0 \mathbf{1}_{3\times 3} + \widehat{\mathcal{M}}^f_{Hier} \ ,$$

The matrix $\widehat{\mathcal{M}}^f_{Hier}$ has two texture zeroes

$$\widehat{\mathcal{M}}^f_{Hier} = \begin{pmatrix} 0 & a^f & 0 \\ a^{f*} & b'^f & c^f \\ 0 & c^{f*} & d'^f \end{pmatrix} = \begin{pmatrix} 0 & a^f & 0 \\ a^{f*} & b^f - \mu^f_0 & c^f \\ 0 & c^{f*} & d^f - \mu^f_0 \end{pmatrix},$$

and eigenvalues denoted as σ^f_i, $i = 1, 2, 3$.

To achieve these textures we

- Perform a rotation
- May choose a particular value for the rotation angle θ, i.e. a particular change of basis
- Mass matrices hermitian or symmetric
- Shift the matrix and use the known two zeroes textures reparameterization

Rodríguez-Jáuregui, Mondragón, 2000;Barranco, González-Canales, Mondragón, 2008

Then, the physical masses m_i^f are related to the shifted masses σ_i^f simply by

$$m_i^f = \mu_0^f + \sigma_i^f .$$

We denote

$$\begin{array}{lll} \mu_1^f \equiv \sqrt{2} Y_2^f v_S, & \mu_2^f \equiv Y_3^f w_2, & \mu_3^f \equiv 2 Y_1^f v_S, \\ \mu_4^f \equiv Y_3^f w_1, & \mu_5^f \equiv \sqrt{2} Y_4^f v_A, & \mu_6^f \equiv \sqrt{2} Y_5^f w_1, \\ \mu_7^f \equiv \sqrt{2} Y_5^f w_2, & \mu_8^f \equiv \sqrt{2} Y_6^f w_1, & \mu_9^f \equiv \sqrt{2} Y_6^f w_2, \\ \nu_3^f \equiv 2 Y_1^f v_A, & & \end{array}$$

$w_{1,2}$ are the vev's of the Higgs doublets, and v_S is the one of the singlet

The shift and the rotation are unobservable as long as we rotate the matrices in the u and d sectors with the same angle θ

$$\tan\theta = w_1 / w_2$$

$\theta = \pi/3$ corresponds to one of the global minima of the Higgs potential

E. Rodríguez-Jáuregui, O. Félix, M.M., 2009; E. Barradas et al, 2014; P. Das et al, 2014

Diagonalization procedure

- Bring the matrices to the general basic S_3 form by means of a rotation and a shift

$$\mathcal{M}^f_{Hier} = \mu^f_0 \mathbf{1}_{3\times 3} + \widehat{\mathcal{M}}^f_{Hier}$$

- Factorize $\mathcal{M}^f_{Hier}$ in polar form in terms of a real symmetric matrix $\bar{\mathcal{M}}^f_{Hier}$ and a diagonal matrix of phases

$$\mathcal{P}_f \equiv \mathrm{diag}[1, e^{i\phi_{1f}}, e^{i(\phi_{1f}+\phi_{2f})}]$$

$$\bar{\mathcal{M}}^f_{Hier} \equiv \mathcal{P}^\dagger_f \frac{\widehat{\mathcal{M}}^f_{Hier}}{\sigma_3} \mathcal{P}_f$$

The CP phase will be a combination of these two phases, so there is only one independent phase in the V_{CKM}

- Normalize by the heaviest shifted mass in the sector

$$\tilde{\sigma}_i = \frac{\tilde{m}_i - \tilde{\mu}_3}{1 - \tilde{\mu}_3}; \quad \tilde{\sigma}_i = \frac{\sigma_i}{\sigma_3}; \quad \tilde{\mu}_3 = \frac{\mu_3}{m_3}; \quad \tilde{m}_i = \frac{m_i}{m_3}$$

i=1,2

- Reparameterize using the three matrix invariants

$$\begin{aligned} \mathrm{Tr}[\bar{\mathcal{M}}^f_{Hier}] &= \widetilde{\sigma}^f_1 - \widetilde{\sigma}^f_2 + 1 \ , \\ \mathrm{Det}[\bar{\mathcal{M}}^f_{Hier}] &= -\widetilde{\sigma}^f_1 \widetilde{\sigma}^f_2 \ , \\ \mathrm{Tr}[(\bar{\mathcal{M}}^f_{Hier})^2] &= (\widetilde{\sigma}^f_1)^2 + (\widetilde{\sigma}^f_2)^2 + 1 \ , \end{aligned}$$

- This allows us to write the mass matrix $\widehat{\mathcal{M}}^f_{S_3}$ in terms of its invariants and just one parameter δ_f

$$\bar{\mathcal{M}}^f_{Hier} = \begin{pmatrix} 0 & \sqrt{\frac{\widetilde{\sigma}^f_1 \widetilde{\sigma}^f_2}{1-\delta_f}} & 0 \\ \sqrt{\frac{\widetilde{\sigma}^f_1 \widetilde{\sigma}^f_2}{1-\delta_f}} & \widetilde{\sigma}^f_1 - \widetilde{\sigma}^f_2 + \delta_f & \sqrt{\frac{\delta_f}{1-\delta_f} \xi^f_1 \xi^f_2} \\ 0 & \sqrt{\frac{\delta_f}{1-\delta_f} \xi^f_1 \xi^f_2} & 1 - \delta_f \end{pmatrix},$$

where

$$\xi^f_1 \equiv 1 - \widetilde{\sigma}^f_1 - \delta_f, \quad \xi^f_2 \equiv 1 + \widetilde{\sigma}^f_2 - \delta_f \ ,$$

- δ_f is a constrained parameter, not entirely free: $0 < \delta_f < 1 - \widetilde{\sigma}^f_1$
- CP phase is only “free free” parameter

The CKM matrix

The V_{CKM} matrix is defined as

$$V_{CKM}^{th} = \mathbf{U}_{u_L}^{\dagger} \mathbf{U}_{d_L} = \mathbf{O}_u^T P^{(u-d)} \mathbf{O}_d,$$

where $P^{(u-d)} = \mathrm{diag}[1, e^{i\phi_1}, e^{i(\phi_1+\phi_2)}]$ with $\phi_i \equiv \phi_{iu} - \phi_{id}$, and $\mathbf{O}_{u,d}$ are the real orthogonal matrices, that diagonalize the mass matrix.

Thus, we can express the V_{CKM} as function of the quark masses:

exact, analytical expressions

$$V^1_{ud} = \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_s\xi^u_1\xi^d_1}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi^u_1\xi^d_1} + \sqrt{\delta_u\delta_d\xi^u_2\xi^d_2}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1},$$

$$V^1_{us} = \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d\xi^u_1\xi^d_2}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} - \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_s}{\mathcal{D}_{1u}\mathcal{D}_{2d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi^u_1\xi^d_2} + \sqrt{\delta_u\delta_d\xi^u_2\xi^d_1}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1},$$

$$V^1_{ub} = -\sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_d\xi^u_1}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} - \sqrt{\frac{\widetilde{\sigma}_u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\delta_d\xi^u_1} - \sqrt{\delta_u\xi^u_2\xi^d_1\xi^d_2}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1},$$

$$V^1_{cd} = \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_s\xi^u_2\xi^d_1}{\mathcal{D}_{2u}\mathcal{D}_{1d}}} - \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi^u_2\xi^d_1} + \sqrt{\delta_u\delta_d\xi^u_1\xi^d_2}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1},$$

$$V^1_{cs} = \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d\xi^u_2\xi^d_2}{\mathcal{D}_{2u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_s}{\mathcal{D}_{2u}\mathcal{D}_{2d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi^u_2\xi^d_2} + \sqrt{\delta_u\delta_d\xi^u_1\xi^d_1}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1},$$

$$V^1_{cb} = -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_d\xi^u_2}{\mathcal{D}_{2u}\mathcal{D}_{3d}}} + \sqrt{\frac{\widetilde{\sigma}_c}{\mathcal{D}_{2u}\mathcal{D}_{3d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\delta_d\xi^u_2} - \sqrt{\delta_u\xi^u_1\xi^d_1\xi^d_2}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1},$$

$$V^1_{td} = -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_s\delta_u\xi^d_1}{\mathcal{D}_{3u}\mathcal{D}_{1d}}} - \sqrt{\frac{\widetilde{\sigma}_d}{\mathcal{D}_{3u}\mathcal{D}_{1d}}}\left(\sqrt{\delta_u(1-\delta_u)(1-\delta_d)\,\xi^d_1} - \sqrt{\delta_d\xi^u_1\xi^u_2\xi^d_2}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1},$$

$$V^1_{ts} = -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_d\delta_u\xi^d_2}{\mathcal{D}_{3u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{\sigma}_s}{\mathcal{D}_{3u}\mathcal{D}_{2d}}}\left(\sqrt{\delta_u(1-\delta_u)(1-\delta_d)\,\xi^d_2} - \sqrt{\delta_d\xi^u_1\xi^u_2\xi^d_1}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1},$$

$$V^1_{tb} = \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_u\delta_d}{\mathcal{D}_{3u}\mathcal{D}_{3d}}} + \left(\sqrt{\frac{\xi^u_1\xi^u_2\xi^d_1\xi^d_2}{\mathcal{D}_{3u}\mathcal{D}_{3d}}} + \sqrt{\frac{\delta_u\delta_d(1-\delta_u)(1-\delta_d)}{\mathcal{D}_{3u}D_{3d}}}e^{i(\phi_2-\phi_1)}\right)e^{i\phi_1}\ ,$$

where

$$\begin{aligned}
\xi_1^{u,d} &= 1-\widetilde{\sigma}_{u,d}-\delta_{u,d}, \quad \xi_2^{u,d}=1+\widetilde{\sigma}_{c,s}-\delta_{u,d}, \\
\mathcal{D}_{1(u,d)} &= (1-\delta_{u,d})(\widetilde{\sigma}_{u,d}+\widetilde{\sigma}_{c,s})(1-\widetilde{\sigma}_{u,d}), \\
\mathcal{D}_{2(u,d)} &= (1-\delta_{u,d})(\widetilde{\sigma}_{u,d}+\widetilde{\sigma}_{c,s})(1+\widetilde{\sigma}_{c,s}), \\
\mathcal{D}_{3(u,d)} &= (1-\delta_{u,d})(1-\widetilde{\sigma}_{u,d})(1+\widetilde{\sigma}_{c,s}).
\end{aligned}$$

where $\phi_1 \neq 0, \ \phi_2 = \phi_1$

Using the most recent data for the quark masses

2013 values and mass ratios (with $\widetilde{m}_s^{th}$)			
m_t	171.8 ± 1.1		
m_b	2.85 ± 0.04		
m_c	0.63 ± 0.025	$\widetilde{m}_c\,(M_Z)$	0.0036 ± 0.00017
m_s	0.059 ± 0.0066	$\widetilde{m}_s\,(M_Z)$	0.021 ± 0.0026
m_d	0.0028 ± 0.0005	$\widetilde{m}_d\,(M_Z)$	0.00097 ± 0.00017
m_u	0.0013 ± 0.0005	$\widetilde{m}_u\,(M_Z)$	0.0000077 ± 0.0000030

We find a very good agreement of the V_{CKM} matrix with the data for appropriate values of the constrained parameters δ_f and a CP violating phase $\sim 80 \sim 100^o$

via a (test of hypothesis) χ^2 analysis

González-Canales, et al; 2014

Leptons

$-$	$+$
$H_S,\ \nu_{3R}$	$H_I,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

- In the leptonic sector we add a Z_2 symmetry
- FCNC's are strongly suppressed by the $S_3 \times Z_2$ symmetry and the mass hierarchy of the charged leptons
- Possible to write the mixing angles in terms of the lepton masses
- Predictions for neutrino masses and mixings
- S_3 gives $\theta_{13} \neq 0$
- If $M_{1R} = M_{2R}$, θ_{13} too small: lower bound, and θ_{12}, θ_{23} within experimental limits

A. Mondragón, M. M., E. Peinado, 2007,2008

- If $M_{1R} \neq M_{2R} \quad \Rightarrow \quad \theta_{12}, \theta_{23}, \theta_{13}$ compatible with recent data

A. Mondragón, M. M., F. González, 2012

Charged leptons

The mass matrix of the charged leptons takes the form

$$\mathbf{M}_e = m_\tau \begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix}. \quad (1)$$

Reparametrized in terms of its eigenvalues and written to order $\left(m_\mu m_e/m_\tau^2\right)^2$ and $x^4 = (m_e/m_\mu)^4$, is

$$\mathbf{M}_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}.$$

Only free parameter is the Dirac phase δ_e

Neutrinos

The $S_3 \times Z_2$ gives the following matrix for Dirac neutrinos

$$\mathbf{M}_{\nu_D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}, \qquad (2)$$

Kubo et al 2003, Felix, Mondragón 2006, Mondragon 2007

and considering the following form to the mass matrix of right-handed neutrinos $\mathbf{M}_{\nu_R} = \mathrm{diag}\{M_1, M_2, M_3\}$ Then, the mass matrix $\mathbf{M}_{\nu_L}$ takes the form

$$\mathbf{M}_{\nu_L} = \begin{pmatrix} \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} \\ \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} \\ \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} & \frac{2\left(\mu_4^\nu\right)^2}{\overline{M}} + \frac{\left(\mu_3^\nu\right)^2}{M_3} \end{pmatrix}, \quad \lambda = \frac{1}{2}\left(\frac{M_2 - M_1}{M_1 + M_2}\right), \text{ and } \overline{M} = 2\frac{M_1 M_2}{M_2 + M_1}.$$

Reparameterized in terms of the neutrino masses, $\mathbf{M}_{\nu_L}$ for a normal [inverted] hierarchy

$$\mathbf{M}_{\nu_L}^{N[I]} = \begin{pmatrix} \mu_0 + d & d & \frac{1}{\sqrt{2}}\left(C^{N[I]} + A^{N[I]}\right) \\ d & \mu_0 + d & \frac{1}{\sqrt{2}}\left(C^{N[I]} - A^{N[I]}\right) \\ \frac{1}{\sqrt{2}}\left(C^{N[I]} + A^{N[I]}\right) & \frac{1}{\sqrt{2}}\left(C^{N[I]} - A^{N[I]}\right) & m_{\nu_1} + m_{\nu_2} + m_{\nu_3} - 2\left(\mu_0 + d\right) \end{pmatrix} \quad (3)$$

with

$$C^{N[I]} = \sqrt{\frac{\left(2d + \mu_0 - m_{\nu_1}\right)\left(2d + \mu_0 - m_{\nu_{2[3]}}\right)\left(m_{\nu_{3[2]}} - \mu_0 - 2d\right)}{2d}}$$

$$A^{N[I]} = \sqrt{\frac{\left(m_{\nu_2} - \mu_0\right)\left(m_{\nu_{3[1]}} - \mu_0\right)\left(\mu_0 - m_{\nu_{1[3]}}\right)}{2d}}$$

$$d = \frac{2\,|\lambda|\,|\mu_2^\nu|^2}{\left|\overline{M}\right|} \qquad \mu_0 = \frac{2\,|\mu_2^\nu|^2}{\left|\overline{M}\right|}$$

The values allowed for the parameters μ_0 and $2d + \mu_0$ are in the following ranges: $m_{\nu_{2[1]}} > \mu_0 > m_{\nu_{1[3]}}$ and $m_{\nu_{3[2]}} > 2d + \mu_0 > m_{\nu_{2[1]}}$.

V_{PMNS} matrix also parameterized in terms of the leptons masses. Possible to write the mixing angles in terms of the lepton masses, and the parameters μ_0 and d, which carry the information of the heavy right-handed neutrinos

A. Mondragón, M. M., E. Peinado, 2007, 2008; F. González, A. Mondragón, M.M., 2012

For the reactor mixing angle θ^l_{13} and for an inverted neutrino mass hierarchy $(m_{\nu_2} > m_{\nu_1} > m_{\nu_3})$ we obtain:

$$\sin^2 \theta^l_{13} \approx \frac{(\mu_0 + 2d - m_{\nu_3})(\mu_0 - m_{\nu_3})}{(m_{\nu_1} - m_{\nu_3})(m_{\nu_2} - m_{\nu_3})}.$$

We have information on the right-handed neutrinos $M_{1,2}$ in the mixing angles

When $d = 0 \Rightarrow M_1 = M_2$ gives lower bound for θ_{13}

recover previous results of A. Mondragón, M.M., E. Peinado

- The case $M_1 = M_2$ gives us a lower bound for θ^l_{13} and sets the mass of the Dirac neutrinos
- Using these results as a starting point, we can do an analysis with μ_0 and d and find the best values for the mixing angles
- For a normal neutrino mass hierarchy
  $m_{\nu_1} = 3.22 \times 10^{-3}$ eV, $m_{\nu_2} = 9.10 \times 10^{-3}$ eV, $m_{\nu_3} = 4.92 \times 10^{-2}$ eV
  and the parameter values
  $\delta_l = \pi/2$, $\mu_0 = 0.049$ eV and $d = 8 \times 10^{-5}$ eV,
  we obtain

$$\sin^2 \theta^l_{13} \approx 0.029 \longrightarrow \theta^l_{13} \approx 10.8^\circ,$$

  in good agreement with experimental data.
- The solar and atmospheric mixing angles:

$$\theta^{l\,th}_{12} = 35^\circ, \quad \theta^{l\,th}_{23} = 46^\circ,$$

- Complete analysis similar to the one presented here with χ^2 and both hierarchies underway

$S_3 \times Z_2$ in both quarks and leptons

- S_3 works very well, what happens when we go to higher energies?
- Could it be a residual symmetry from a larger broken one?
- Study details and variations of the symmetry
- We look at $S_3 \times Z_2$ in both sectors, with different assignments of the Higgs singlet
- The symmetry gives NNI textures
- Leads to different models with different features
- In some models mass matrices similar to the Q_6 case

(Kawashima, Kubo, Lenz, 2009; see Félix González's talk)

Some lost textures

Two examples, with simplfiying assumptions $w_1 = w_2$ and $M_{1R} = M_{2R}$

Matter	L_I	L_3	ℓ_{IR}	ℓ_{3R}	Q_I	Q_3	d_{IR}	d_{3R}	u_{IR}	u_{3R}	N_I	N_3	H_I	H_3
$\mathbf{S}_3$	$\mathbf{2}$	$\mathbf{1}_S$	$\mathbf{2}$	$\mathbf{1}_S$	$\mathbf{2}$	$\mathbf{1}_S$	$\mathbf{2}$	$\mathbf{1}_A$	$\mathbf{2}$	$\mathbf{1}_A$	$\mathbf{2}$	$\mathbf{1}_A$	$\mathbf{2}$	$\mathbf{1}_A$
$\mathbf{Z}_2$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1

Table: Assignment for scenario C

Matter	L_I	L_3	ℓ_{IR}	ℓ_{3R}	Q_I	Q_3	d_{IR}	d_{3R}	u_{IR}	u_{3R}	N_I	N_3	H_I	H_3
S_3	$\mathbf{2}$	$\mathbf{1}_S$	$\mathbf{2}$	$\mathbf{1}_A$	$\mathbf{2}$	$\mathbf{1}_S$	$\mathbf{2}$	$\mathbf{1}_A$	$\mathbf{2}$	$\mathbf{1}_A$	$\mathbf{2}$	$\mathbf{1}_S$	$\mathbf{2}$	$\mathbf{1}_A$
Z_2	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1

Table: Assignment for scenario D

Both give mass matrices for the quarks which lead to NNI after a $\pi/4$ rotation

$$\mathbf{M}_{(u,d)} = \begin{pmatrix} a_{(u,d)} & 0 & b_{(u,d)} \\ 0 & a_{(u,d)} & -b_{(u,d)} \\ c_{(u,d)} & c_{(u,d)} & d_{(u,d)} \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & \hat{a}_f & 0 \\ \hat{a}_f & 0 & \hat{B}_f \\ 0 & \hat{C}_f & \hat{d}_f \end{pmatrix}; \quad \mathbf{U}_{\pi/4} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In the leptonic sector the chaged mass matrix for D is the same as for the quarks, but in C it is different

$$\mathbf{M}_\ell = \begin{pmatrix} a_\ell & 0 & b_\ell \\ 0 & a_\ell & b_\ell \\ c_\ell & c_\ell & 0 \end{pmatrix}$$

The effective neutrino mass matrix $\mathbf{M}_\nu = \mathbf{M_D M_R^{-1} M_D^T}$ for C gives

$$\mathbf{M}_\nu = \begin{pmatrix} A_\nu & -B_\nu & C_\nu \\ -B_\nu & A_\nu & D_\nu \\ C_\nu & D_\nu & E_\nu \end{pmatrix}$$

For scenario D in the neutrino sector, the different assignment leads to

$$\mathbf{M}_\nu = \begin{pmatrix} A_\nu & B_\nu & C_\nu \\ B_\nu & A_\nu & C_\nu \\ D_\nu & D_\nu & E_\nu \end{pmatrix}$$

where the entries in the two matrices are different combinations of the vevs and the parameters $x = 1/M_1$ and $y = 1/M_2$.

CKM matrices

Now we can express the V_{CKM} matrix in terms of only three parameters, δ_u, δ_d and a phase. The V_{CKM} elements look like

$$V_{us} \approx \sqrt{\frac{\tilde{m}_d \delta_d}{\tilde{m}_s}} - \sqrt{\frac{\tilde{m}_u \delta_u}{\tilde{m}_c}} \exp(i\bar{\eta}_{q_2}), \quad V_{cd} \approx \sqrt{\frac{\tilde{m}_u \delta_u}{\tilde{m}_c}} - \sqrt{\frac{\tilde{m}_d \delta_d}{\tilde{m}_s}} \exp(i\bar{\eta}_{q_2}). \qquad (4)$$

where δ_f are constrained parameters $1 > \delta_f > \tilde{m}_{f_2} > \tilde{m}_{f_1}$

Then the Jarlskog invariant comes out as a prediction

A previous analysis of this type of matrices in S_3 gives good fit to the data, work in progress

J. Barranco, F. González, A. Mondragón, 2009

PMNS matrices

The orthogonal matrix O_ν that appears in the V_{PMNS} is,

$$\mathbf{O}_\nu \begin{pmatrix} \sqrt{\frac{e^\nu_- p^\nu_1 k^\nu_2}{r^\nu_1}} & -\sqrt{\frac{e^\nu_- p^\nu_2 k^\nu_1}{r^\nu_2}} & \sqrt{\frac{e^\nu_+ k^\nu_1 k^\nu_2}{r^\nu_3}} \\ \sqrt{\frac{p^\nu_1 k^\nu_1}{r^\nu_1}} & \sqrt{\frac{p^\nu_2 k^\nu_2}{r^\nu_2}} & \sqrt{\frac{p^\nu_3 k^\nu_3}{r^\nu_3}} \\ -\sqrt{\frac{e^\nu_+ p^\nu_2 k^\nu_1}{r^\nu_1}} & -\sqrt{\frac{e^\nu_+ p^\nu_1 k^\nu_2}{r^\nu_2}} & \sqrt{\frac{e^\nu_- p^\nu_1 p^\nu_2}{r^\nu_3}} \end{pmatrix} \tag{5}$$

with

$$r^\nu_{(1,2)} \equiv (1 - \tilde{m}_{\nu_{(1,2)}})(\tilde{m}_{\nu_2} - \tilde{m}_{\nu_1}), r^\nu_3 \equiv (1 - \tilde{m}_{\nu_2})(1 - \tilde{m}_{\nu_1}), \quad p^\nu_{(1,2)} \equiv |\tilde{x}_+| - \tilde{m}_{\nu_{(1,2)}};$$

$$p^\nu_3 \equiv 1 - |\tilde{x}_+|, \quad k^\nu_1 \equiv |\tilde{x}_-| - \tilde{m}_{\nu_1}, k^\nu_2 \equiv \tilde{m}_{\nu_2} - |\tilde{x}_-|, \quad k^\nu_3 \equiv 1 - |\tilde{x}_-|, \quad e^\nu_\mp = \frac{1 - |\tilde{x}_\mp|}{|\tilde{x}_+| - |\tilde{x}_-|}.$$

Here, $|\tilde{x}_+|$ and $|\tilde{x}_-|$ are two free parameters which cannot be fixed in terms of the physical masses, and $1 > |\tilde{x}_+| > \tilde{m}_{\nu_2} > |\tilde{x}_-| > \tilde{m}_{\nu_1}$.

For scenario C, making the following approximations $|\tilde{x}_+| \approx 2\tilde{m}_{\nu_2}$ and $|\tilde{x}_-| \approx 2\tilde{m}_{\nu_1}$

$$O^{\nu}_{11} \approx \mathcal{O}(1), \quad O^{\nu}_{21} \approx \sqrt{\frac{\tilde{m}_{\nu_1}}{2\tilde{m}_{\nu_2}}}, \quad O^{\nu}_{31} \approx -\sqrt{\frac{\tilde{m}_{\nu_1}(\tilde{m}^{-}_{\nu_2} - \tilde{m}_{\nu_2})}{2\tilde{m}_{\nu_2}}}, \quad O^{\nu}_{12} \approx -\sqrt{\frac{\tilde{m}_{\nu_1}\tilde{m}^{-}_{\nu_2}}{\tilde{m}_{\nu_2}}};$$

$$O^{\nu}_{22} \approx \sqrt{\frac{\tilde{m}^{-}_{\nu_2}}{2}}, \quad O^{\nu}_{32} \approx -\sqrt{\frac{\tilde{m}^{+}_{\nu_2}}{2}}, \quad O^{\nu}_{13} \approx \sqrt{\tilde{m}_{\nu_1}\tilde{m}_{\nu_2}\tilde{m}^{+}_{\nu_2}}, \quad O^{\nu}_{23} \approx \sqrt{\frac{\tilde{m}^{+}_{\nu_2}}{2}}, \quad O^{\nu}_{33} \approx \sqrt{\frac{\tilde{m}^{-}_{\nu_2}}{2}}.$$

where $\tilde{m}^{\pm}_{\nu_2} \equiv 1 \pm \tilde{m}_{\nu_2}$. With these particular values for the free parameters, one obtains

$$\sin\theta_{13} \approx \sqrt{\tilde{m}^{+}_{\nu_2}/2}\left[\sqrt{2\tilde{m}_{\nu_1}\tilde{m}_{\nu_2}} - \tilde{m}_{\mu}\exp i\eta_{\nu}\right]; \sin\theta_{23} \approx \frac{\exp i\eta_{\nu_3}\sqrt{\tilde{m}^{-}_{\nu_2}/2}}{\sqrt{1 - |\sin\theta_{13}|^2}};$$

$$\sin\theta_{12} \approx \frac{-\sqrt{\tilde{m}^{-}_{\nu_2}/2}}{\sqrt{1 - |\sin\theta_{13}|^2}}\left[\sqrt{2\tilde{m}_{\nu_1}/\tilde{m}_{\nu_2}} + \tilde{m}_{\mu}\exp i\eta_{\nu}\right]$$

where $\cos\theta_{\ell} \approx 1 - \tilde{m}^2_{\mu}/2$ and $\sin\theta_{\ell} \approx \tilde{m}_{\mu}$.

We have three equations with three variables.

Scenario D is the same as $SU(5) \times Q_6$

And now?

- The details of the Higgs potential are crucial to distinguish between scenarios
- Flavour symmetry breaking related directly to the electroweak breaking
- In S_3 models in the quark sector at least one of the global minima is compatible with good phenomenology $\theta = \pi/6$
- Other minima have to be analysed
- In the $S_3 \times Z_2$ models the more general models have to be studied ($w_1 \neq w_2$ and $M_1 \neq M_2$), together with the potential
- Try to embed in a Grand Unified Theory

  e.g. Antusch et al, 2011; Cooper et al, 2010, 2012; Raby et al 2011; and many more...

- One $S_3 \times Z_2$ gives same matrices as $SU(5) \times Q_6$
  $\Rightarrow$ NNI form with interesting neutrino sector

  J.C. Gómez Izquierdo, F. González Canales, M.M.

- Full analysis of leptonic and Higgs sectors underway

Conclusions

- The permutational symmetry S_3 is the smallest non-abelian discrete symmetry suggested by data
- Used as a flavour symmetry in the quark, lepton and Higgs sectors allows for a "unified" treatment of fermion masses
- Interesting new textures with $S_3 \times Z_2$ also in quark sector
- Possible to find analytical expressions for mixing matrices of both quarks and leptons in terms of mass ratios
- Good phenomenology **both** in the quark and lepton sectors
- In the quark sector $\Rightarrow$ Fritzsch mass textures and the NNI form, both of which give good phenomenology $\Rightarrow$ fitting V_{CKM} with only three parameters
- In the leptonic sector $\Rightarrow$ the neutrino masses and mixing angles compatible with experimental data, related to the lepton and right-handed neutrino masses