

Yukawa Coupling Ratios in $SU(5)$ GUTs

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Based on collaborations with:

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FLASY 2014

Outline

- Introduction
- Yukawa Coupling Ratios
 - Theoretical Considerations
 - Comparison to Data & Applications
 - GUT Breaking
- Summary and Conclusions

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Why Grand Unification?

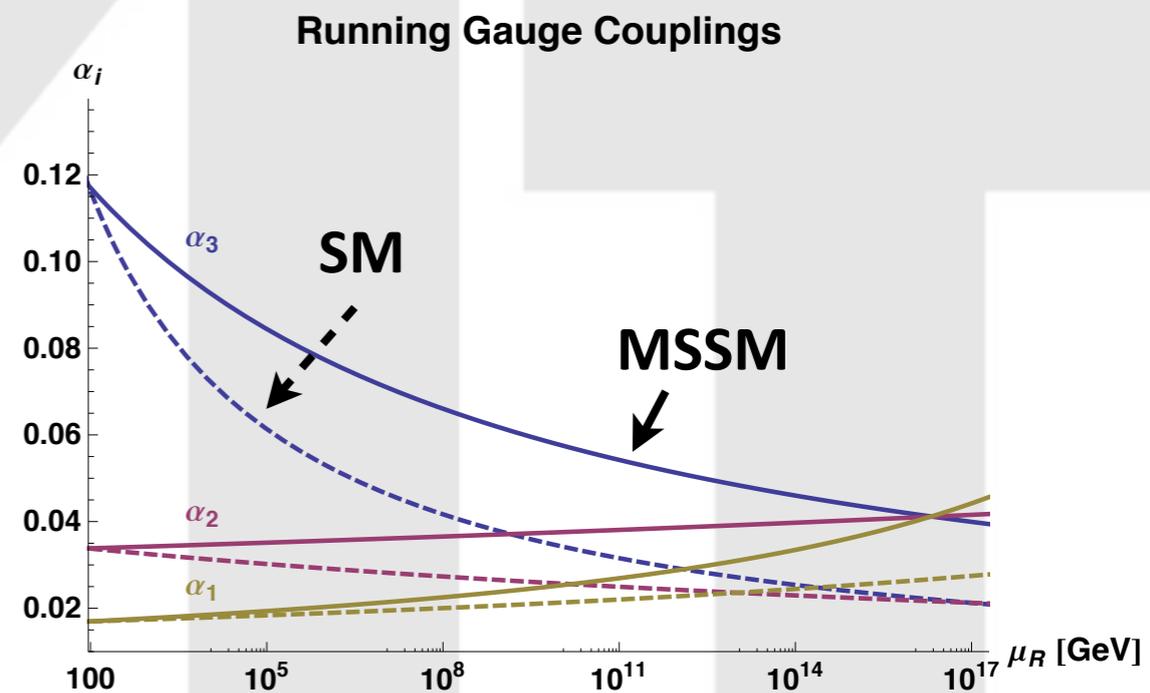
- Simplification (unify $SU(3)_C \times SU(2)_L \times U(1)_Y$)
- Solves one naturalness problem
 - ➔ The cancellation of the electron and proton charge in H
- Relates fermion masses (Yukawa couplings)
- Small neutrino masses due to the seesaw

Why SUSY GUTs?

- What is SUSY?



- What is SUSY good for?
 - Gives dark matter candidate
 - Solves the hierarchy problem
 - Gauge coupling unification



Why this talk?

- What do GUTs tell us about relations among fermion masses?
- Are these relations realistic?
- Can we learn more from that?
- Implications from/for GUT breaking?

Outline

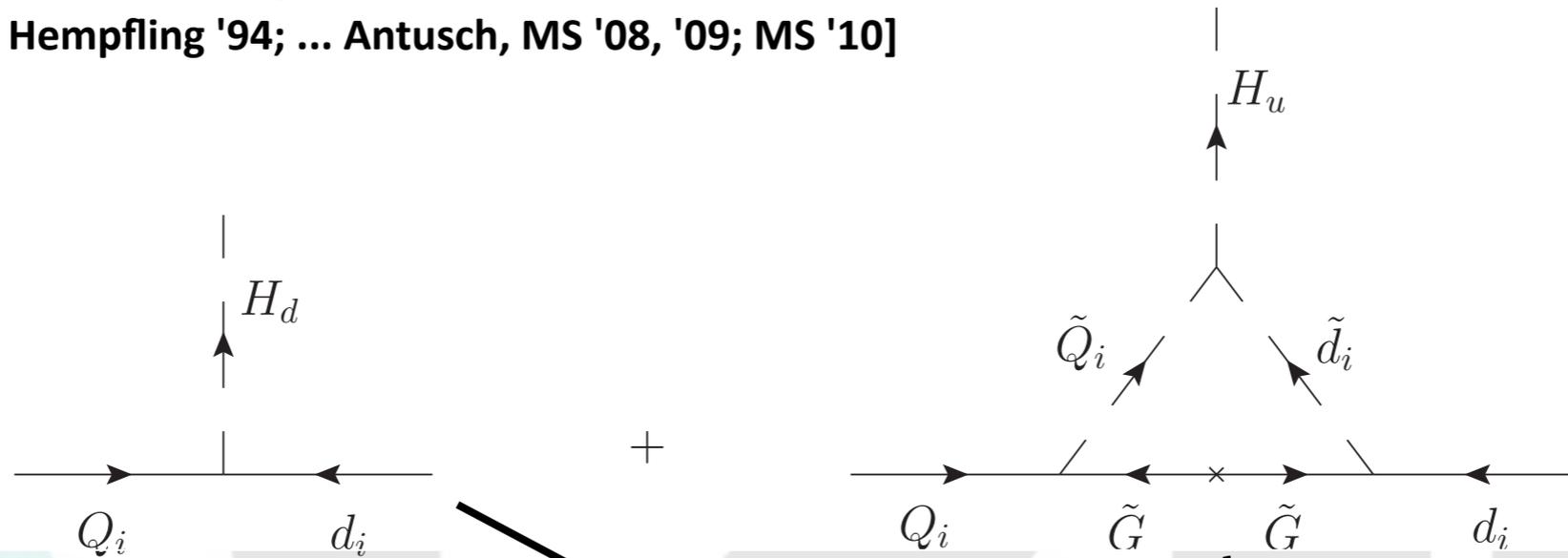
- Introduction
- **Yukawa Coupling Ratios**
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SUSY Threshold Corrections I

[Hall, Rattazzi, Sarid '94; Blazek, Raby, Pokorski '95; Carena, Olechowski, Pokorski, Wagner '94; Hempfling '94; ... Antusch, MS '08, '09; MS '10]



Matching:

$$\mathcal{L}_{\text{eff}}^{\text{MSSM}} = -y_b^{\text{MSSM}} (\bar{b}_R Q_L H_d + \varepsilon_b \bar{b}_R Q_L H_u^c) + \text{H.c.}$$

to:

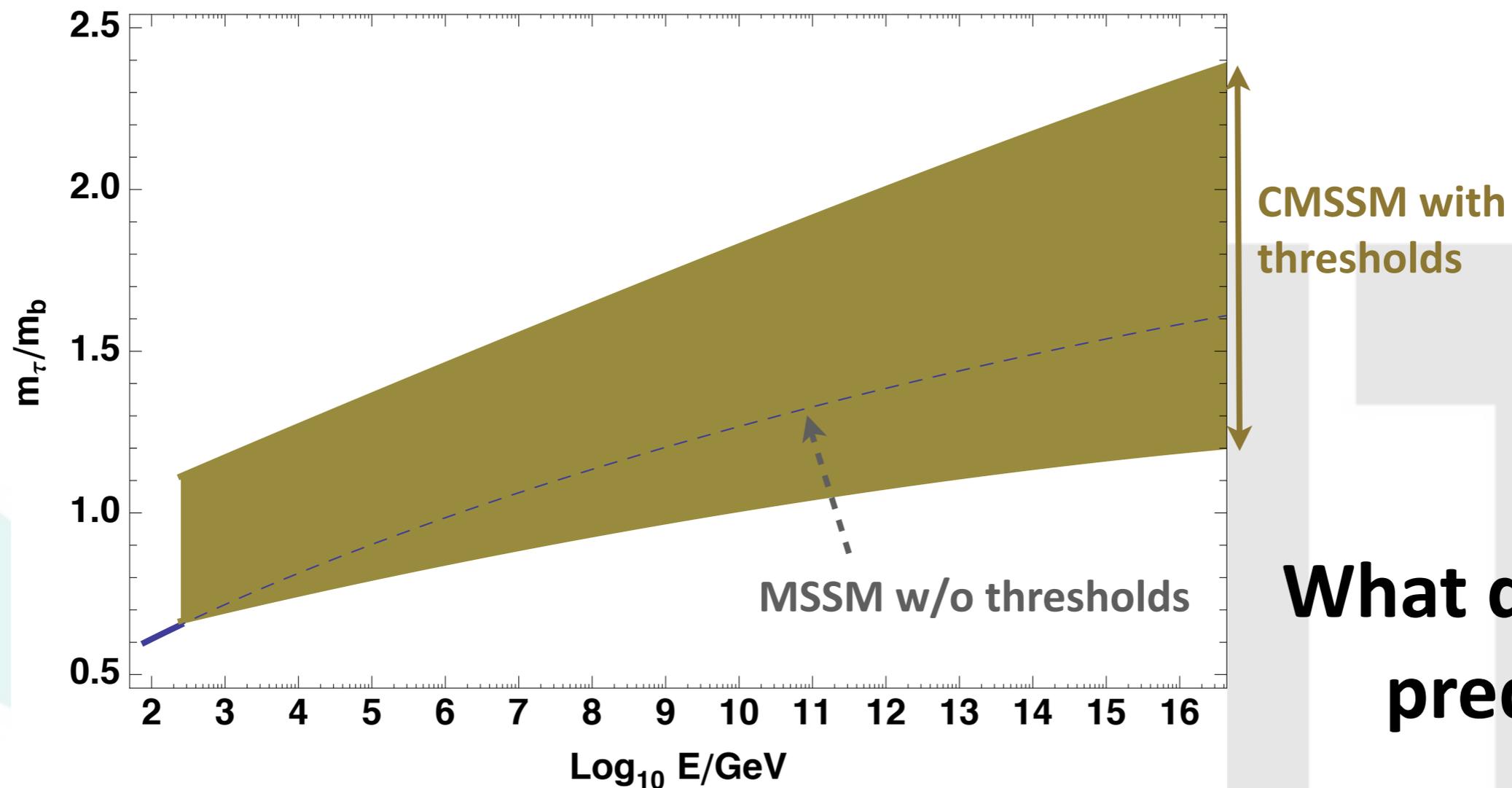
$$\mathcal{L}^{\text{SM}} = -y_b^{\text{SM}} \bar{b}_R Q_L \epsilon H + \text{H.c.}$$

gives:

$$y_b^{\text{SM}} = y_b^{\text{MSSM}} \cos \beta (1 + \varepsilon_b \tan \beta)$$

$\tan \beta$ enhanced!!!

SUSY Threshold Corrections II



What do GUTs predict?

Alternative Ratios I

Consider first the most minimal case:

$$F_3 T_3 \langle \bar{H}_5 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} b_R^c \\ b_B^c \\ b_G^c \\ \tau \\ -\nu_\tau \end{pmatrix} \begin{pmatrix} 0 & -t_G^c & t_B^c & -t_R & -b_R \\ t_G^c & 0 & -t_R^c & -t_B & -b_B \\ -t_B^c & t_R^c & 0 & -t_G & -b_G \\ t_R & t_B & t_G & 0 & -\tau^c \\ b_R & b_B & b_G & \tau^c & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}$$

$$\xrightarrow{SSB} -\frac{v}{\sqrt{2}} (\bar{b}b + \bar{\tau}\tau) \longrightarrow y_\tau / y_b = 1$$

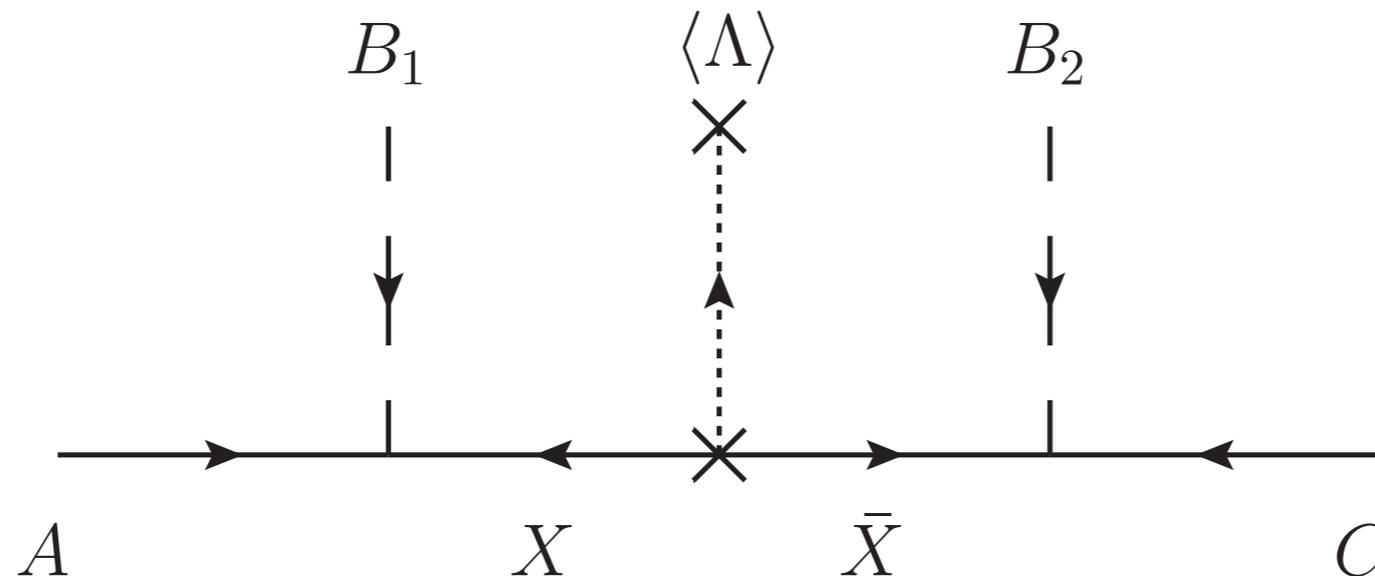
[Georgi, Glashow '74, see also talk by Q. Shafi]

Using a 45-dim. Higgs gives a ratio of -3

[Georgi, Jarlskog '79]

Alternative Ratios II

[Antusch, MS '09; MS '10; Antusch, King, MS '13]



- Take $A = F_2$, $B_1 = H_{24}$, $B_2 = H_{45}$, $C = T_2$, $X = 5$, $\Lambda = 1$
- From $\langle H_{24} \rangle$: $\gamma_\mu/\gamma_s = -3/2$
- From $\langle H_{45} \rangle$: $\gamma_\mu/\gamma_s = -3$
- From both combined: $\gamma_\mu/\gamma_s = 9/2$

Alternative Ratios III

[Antusch, MS '09; MS '10; Antusch, King, MS '13;
dim. 6 SU(5) operators Antusch *et al.* '14]

Operator Dimension	$(Y_e)_{ji}/(Y_d)_{ij}$
4	1 -3
5	$1/6$ $-1/2$ $-2/3$ 1 $\pm 3/2$ -3 $9/2$ 6 -18

[Antusch, King, MS '13] (red arrows pointing to $1/6$, $-1/2$, $-2/3$)

[Antusch, MS '09; MS '10] (green arrows pointing to $1/6$, $-1/2$, $-2/3$, 1 , $\pm 3/2$, -3 , $9/2$, 6 , -18)

Summary of possible predictions from SU(5)

Operator Dimension	$((Y_e)_{ij}/(Y_d)_{ij}, (Y_u)_{ij}/(Y_d)_{ij})$
4	(1, 1) (-3, 1)
5	$(0, \pm 1)$ $(-1/3, \pm 1)$ $(1, \pm 1)$ $(3/2, \pm 1)$ $(-3, \pm 1)$ $(9, \pm 1)$

[Antusch, King, MS '13] (red arrows pointing to $(0, \pm 1)$, $(-1/3, \pm 1)$, $(1, \pm 1)$, $(3/2, \pm 1)$)

[Antusch, MS '09; MS '10] (green arrow pointing to $(9, \pm 1)$)

Summary of possible predictions from Pati-Salam

Which ratios are realistic?

Outline

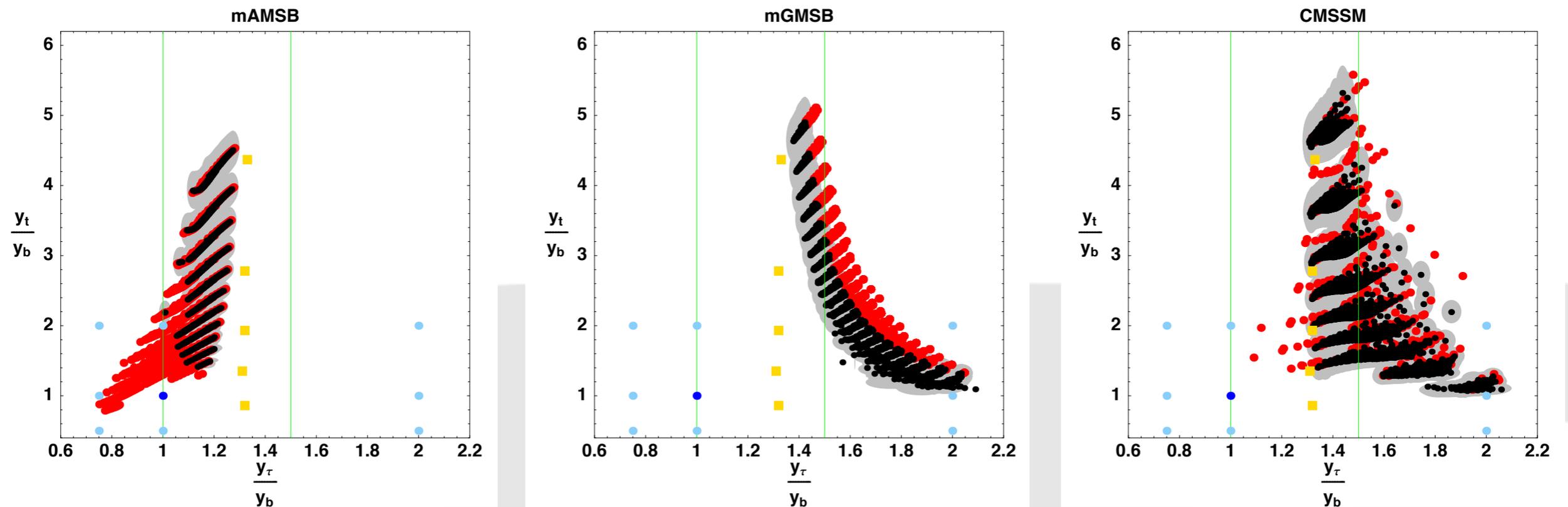
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Used Data in the 2009 study

- Direct LEP searches
- Electroweak precision observables
- B-physics:
 - $\text{BR}(b \rightarrow s\gamma) = (3.55 \pm 0.36) \times 10^{-4}$
 - $\text{BR}(B_s \rightarrow \mu\mu) \leq 5.8 \times 10^{-8}$
- Anomalous magnetic moment of the muon

Presented data

Results for the 3rd generation



exp. allowed (2009)

exp. disavored (2009)

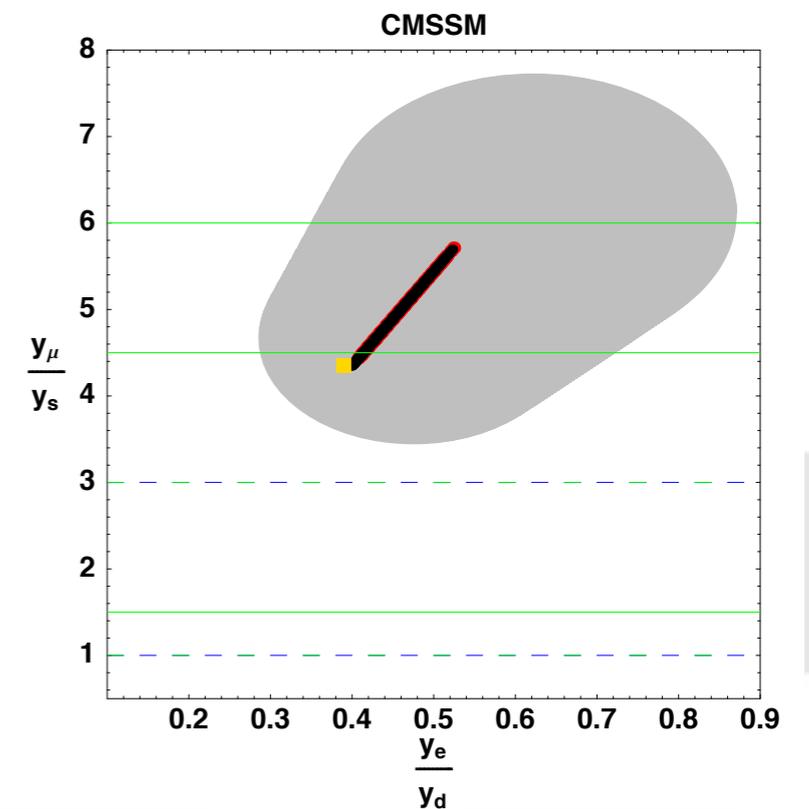
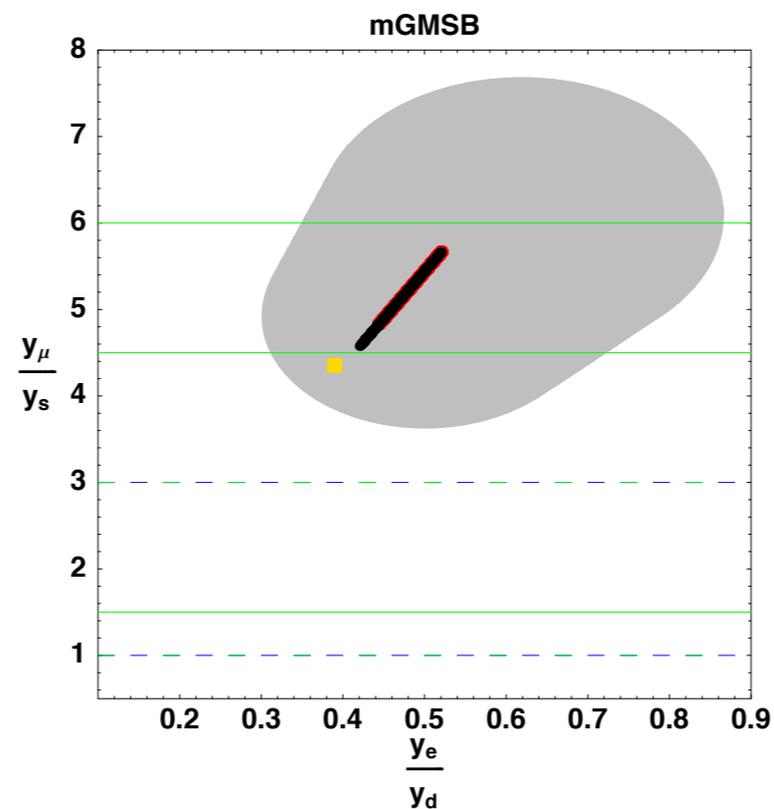
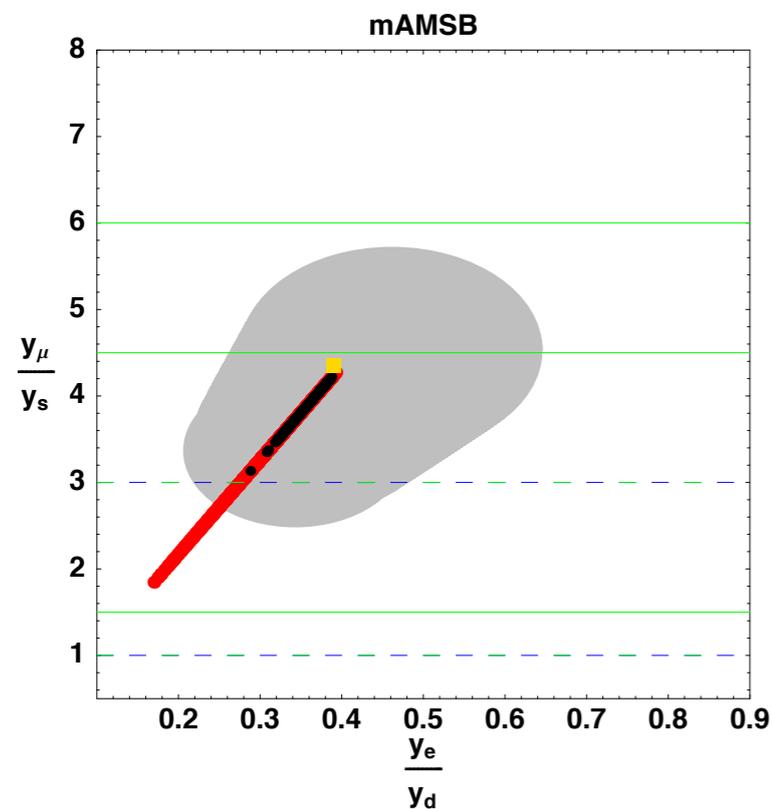
mass uncertainties

w/o thresholds

SU(5) predictions

Pati-Salam predictions

Results for the 1st and 2nd generation



exp. allowed (2009)

mass uncertainties

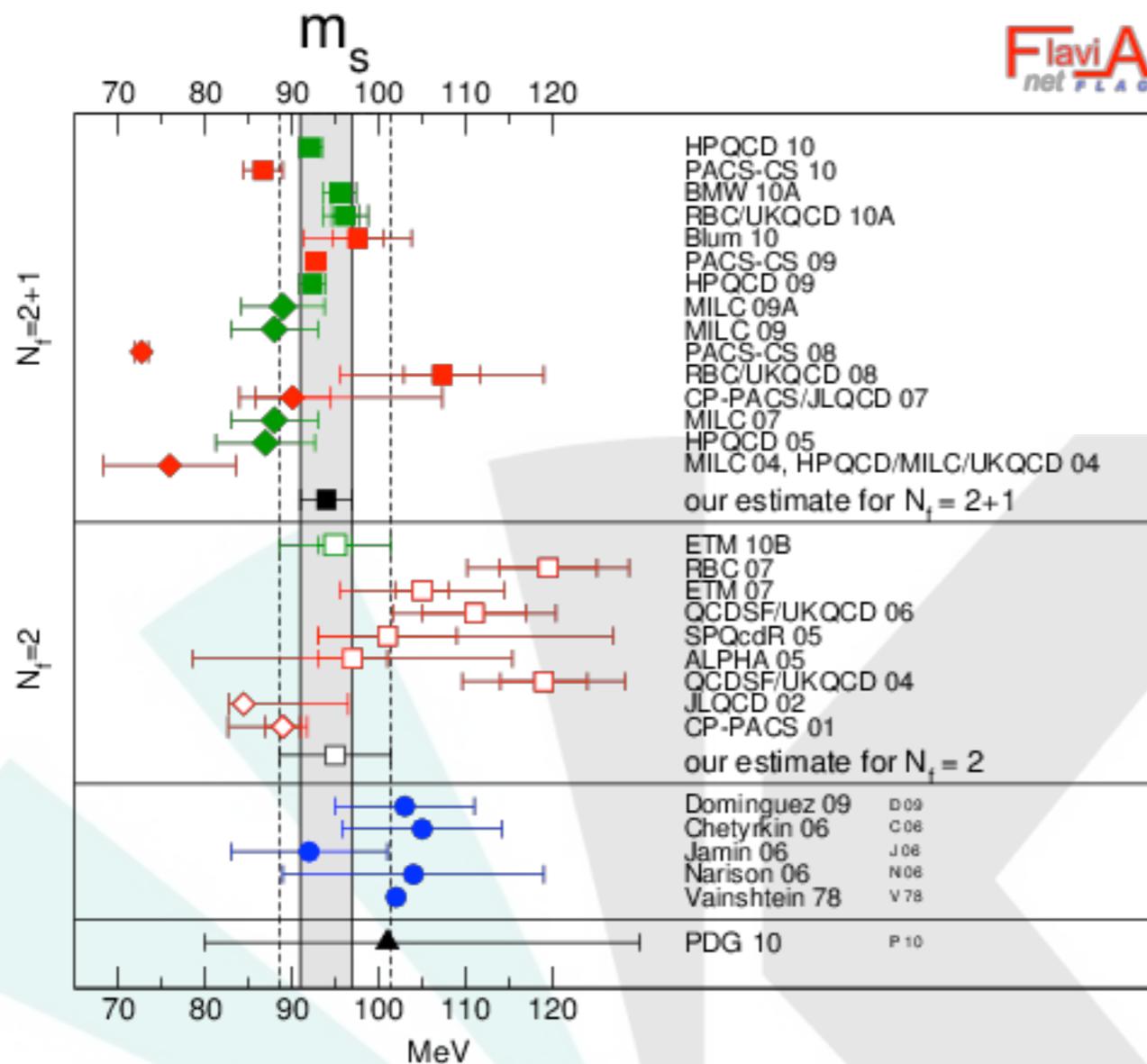
SU(5) predictions

exp. disfavored (2009)

w/o thresholds

Pati-Salam predictions

Regarding mass uncertainties...

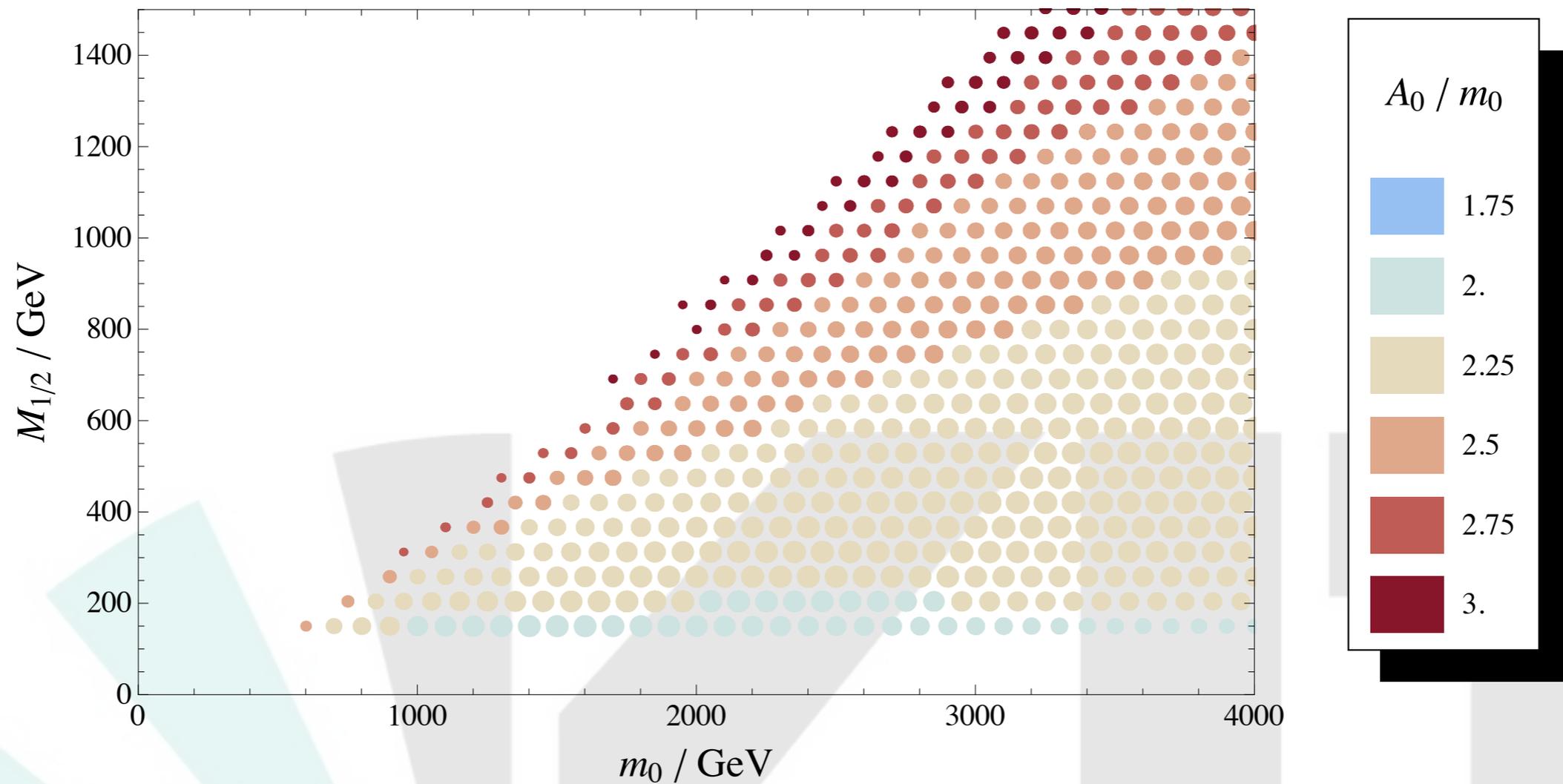


we took this number
in '09

[taken from http://itpwiki.unibe.ch/flag/index.php/Quark_masses on 14/01/13]

Implications for SUSY Spectrum

[Antusch, Calibbi, Maurer, MS '11]

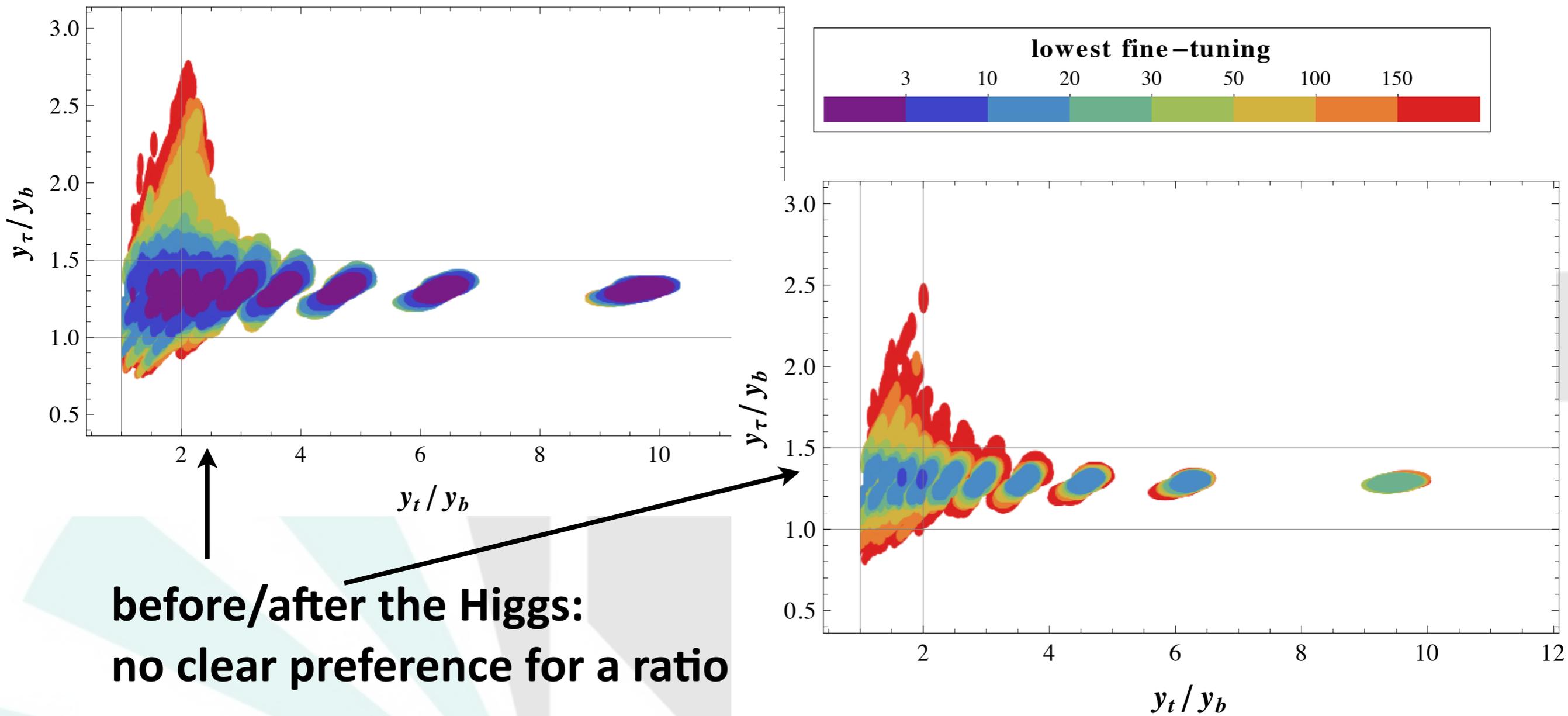


Constrained Spectrum from a CMSSM scan

$$(\gamma_\tau / \gamma_b = -3/2, \gamma_\mu / \gamma_s = 6, \tan \beta = 30)$$

Naturalness of Fermion Mass Ratios

[Antusch, Calibbi, Maurer, Monaco, MS '12]



The θ_{13} study - Setup

[Marzocca *et al.* '11, see also Antusch, Maurer '11]

- Assume SU (5) relations:

$$\hat{\lambda}_{[12]}^D = \begin{pmatrix} a & b' \\ b & c \end{pmatrix} \quad \hat{\lambda}_{[12]}^E = \begin{pmatrix} \alpha a & \beta b \\ \beta' b' & \gamma c \end{pmatrix}$$

- No 1-3 mixing in the neutrino and only 1-2 mixing in the charged lepton sector:

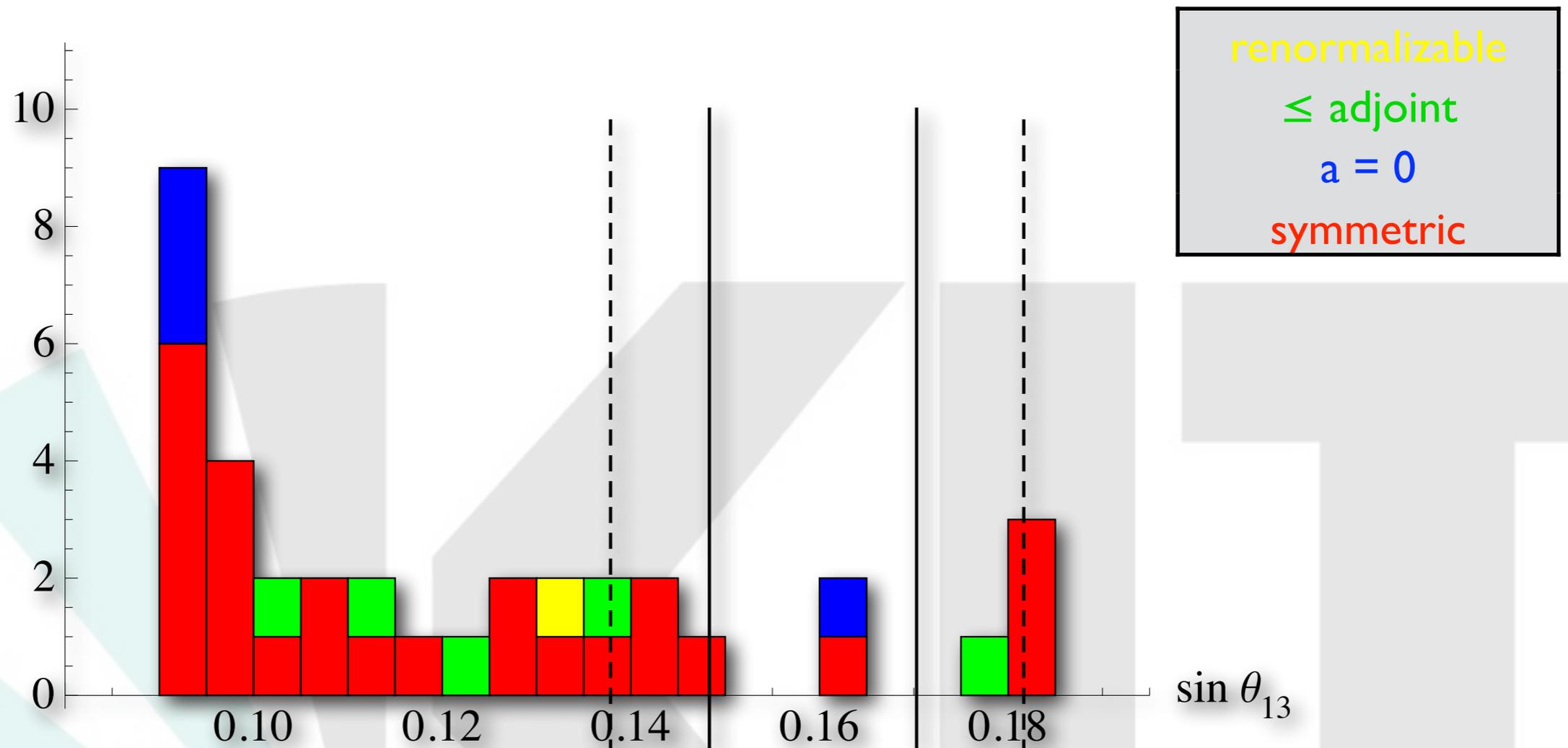
$$\sin \theta_{13} \approx \sin \theta_{12}^e \sin \theta_{23}^\nu \approx \frac{1}{\sqrt{2}} \frac{\beta' b'}{\gamma c}$$

- $\alpha, \beta, \beta', \gamma$ are SU(5) Clebsch-Gordan coefficients (1, -3/2, -3, 9/2, 6, ...)

[for lists see Antusch, MS '09; Antusch, King, MS '13; Antusch *et al.* '14]

Scan results with free

θ_{13} (May '12)



[Based on the global fit by Forero, Tortola, Valle '12, Thanks to D. Marzocca for providing this plot]

The Good Cases

- The Georgi-Jarlskog structure would predict

$$\sin \theta_{13} \approx \frac{1}{3\sqrt{2}} \theta_C \approx 3^\circ$$

- The good cases feature instead

$$\sin \theta_{13} \approx \frac{1}{\sqrt{2}} \theta_C \approx 9^\circ$$

- Successful models realised

[1205.5241 (SISSA); 1305.6612, 1306.3984 (Basel)]

- Can improve as well on

$$\left| \frac{y_\mu}{y_s} \frac{y_d}{y_e} \right| = 10.7^{+1.8}_{-0.8}$$

[Antusch, Maurer '13]

Corrections to other Mixing Angles

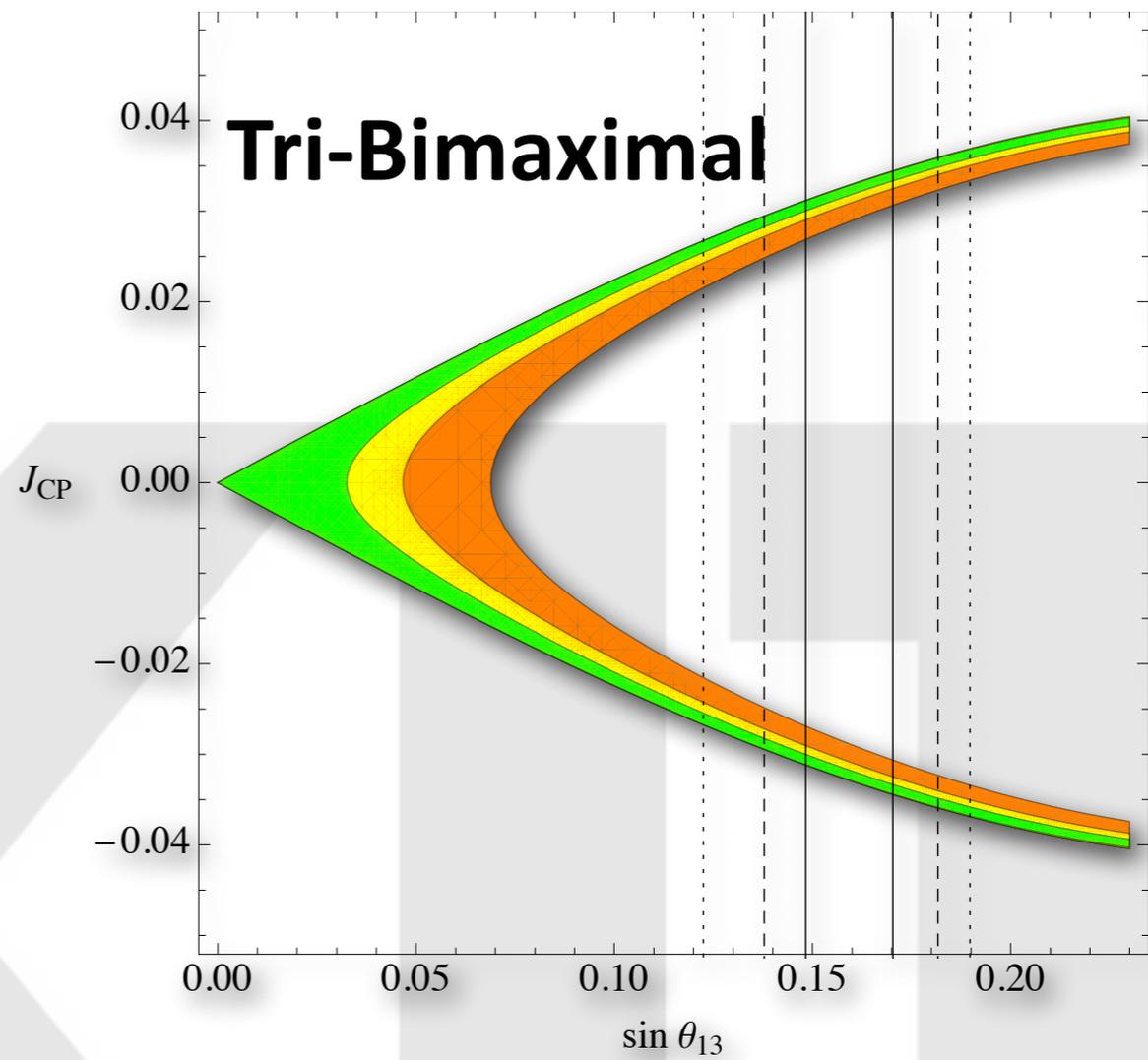
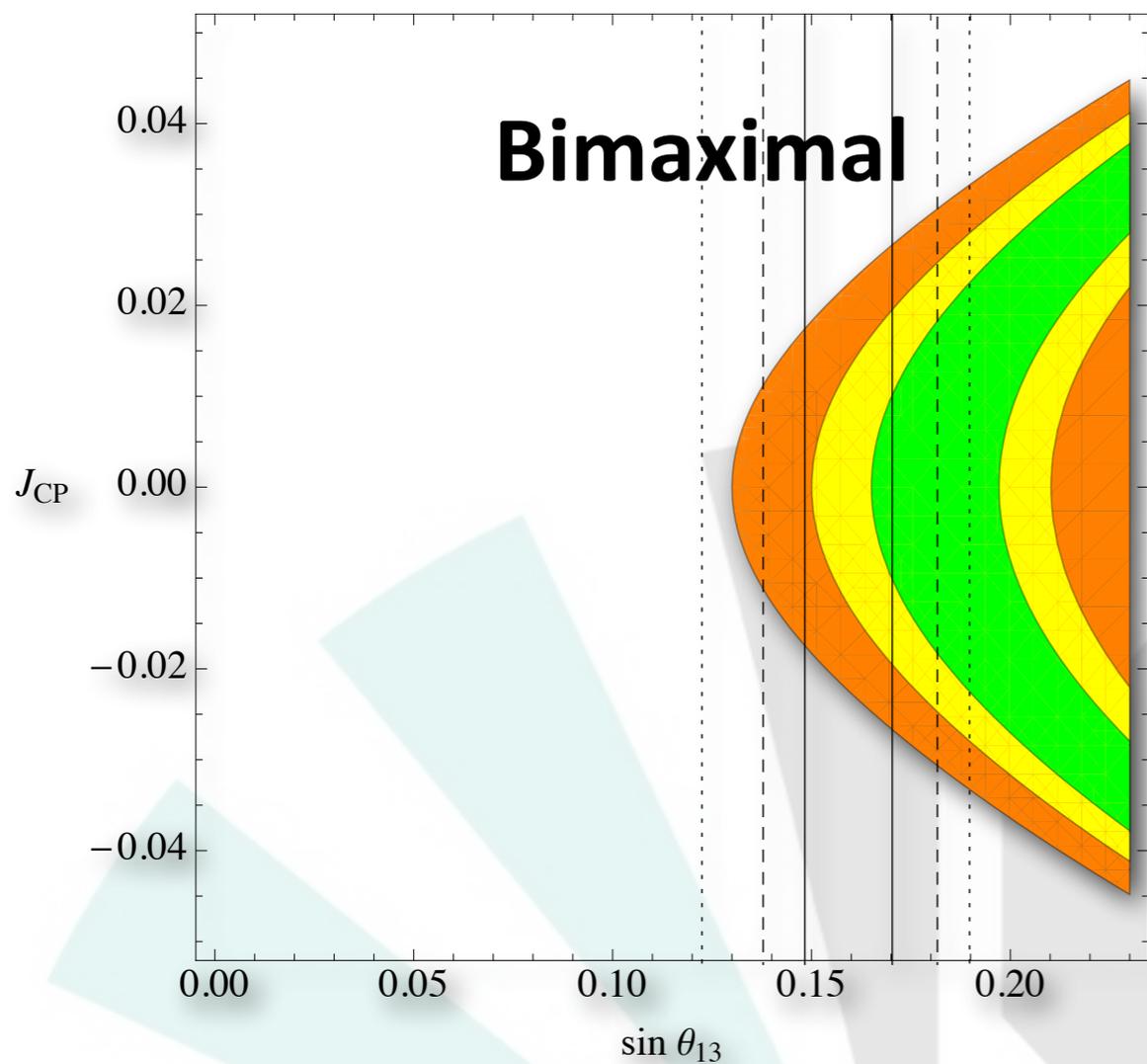
- For Bimaximal mixing:

$$\sin^2 \theta_{12} \approx \frac{1}{2} + \sin \theta_{13} \cos \delta$$

- For Tri-Bimaximal mixing:

$$\sin^2 \theta_{12} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3} \sin \theta_{13} \cos \delta$$

Implications for J_{CP} (May '12)



[Based on the global fit by Forero, Tortola, Valle '12, Thanks to D. Marzocca for providing this update]

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Two Questions

1. Can the doublet-triplet-splitting problem be solved in this models?
2. How does the potential for the adjoints look like?

Double Missing Partner Mechanism

- Coloured triplets projected out by missing partners in **50** (MPM)

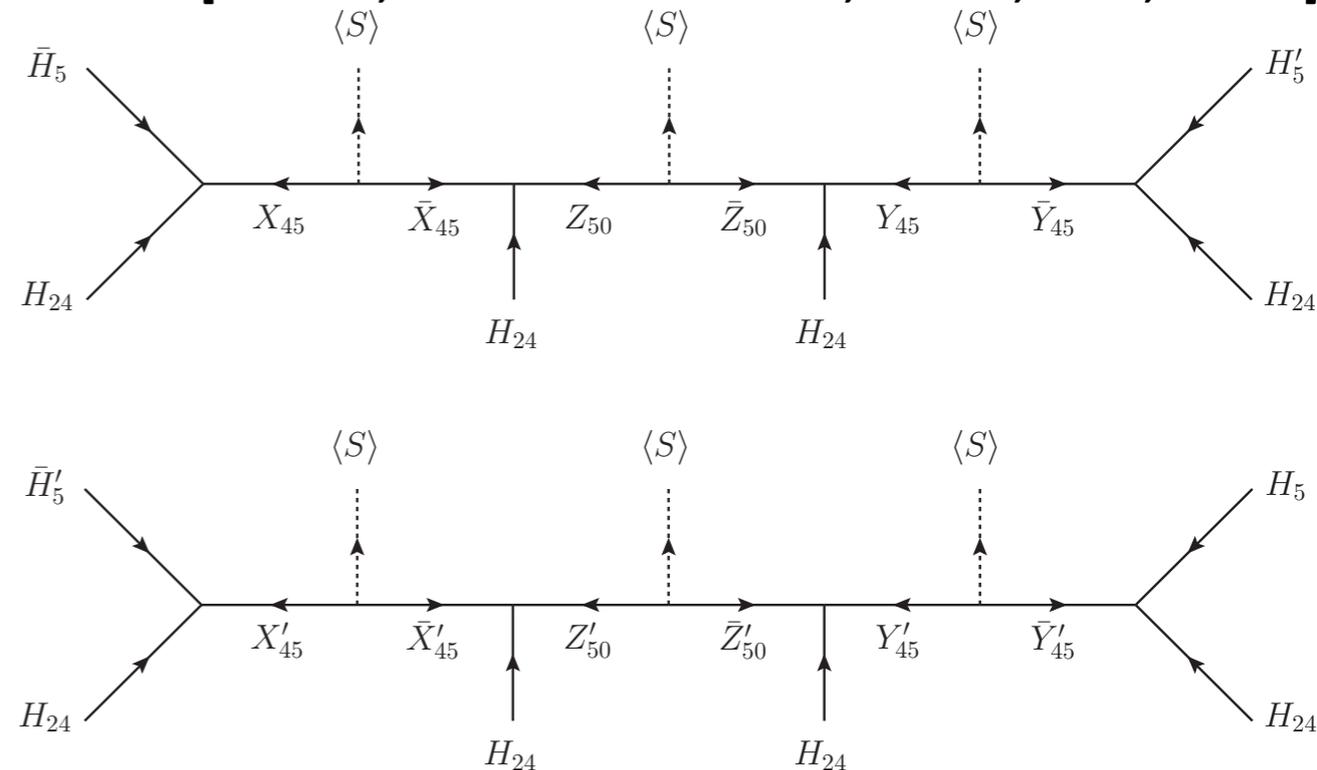
[Masiero, Nanopoulos, Namvakis, Yanagida '82; Grinstein '82]

- The DMPM keeps gauge couplings perturbative up to the Planck scale

[Hisano, Moroi, Tobe, Yanagida '95]

- We replaced the **75** with two **24**'s to allow charges for **24**'s

[Antusch, de Medeiros-Varzielas, Maurer, Sluka, MS '14]



GUT breaking Potentials

[Antusch, de Medeiros-Varzielas, Maurer, Sluka, MS '14]

- Four different superpotentials:

(a) $W = M_{24} \text{tr} H_{24}^2 + M'_{24} \text{tr} H_{24}'^2 + \kappa' \text{tr} H_{24} H_{24}'^2 + \lambda \text{tr} H_{24}^3$, with \mathbb{Z}_2

(b) $W = \tilde{M}_{24} \text{tr} H_{24} H_{24}' + \lambda \text{tr} H_{24}^3 + \lambda' \text{tr} H_{24}'^3$, with \mathbb{Z}_3

(c) $W = \tilde{M}_{24} \text{tr} H_{24} H_{24}' + \lambda \text{tr} H_{24}^3 + \kappa' \text{tr} H_{24} H_{24}'^2$, with \mathbb{Z}_4^R

(d) no symmetry

- Different GUT scales and effective triplet masses (p decay sufficiently suppressed)

➡ Talk of Vinzenz Maurer on wednesday (session IIB)

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Summary and Conclusions

- There are interesting GUT ratios beyond the standard ones, e.g.
 - $y_\mu/y_s = 9/2, 6$ and $y_\tau/y_b = -3/2$
- Low energy data constrains GUT models, e.g.
 - LHC constrains the SUSY spectrum
 - Information even from neutrinos
- Viable GUT breaking Higgs potentials