Multi-Higgs models with Abelian symmetries (The Yukawa sector)

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Based on the work:

H.S., Phys.Rev.D88 (2013)

Outline

Abelian symmetries in the Yukawa sector

Yukawa sector in 2HDM Ferreira, Silva (2011)
Scalar sector in NHDM Ivanov, Keus, Vdovin (2012)

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Yukawa sector

$$-\mathcal{L}_{\text{Yuk}} = (\Gamma_a)_{\alpha\beta}\,\overline{Q_{L\alpha}}\,\Phi_a\,n_{R\beta} + (\Delta_a)_{\alpha\beta}\,\overline{Q_{L\alpha}}\,\tilde{\Phi}_a\,p_{R\beta} + \text{H.c.}$$

Symmetry transformation

$$Q_L \to \mathcal{S}_L\,Q_L\,, \quad n_R \to \mathcal{S}_R^n\,n_R\,, \quad p_R \to \mathcal{S}_R^p\,p_R\,, \quad \Phi \to \mathcal{S}_H\,\Phi$$

$$\mathcal{S}_L^\dagger\,\mathcal{A}_b\,\mathcal{S}_R\,(\mathcal{S}_H)_{ba} = \mathcal{A}_a\,, \quad \mathcal{A}_a = \{\Gamma_a, \Delta_a\}$$

Abelian symmetries in the Yukawa sector

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$$\mathcal{S}_L^\dagger\, \mathcal{A}_b\, \mathcal{S}_R\, (\mathcal{S}_H)_{ba} = \mathcal{A}_a\,, \quad \mathcal{A}_a = \{\Gamma_a, \Delta_a\}$$

Abelian symmetries

$$\mathcal{S}_L = \text{diag}\left(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}\right)\,, \mathcal{S}_R = \text{diag}\left(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}\right)\,,$$
$$\mathcal{S}_H = \text{diag}\left(e^{i\theta_1}, e^{i\theta_2}, \cdots, e^{i\theta_N}\right)$$

Abelian symmetries in the Yukawa sector

Hermitian combinations $\mathcal{H}_L^a = \mathcal{A}_a \mathcal{A}_a^\dagger$ and $\mathcal{H}_R^a = \mathcal{A}_a^\dagger \mathcal{A}_a$

$$\mathcal{S}_L^\dagger \mathcal{H}_L^a \mathcal{S}_L = \mathcal{H}_L^a\,, \quad \mathcal{S}_R^\dagger \mathcal{H}_R^a \mathcal{S}_R = \mathcal{H}_R^a$$

(1) $\mathcal{S}$ has a full degeneracy,

(2) $\mathcal{S}$ has two-fold degeneracy,

(3) $\mathcal{S}$ is nondegenerate

Abelian symmetries in the Yukawa sector

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$$\mathcal{S}_L^\dagger \mathcal{H}_L^a \mathcal{S}_L = \mathcal{H}_L^a\,, \quad \mathcal{S}_R^\dagger \mathcal{H}_R^a \mathcal{S}_R = \mathcal{H}_R^a$$

(1) $\mathcal{S}$ has a full degeneracy,

(2) $\mathcal{S}$ has two-fold degeneracy,

(3) $\mathcal{S}$ is nondegenerate

$$\mathcal{P}'\left\{ A_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, A_2 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \times \end{pmatrix}, A_3 = \begin{pmatrix} & & \times \\ & & \times \\ \times & \times & \end{pmatrix}, A_4 = \begin{pmatrix} & & \times \\ & & \times \\ & \times & \end{pmatrix}, A_5 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ & & \end{pmatrix}, \right.$$

$$A_6 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ \times & \times & \end{pmatrix}, A_7 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \end{pmatrix}, A_8 = \begin{pmatrix} & & \times \\ & & \times \\ & & \end{pmatrix}, A_9 = \begin{pmatrix} & & \\ & & \\ \times & \times & \times \end{pmatrix}, A_{10} = \begin{pmatrix} & & \times \\ & & \times \\ & & \times \end{pmatrix},$$

$$\left. A_{11} = \begin{pmatrix} & & \\ & & \\ \times & \times & \end{pmatrix}, A_{12} = \begin{pmatrix} & & \\ & & \\ & & \times \end{pmatrix}, A_{13} = \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix}, A_{14} = \begin{pmatrix} & & \\ \times & \times & \\ & & \times \end{pmatrix}, A_{15} = \begin{pmatrix} & & \\ & \times & \\ & & \times \end{pmatrix} \right\} \mathcal{P}$$

Abelian symmetries in the Yukawa sector

Group the textures into classes

$\mathcal{H}_L^a$ \ $\mathcal{H}_R^a$	$\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\mathcal{P}^R \begin{pmatrix} \bullet & \bullet & \\ \bullet & \bullet & \\ & & \bullet \end{pmatrix} \mathcal{P}^R$	$\begin{pmatrix} \bullet & & \\ & \bullet & \\ & & \bullet \end{pmatrix}$
$\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$	A_1	$\{A_6, A_{10}\}\mathcal{P}^R$	$\{A_{10}\}\mathcal{P}$
$\mathcal{P}^L \begin{pmatrix} \bullet & \bullet & \\ \bullet & \bullet & \\ & & \bullet \end{pmatrix} \mathcal{P}^L$	$\mathcal{P}^L\{A_5, A_9\}$	$\mathcal{P}^L\{A_2, A_3, A_7, A_8, A_{11}, A_{12}\}\mathcal{P}^R$	$\mathcal{P}^L\{A_4, A_8, A_{12}\}\mathcal{P}$
$\begin{pmatrix} \bullet & & \\ & \bullet & \\ & & \bullet \end{pmatrix}$	$\mathcal{P}'\{A_9\}$	$\mathcal{P}'\{A_{11}, A_{12}, A_{14}\}\mathcal{P}^R$	$\mathcal{P}'\{A_{12}, A_{13}, A_{15}\}\mathcal{P}$

$(\mathbf{i}, \mathbf{j})$, with i and j the number of different phases in the left and right generator, respectively

Abelian symmetries, chains, and charge vector

Chain $(C_n^{(\mathbf{i},\mathbf{j})})$: For example

$$\begin{pmatrix} \color{blue}\times & \color{red}\times & \color{blue}\times \\ \color{red}\times & \color{green}\times & \color{red}\times \\ \color{blue}\times & \color{red}\times & \color{blue}\times \end{pmatrix} : \left\{ \begin{pmatrix} \color{blue}\times & & \color{blue}\times \\ & & \\ \color{blue}\times & & \color{blue}\times \end{pmatrix} , \begin{pmatrix} & \color{red}\times & \\ \color{red}\times & & \color{red}\times \\ & \color{red}\times & \end{pmatrix} , \begin{pmatrix} & & \\ & \color{green}\times & \\ & & \end{pmatrix} \right\}$$

Using discrete charges

$$\mathcal{S}_L = \text{diag}\left(1, \omega_n^{k_{L1}}, \omega_n^{k_{L2}}\right) \ , \quad \mathcal{S}_R = \text{diag}\left(\omega_n^{k_1}, \omega_n^{k_2}, \omega_n^{k_3}\right) \ .$$

Abelian symmetries, chains, and charge vector

Chain ($C_n^{(\mathbf{i},\mathbf{j})}$): For example

$$\begin{pmatrix} \color{blue}{\times} & \color{red}{\times} & \color{blue}{\times} \\ \color{red}{\times} & \color{green}{\times} & \color{red}{\times} \\ \color{blue}{\times} & \color{red}{\times} & \color{blue}{\times} \end{pmatrix} : \left\{ \begin{pmatrix} \color{blue}{\times} & & \color{blue}{\times} \\ & & \\ \color{blue}{\times} & & \color{blue}{\times} \end{pmatrix} , \begin{pmatrix} & \color{red}{\times} & \\ \color{red}{\times} & & \color{red}{\times} \\ & \color{red}{\times} & \end{pmatrix} , \begin{pmatrix} & & \\ & \color{green}{\times} & \\ & & \end{pmatrix} \right\}$$

Using discrete charges

$$\mathcal{S}_L = \text{diag}\left(1, \omega_n^{k_{L1}}, \omega_n^{k_{L2}}\right) \ , \quad \mathcal{S}_R = \text{diag}\left(\omega_n^{k_1}, \omega_n^{k_2}, \omega_n^{k_3}\right) \ .$$

Size of chain = smallest $|G|$

Z_{kn}: the order of the group has to belong to $k\mathbb{Z}$

$Z_{n\geq k}$: the group could be a Z_k, Z_{k+1} or even a $U(1)$

Abelian symmetries, chains, and charge vector

Let us choose class $(\mathbf{2}, \mathbf{2})$: $\mathcal{P}^L \{A_2\,, \ A_3\,, \ A_7\,, \ A_8\,, A_{11}\,, \ A_{12}\} \,\mathcal{P}^R$

$$A_2 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \times \end{pmatrix}, A_3 = \begin{pmatrix} & & \times \\ & & \times \\ \times & \times & \end{pmatrix}, A_7 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \end{pmatrix},$$

$$A_8 = \begin{pmatrix} & & \times \\ & & \times \\ & & \end{pmatrix}, A_{11} \begin{pmatrix} & & \\ & & \\ \times & \times & \end{pmatrix}, A_{12} = \begin{pmatrix} & & \\ & & \\ & & \times \end{pmatrix}$$

Abelian symmetries, chains, and charge vector

Let us choose class $(\mathbf{2},\mathbf{2})$: $\mathcal{P}^L\,\{A_2\,,\;A_3\,,\;A_7\,,\;A_8\,,A_{11}\,,\;A_{12}\}\,\mathcal{P}^R$

$$A_2 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \times \end{pmatrix}, A_3 = \begin{pmatrix} & & \times \\ & & \times \\ \times & \times & \end{pmatrix}, A_7 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \end{pmatrix},$$

$$A_8 = \begin{pmatrix} & & \times \\ & & \times \\ & & \end{pmatrix}, A_{11} \begin{pmatrix} & & \\ & & \\ \times & \times & \end{pmatrix}, A_{12} = \begin{pmatrix} & & \\ & & \\ & & \times \end{pmatrix}$$

Two cases: $\begin{cases} \text{with } \mathcal{P}^L A_2 \mathcal{P}^R \\ \\ \text{without } \mathcal{P}^L A_2 \mathcal{P}^R \end{cases}$

Abelian symmetries, chains, and charge vector

- **With a texture $\mathcal{P}^{\mathbf{L}}\mathbf{A_2}\mathcal{P}^{\mathbf{R}}$**

$$\begin{cases} \mathcal{S}_R = \mathcal{P}^R \text{diag}\left(1,\ 1,\ \omega_n^{k_{L2}}\right)\mathcal{P}^R \\ \\ \mathcal{S}_L = \mathcal{P}^L \text{diag}\left(1,\ 1,\ \omega_n^{k_{L2}}\right)\mathcal{P}^L \end{cases}$$

leading to the phase transformation matrix $\Theta_{\mathcal{P}^L A_2 \mathcal{P}^R}$

$$\frac{2\pi}{n}.\mathcal{P}^L \begin{pmatrix} 0 & 0 & k_{L2} \\ 0 & 0 & k_{L2} \\ -k_{L2} & -k_{L2} & 0 \end{pmatrix} \mathcal{P}^R$$

Abelian symmetries, chains, and charge vector

In this case we have two possibilities:

(i) $k_{L2} \neq -k_{L2}$

This implies $k_{L2} \neq n/2$. The order of the group has to be $n \geq 3$, leading to the chain

$$Z_{n\geq 3}: \mathcal{P}^L \left\{ A_2 \oplus A_8 \oplus A_{11} \right\} \mathcal{P}^R .$$

$$\left\{ \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \times \end{pmatrix}, \begin{pmatrix} & & \times \\ & & \times \\ & & \end{pmatrix}, \begin{pmatrix} & & \\ & & \\ \times & \times & \end{pmatrix} \right\}$$

The associated charge vector is

$$\left(1, \omega_n^{-k_{L2}}, \omega_n^{k_{L2}}\right) .$$

Abelian symmetries, chains, and charge vector

(ii) $k_{L2} = -k_{L2}$

This implies $k_{L2} = n/2$. The order of the group has to be $n \in 2\,\mathbb{Z}$, leading to the chain

$$Z_{2n} : \mathcal{P}^L \{A_2 \oplus A_3\} \mathcal{P}^R .$$

$$\left\{ \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \times \end{pmatrix}, \begin{pmatrix} & & \times \\ & & \times \\ \times & \times & \end{pmatrix} \right\}$$

We have made the redefinition $n \to 2n$. The associated charge vector is

$$(1, \omega_{2n}^n) \,, \quad k_{L2} = n \,.$$

Abelian symmetries, chains, and charge vector

- **Without the texture $\mathcal{P}^{\mathbf{L}}\mathbf{A_2}\mathcal{P}^{\mathbf{R}}$**

$$\begin{cases} \mathcal{S}_R = \mathcal{P}^R \mathrm{diag}\left(1,\ 1,\ \omega_n^{k_3}\right) \mathcal{P}^R\,, \\ \\ \mathcal{S}_L = \mathcal{P}^L \mathrm{diag}\left(1,\ 1,\ \omega_n^{k_{L2}}\right) \mathcal{P}^L\,, \end{cases}$$

leading to the phase transformation matrix $\Theta_{\mathcal{P}^L A_7 \mathcal{P}^R}$

$$\frac{2\pi}{n}.\mathcal{P}^L \begin{pmatrix} 0 & 0 & k_3 \\ 0 & 0 & k_3 \\ -k_{L2} & -k_{L2} & k_3 - k_{L2} \end{pmatrix} \mathcal{P}^R\,.$$

Abelian symmetries, chains, and charge vector

In this case we have the following possibilities:

(i) $k_3 = -k_{L2}$

This implies $k_{L2} \neq n/2$. The group order has to be $n \geq 3$, leading to the chain

$$Z_{n\geq 3}: \mathcal{P}^L \left\{A_7 \oplus A_3 \oplus A_{12}\right\} \mathcal{P}^R ,$$

$$\left\{ \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \end{pmatrix}, \begin{pmatrix} & & \times \\ & & \times \\ \times & \times & \end{pmatrix}, \begin{pmatrix} & & \\ & & \\ & & \times \end{pmatrix} \right\}$$

with the associated charge vector

$$\left(1, \omega_n^{k_{L2}}, \omega_n^{2k_{L2}}\right) .$$

Abelian symmetries, chains, and charge vector

(ii) $k_3 \neq -k_{L2}$

The group order has to be $n \geq 4$, leading to the chain

$$Z_{n\geq 4}:\ \mathcal{P}^L \left\{A_7 \oplus A_8 \oplus A_{11} \oplus A_{12}\right\} \mathcal{P}^R \,.$$

$$\left\{ \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \end{pmatrix}, \begin{pmatrix} & & \times \\ & & \times \\ & & \end{pmatrix}, \begin{pmatrix} & & \\ & & \\ \times & \times & \end{pmatrix}, \begin{pmatrix} & & \\ & & \\ & & \times \end{pmatrix} \right\}$$

The associated charge vector is

$$\left(1, \omega_n^{-k_3}, \omega_n^{k_{L2}}, \omega_n^{k_{L2}-k_3}\right) \,.$$

List of chains and charge vector for the class (2,i)

$C_1^{(2,1)}$	$\mathcal{P}^L\{A_5 \oplus A_9\}$	$(1,\, \omega_n^{k_{L2}})$
$C_1^{(2,2)}$	$\mathcal{P}^L\{A_2 \oplus A_3\}\mathcal{P}^R$	$(1,\, \omega_{2n}^{n}),\; k_{L2} = n$
$C_2^{(2,2)}$	$\mathcal{P}^L\{A_2 \oplus A_8 \oplus A_{11}\}\mathcal{P}^R$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{k_{L2}})$
$C_3^{(2,2)}$	$\mathcal{P}^L\{A_7 \oplus A_3 \oplus A_{12}\}\mathcal{P}^R$	$(1,\, \omega_n^{k_{L2}},\, \omega_n^{2k_{L2}})$
$C_4^{(2,2)}$	$\mathcal{P}^L\{A_7 \oplus A_8 \oplus A_{11} \oplus A_{12}\}\mathcal{P}^R$	$(1,\, \omega_n^{-k_3},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_3})$
$C_1^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{321} \oplus A_4\mathcal{P}_{123}\}$	$(1,\, \omega_{3n}^{2n},\, \omega_{3n}^{n}),\; k_{L2} = 2n$
$C_2^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_{2(n+1)}^{n+1},\, \omega_{2(n+1)}^{-k_1},\, \omega_{2(n+1)}^{-k_1+n+1}),\; k_{L2} = n+1$
$C_3^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{123} \oplus A_8\mathcal{P}_{13} \oplus A_{12}\}$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{-2k_{L2}},\, \omega_n^{k_{L2}})$
$C_4^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{321} \oplus A_{12}\mathcal{P}_{13} \oplus A_8\mathcal{P}_{23}\}$	$(1,\, \omega_n^{k_{L2}},\, \omega_n^{2k_{L2}},\, \omega_n^{-k_{L2}})$
$C_5^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_8\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{-k_1},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_1})$
$C_6^{(2,3)}$	$\mathcal{P}^L\{A_8 \oplus A_8\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12} \oplus A_{12}\mathcal{P}_{23} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_n^{-k_2},\, \omega_n^{-k_1},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_2},\, \omega_n^{k_{L2}-k_1})$

Abelian symmetries, chains, and charge vector

List of chains and charge vector for the class (2,i)

$C_1^{(2,1)}$	$\mathcal{P}^L\{A_5 \oplus A_9\}$	$(1,\, \omega_n^{k_{L2}})$
$C_1^{(2,2)}$	$\mathcal{P}^L\{A_2 \oplus A_3\}\mathcal{P}^R$	$(1,\, \omega_{2n}^{n}),\, k_{L2} = n$
$C_2^{(2,2)}$	$\mathcal{P}^L\{A_2 \oplus A_8 \oplus A_{11}\}\mathcal{P}^R$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{k_{L2}})$
$C_3^{(2,2)}$	$\mathcal{P}^L\{A_7 \oplus A_3 \oplus A_{12}\}\mathcal{P}^R$	$(1,\, \omega_n^{k_{L2}},\, \omega_n^{2k_{L2}})$
$C_4^{(2,2)}$	$\mathcal{P}^L\{A_7 \oplus A_8 \oplus A_{11} \oplus A_{12}\}\mathcal{P}^R$	$(1,\, \omega_n^{-k_3},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_3})$
$C_1^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{321} \oplus A_4\mathcal{P}_{123}\}$	$(1,\, \omega_{3n}^{2n},\, \omega_{3n}^{n}),\, k_{L2} = 2n$
$C_2^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_{2(n+1)}^{n+1},\, \omega_{2(n+1)}^{-k_1},\, \omega_{2(n+1)}^{-k_1+n+1}),\, k_{L2} = n+1$
$C_3^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{123} \oplus A_8\mathcal{P}_{13} \oplus A_{12}\}$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{-2k_{L2}},\, \omega_n^{k_{L2}})$
$C_4^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{321} \oplus A_{12}\mathcal{P}_{13} \oplus A_8\mathcal{P}_{23}\}$	$(1,\, \omega_n^{k_{L2}},\, \omega_n^{2k_{L2}},\, \omega_n^{-k_{L2}})$
$C_5^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_8\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{-k_1},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_1})$
$C_6^{(2,3)}$	$\mathcal{P}^L\{A_8 \oplus A_8\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12} \oplus A_{12}\mathcal{P}_{23} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_n^{-k_2},\, \omega_n^{-k_1},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_2},\, \omega_n^{k_{L2}-k_1})$

Abelian symmetries, chains, and charge vector

List of chains and charge vector for the class (2,i)

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$C_1^{(2,2)}$	$\mathcal{P}^L\{A_2 \oplus A_3\}\mathcal{P}^R$	$(1,\, \omega_{2n}^{n}),\ k_{L2} = n$
$C_2^{(2,2)}$	$\mathcal{P}^L\{A_2 \oplus A_8 \oplus A_{11}\}\mathcal{P}^R$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{k_{L2}})$
$C_3^{(2,2)}$	$\mathcal{P}^L\{A_7 \oplus A_3 \oplus A_{12}\}\mathcal{P}^R$	$(1,\, \omega_n^{k_{L2}},\, \omega_n^{2k_{L2}})$
$C_4^{(2,2)}$	$\mathcal{P}^L\{A_7 \oplus A_8 \oplus A_{11} \oplus A_{12}\}\mathcal{P}^R$	$(1,\, \omega_n^{-k_3},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_3})$
$C_1^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{321} \oplus A_4\mathcal{P}_{123}\}$	$(1,\, \omega_{3n}^{2n},\, \omega_{3n}^{n}),\ k_{L2} = 2n$
$C_2^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_{2(n+1)}^{n+1},\, \omega_{2(n+1)}^{-k_1},\, \omega_{2(n+1)}^{-k_1+n+1}),\ k_{L2} = n+1$
$C_3^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{123} \oplus A_8\mathcal{P}_{13} \oplus A_{12}\}$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{-2k_{L2}},\, \omega_n^{k_{L2}})$
$C_4^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_4\mathcal{P}_{321} \oplus A_{12}\mathcal{P}_{13} \oplus A_8\mathcal{P}_{23}\}$	$(1,\, \omega_n^{k_{L2}},\, \omega_n^{2k_{L2}},\, \omega_n^{-k_{L2}})$
$C_5^{(2,3)}$	$\mathcal{P}^L\{A_4 \oplus A_8\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_n^{-k_{L2}},\, \omega_n^{-k_1},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_1})$
$C_6^{(2,3)}$	$\mathcal{P}^L\{A_8 \oplus A_8\mathcal{P}_{23} \oplus A_8\mathcal{P}_{13} \oplus A_{12} \oplus A_{12}\mathcal{P}_{23} \oplus A_{12}\mathcal{P}_{13}\}$	$(1,\, \omega_n^{-k_2},\, \omega_n^{-k_1},\, \omega_n^{k_{L2}},\, \omega_n^{k_{L2}-k_2},\, \omega_n^{k_{L2}-k_1})$

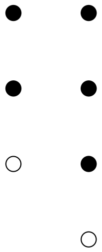
Total of 110 different chains, without counting ${}^0C^{(\mathbf{i,j})}$ and right permutations

Building models: 2HDM example

(1) Choose two chains: ${}^{0}C_1^{\mathbf{2},1}$ and ${}^{0}C_3^{\mathbf{2},\mathbf{2}}$.

${}^{0}C_1^{\mathbf{2},1}: \quad \mathcal{P}^L \{A_5 \oplus A_9 \oplus A_0\}$ (down sector)

${}^{0}C_3^{\mathbf{2},\mathbf{2}}: \quad \mathcal{P}^L \{A_7 \oplus A_3 \oplus A_{12} \oplus A_0\} \mathcal{P}^R$ (up sector)

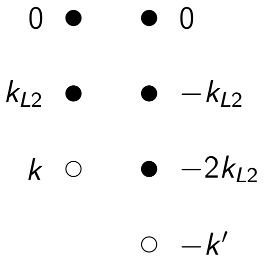


Building models: 2HDM example

(2) Write the associated charges. Conjugate the right column.

$${}^{0}C_1^{\mathbf{2},1}: \quad (1, \omega_n^{k_{L2}}, \omega_n^{k})$$

$${}^{0}C_3^{\mathbf{2},2}: \quad (1, \omega_n^{k_{L2}}, \omega_n^{2k_{L2}}, \omega_n^{k'})$$



Building models: 2HDM example

(3) Connect the nodes.

$^{0}C_{1}^{\mathbf{2},1}$: $\det(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

$^{0}C_{3}^{\mathbf{2},2}$: $\det(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$

0	$\bullet$	$\bullet$	0
k_{L2}	$\bullet$	$\bullet$	$-k_{L2}$
k	$\circ$	$\bullet$	$-2k_{L2}$
		$\circ$	$-k'$

Building models: 2HDM example

(3) Connect the nodes.

${}^{0}C_1^{\mathbf{2,1}}$: det$(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

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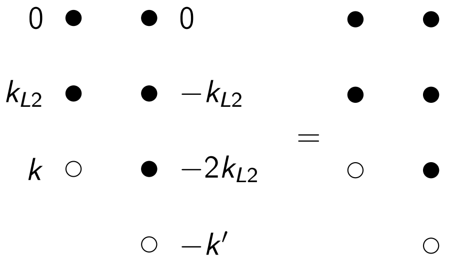

Building models: 2HDM example

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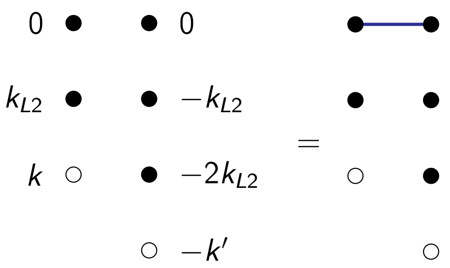
$^{0}C_3^{\mathbf{2},\mathbf{2}}$: $\det(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$



Building models: 2HDM example

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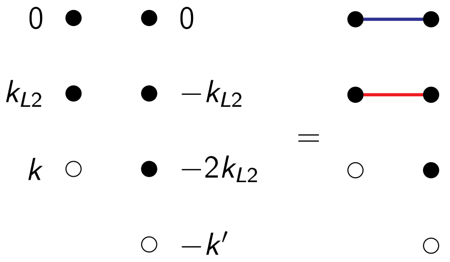

Building models: 2HDM example

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$^0C_1^{\mathbf{2,1}}$: $\det(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

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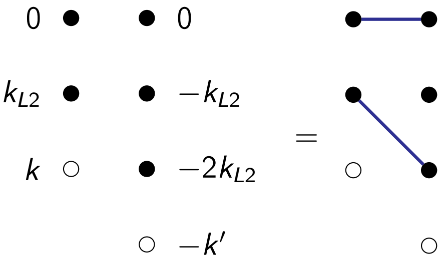

Building models: 2HDM example

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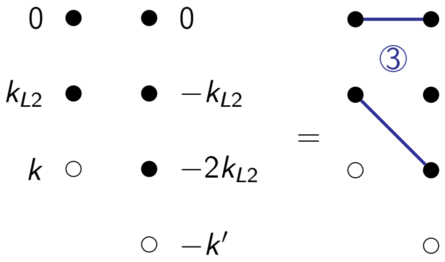

Building models: 2HDM example

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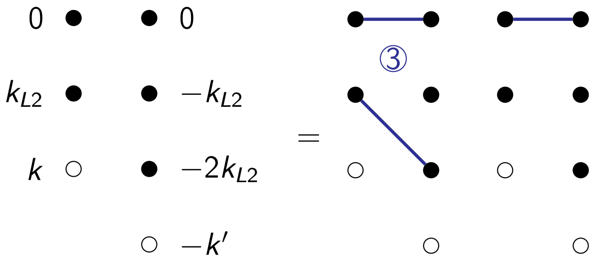

Building models: 2HDM example

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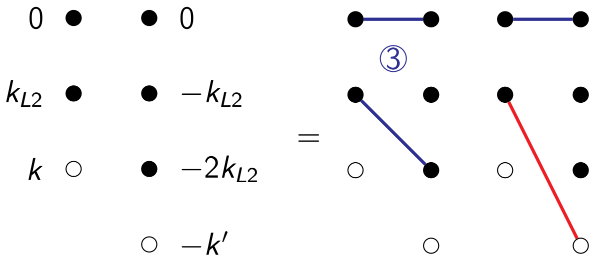
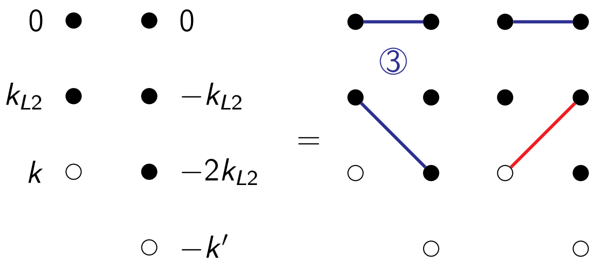


Building models: 2HDM example

(3) Connect the nodes.

$^{0}C_{1}^{\mathbf{2},1}$: $\det(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

$^{0}C_{3}^{\mathbf{2},2}$: $\det(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$

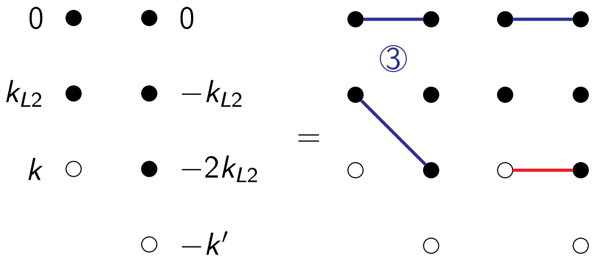


Building models: 2HDM example

(3) Connect the nodes.

$^0C_1^{\mathbf{2,1}}$: det$(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

$^0C_3^{\mathbf{2,2}}$: det$(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$



Building models: 2HDM example

(3) Connect the nodes.

$^{0}C_{1}^{\mathbf{2},1}$: det$(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

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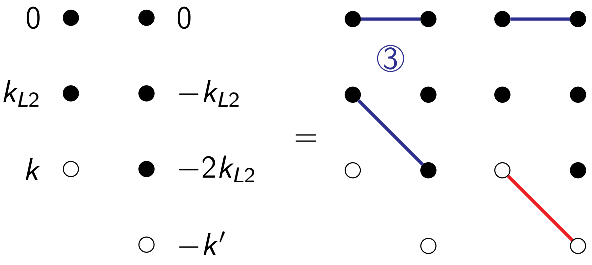

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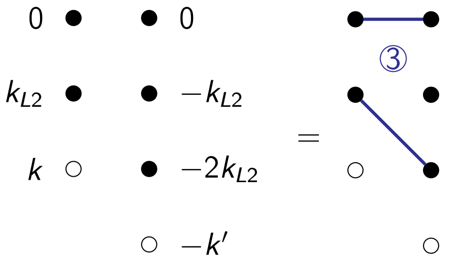


Building models: 2HDM example

(4) Global phase rotation.

$^{0}C_{1}^{\mathbf{2},1}$: $\det(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

$^{0}C_{3}^{\mathbf{2},2}$: $\det(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$

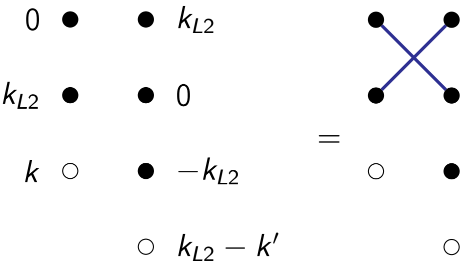
0	●	●	k_{L2}		●	●
k_{L2}	●	●	0		●	●
k	○	●	$-k_{L2}$	$=$	○	●
		○	$k_{L2} - k'$			○

Building models: 2HDM example

(4) Global phase rotation.

$^0C_1^{\mathbf{2},1}$: $\det(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

$^0C_3^{\mathbf{2},2}$: $\det(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$

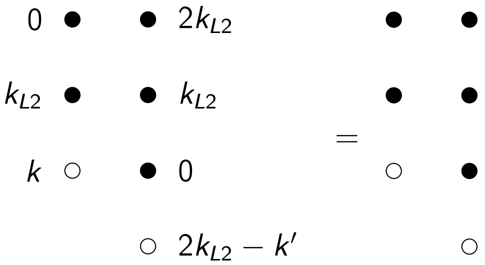


Building models: 2HDM example

(4) Global phase rotation.

${}^{0}C_{1}^{\mathbf{2},1}$: $\det(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

${}^{0}C_{3}^{\mathbf{2},2}$: $\det(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$

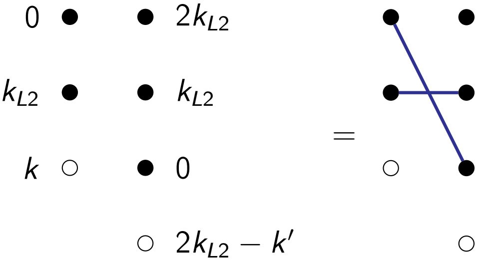


Building models: 2HDM example

(4) Global phase rotation.

$^{0}C_1^{\mathbf{2},1}$: $\det(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

$^{0}C_3^{\mathbf{2},2}$: $\det(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$



Building models: 2HDM example

(4) Global phase rotation.

$$
\begin{array}{lll}
{}^{0}C_1^{\mathbf{2},1} : & \det(M) \neq 0,\ A_5 \oplus A_9\,(3 \text{ mixing angles}) \\
{}^{0}C_3^{\mathbf{2},2} : & \det(M) \neq 0,\ A_7 \oplus A_3(3 \text{ mixing angles}),\ A_7 \oplus A_{12}
\end{array}
$$

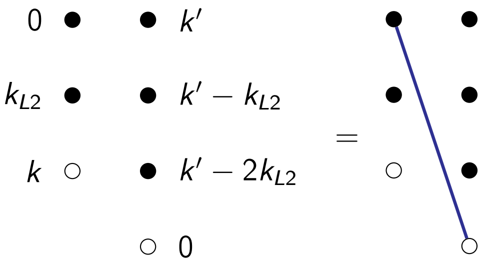
$$
\begin{array}{cccl}
0 & \bullet & \bullet & k' \\
k_{L2} & \bullet & \bullet & k' - k_{L2} \\
k & \circ & \bullet & k' - 2k_{L2} \\
 & & \circ & 0
\end{array}
\;=\;
\begin{array}{cc}
\bullet & \bullet \\
\bullet & \bullet \\
\circ & \bullet \\
 & \circ
\end{array}
$$

Building models: 2HDM example

(4) Global phase rotation.

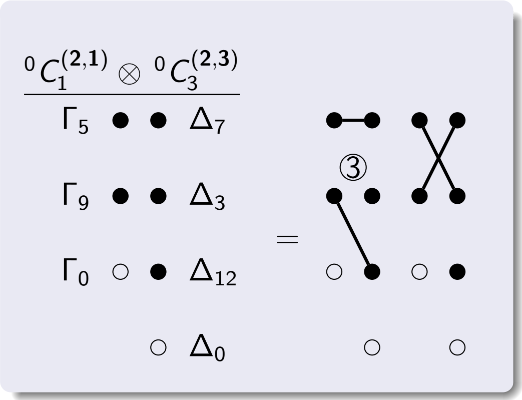
$^{0}C_{1}^{\mathbf{2},1}$: det$(M) \neq 0$, $A_5 \oplus A_9$ (3 mixing angles)

$^{0}C_{3}^{\mathbf{2},2}$: det$(M) \neq 0$, $A_7 \oplus A_3$(3 mixing angles), $A_7 \oplus A_{12}$



Building models: 2HDM example

Putting all together



Model 1

$$\overbrace{\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}}^{\Gamma_1} \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}}^{\Gamma_2}$$

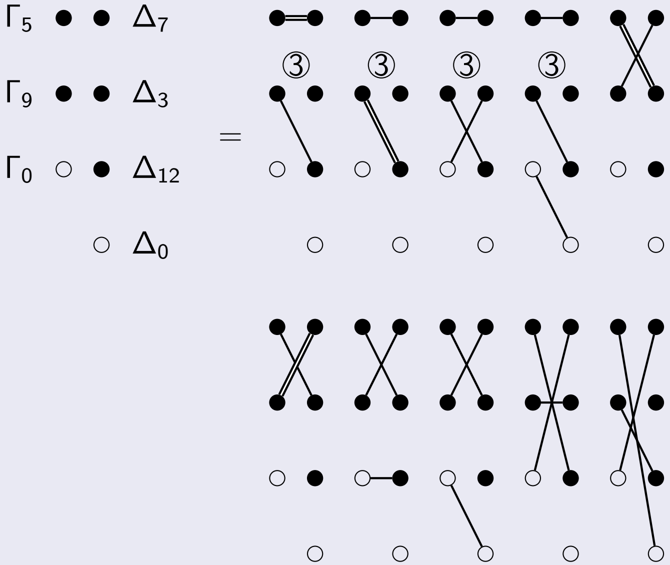
$$\underbrace{\begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\Delta_1} \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}}_{\Delta_2}$$

Model 2

$$\overbrace{\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}}^{\Gamma_1} \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}}^{\Gamma_2}$$

$$\underbrace{\begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}}_{\Delta_1} \underbrace{\begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\Delta_2}$$

Building models: 3HDM example



Bottom-Up approach

- Decompose the Yukawa texture in the largest set of disjoint textures, say k. This implementation will need k Higgs fields.
- Contract two textures. This will reduce the set to $k-1$. Do this for all possible combinations. Exclude the cases which:
  (1) Not all textures belong to the same class;
  (2) One texture is not in the set of A_i textures;
- Continue contracting until you only have one texture.

Bottom-Up approach

- Decompose the Yukawa texture in the largest set of disjoint textures, say k. This implementation will need k Higgs fields.
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  (1) Not all textures belong to the same class;
  (2) One texture is not in the set of A_i textures;
- Continue contracting until you only have one texture.

- Check if you have any chain that contain those set of textures
- Use the Smith Normal Form method
  GUTs Petersen, Ratz, Schieren (2009)
  NHDM scalar Ivanov, Keus, Vdovin (2012)
  NHDM Yukawa Ivanov, Nishi (2013)

Exampte: Nearest-neighbour-interaction

Up and down **M** textures

$$\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} = \begin{pmatrix} & & \\ \times & & \\ & & \end{pmatrix} + \begin{pmatrix} & \times & \\ & & \\ & & \end{pmatrix} + \begin{pmatrix} & & \\ & & \\ & \times & \end{pmatrix} + \begin{pmatrix} & & \\ & & \times \\ & & \end{pmatrix} + \begin{pmatrix} & & \\ & & \\ & & \times \end{pmatrix}$$

$\{\mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ \mathcal{P}_{13}A_{12}\mathcal{P}_{23},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12},\ A_{12}\} \in (\mathbf{3},\mathbf{3}).\ Z_8$

Exampte: Nearest-neighbour-interaction

Up and down **M** textures

$$\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} = \begin{pmatrix} & & \\ \times & & \\ & & \end{pmatrix} + \begin{pmatrix} & \times & \\ & & \\ & & \end{pmatrix} + \begin{pmatrix} & & \\ & & \\ & \times & \end{pmatrix} + \begin{pmatrix} & & \\ & & \times \\ & & \end{pmatrix} + \begin{pmatrix} & & \\ & & \\ & & \times \end{pmatrix}$$

$$\{\mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ \mathcal{P}_{13}A_{12}\mathcal{P}_{23},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12},\ A_{12}\} \in (\mathbf{3},\mathbf{3}).\ \ Z_8$$

Contracting two textures

$$\overbrace{\begin{pmatrix} 0 & 0 & 0 \\ \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{23}A_{12}\mathcal{P}_{13}} \cup \overbrace{\begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{13}A_{12}\mathcal{P}_{23}} = \overbrace{\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{321}A_{15}\mathcal{P}_{13}} \in (\mathbf{3},\mathbf{3})$$

Exampte: Nearest-neighbour-interaction

Up and down **M** textures

$$\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} = \begin{pmatrix} & & \\ \times & & \\ & & \end{pmatrix} + \begin{pmatrix} & \times & \\ & & \\ & & \end{pmatrix} + \begin{pmatrix} & & \\ & & \\ & \times & \end{pmatrix} + \begin{pmatrix} & & \\ & & \times \\ & & \end{pmatrix} + \begin{pmatrix} & & \\ & & \\ & & \times \end{pmatrix}$$

$$\{\mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ \mathcal{P}_{13}A_{12}\mathcal{P}_{23},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12},\ A_{12}\} \in (\mathbf{3},\mathbf{3}).\ \ Z_8$$

Contracting two textures

$$\overbrace{\begin{pmatrix} 0 & 0 & 0 \\ \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{23}A_{12}\mathcal{P}_{13}} \cup \overbrace{\begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{13}A_{12}\mathcal{P}_{23}} = \overbrace{\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{321}A_{15}\mathcal{P}_{13}} \in (\mathbf{3},\mathbf{3})$$

However, the union

$$\overbrace{\begin{pmatrix} 0 & 0 & 0 \\ \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{23}A_{12}\mathcal{P}_{13}} \cup \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{23}A_{12}} = \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ \times & 0 & \times \\ 0 & 0 & 0 \end{pmatrix}}^{\mathcal{P}_{23}A_{11}\mathcal{P}_{12}} \in \mathcal{P}_{23}(\mathbf{2},\mathbf{2}) \text{ or } (\mathbf{3},\mathbf{2})$$

Nearest-neighbour-interaction

Doing this procedure for all combinations one finds five distinct cases

$$
\begin{aligned}
(1): &\quad \{\mathcal{P}_{321}A_{15}\mathcal{P}_{13},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12},\ A_{12}\} \\
(2): &\quad \{A_{15}\mathcal{P}_{12},\ \mathcal{P}_{13}A_{12}\mathcal{P}_{23},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12}\} \\
(3): &\quad \{\mathcal{P}_{321}A_{15},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ A_{12}\} \\
(4): &\quad \{\mathcal{P}_{12}A_{15},\ \mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ \mathcal{P}_{23}A_{12},\ A_{12}\mathcal{P}_{23}\} \\
(5): &\quad \{A_{15}\mathcal{P}_{23},\ \mathcal{P}_{13}A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ A_{12}\}
\end{aligned}
$$

Nearest-neighbour-interaction

Doing this procedure for all combinations one finds five distinct cases

$$\begin{aligned}
(1):&\quad \{\mathcal{P}_{321}A_{15}\mathcal{P}_{13},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12},\ A_{12}\}\\
(2):&\quad \{A_{15}\mathcal{P}_{12},\ \mathcal{P}_{13}A_{12}\mathcal{P}_{23},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12}\}\\
(3):&\quad \{\mathcal{P}_{321}A_{15},\ A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ A_{12}\}\\
(4):&\quad \{\mathcal{P}_{12}A_{15},\ \mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ \mathcal{P}_{23}A_{12},\ A_{12}\mathcal{P}_{23}\}\\
(5):&\quad \{A_{15}\mathcal{P}_{23},\ \mathcal{P}_{13}A_{12}\mathcal{P}_{23},\ \mathcal{P}_{23}A_{12}\mathcal{P}_{13},\ A_{12}\}
\end{aligned}$$

Therefore, the diagram for case (2) is

$$\begin{array}{rcl}
\Gamma_{15}\mathcal{P}_{12} & \bullet\ \bullet & \Delta_{15}\mathcal{P}_{12}\\
\mathcal{P}_{321}\Gamma_{15}\mathcal{P}_{12} & \bullet\ \bullet & \mathcal{P}_{321}\Delta_{15}\mathcal{P}_{12}\\
\mathcal{P}_{13}\Gamma_{15}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{13}\Delta_{15}\mathcal{P}_{13}\\
\mathcal{P}_{13}\Gamma_{12}\mathcal{P}_{23} & \bullet\ \bullet & \mathcal{P}_{13}\Delta_{12}\mathcal{P}_{23}\\
\mathcal{P}_{23}\Gamma_{12} & \bullet\ \bullet & \mathcal{P}_{23}\Delta_{12}\\
\Gamma_{12}\mathcal{P}_{23} & \bullet\ \bullet & \Delta_{12}\mathcal{P}_{23}
\end{array} \quad =$$

Nearest-neighbour-interaction

Two Higgs doublets

$\{A_{13}\mathcal{P}_{12},\, A_{15}\mathcal{P}_{23}\}$

$$\left\{\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix},\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}\right\}$$

$$\begin{array}{rcl} \Gamma_{13}\mathcal{P}_{12} & \bullet\;\bullet & \Delta_{13}\mathcal{P}_{12} \\ \mathcal{P}_{123}\Gamma_{15}\mathcal{P}_{13} & \bullet\;\bullet & \mathcal{P}_{123}\Delta_{15}\mathcal{P}_{13} \\ \Gamma_{15}\mathcal{P}_{23} & \bullet\;\bullet & \Delta_{15}\mathcal{P}_{23} \\ \mathcal{P}_{13}\Gamma_{15}\mathcal{P}_{13} & \bullet\;\bullet & \mathcal{P}_{13}\Delta_{15}\mathcal{P}_{13} \end{array} =$$

Nearest-neighbour-interaction

Two Higgs doublets

$\{A_{13}\mathcal{P}_{12},\ A_{15}\mathcal{P}_{23}\}$

$$\left\{\begin{pmatrix}0 & \times & 0\\ \times & 0 & 0\\ 0 & 0 & \times\end{pmatrix},\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \times\\ 0 & \times & 0\end{pmatrix}\right\}$$

$\Gamma_{13}\mathcal{P}_{12}$ • • $\Delta_{13}\mathcal{P}_{12}$

$\mathcal{P}_{123}\Gamma_{15}\mathcal{P}_{13}$ • • $\mathcal{P}_{123}\Delta_{15}\mathcal{P}_{13}$ $=$

$\Gamma_{15}\mathcal{P}_{23}$ • • $\Delta_{15}\mathcal{P}_{23}$

$\mathcal{P}_{13}\Gamma_{15}\mathcal{P}_{13}$ • • $\mathcal{P}_{13}\Delta_{15}\mathcal{P}_{13}$

Branco, Emmanuel-Costa, Simoes (2010) Z_{4n}

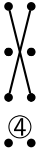
Nearest-neighbour-interaction

Two Higgs doublets

$\{A_{13}\mathcal{P}_{12},\ A_{15}\mathcal{P}_{23}\}$

$$\left\{\begin{pmatrix}0 & \times & 0\\ \times & 0 & 0\\ 0 & 0 & \times\end{pmatrix},\ \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \times\\ 0 & \times & 0\end{pmatrix}\right\}$$

$$\begin{array}{ccc} \Gamma_{13}\mathcal{P}_{12} & \bullet\ \bullet & \Delta_{13}\mathcal{P}_{12}\\ \mathcal{P}_{123}\Gamma_{15}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{123}\Delta_{15}\mathcal{P}_{13}\\ \Gamma_{15}\mathcal{P}_{23} & \bullet\ \bullet & \Delta_{15}\mathcal{P}_{23}\\ \mathcal{P}_{13}\Gamma_{15}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{13}\Delta_{15}\mathcal{P}_{13} \end{array} =$$



Branco, Emmanuel-Costa, Simoes (2010) Z_{4n}

$$\begin{array}{ccc} \Gamma_{13}\mathcal{P}_{12} & \bullet\ \bullet & \Delta_{13}\mathcal{P}_{12}\\ \mathcal{P}_{123}\Gamma_{15}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{123}\Delta_{15}\mathcal{P}_{13}\\ \Gamma_{15}\mathcal{P}_{23} & \bullet\ \bullet & \Delta_{15}\mathcal{P}_{23}\\ \mathcal{P}_{13}\Gamma_{12}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{13}\Delta_{12}\mathcal{P}_{13}\\ \mathcal{P}_{23}\Gamma_{12}\mathcal{P}_{23} & \bullet\ \bullet & \mathcal{P}_{23}\Delta_{12}\mathcal{P}_{23} \end{array} =$$

Nearest-neighbour-interaction

Two Higgs doublets

$\{A_{13}\mathcal{P}_{12},\ A_{15}\mathcal{P}_{23}\}$

$$\left\{ \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} \right\}$$

$$\begin{array}{rcl} \Gamma_{13}\mathcal{P}_{12} & \bullet\ \bullet & \Delta_{13}\mathcal{P}_{12} \\ \mathcal{P}_{123}\Gamma_{15}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{123}\Delta_{15}\mathcal{P}_{13} \\ \Gamma_{15}\mathcal{P}_{23} & \bullet\ \bullet & \Delta_{15}\mathcal{P}_{23} \\ \mathcal{P}_{13}\Gamma_{15}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{13}\Delta_{15}\mathcal{P}_{13} \end{array} =$$

Branco, Emmanuel-Costa, Simoes (2010) Z_{4n}

$Z_{n\geq 5}$

$$\begin{array}{rcl} \Gamma_{13}\mathcal{P}_{12} & \bullet\ \bullet & \Delta_{13}\mathcal{P}_{12} \\ \mathcal{P}_{123}\Gamma_{15}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{123}\Delta_{15}\mathcal{P}_{13} \\ \Gamma_{15}\mathcal{P}_{23} & \bullet\ \bullet & \Delta_{15}\mathcal{P}_{23} \\ \mathcal{P}_{13}\Gamma_{12}\mathcal{P}_{13} & \bullet\ \bullet & \mathcal{P}_{13}\Delta_{12}\mathcal{P}_{13} \\ \mathcal{P}_{23}\Gamma_{12}\mathcal{P}_{23} & \bullet\ \bullet & \mathcal{P}_{23}\Delta_{12}\mathcal{P}_{23} \end{array} =$$

Nearest-neighbour-interaction

Two Higgs doublets

$\{A_{13}\mathcal{P}_{12},\, A_{15}\mathcal{P}_{23}\}$

$$\left\{\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix},\, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}\right\}$$

$$\begin{array}{rcl} \Gamma_{13}\mathcal{P}_{12} & \bullet\;\bullet & \Delta_{13}\mathcal{P}_{12} \\ \mathcal{P}_{123}\Gamma_{15}\mathcal{P}_{13} & \bullet\;\bullet & \mathcal{P}_{123}\Delta_{15}\mathcal{P}_{13} \\ \Gamma_{15}\mathcal{P}_{23} & \bullet\;\bullet & \Delta_{15}\mathcal{P}_{23} \\ \mathcal{P}_{13}\Gamma_{15}\mathcal{P}_{13} & \bullet\;\bullet & \mathcal{P}_{13}\Delta_{15}\mathcal{P}_{13} \end{array} =$$

Branco, Emmanuel-Costa, Simoes (2010) Z_{4n}

$Z_{n\geq 5}$

$$\begin{array}{rcl} \Gamma_{13}\mathcal{P}_{12} & \bullet\;\bullet & \Delta_{13}\mathcal{P}_{12} \\ \mathcal{P}_{123}\Gamma_{15}\mathcal{P}_{13} & \bullet\;\bullet & \mathcal{P}_{123}\Delta_{15}\mathcal{P}_{13} \\ \Gamma_{15}\mathcal{P}_{23} & \bullet\;\bullet & \Delta_{15}\mathcal{P}_{23} \\ \mathcal{P}_{13}\Gamma_{12}\mathcal{P}_{13} & \bullet\;\bullet & \mathcal{P}_{13}\Delta_{12}\mathcal{P}_{13} \\ \mathcal{P}_{23}\Gamma_{12}\mathcal{P}_{23} & \bullet\;\bullet & \mathcal{P}_{23}\Delta_{12}\mathcal{P}_{23} \end{array} =$$

Ivanov, Nishi (2013)

These textures imply the existence of a $U(1)$ symmetry in the Yukawa sector of 2HDM

Conclusions

- The presence of Abelian symmetries in the Yukawa sector leads to a finite set of possible textures
- Organizing these textures into chains allow us to convert this analysis into a combinatorial problem
- Bottom-Up approach can be advantageous in model building
- Information on FCNCs can also be extracted in this way