Neutrino Mass Textures

- Impact of CP phase observation on Neutrino Mass Textures -

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1 Introduction

FLASY predicts Lepton Mass Matrices,

which can be tested by Neutrino Masses and Mixing Angles.

Especially, experimental data of CP violating phase will give us severe tests for

Lepton Mass Matrices

FTY texture for Lepton

Fukugita, M.T, Yanagida, 2003

$$m_E = \begin{pmatrix} 0 & A_\ell & 0 \\ A_\ell & 0 & B_\ell \\ 0 & B_\ell & C_\ell \end{pmatrix}, \qquad m_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & 0 & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix}$$

$$M_R = M_0 \mathbf{1}$$

$$U_{e2} \simeq -\left(\frac{m_1}{m_2}\right)^{1/4} + \left(\frac{m_e}{m_\mu}\right)^{1/2} e^{i\sigma}$$

$$U_{e3} \simeq \left(\frac{m_e}{m_\mu}\right)^{1/2} U_{\mu 3} + \left(\frac{m_2}{m_3}\right)^{1/2} \left(\frac{m_1}{m_3}\right)^{1/4}$$

$$U_{\mu 3} \simeq \left(\frac{m_2}{m_3}\right)^{1/4} e^{i\sigma} - \left(\frac{m_\mu}{m_\tau}\right)^{1/2} e^{i\tau}$$

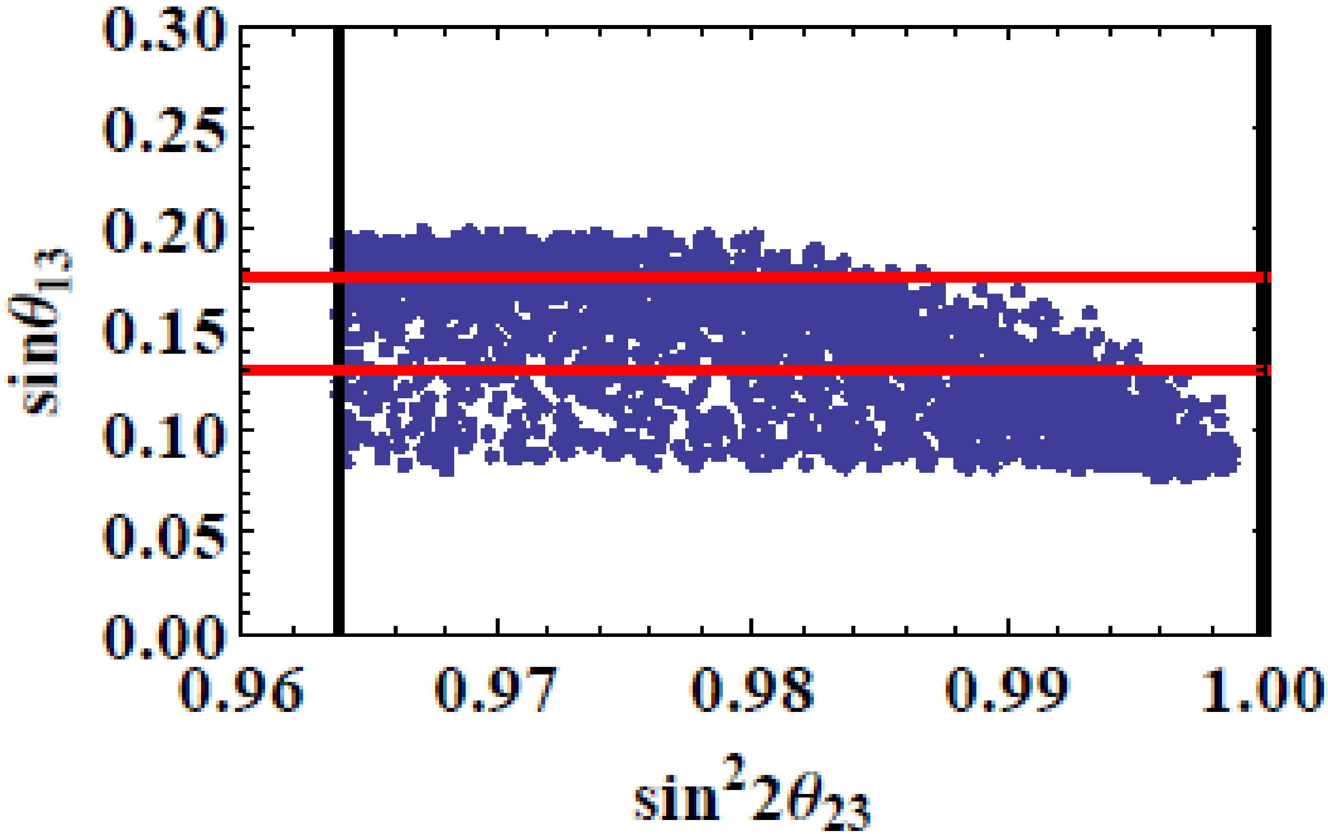
$$\sin\theta_{23} \simeq \sqrt[8]{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.63 \sim 0.66 = \mathcal{O}(\sqrt[4]{\lambda})$$

Mixing Sum Rule

$$|U_{e3}| \approx |U_{\mu 3}|^2 |U_{e2} U_{\mu 3}| = |U_{\mu 3}|^3 |U_{e2}|$$

FTY texture for Lepton

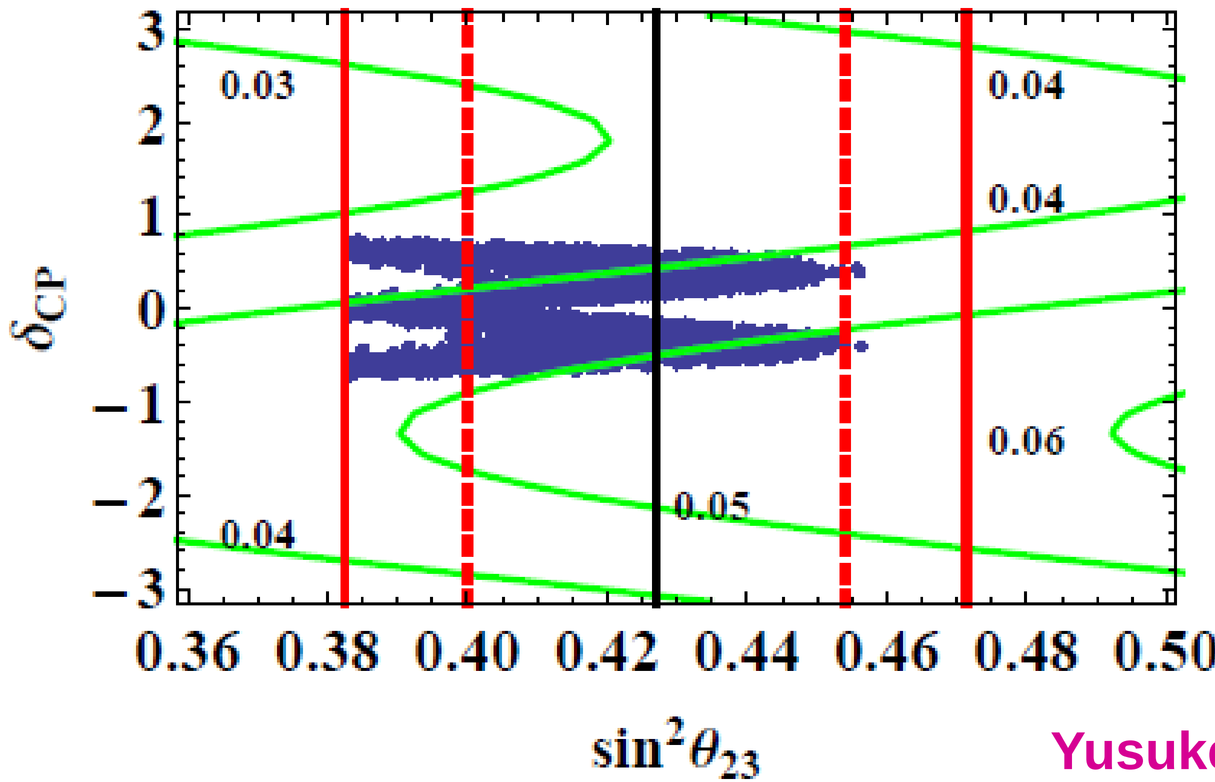
Fukugita, Shimizu, M.T, Yanagida, 2012



$\Theta_{23}=\pi/4$ is excluded.

CP phase is predicted !

1 σ 90% C.L.



Yusuke Shimizu 2013

$$P(\nu_\mu \to \nu_e) = 0.03,\ 0.04,\ 0.05,\ 0.06$$

If $|\delta_{CP}|=\pi/2$, this texture is excluded.

Let us examine the CP phase in the FLASY motivated Texture of Neutrino Masses focusing on T2K data.

Conventional definition of Mixing Matrix

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij},\ s_{ij} \equiv \sin\theta_{ij}$$

2 FLASY motivated Textures

Before 2012 (no data for Θ_{13})
Neutrino Data suggested
Tri-bimaximal Mixing of Neutrinos

$$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$$

$$U_{\text{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Harrison, Perkins, Scott (2002)

Tri-bimaximal Mixing (TBM) is realized by

$$m_{TBM} = \frac{m_1+m_3}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

in the diagonal basis of charged leptons.

Mixing angles are independent of neutrino masses.

Integer (inter-family related) matrix elements suggest Non-Abelian Discrete Flavor Symmetry.

In 2012

Reactor angle θ_{13} was measured by T2K, Daya Bay, MINOS, RENO, Double Chooz

$$\theta_{13} \simeq 9^\circ \simeq \theta_c/\sqrt{2}$$

Tri-bimaximal mixing was ruled out !

- Deviation from Tri-bimaximal mixing ?
- Different Anzatz ?
  Tri-maximal mixing, Tri-bimaximal Cabibbo ....

Indirect Approach : A_4 Model to realize large θ_{13}

Modify G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ	ξ'
$SU(2)$	2	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω

Y. Simizu, M. Tanimoto, A. Watanabe, PTP 126, 81(2011)

$$\mathbf{3}\times\mathbf{3}\Rightarrow\mathbf{1} \quad = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$
$$\mathbf{3}\times\mathbf{3}\Rightarrow\mathbf{1}' \quad = a_1 * b_2 + a_2 * b_1 + a_3 * b_3$$
$$\mathbf{3}\times\mathbf{3}\Rightarrow\mathbf{1}'' \quad = a_1 * b_3 + a_2 * b_2 + a_3 * b_1$$

ξ $\qquad\qquad$ ξ'

$$\mathbf{1}\times\mathbf{1}\Rightarrow\mathbf{1} \quad , \qquad \mathbf{1}''\times\mathbf{1}'\Rightarrow\mathbf{1}$$

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} \qquad \begin{pmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}$$

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_\xi \alpha_\xi v_u^2}{\Lambda}, \qquad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda}$$

$$a = -3b$$

Both normal and inverted mass hierarchies are possible.

After rotaing it by Tri-bimaximal mixing matrix, we get

$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix} a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d \\ 0 & a+3b+c+d & 0 \\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix} V_{\text{tri-bi}}^T$$

Tri-maximal mixing: TM2

Direct Approach of FLASY

A_4 has 12 elements and subgroups:

three Z_2, four Z_3, one $Z_2 \times Z_2$ (klein four-group)

Z_2: {1,S}, {1,T²ST}, {1,TST²}

Z_3: {1,T,T²}, {1,ST,T²S}, {1,TS, ST²}, {1,STS,ST²S}

K_4: {1,S,T²ST,TST²}

Suppose A_4 is spontaneously broken to subgroups:

Neutrino sector preserves Z_2: {1,S}

Charged lepton sector preserves Z_3: {1,T,T²}

$$S^T m^\nu_{LL} S = m^\nu_{LL}, \quad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

⇓

$$[S, m^\nu_{LL}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

Mixing matrices diagonalise m^ν_{LL}, $Y_e Y_e^\dagger$ also diagonalize S and T, respectively !

For the triplet representation

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$

$$m_{\nu LL} = \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

Ding, King, Luhn, Stuart, JHEP1305, arXiv:1303.6180

This matrix respects $G_\nu = \{1, S\}$.

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}-c/\sqrt{2} \\ -c/\sqrt{6}-s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}+c/\sqrt{2} \end{pmatrix}$$

$$c = \cos\theta, \;\; s = \sin\theta$$

1-3 mixing in TBM basis

Tri-maximal mixing : TM2

Freedom of the rotation between 1st and 3rd column.

θ is fixed by the experimental data.

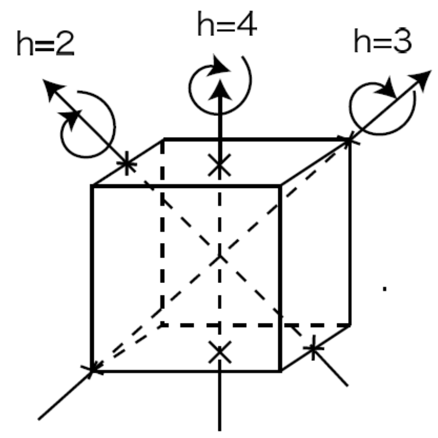
Another example with S_4 group

All permutations among four objects, 4 ! =24 elements

24 elements can be generated by S,T and U:

$S^2=T^3=U^2=1,\ ST^3=(SU)^2=(TU)^2=(STU)^4=1$

Irreducible representations: 1, 1', 2, 3, 3'



Symmetry of a cube

$$U=\mp\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

$$S=\frac{1}{3}\begin{pmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{pmatrix},\quad T=\begin{pmatrix}1&0&0\\0&\omega^2&0\\0&0&\omega\end{pmatrix};\quad \omega=e^{2\pi i/3}$$

S_4 has subgroups

nine Z_2, four Z_3, three Z_4, four $Z_2 \times Z_2$ (K_4)

Suppose S_4 is spontaneously broken to subgroups:

Neutrino sector preserves SU (Z_2)

Charged lepton sector preserves T (Z_3)

$$(SU)^T m_{LL}^{\nu} SU = m_{LL}^{\nu}, \quad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

⬇

$$[SU, m_{LL}^{\nu}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

$$m_{\nu LL} = \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & -2 \end{pmatrix}$$

Rodejohann, Zhang, arXiv:1207.1225, Li, Ding, arXiv1312.4401

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & c/\sqrt{3} & s/\sqrt{3} \\ -1/\sqrt{6} & c/\sqrt{3} - s/\sqrt{2} & -s/\sqrt{3} - c/\sqrt{2} \\ -1/\sqrt{6} & c/\sqrt{3} + s/\sqrt{2} & -s/\sqrt{3} + c/\sqrt{2} \end{pmatrix}$$

$$c = \cos\theta, \;\; s = \sin\theta$$

2nd-3rd column mixing in TBM basis

Tri-maximal mixing TM1

3 Predicting CP phase

Mixing sum rules

TM2
A_4, S_4

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}-c/\sqrt{2} \\ -c/\sqrt{6}-s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}+c/\sqrt{2} \end{pmatrix}$$

TM1
S_4

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & c/\sqrt{3} & s/\sqrt{3} \\ -1/\sqrt{6} & c/\sqrt{3}+s/\sqrt{2} & s/\sqrt{3}-c/\sqrt{2} \\ -1/\sqrt{6} & c/\sqrt{3}-s/\sqrt{2} & s/\sqrt{3}+c/\sqrt{2} \end{pmatrix}$$

$$c = \cos\theta, \;\; s = \sin\theta$$

TM2: Including CP phase

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2se^{-i\sigma}/\sqrt{6} \\ -c/\sqrt{6}+se^{i\sigma}/\sqrt{2} & 1/\sqrt{3} & -se^{-i\sigma}/\sqrt{6}-c/\sqrt{2} \\ -c/\sqrt{6}-se^{i\sigma}/\sqrt{2} & 1/\sqrt{3} & -se^{-i\sigma}/\sqrt{6}+c/\sqrt{2} \end{pmatrix}$$

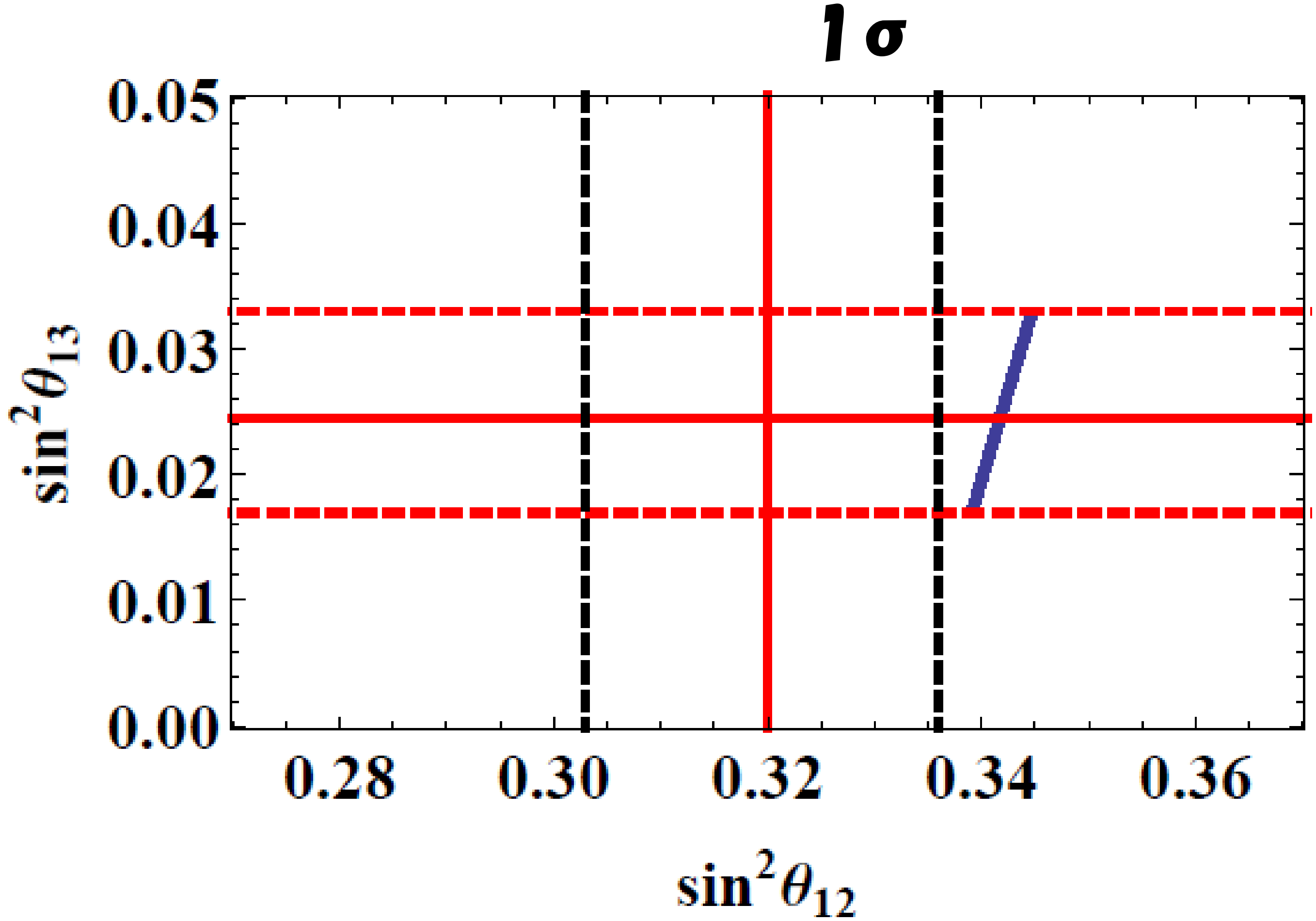
$$\sin^2\theta_{13} = \tfrac{2}{3}\sin^2\theta, \quad \sin^2\theta_{12} = \tfrac{1}{1+2\cos^2\theta}$$

$$\sin^2\theta_{23} = \tfrac{1}{2}\left(1 + \tfrac{\sqrt{6}\sin 2\theta\cos\sigma}{1+2\cos^2\theta}\right)$$

$$\sin^2\theta_{12} > \frac{1}{3}$$

$$\sin\delta_{CP} = -\frac{(2+\cos 2\theta)\sin\sigma}{\sqrt{[(2+\cos 2\theta)^2 - 3\sin^2 2\theta\cos^2\sigma]}}$$

TM2

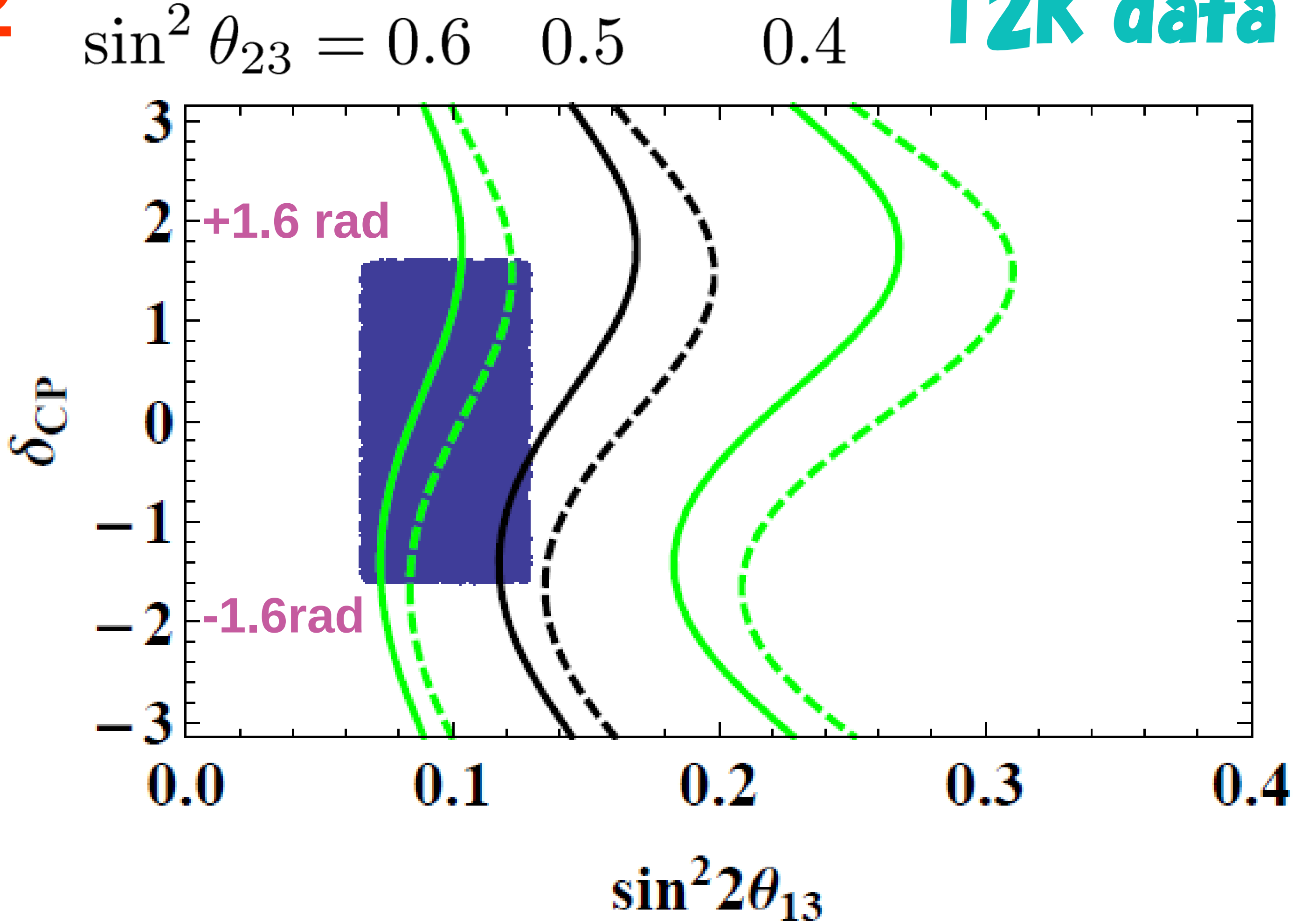


Y.Shimizu, M.T, arXiv 1405.1521

NH: solid curves IH: dashed curves

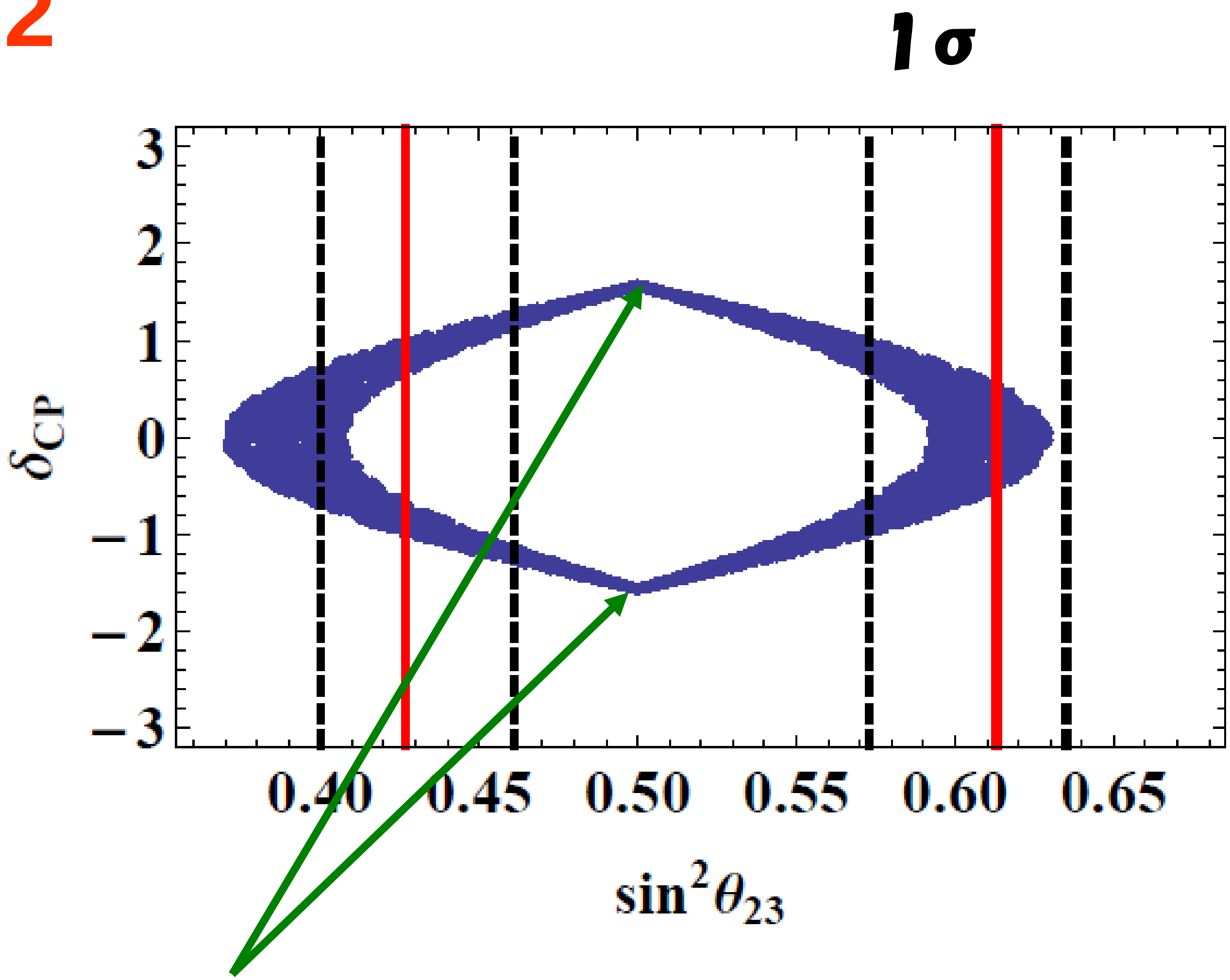
TM2

T2K data



Y.Shimizu, M.T, arXiv 1405.1521

TM2



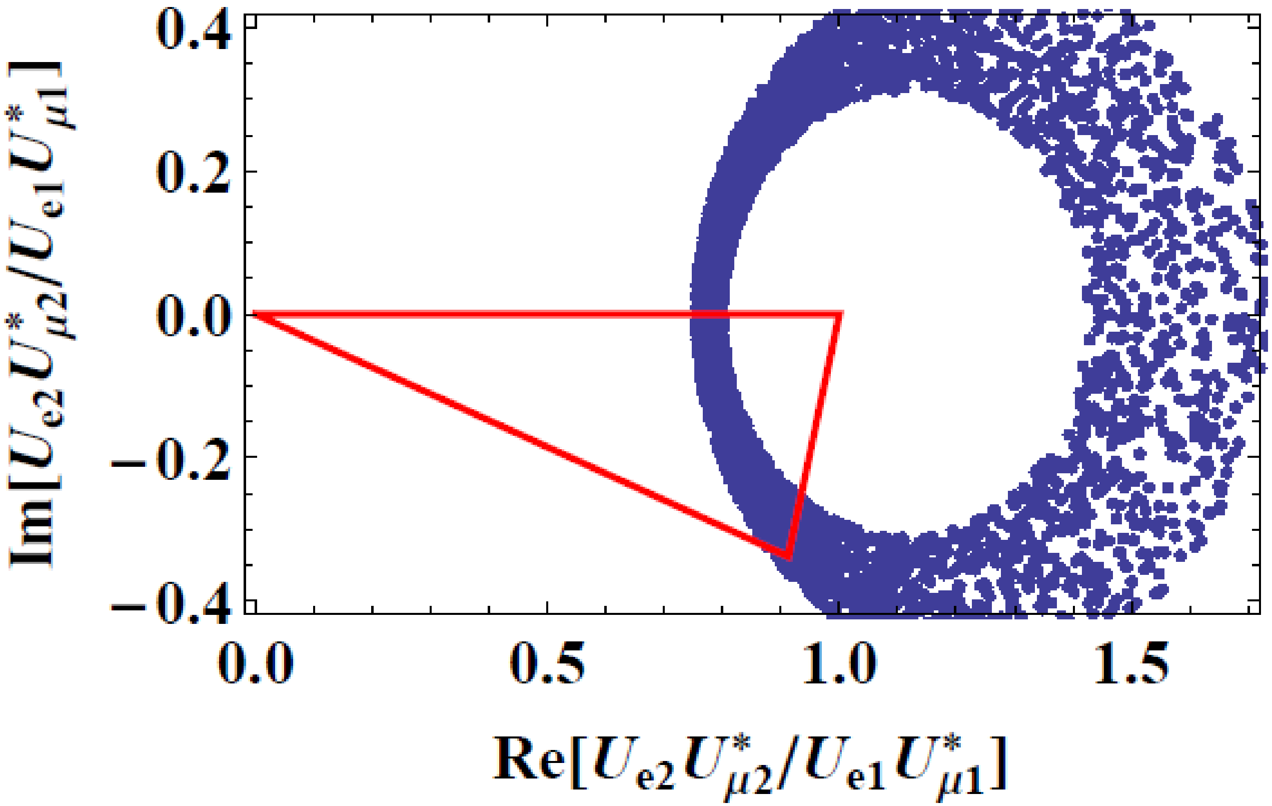
Generalized CP : **maximal CP phase**

Y.Shimizu, M.T, arXiv 1405.1521

Unitarity Triangle

$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$$

TM2



Reference Triangle

$$\delta_{CP} = -\frac{\pi}{2}, \quad \sin^2\theta_{13} = 0.0251, \quad \sin^2\theta_{12} = 0.312, \quad \sin^2\theta_{23} = 0.514$$

TM1: Including CP phase

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & c/\sqrt{3} & se^{-i\sigma}/\sqrt{3} \\ -1/\sqrt{6} & c/\sqrt{3} + se^{i\sigma}/\sqrt{2} & se^{-i\sigma}/\sqrt{3} - c/\sqrt{2} \\ -1/\sqrt{6} & c/\sqrt{3} - se^{i\sigma}/\sqrt{2} & se^{-i\sigma}/\sqrt{3} + c/\sqrt{2} \end{pmatrix}$$

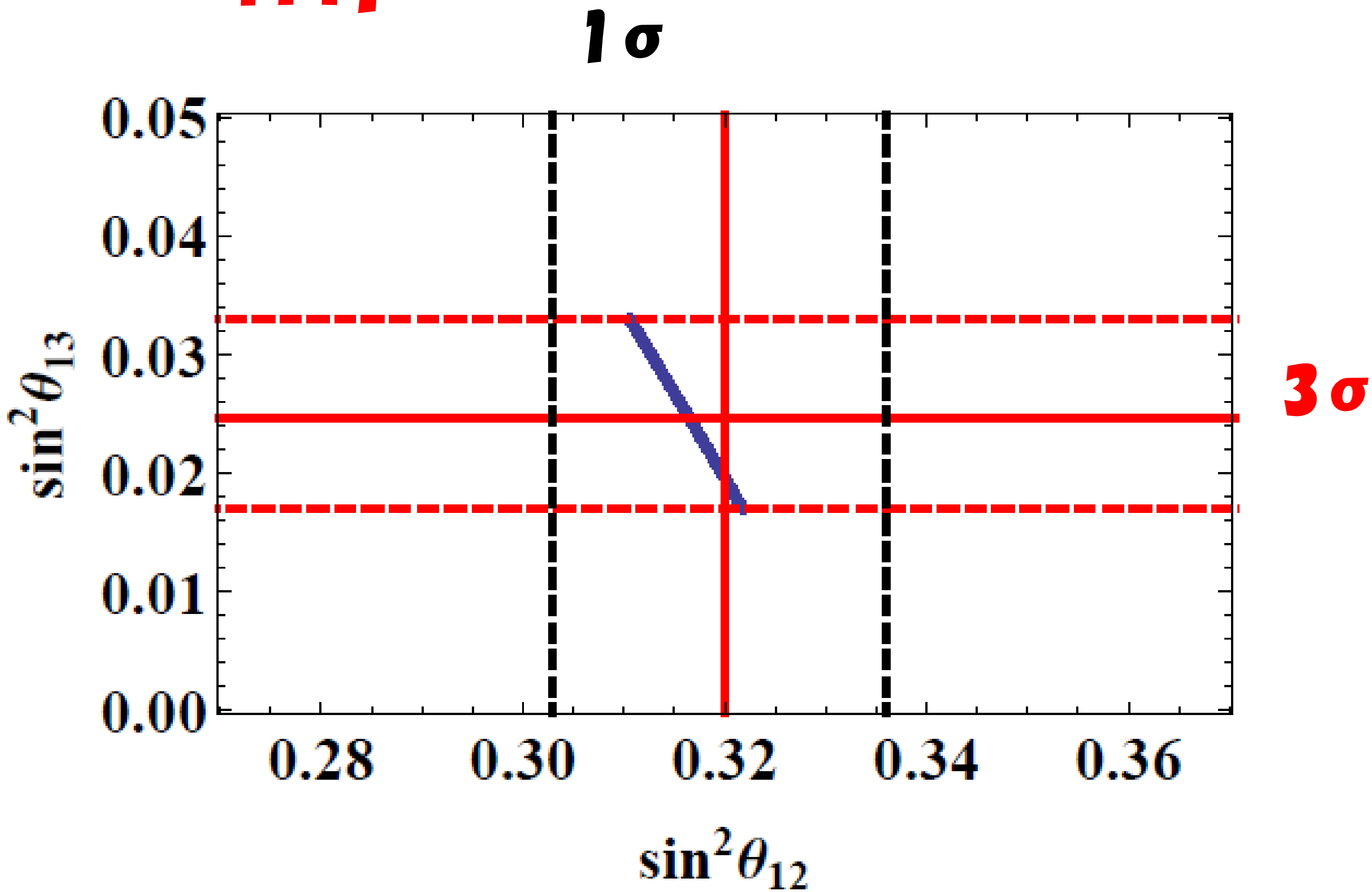
$$\sin^2\theta_{13} = \tfrac{1}{3}\sin^2\theta, \quad \sin^2\theta_{12} = 1 - \tfrac{1}{2+\cos^2\theta}$$

$$\sin^2\theta_{23} = \tfrac{1}{2}\left(1 - \tfrac{\sqrt{6}\sin 2\theta\cos\sigma}{2+\cos^2\theta}\right)$$

$$\sin^2\theta_{12} < \tfrac{1}{3}$$

$$\sin\delta_{CP} = -\frac{\sin 2\theta(5+\cos 2\theta)\sin\sigma}{\sqrt{\sin^2 2\theta[(5+\cos 2\theta)^2 - 24\sin^2 2\theta\cos^2\sigma]}}$$

TM1

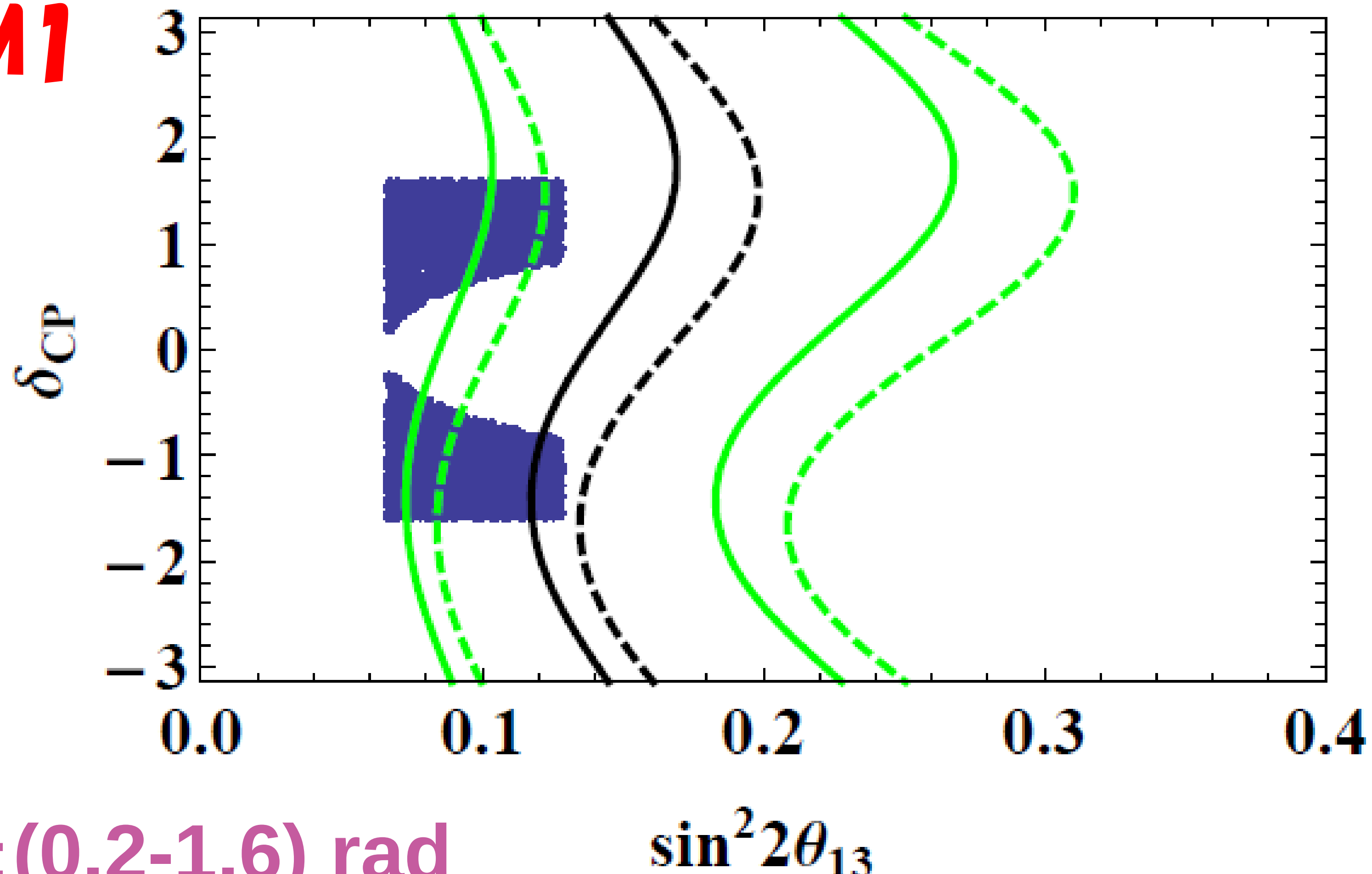


Y.Shimizu, M.T, arXiv 1405.1521

NH: solid curves **IH: dashed curves**

$\sin^2\theta_{23} = 0.6 \quad 0.5 \quad 0.4$ **T2K data**

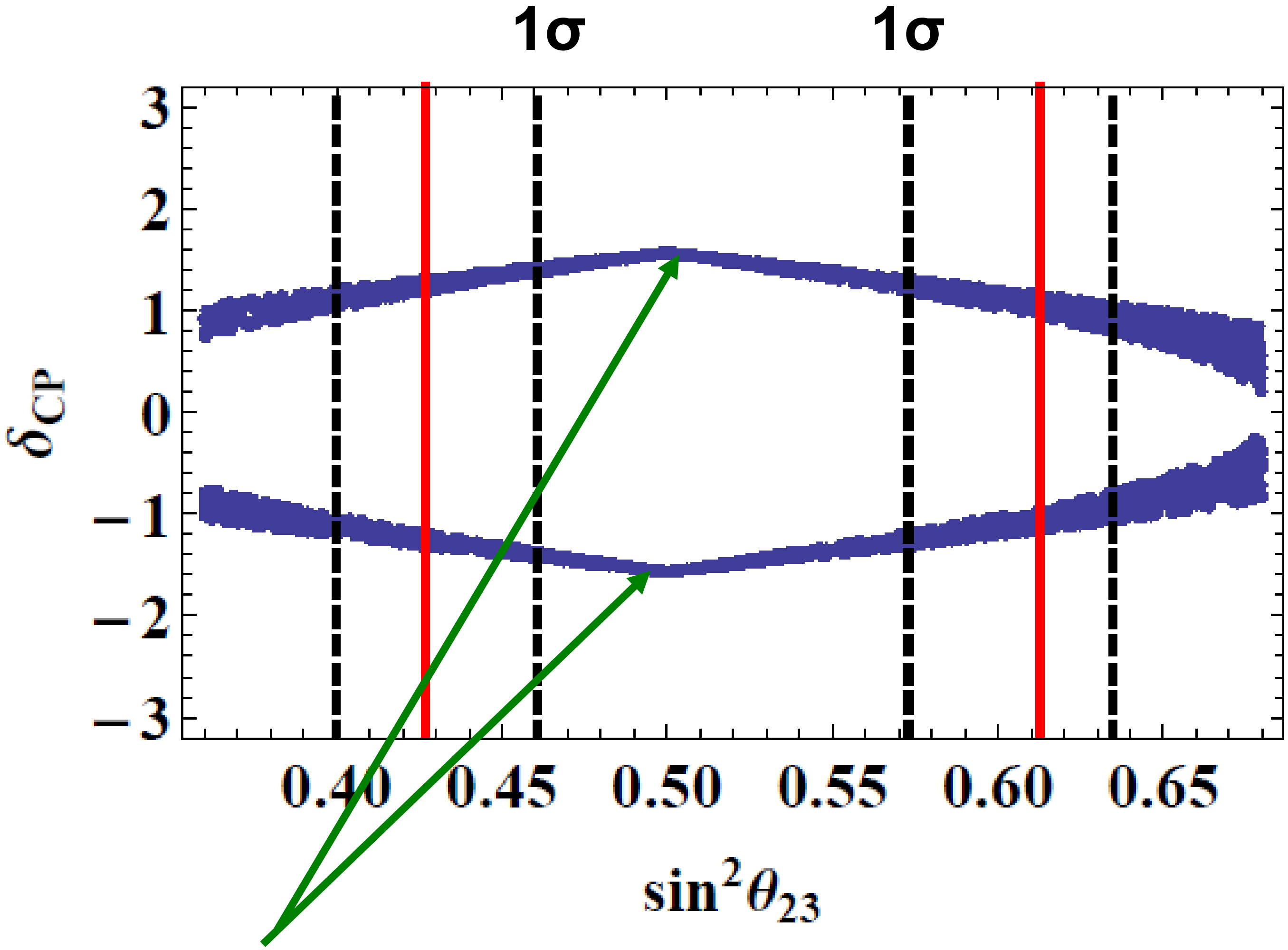
TM1



±(0.2-1.6) rad

Y.Shimizu, M.T, arXiv 1405.1521

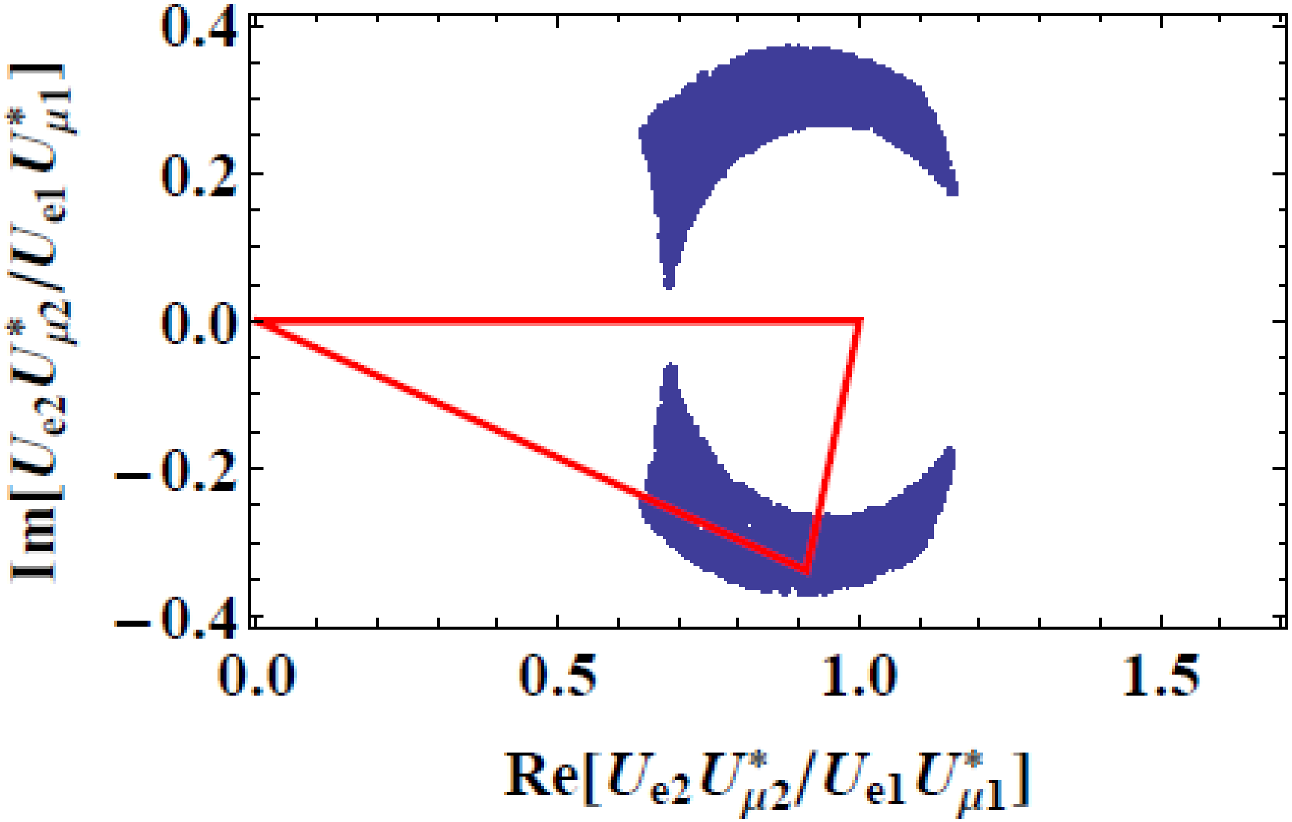
TM1



Generalized CP : maximal CP phase

Unitarity Triangle $U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$

TM1



Reference Triangle

$$\delta_{CP} = -\frac{\pi}{2}, \quad \sin^2\theta_{13} = 0.0251, \quad \sin^2\theta_{12} = 0.312, \quad \sin^2\theta_{23} = 0.514$$

We consider another case:
Rotation of charged lepton mass matrix

$$U_{\rm PMNS} = \begin{pmatrix} \cos\phi & 0 & -e^{-i\sigma}\sin\phi \\ 0 & 1 & 0 \\ e^{i\sigma}\sin\phi & 0 & \cos\phi \end{pmatrix} V_{\rm TBM}$$

$$\sin^2\theta_{12} = \frac{2(1-\sin 2\phi\cos\sigma)}{3(2-\sin^2\phi)}, \quad \sin^2\theta_{13} = \frac{1}{2}\sin^2\phi, \quad \sin^2\theta_{23} = \frac{1}{2-\sin^2\phi}$$

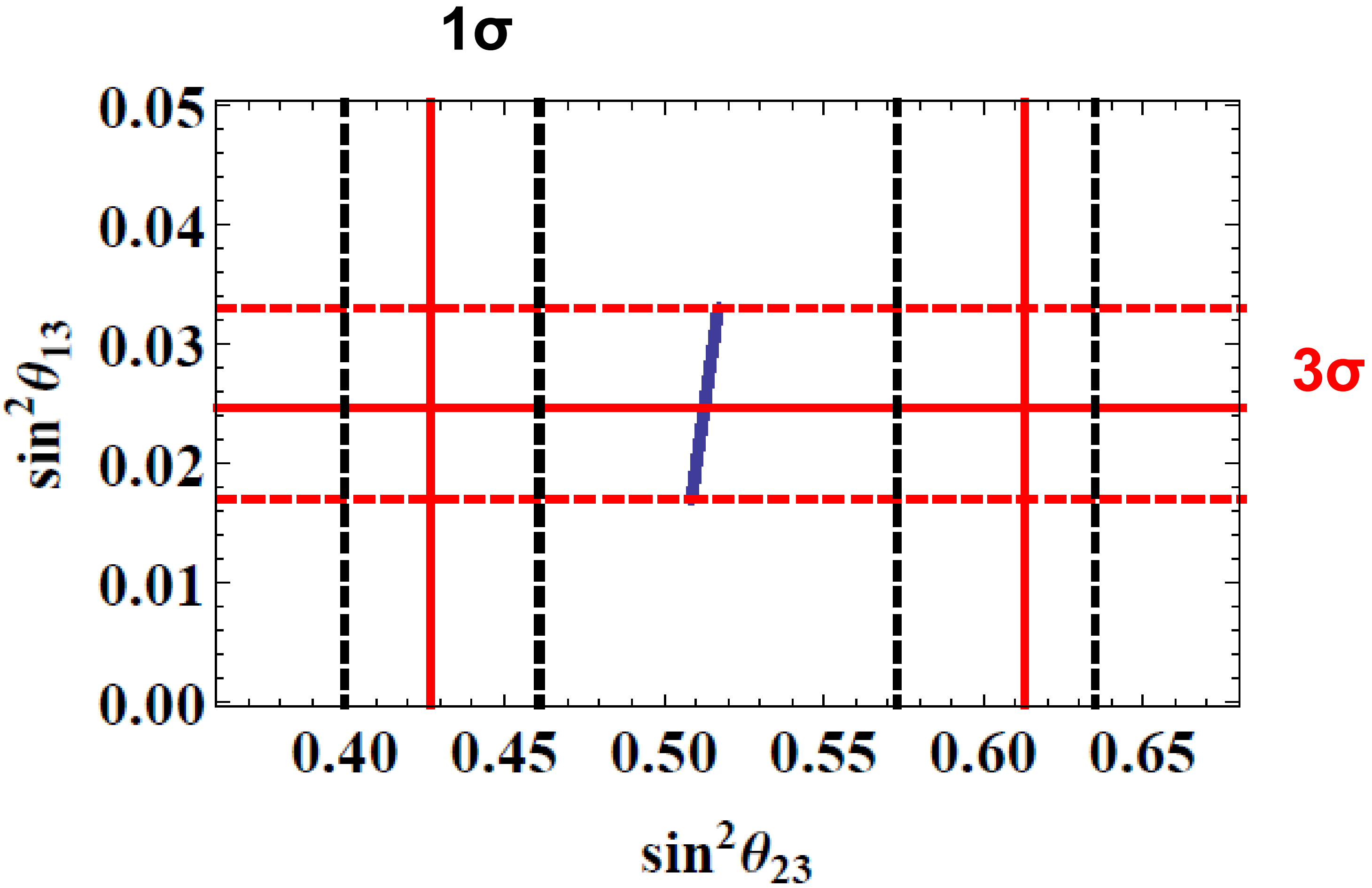
$$\sin\delta_{CP} = \frac{\sin 2\phi(2-\sin^2\phi)\sin\sigma}{\sqrt{\sin^2 2\phi(4-3\sin^2\phi+2\sin 2\phi\cos\sigma)(1-\sin 2\phi\cos\sigma)}}$$

$\sin^2\theta_{23} > \frac{1}{2}$

for 1-3 mixing

Marzocca, Petcov, Romanio, Sevilla, arXiv 1302.0423
Petcov, arXiv 1405.6006

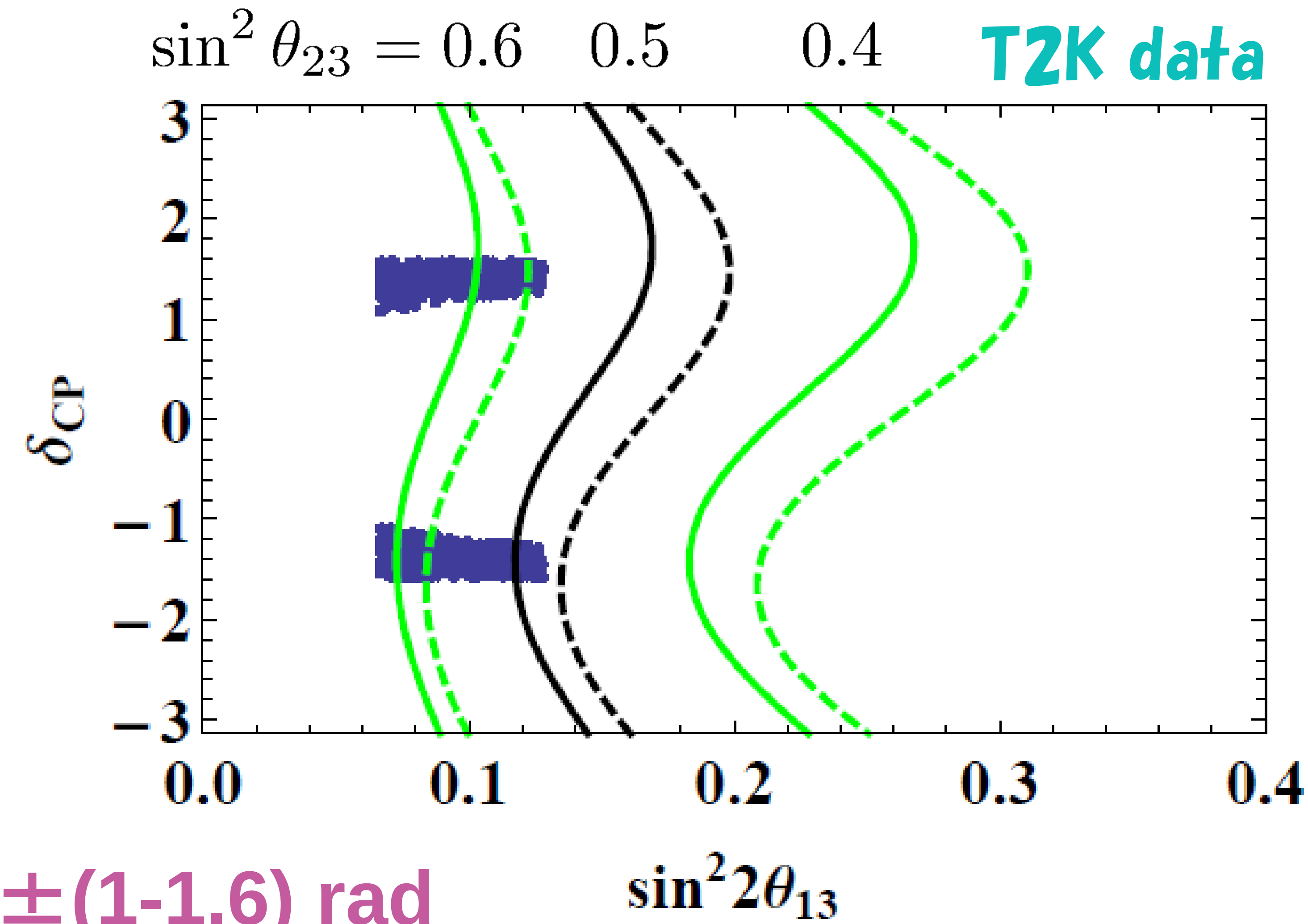
Rotation of Charged Lepton



Y.Shimizu, M.T, arXiv:1405.1521

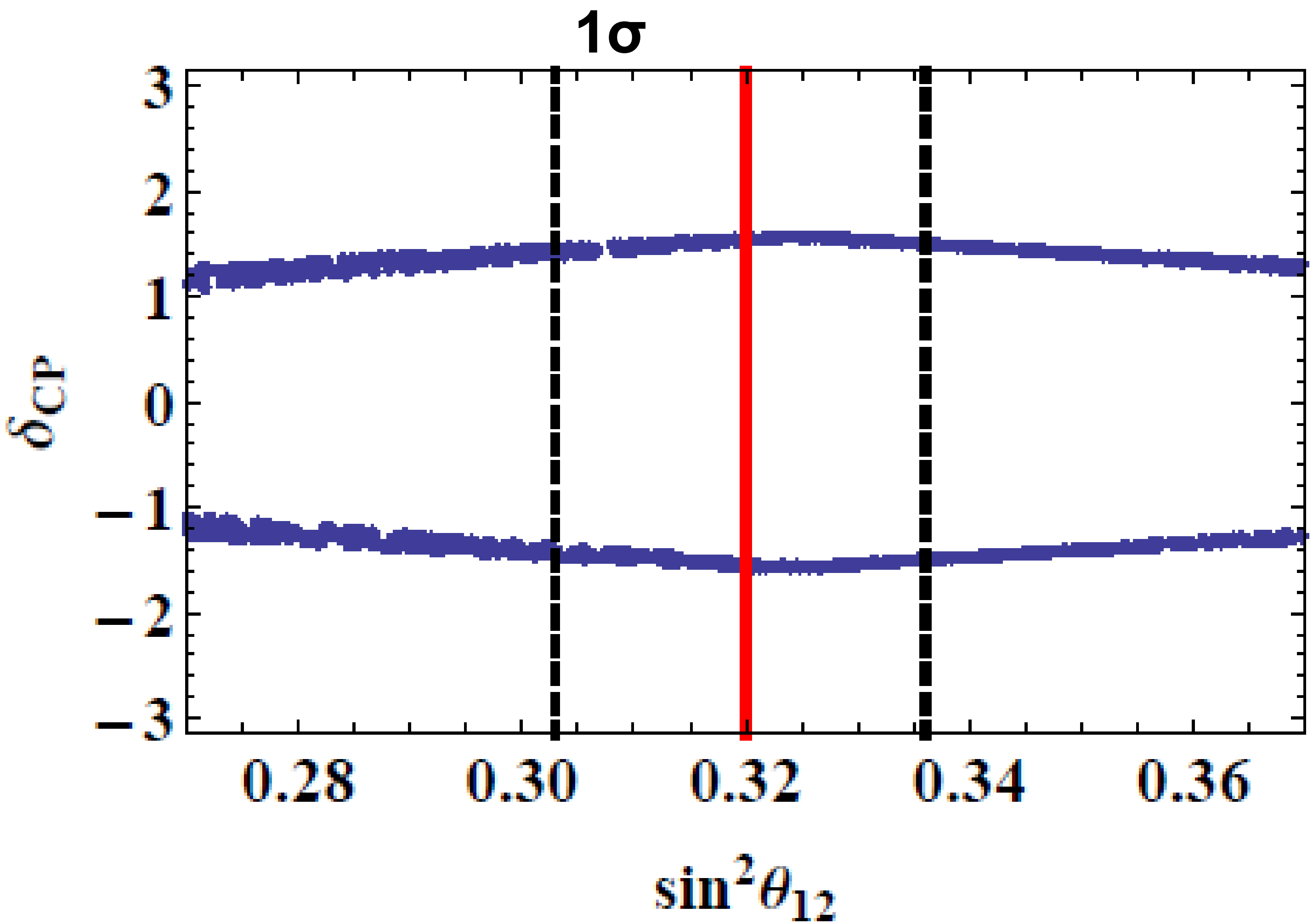
Rotation of Charged Lepton

NH: solid curves IH: dashed curves

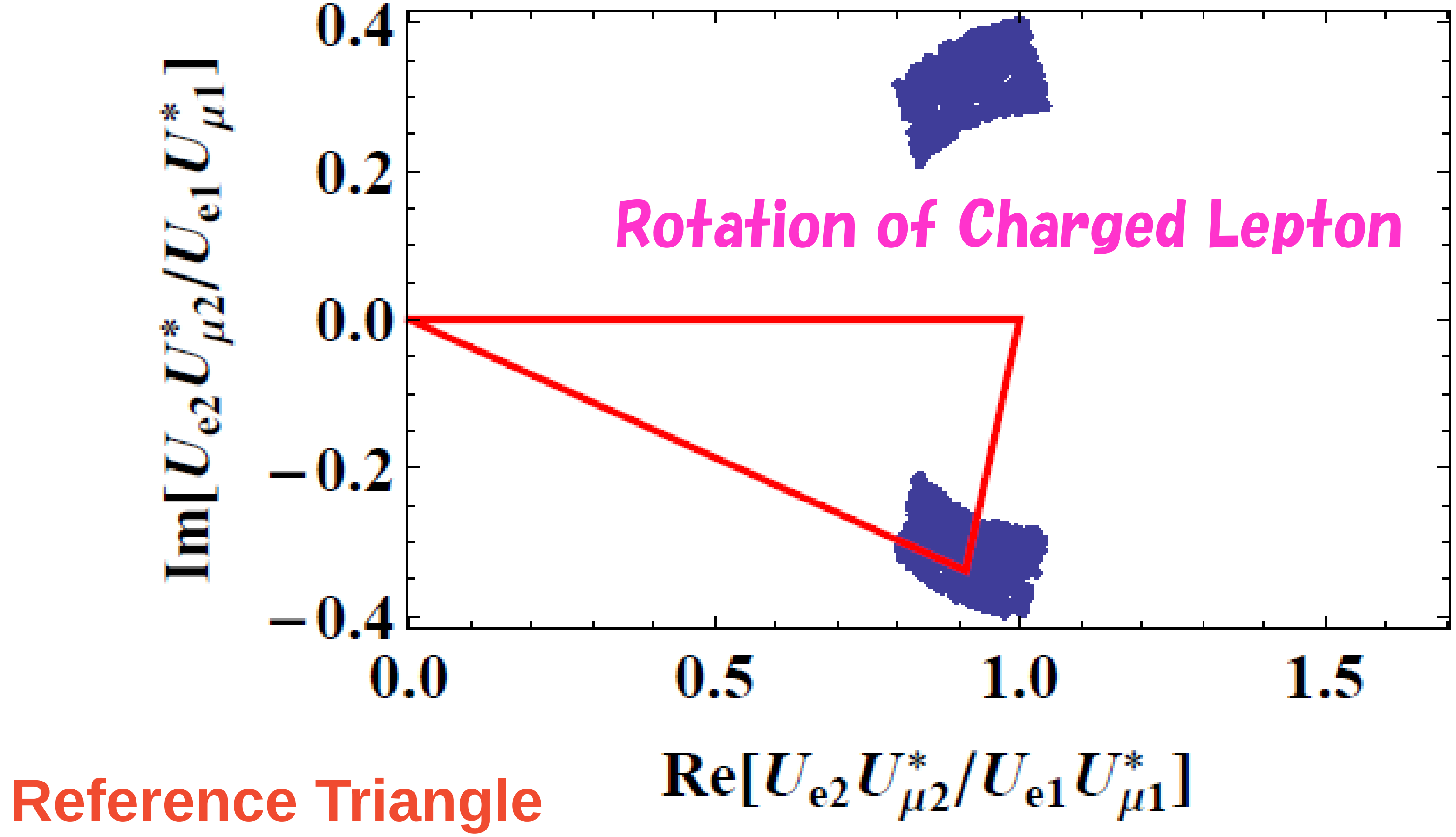


±(1-1.6) rad

Rotation of Charged Lepton



Unitarity Triangle $U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3} = 0$



$$\delta_{CP} = -\frac{\pi}{2}, \quad \sin^2\theta_{13} = 0.0251, \quad \sin^2\theta_{12} = 0.312, \quad \sin^2\theta_{23} = 0.514$$

4 Summary

- CP phase can test
  the neutrino mass textures
  with combination of the mixing angles.

- Require the precise determination of θ_{23}

- Wait improved T2K data and NOvA new data !