Symmetries in multi-Higgs models and their quark sector consequences

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based on:

Gonzalez Felipe, Ivanov, Nishi, Serodio, Silva, arXiv:1401.5807 [hep-ph].

Outline

Flavour puzzle and symmetries

- Search for a symmetry-based resolution of the flavour puzzle: a bSM construction based on flavour group G and as few assumptions as possible which would naturally lead to quark and neutrino masses and mixing.
- Many models with various groups G have been suggested but no clear winner yet.
- Traditional approach: impose a group G, introduce new fields when needed. Complementary approach: deduce what you can get, in terms of symmetries and consequences, with a given new field content.
- Our focus: interplay between the group G and the quark mass matrices within pure NHDM (no new fields beyond N Higgs doublets).

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Yukawa interactions in NHDM

In the SM with three quark generations Q_{Li}, d_{Ri}, u_{Ri}, $i = 1, 2, 3$, the quark mass matrices come from

$$-\mathcal{L}_Y = \bar{Q}_{Li}\Gamma_{ij}\phi d_{Rj} + \bar{Q}_{Li}\Delta_{ij}\tilde{\phi}u_{Rj} + h.c. \to (M_d)_{ij}\bar{d}_{Li}d_{Rj} + (M_u)_{ij}\bar{u}_{Li}u_{Rj} + h.c.$$

In models with N Higgs doublets ϕ_k, $k = 1, \dots, N$, we get

$$-\mathcal{L}_Y = \bar{Q}_L\Gamma_k\phi_k d_R + \bar{Q}_L\Delta_k\tilde{\phi}_k u_R + h.c.$$

After EWSB, ϕ_k acquire some vevs: $\langle\phi_k^0\rangle = (v_1, v_2, \dots, v_N)/\sqrt{2}$, where v_k are in general complex, and generate M_d and M_u:

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Matrices M_d and M_u can be diagonalized via

$$V_{dL}^{\dagger} M_d V_{dR} = D_d \,, \quad V_{uL}^{\dagger} M_u V_{uR} = D_u \,,$$

where $D_d = \mathrm{diag}(m_d, m_s, m_b)$ and $D_u = \mathrm{diag}(m_u, m_c, m_t)$. Focusing on the left fields participating in $SU(2)_L$, we define $H_d = M_d M_d^{\dagger}$, $H_u = M_u M_u^{\dagger}$, which are invariant under V_{qR}. Then,

$$H_d = V_{dL} D_d^2 V_{dL}^{\dagger} \,, \quad H_u = V_{uL} D_u^2 V_{uL}^{\dagger} \,.$$

$SU(2)_L$ gauge interactions induce vertices $\bar{u}_L W^+ d_L$, which becomes non-diagonal if $V_{dL} \neq V_{uL}$. This is quantified with the CKM matrix:

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Yukawa interactions in NHDM

We want the quark sector to be physically viable:

- quark masses are non-zero and non-degenerate, for down and up-quarks, separately;
- non-block-diagonal CKM matrix V, to allow for three mixing angles,
- non-zero CP-violating phase in V: $J = \det(H_d H_u - H_u H_d) \neq 0$.

Suppose we have a specific NHDM implementing a flavour symmetry group G. We have the Higgs potential $\to$ vev alignment v_k, and we know Γ_k and Δ_k $\to$ quark masses and V.

Our goal

We want to know which flavour symmetry groups G in which NHDMs can lead to a minimally viable quark sector.

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Yukawa interactions in NHDM

In *NPB398 (1993) 319*, Leurer, Nir, and Seiberg (LNS) studied essentially the same problem and arrived at a no-go theorem which forbids flavour symmetry to remain after EWSB.

In [Gonzalez Felipe, Ivanov, Nishi, Serodio, Silva, arXiv:1401.5807], we found a loophole in the LNS formulation of the theorem, we refined it, and explored further its implications.

Flavour symmetries

Consider a flavour group G acting by 3D representations $D(Q_L)$, $D(d_R)$, $D(u_R)$:

$$Q_{L\alpha} \mapsto (\mathcal{G}_{Lg})_{\alpha\beta}\, Q_{L\beta}\,, \quad d_{R\alpha} \mapsto (\mathcal{G}^d_{Rg})_{\alpha\beta}\, d_{R\beta}\,, \quad u_{R\alpha} \mapsto (\mathcal{G}^u_{Rg})_{\alpha\beta}\, u_{R\beta}\,,$$

and by an N-dimensional representation $D(\phi)$: $\phi_k \mapsto (\mathcal{G}_g)_{kl}\,\phi_l$.

If $\mathcal{L}_Y$ is invariant under G, then Γ_k and Δ_k satisfy

$$\mathcal{G}^\dagger_{Lg}\,\Gamma^k\,\mathcal{G}^d_{Rg}\,(\mathcal{G}_g)_{kl} = \Gamma^l\,, \quad \mathcal{G}^\dagger_{Lg}\,\Delta^k\,\mathcal{G}^u_{Rg}\,(\mathcal{G}^*_g)_{kl} = \Delta^l \quad \text{for all } g \in G.$$

Clearly, if $\mathcal{G}_{Lg} = \mathcal{G}_{Rg} = e^{i\theta}\mathbb{1}$ and $\mathcal{G}_g = \mathbb{1}$, these eqns are satisfied. It reflects baryon number conservation, and we do not include $U(1)_B$ in the group G.

However, we do include $U(1)_Y$, $\mathcal{G}_{ig} = e^{i\theta Y_i}\mathbb{1}$, in the definition of the full flavour symmetry group G.

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No-go theorem

After EWSB, the flavour symmetry group G can either

- break completely,
- partially break to a proper subgroup of G,
- be conserved.

Question 1: which of these options lead to a viable quark sector?

Question 2: which options are actually possible for a given G?

No-go theorem answers question 1:
the only way to obtain a non-block-diagonal CKM mixing matrix and, simultaneously, non-degenerate and non-zero quark masses, is that $\langle\phi_k\rangle$ break completely the flavour group G, except possibly for some symmetry belonging to baryon number.

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No-go theorem

Suppose that G is (partially) conserved after EWSB: there exists $g \in G$ such that

$$(\mathcal{G}_g)_{kl}\ v_l = v_k\,.$$

Then, $\mathcal{G}_{Lg}^\dagger\,\Gamma^k\,\mathcal{G}_{Rg}^d\,(\mathcal{G}_g)_{kl}\ v_l = \Gamma^l v_l$, so that

$$\mathcal{G}_{Lg}^\dagger M_d\ \mathcal{G}_{Rg}^d = M_d\,, \quad \mathcal{G}_{Lg}^\dagger M_u\ \mathcal{G}_{Rg}^u = M_u\,.$$

Switching to matrices H_d and H_u:

$$\mathcal{G}_{Lg}^\dagger H_d\ \mathcal{G}_{Lg} = H_d\,, \quad \mathcal{G}_{Lg}^\dagger H_u\ \mathcal{G}_{Lg} = H_u\,.$$

If $\mathcal{G}_{Lg}$ commutes with both H_d and H_u, something bad must happen to CKM!

No-go theorem

Indeed, suppose $\mathcal{G}_{Lg}$ is not $\propto \mathbb{1}$. Then, being a 3×3 unitary matrix, it contains a non-degenerate eigenvalue $e^{i\theta}$, and the corresponding eigenvector w. Then,

$$\mathcal{G}_{Lg}(H_d\, w) = H_d\, \mathcal{G}_{Lg} w = e^{i\theta}(H_d\, w)\,,$$

so that $H_d\, w \propto w \to$ w is also an eigenvector of H_d and, therefore, it fills one column of V_{dL}.

But w is also an eigenvector of H_u, and a column in V_{uL}. Therefore, $V = V_{uL}^\dagger V_{dL}$ has a block-diagonal form $\to$ violates physical requirements.

Thus, the only viable possibility is to take $\mathcal{G}_{Lg} = e^{i\theta}\mathbb{1}$, which places no restrictions on H_u, H_d.

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If $\mathcal{G}_{Lg} = e^{i\theta}\mathbb{1}$, look again at $\mathcal{G}_{Lg}^\dagger M_d\, \mathcal{G}_{Rg}^d = M_d$ and $\mathcal{G}_{Lg}^\dagger M_u\, \mathcal{G}_{Rg}^u = M_u$. We require no massless quarks $\to$ M_d and M_u are invertible, therefore

$$\mathcal{G}_{Lg} = \mathcal{G}_{Rg}^d = \mathcal{G}_{Rg}^u = e^{i\theta}\,.$$

Return now to $\mathcal{G}_{Lg}^\dagger \Gamma^k \mathcal{G}_{Rg}^d (\mathcal{G}_g)_{kl} = \Gamma^l$ and $\mathcal{G}_{Lg}^\dagger \Delta^k \mathcal{G}_{Rg}^u (\mathcal{G}_g^*)_{kl} = \Delta^l$. From here we conclude that

$$\Gamma^k (\mathcal{G}_g - \mathbb{1})_{kl} = \mathbf{0}\,, \quad \Delta^k (\mathcal{G}_g - \mathbb{1})^*{}_{kl} = \mathbf{0},$$

If we talk about Higgses which are coupled at least to some quarks, then

$$\mathcal{G}_g = \mathbb{1}_{N\times N}\,.$$

Thus, the only viable residual symmetry is $\in U(1)_B$.

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Historical remark

In *NPB398 (1993) 319*, Leurer, Nir, and Seiberg state:

> *... if symmetry is not broken, there are either degenerate quarks or vanishing mixing angles.*

The difference with our formulation is that we require, in addition, that there be no massless quarks. This condition involves the right quark space, which was disregarded in the LNS paper.

It allows us to construct a counterexample to the LNS statement: a model with a residual symmetry, with non-degenerate quarks, and non-vanishing mixing angles — but with massless quarks, in agreement with our theorem.

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Historical remark

Consider three-Higgs-doublet model with the symmetry group A_4, with

$$Q_L \sim (1,\, 1',\, 1'')\,, \quad \phi \sim d_R \sim u_R \sim 3\,.$$

There are four types of the global minima. One of them, $(1,\, e^{i\alpha},\, 0)$, is invariant under a residual $\mathbb{Z}_2$:

$$\mathcal{G}_L = \mathbb{1}\,, \quad \mathcal{G}_R^d = \mathcal{G}_R^u = \mathcal{G} = \mathrm{diag}(1,\, 1,\, -1)\,.$$

The mass matrices

$$\begin{pmatrix} a & a\, e^{i\alpha} & 0 \\ b & b\,\omega\, e^{i\alpha} & 0 \\ c & c\,\omega^2\, e^{i\alpha} & 0 \end{pmatrix}.$$

After diagonalization, we get non-degenerate quarks, three mixing angles, and non-zero CP-violation (in contradiction to LNS!) — but with one massless quark in the up and down sectors.

How generic is residual symmetry?

The no-go theorem states that only the complete breaking can lead to a viable quark sector. The question is whether the complete breaking of the chosen flavour group G is actually possible.

Ideally, when imposing a group G in NHDM, we should

- know how many N we need for the group G,
- know the most general form of the G-symmetric potential,
- know all possible vev alignments of the global minimum.

Only then we can directly check whether any of these minima breaks the group completely. Answering these questions on the case-by-case basis is challenging, and general results are needed.

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How generic is residual symmetry?

A example: 3HDM with discrete groups $G_H = G/U(1)_Y$, and with Higgses in a faithful 3D irrep.

- Only four such groups are possible [Ivanov, Vdovin, *EPJ* C73 (2013) 2309]: $G_H = A_4,\ S_4,\ \Delta(54)/\mathbb{Z}_3,\ \Sigma(36)$. Any other group will lead to an extra continuous symmetry.
- For each group, the general form of the potential is known, and the minimization problem in the entire free parameter space was solved.
- In none of these cases, the vev alignment breaks the group G completely.

Therefore, whatever irrep assignments for Q_L, d_R, u_R we use, we can never get the viable quark sector in any of these models.

To avoid it, one needs (1) yet more fields, (2) higher-order G-symmetric terms, (3) explicit symmetry breaking.

How generic is residual symmetry?

This 3HDM example hints at a general rule within pure NHDM: sufficiently large symmetry groups cannot be broken completely and, therefore, cannot lead to viable quark sector.

The only question is the exact definition of "sufficiently large symmetry".

Not yet solved. The answer must depend not only on the group G, but also on the algebraic properties of the potential.

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- If G is broken completely, the potential has $|G_H|$ degenerate global minima. But a polynomial of a given degree (say, quadratic + quartic) cannot have too many isolated minima, say, n_{max} at most. If $|G_H| > n_{max}$, then the group cannot be broken completely.

  For 2HDM, $n_{max} = 2$; for 3HDM, $n_{max} = 8$ (tentatively); the problem is to find n_{max} for general NHDM.

- With higher-order (e.g. sextic) G-symmetric interaction terms included, the degree of the potential increases $\to$ n_{max} grows. It can lead to a situation when it is the presence of these higher-order terms that makes the complete breaking of the group G possible. An example: $\Delta(27)$ 3HDM with sextic terms

$$\frac{1}{\Lambda^2}\left[(\phi_1^\dagger\phi_2)^2(\phi_3^\dagger\phi_1) + \text{cyclic} + h.c.\right]$$

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Conclusions

- Investigating general symmetry-related issues with a given new field content is a useful tool complementary to the mainstream bSM flavour model building. We studied a particular question: which symmetry groups in pure G can lead to viable quark sectors.
- Refining [Leurer, Nir, and Seiberg, 1993], we proved a no-go theorem: the only way to a viable CKM and quark masses is to completely break G upon EWSB.
- However, not every group can be completely broken! I illustrated that with 3HDM with Higges in faithful triplet irrep and gave an outlook for the anticipated general results.