Using Neutrino Mass Sum Rules to ***realistically*** probe Flavour Models

Alexander Merle

University of Southampton, U.K.

E-mail: A.Merle (AT) soton.ac.uk

<u>*Based on:*</u>

JHEP **1312** (2013) 005, with Steve King & Alex Stuart

FLASY 2014, Brighton, 20-06-2014

Contents:

1. Mass Sum Rules: Origin & Anatomy

1. Mass Sum Rules: Origin & Anatomy

- **BASIC PROBLEM FOR FLAVOUR MODELS**:

  if the mixing angles are not correctly predicted, we classify a model as "excluded"

  ➔ *thus*: mixing angles are not well suited to distinguish models without precision

- an alternative observable would be the $|m_{ee}|$ in neutrinoless double beta decay, which depends on the mass scale and on the phases

  ➔ *BUT*: suffers from nuclear physics unknowns

☺ Fortunately, some models yield **SUM RULES**!!!

1. Mass Sum Rules: Origin & Anatomy

- initial observation: if M_ν constains two decisive parameters (typically by two flavon couplings), then it yields a mass sum rule
- known example based on A_4:

$$M_R = \begin{pmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{pmatrix} \Lambda \qquad M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} yv$$

1. Mass Sum Rules: Origin & Anatomy

- initial observation: if M_ν constains two decisive parameters (typically by two flavon couplings), then it yields a mass sum rule
- known example based on A_4:

trivial

$$M_R = \begin{pmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{pmatrix} \Lambda \qquad M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} yv$$

1. Mass Sum Rules: Origin & Anatomy

- initial observation: if M_ν constains two decisive parameters (typically by two flavon couplings), then it yields a mass sum rule
- known example based on A_4:

trivial

$$M_R = \begin{pmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{pmatrix} \Lambda \qquad M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} yv$$

- (complex) masses & sum rule:

$$(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3) = \left(\frac{1}{3\alpha_s + \alpha_0}, \frac{1}{\alpha_0}, \frac{1}{3\alpha_s - \alpha_0} \right) \frac{y^2 v^2}{\Lambda}$$

$$\rightarrow \boxed{\frac{1}{\tilde{m}_1} - \frac{1}{\tilde{m}_3} = \frac{2}{\tilde{m}_2}}$$

MASS SUM RULE

2. Analysis & example cases

DISCLAIMER:

• since we wanted to perform a complete study, we had to revisit cases that had already been studied
• we had to partially update (to be frank: *correct*) these discussions
• nearly all the authors of these partially incorrect results are here...

➔ ***Please don't throw any rotten eggs or tomatoes at me, we did it only in the best interest of science!!!***

2. Analysis & example cases

- **EARLIER WORKS BY**:

  Altarelli, Barry, Bazzocchi, Chen, Dorame, Feruglio, Hagedorn, Hirsch, Meloni, Morisi, Peinado, Valle,...

2. Analysis & example cases

- **EARLIER WORKS BY**:

  Altarelli, Barry, Bazzocchi, Chen, Dorame, Feruglio, Hagedorn, Hirsch, Meloni, Morisi, Peinado, Valle,...

- AND, IN PARTICULAR, SINCE HE COMPLAINED THAT I WOULD NOT MENTION HIM IN MY TALK:

WERNER RODEJOHANN

➔ *PIONEERING SYSTEMATIC PAPER ON MASS SUM RULES*:
Barry & Rodejohann, Nucl. Phys. **B842**, 33 (2011)

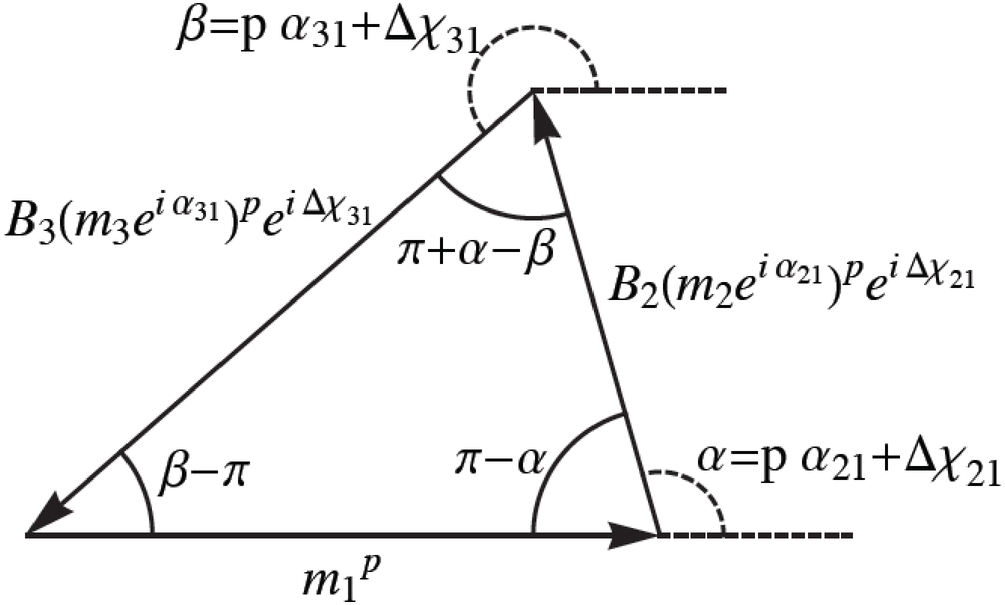
2. Analysis & example cases

- any mass sum rule can be written as follows:

$$m_1^p + B_2\left(m_2 e^{i\alpha_{21}}\right)^p e^{i\Delta\chi_{21}} + B_3\left(m_3 e^{i\alpha_{31}}\right)^p e^{i\Delta\chi_{31}} = 0$$

➔ complex equation that yields **TWO** conditions

➔ geometrical interpretation as triangle:



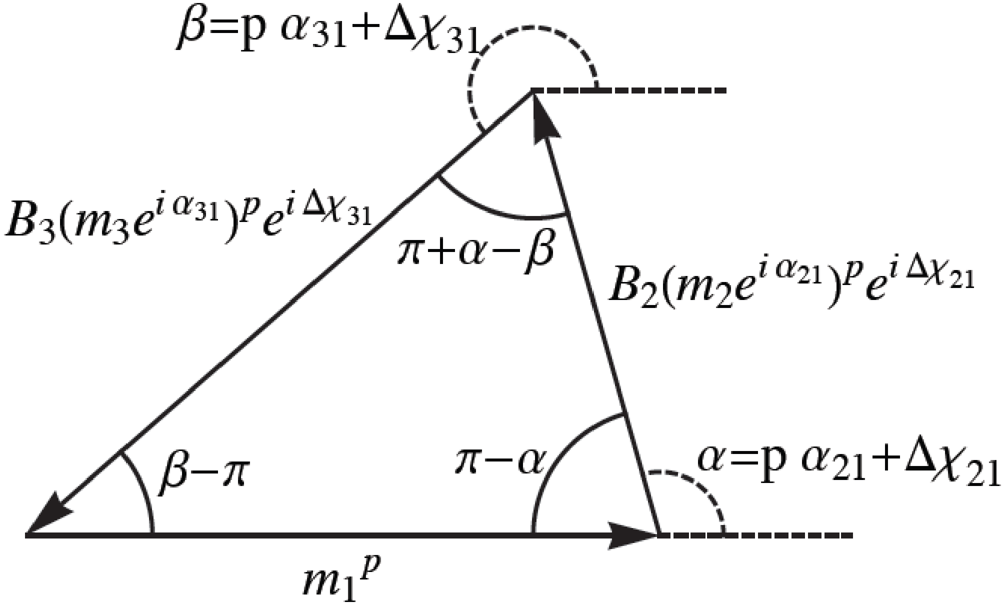
2. Analysis & example cases

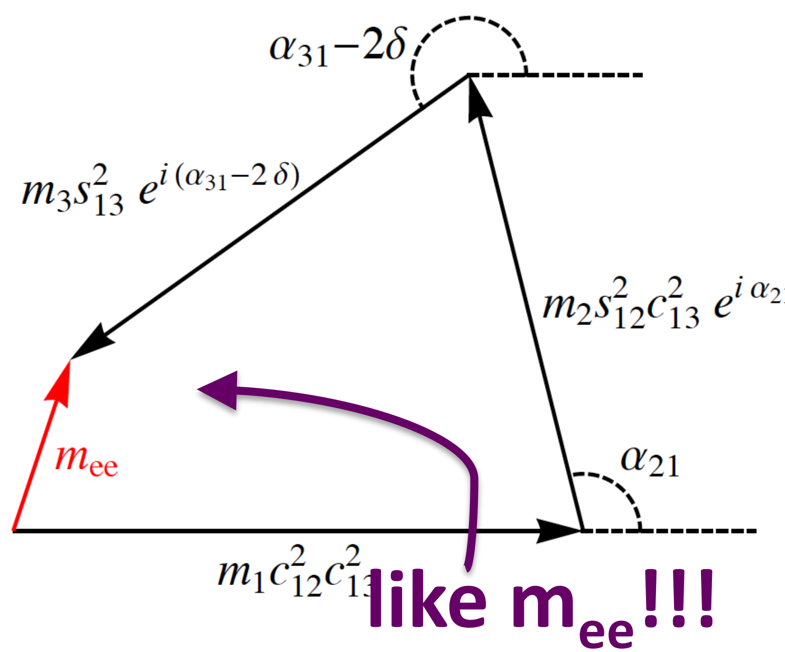
- any mass sum rule can be written as follows:

$$m_1^p + B_2\left(m_2 e^{i\alpha_{21}}\right)^p e^{i\Delta\chi_{21}} + B_3\left(m_3 e^{i\alpha_{31}}\right)^p e^{i\Delta\chi_{31}} = 0$$

➔ complex equation that yields **TWO** conditions

➔ geometrical interpretation as triangle:





2. Analysis & example cases

- sum rule based on S_4 with inverse seesaw:

$$\boxed{\frac{1}{\sqrt{\tilde{m}_1}} + \frac{1}{\sqrt{\tilde{m}_2}} = \frac{2}{\sqrt{\tilde{m}_3}}}$$

Dorame *et al.*, Phys. Rev. **D86**, 056001 (2012)

- in terms of our general parameters:

$$p = -1/2, \quad B_2 = 1, \quad B_3 = 2, \quad \Delta\chi_{21} = 0, \quad \text{and} \quad \Delta\chi_{31} = \pi$$

2. Analysis & example cases

- sum rule based on S_4 with inverse seesaw:

$$\boxed{\frac{1}{\sqrt{\tilde{m}_1}} + \frac{1}{\sqrt{\tilde{m}_2}} = \frac{2}{\sqrt{\tilde{m}_3}}}$$

Dorame *et al.*, Phys. Rev. **D86**, 056001 (2012)

- in terms of our general parameters:

$$p = -1/2, \quad B_2 = 1, \quad B_3 = 2, \quad \Delta\chi_{21} = 0, \quad \text{and} \quad \Delta\chi_{31} = \pi$$

- one can show that $m_3 < m_1 < m_2$ implies:

$$\underline{|\text{LHS}|} = \left|\frac{1}{\sqrt{\tilde{m}_1}} + \frac{1}{\sqrt{\tilde{m}_2}}\right| = \left|\frac{1}{\sqrt{m_1}} + \frac{e^{i(\phi_1-\phi_2)/2}}{\sqrt{m_2}}\right| \underline{\leq} \frac{1}{\sqrt{m_1}} + \frac{1}{\sqrt{m_2}}$$

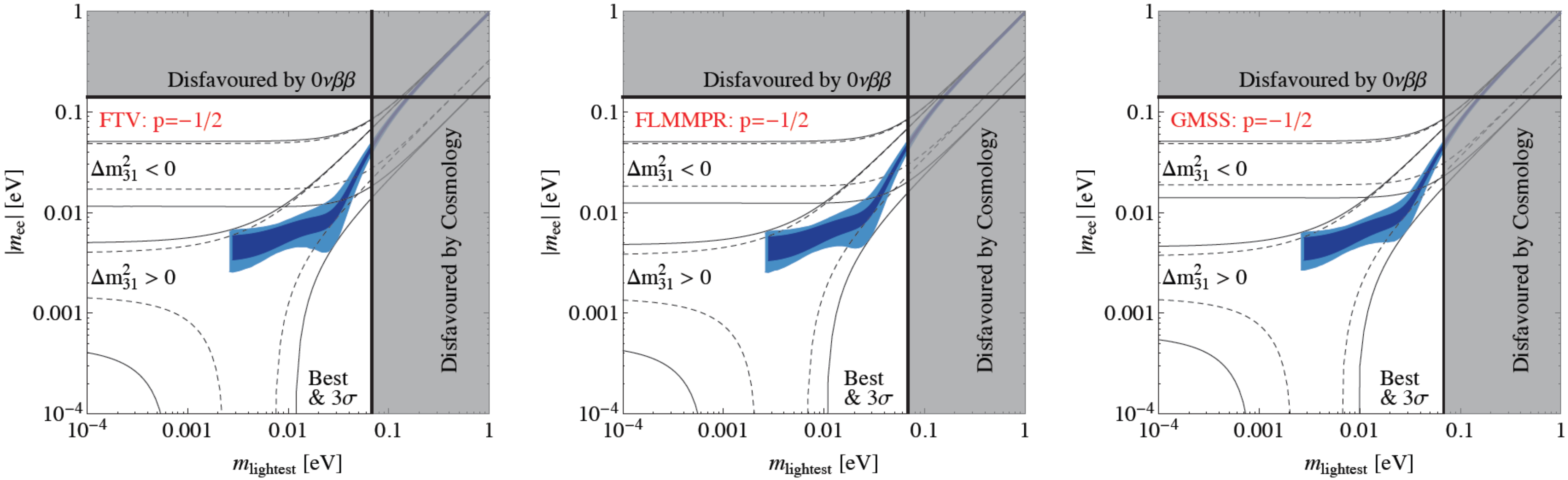
$$\underline{<} \frac{1}{\sqrt{m_3}} + \frac{1}{\sqrt{m_3}} = \frac{2}{\sqrt{m_3}} = \underline{|\text{RHS}|}$$

➔ **|LHS| < |RHS|**

➔ ***forbids inverted mass ordering***

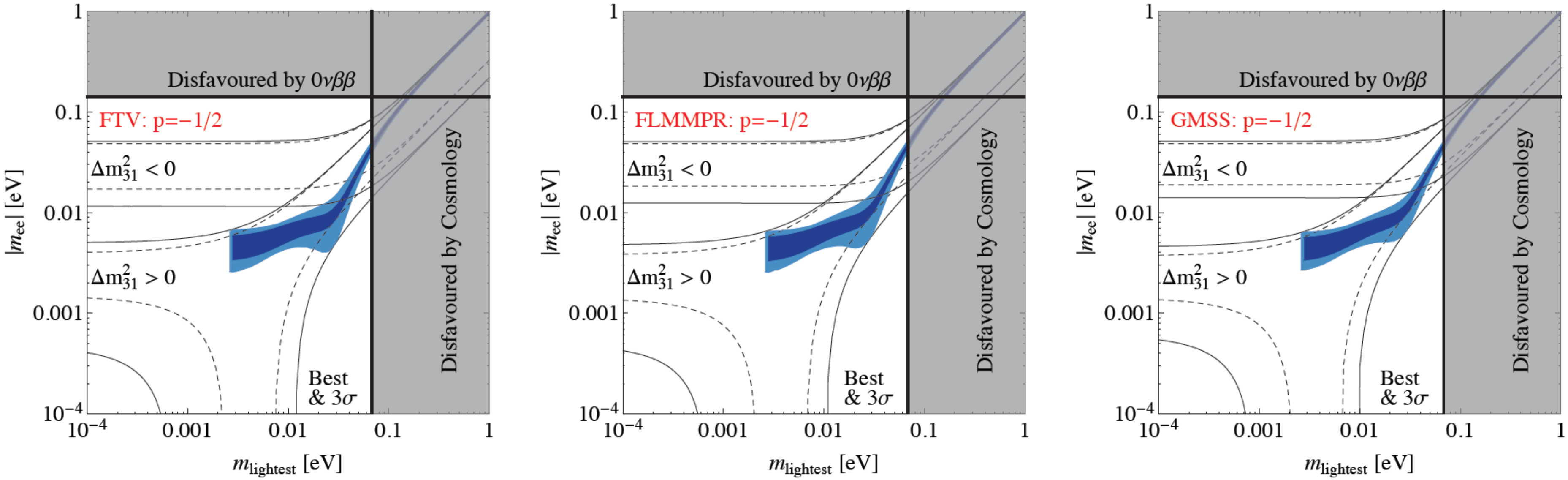
2. Analysis & example cases

- we did our numerics for all 3 global fits:



2. Analysis & example cases

- we did our numerics for all 3 global fits:



➔ ***plots look very similar for all three fits***

➔ ***plots confirm our analytical results***

➔ ***this is why we are so confident that we did things correctly***

Advertisement: *More sum rules*

Advertisement: *More sum rules*

$$\frac{1}{\tilde{m}_3} + \frac{2i(-1)^\eta}{\tilde{m}_2} = \frac{1}{\tilde{m}_1}$$

King, Luhn, Stuart: Nucl. Phys. **B867**, 203 (2013)

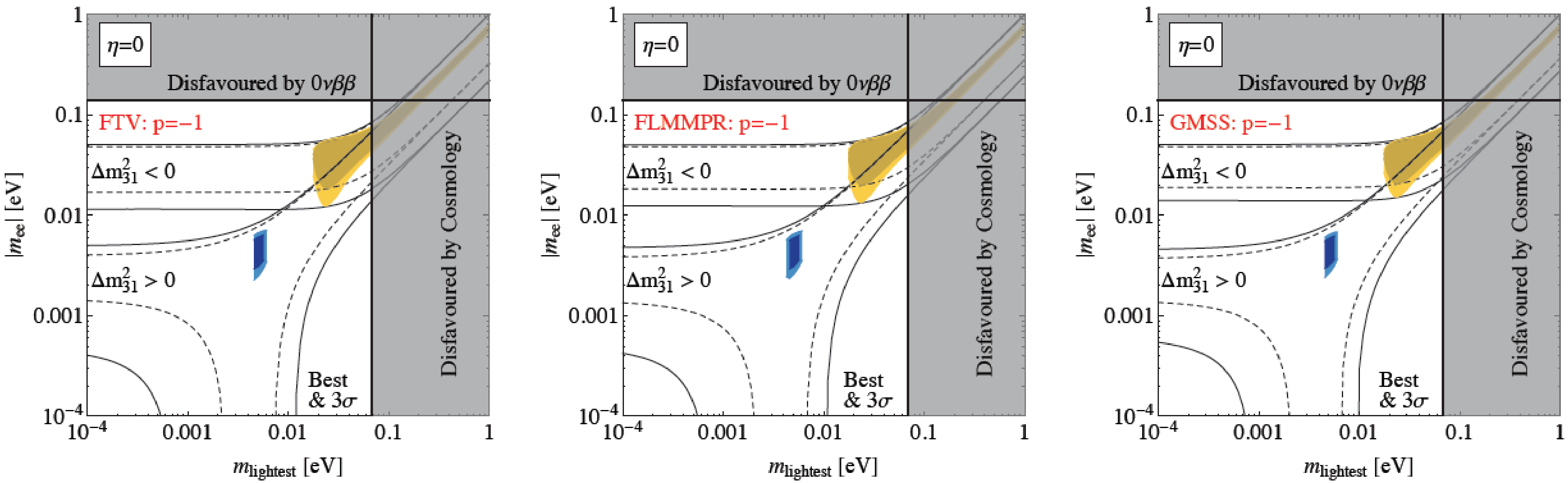
model: Δ(96) & seesaw type I

Advertisement: ***More sum rules***

$$\frac{1}{\tilde{m}_3} + \frac{2i(-1)^\eta}{\tilde{m}_2} = \frac{1}{\tilde{m}_1}$$

King, Luhn, Stuart: Nucl. Phys. **B867**, 203 (2013)

model: Δ(96) & seesaw type I



$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	NO, 3σ	IO, 3σ
m_{lightest}	$\sim [0.0045, 0.0061]$	$\gtrsim 0.018$
$\lvert m_{ee}\rvert$ (FTV)	$[0.0023, 0.0071]$	> 0.012
$\lvert m_{ee}\rvert$ (FLMMPR)	$[0.0024, 0.0070]$	> 0.013
$\lvert m_{ee}\rvert$ (GMSS)	$[0.0024, 0.0068]$	> 0.015

Advertisement: ***More sum rules***

$$\tilde{m}_1^{-1} - 2\tilde{m}_2^{-1} - \tilde{m}_3^{-1} = 0$$

Altarelli, Barry, Chen, Feruglio, Hagedorn, King, Mahantappa, Meloni, Rodejohann, Yu,...

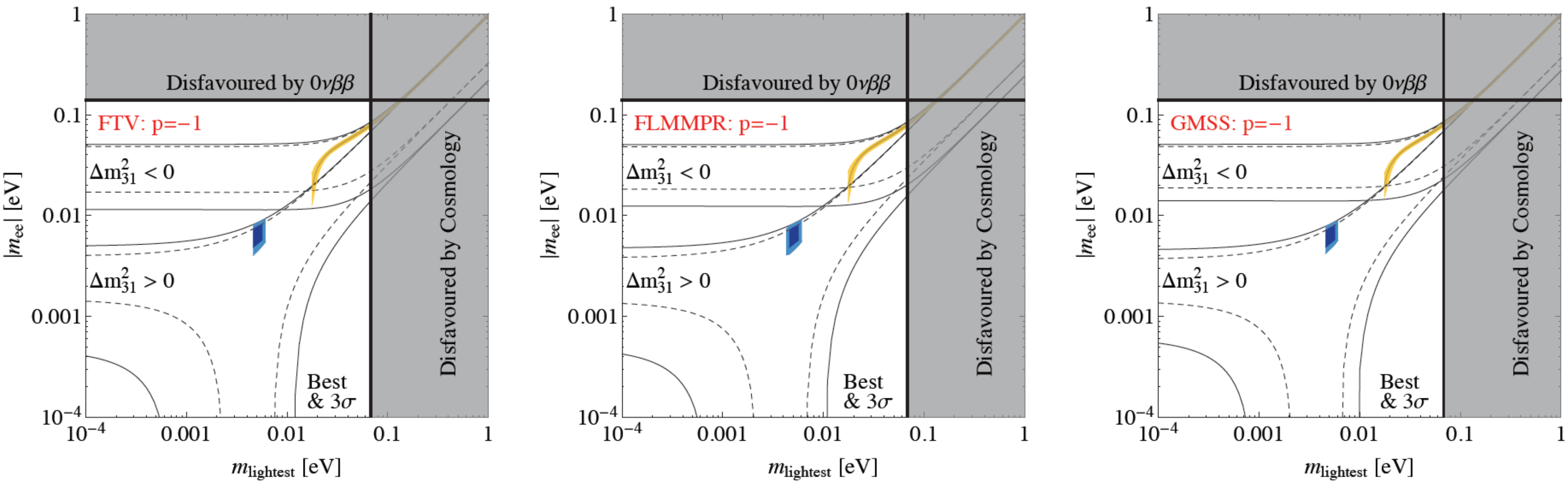
models: A_4 or T‘ & seesaw type I

Advertisement: ***More sum rules***

$$\tilde{m}_1^{-1} - 2\tilde{m}_2^{-1} - \tilde{m}_3^{-1} = 0$$

Altarelli, Barry, Chen, Feruglio, Hagedorn, King, Mahantappa, Meloni, Rodejohann, Yu,...

models: A_4 or T‘ & seesaw type I



$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$	NO, 3σ	IO, 3σ
m_{lightest}	$\sim [0.0045, 0.0061]$	$\gtrsim 0.018$
$\lvert m_{ee} \rvert$ (FTV)	$[0.0040, 0.0092]$	> 0.013
$\lvert m_{ee} \rvert$ (FLMMPR)	$[0.0041, 0.0091]$	> 0.014
$\lvert m_{ee} \rvert$ (GMSS)	$[0.0042, 0.0089]$	> 0.015

Advertisement: ***More sum rules***

$$\boxed{\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3}$$

Lindner, Merle, Niro:
JCAP **1101**, 034 (2011)

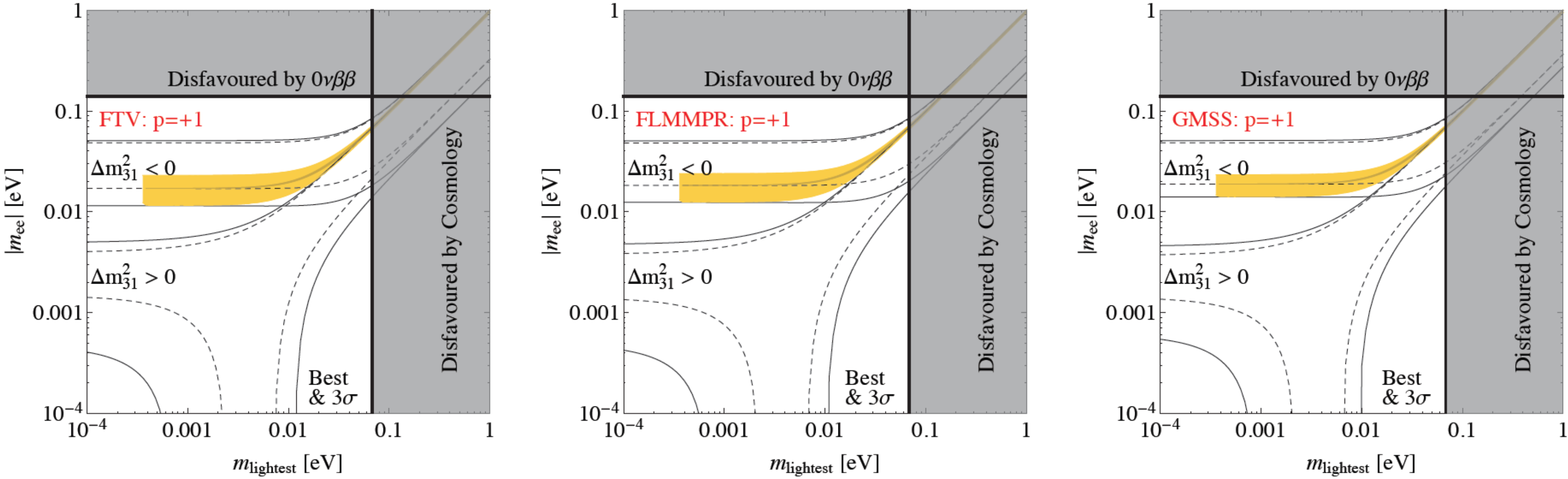
model: L_e-L_μ-L_τ & seesaw type II

Advertisement: *More sum rules*

$$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$$

Lindner, Merle, Niro:
JCAP **1101**, 034 (2011)

model: L_e-L_μ-L_τ & seesaw type II



$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	NO, 3σ	IO, 3σ
$m_{\rm lightest}$	forbidden	$\gtrsim 0.00036$
$\|m_{ee}\|$ (FTV)	forbidden	> 0.012
$\|m_{ee}\|$ (FLMMPR)	forbidden	> 0.013
$\|m_{ee}\|$ (GMSS)	forbidden	> 0.015

3. Global Results

- general findings:
  - neutrino mass sum rules probe ***whole classes of flavour models***

3. Global Results

- general findings:
  - neutrino mass sum rules probe ***whole classes of flavour models***
  - phases **DO** play an important role:
    - δ ***cannot*** be neglected
    - phase re-definitions must be done consistently in both sum rule AND m_{ee}

3. Global Results

- general findings:
  - neutrino mass sum rules probe ***whole classes of flavour models***
  - phases **DO** play an important role:
    - δ ***cannot*** be neglected
    - phase re-definitions must be done consistently in both sum rule AND m_{ee}
  - sum rules generically yield **TWO** conditions

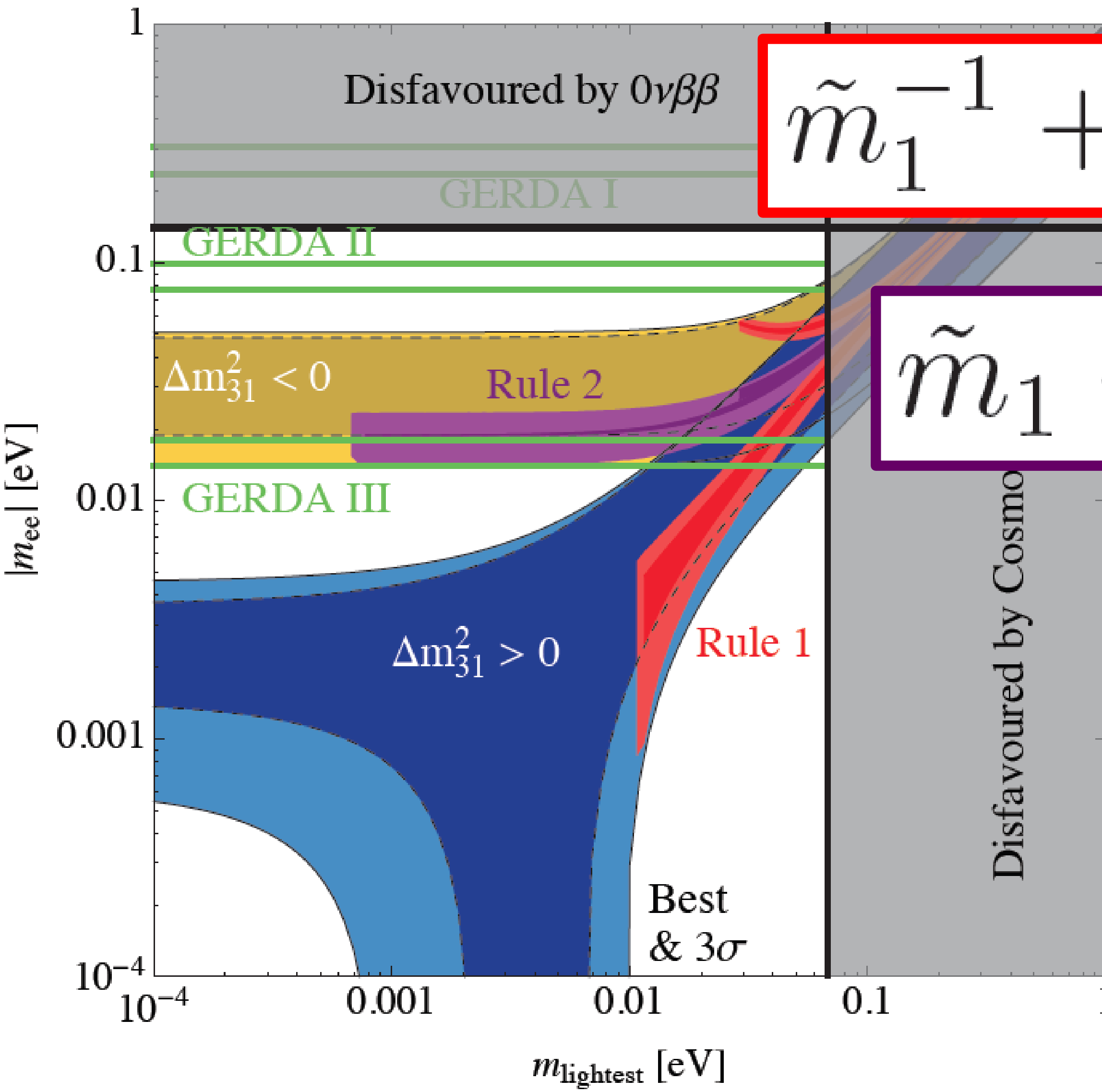
  ➔ ordering and phase relations restricted

  ➔ characteristic "signatures" in $|m_{ee}|$

  ➔ ***restrictions can be so strong that they even may overcome the nuclear physics uncertainties***

3. Global Results

- model distinction:



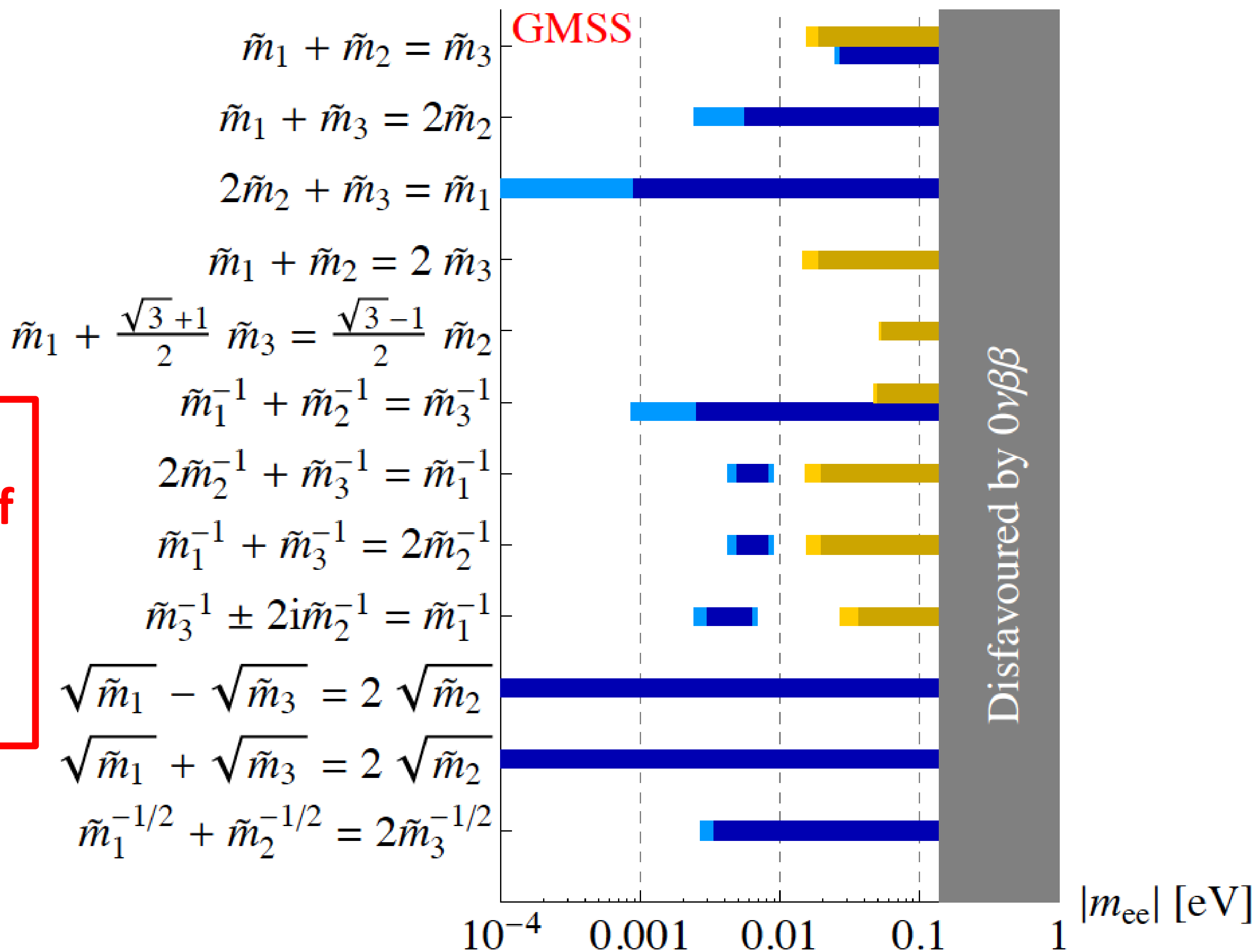
$$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$$

$$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$$

Distinctive power in spite of nuclear physics uncertainties!!!

3. Global Results

- global analysis:



Complete classification of more than 50 known flavour models

4. Conclusions

4. Conclusions

- *often not sufficiently appreciated*: neutrino mass sum rules are among the **most important tools to distinguish flavour models** (in the absence of precision measurements of mixings & phases)

4. Conclusions

- *often not sufficiently appreciated*: neutrino mass sum rules are among the **most important tools to distinguish flavour models** (in the absence of precision measurements of mixings & phases)
- we have done the most complete classification:
  - ***more than 50 models leading to 12 sum rules***
  - ***all three global fits***
  - ***most recent experimental bounds & NME uncertainties***

4. Conclusions

- *often not sufficiently appreciated*: neutrino mass sum rules are among the **most important tools to distinguish flavour models** (in the absence of precision measurements of mixings & phases)
- we have done the most complete classification:
  - ***more than 50 models leading to 12 sum rules***
  - ***all three global fits***
  - ***most recent experimental bounds & NME uncertainties***
- if **your** model predicts a sum rule and **you** want it analysed (pick your fit, get a nice plot, etc.)...

4. Conclusions

- *often not sufficiently appreciated*: neutrino mass sum rules are among the **most important tools to distinguish flavour models** (in the absence of precision measurements of mixings & phases)
- we have done the most complete classification:
  - ***more than 50 models leading to 12 sum rules***
  - ***all three global fits***
  - ***most recent experimental bounds & NME uncertainties***
- if **your** model predicts a sum rule and **you** want it analysed (pick your fit, get a nice plot, etc.)...

... I am happy to serve as co-author!!! :-p

THANK YOU!!!

Effective theory of a doubly charged singlet scalar: complementarity of neutrino physics and the LHC

Stephen F. King*, Alexander Merle†, and Luca Panizzi‡

Physics and Astronomy, University of Southamp

Southampton, SO17 1BJ, United Kingdom

June 18, 2014

Abstract

We consider a rather minimal extension of the Standard Model involving j tra particle, namely a single $SU(2)_L$ singlet scalar S^{++} and its antiparticle . We propose a model independent effective operator, which yields an effective coupling of $S^{\pm\pm}$ to pairs of same sign weak gauge bosons, $W^{\pm}W^{\pm}$. We also allow tree-level couplings of $S^{\pm\pm}$ to pairs of same sign right-handed charged leptons $l_R^{\pm}l_R'^{\pm}$ of the same or different flavour. We calculate explicitly the resulting two-loop diagrams in the effective theory responsible for neutrino mass and mixing. We propose sets of benchmark points for various $S^{\pm\pm}$ masses and couplings which can yield successful neutrino masses and mixing, consistent with limits on charged lepton flavour viola-

uble beta decay. We discuss the prospects for $S^{\pm\pm}$ benchmark points, including single and pair pro- gn leptons plus jets and missing energy. The model of the complementarity between neutrino physics (including LFV) and the LHC, involving just one new particle, the $S^{\pm\pm}$.