Lepton Mixing Predictions from $\Delta(6n^2)$ Groups and Generalised CP (S.F.King,A.J.Stuart,TN,arxiv:1305.3200 and S.F.King,TN,arXiv:1403.1758)

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Outline

- Flavour Symmetries
- $\Delta(6n^2)$ Groups
- Generalised CP
- Combining Flavour Symmetry and Generalised CP
- Conclusions/Future Work

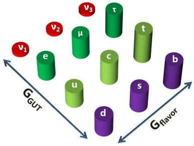
Some Questions Flavour Symmetries Try to Answer

(http://quovadisblog.com/wp-content/uploads/2009/08/Question-sign.jpg)

- Why do particles mix as they do? ($\rightarrow$ this talk)
- Why are there 3 families of particles?
- Why are the masses of the particles what they are?
- Is there CP violation in the lepton sector? ($\rightarrow$ this talk)

Flavour Symmetries (1)

- Flavour symmetries extend the symmetry group of the Standard Model by an additional ("horizontal") symmetry that connects different flavours of particles: $G_{\text{gauge}} \times G_{\text{Flavour}}$.
- The invariance of the Lagrangian under G_{Flavour} restricts the possible couplings and thus the possible mixing matrices.



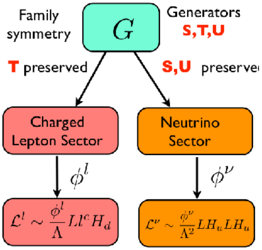
Picture from

http://theophys.kth.se/tepp/Flavor.jpg

Flavour Symmetries (2)

- For Example, have left-handed doublets transform under a 3-dim representation: $\binom{\nu_L}{e_L} \mapsto \rho_r(g) \binom{\nu_L}{e_L}$
- Under such transformations, Majorana Neutrinos with a mass term $\nu_L^T M^\nu \nu_L \neq 0$ have a maximal symmetry of $K = Z_2 \times Z_2$.
- $\Rightarrow$ The flavour group G_{Flavour} must be broken to $G^\nu = K, Z_2$ or $\{1\}$ in the neutrino sector.
- Models with $G^\nu = K = Z_2 \times Z_2$ are called "Direct Models".
- Dirac fields can accomodate a maximal symmetry of $U(1) \times U(1)$.

Direct Models



(From Steve F. King, Christoph Luhn, Neutrino Mass and Mixing with Discrete Symmetry, 1303.6180)

- In direct models, the mixing matrix is fixed just from the choice of the $Z_2 \times Z_2$ subgroup of G_{Flavour}.
- Call the matrices that diagonalise M^e and M^ν, U^e and U^ν resp. $\Rightarrow U^{\text{PMNS}} = U^{e\dagger} U^\nu$
- Let $G^\nu = \{1, G_1^\nu, G_2^\nu, G_3^\nu\}$. Then $[(M^\nu)^\dagger M^\nu, G_i^\nu] = 0$ and $G_i^\nu U_i^\nu = U_i^\nu$.
- One can systematically list all mixing matrices allowed in a direct model by a flavour group via its subgroups.
- For Reviews see e.g. S. F. King and C. Luhn, Rept. Prog. Phys. 76 (2013) 056201 and S. F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, arXiv:1402.4271

$\Delta(6n^2)$ Groups (1)

- The groups $\Delta(6n^2)$ are non-abelian discrete subgroups of $U(3)$ of order $6n^2$ and are isomorphic to a semidirect product: $\Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3$
- The best-known member of the series $\Delta(6n^2)$ is $S_4 = \Delta(24)$.
- The left-handed leptons transform (without loss of generality) under a 3-dimensional representation with the generators a, b, c, d (where $\eta = e^{2\pi i/n}$).
- $U(3) \supset \Delta(6n^2) \supset \Delta(3n^2)$
- $\Delta(6 \times 1^2) = S_3$, $\Delta(6 \times 2^2) = S_4$, $\Delta(6 \times 3^2) = \Delta(54)$, $\Delta(6 \times 4^2) = \Delta(96)$, $\Delta(3 \times 1^2) = Z_3$, $\Delta(3 \times 2^2) = A_4$, $\Delta(3 \times 3^2) = \Delta(27)$
- The group theory of $\Delta(6n^2)$: J. A. Escobar and C. Luhn, J. Math. Phys. 50 (2009) 013524

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$
$$b = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$
$$c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta^{-1} \end{pmatrix}$$

$\Delta(6n^2)$ Groups (2)

- Searches in GAP for allowed mixing matrices in direct models $\Rightarrow$ Only $\Delta(6n^2)$ groups remain viable
  (C. S. Lam, Phys. Rev. D 87 (2013) 013001; M. Holthausen, K. S. Lim and M. Lindner, Phys. Lett. B 721 (2013) 61)
- Models with examples of $\Delta(6n^2)$ groups are popular and succesful:
  (R. d. A. Toorop, F. Feruglio and C. Hagedorn, Phys. Lett. B 703, 447 (2011);
  R. de Adelhart Toorop, F. Feruglio and C. Hagedorn, Nucl. Phys. B 858, 437 (2012);
  G. -J. Ding, Nucl. Phys. B 862, 1 (2012);
  S. F. King, C. Luhn and A. J. Stuart, Nucl. Phys. B 867, 203 (2013);
  C. S. Lam, Phys. Rev. D 87, no. 5, 053012 (2013);
  S. F. King and C. Luhn, JHEP 0910, 093 (2009);
  I. de Medeiros Varzielas and G. G. Ross, JHEP 1212, 041 (2012);
  R. Krishnan, J. Phys. Conf. Ser. 447, 012043 (2013);
  M. Holthausen and K. S. Lim, Phys. Rev. D 88, 033018 (2013))

Mixing Results from $\Delta(6n^2)$

- Only for even n does $\Delta(6n^2)$ always contain phenomenologically viable Klein subgroups
- In terms of the generators a, b, c, the (relevant) Klein groups are given by

$$K = \{1, c^{n/2}, abc^{\gamma}, abc^{\gamma+n/2}\} \text{ with } \gamma = 0, \ldots, n/2$$

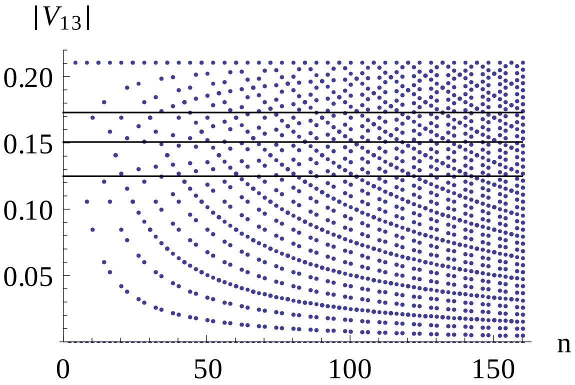
- All mixing matrices for even n have the form (up to ordering and without Majorana phases) where ϑ depends on the subgroup and takes values $\vartheta = \pi\gamma/n, \gamma = 0, \ldots, n/2$

$$V = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos(\vartheta) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin(\vartheta) \\ -\sqrt{\frac{2}{3}}\sin\left(\frac{\pi}{6}+\vartheta\right) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{6}+\vartheta\right) \\ \sqrt{\frac{2}{3}}\sin\left(\frac{\pi}{6}-\vartheta\right) & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{6}-\vartheta\right) \end{pmatrix}$$

- This lepton mixing matrix is trimaximal with θ_{13} fixed up to a discrete choice and CP-phase 0 or π.
- Each value of V_{13} corresponds to two possible values of θ_{23} with $\delta_{CP} = 0$ and $\delta_{CP} = \pi$, leading to the sum rule $\theta_{23} = 45^\circ \mp \theta_{13}/\sqrt{2}$.

Mixing Results (contd.)

One can plot the possible values of $|V_{13}|$, the lines denote the present approximate 3σ range of $|V_{13}|$ (from Fogli et al,1205.5254):



Examples include $|V_{13}| = 0.211, 0.170, 0.160, 0.154$ for $n = 4, 10, 16, 22$, respectively.

Generalised CP (1)

- In a direct model, all mixing angles and the Dirac phase are purely predicted from symmetry.
- In the Standard Model, violation of CP occurs in the flavour sector.
- Promoting CP to a symmetry at high energies which is then broken allows to impose further constraints on low energy mass matrices.
- For direct models and especially with $\Delta(6n^2)$ groups, CP symmetries had not been studied in detail yet.

Generalised CP (2)

- Examples for models employing generalised CP:
  I. Girardi, A. Meroni, S. T. Petcov and M. Spinrath, JHEP **1402**, 050 (2014)
  G. -J. Ding and Y. -L. Zhou, arXiv:1312.5222
  F. Feruglio, C. Hagedorn and R. Ziegler, Eur. Phys. J. C **74**, 2753 (2014)
  C. Luhn, Nucl. Phys. B **875**, 80 (2013)
  G. -J. Ding, S. F. King and A. J. Stuart, JHEP **1312**, 006 (2013)
  G. -J. Ding, S. F. King, C. Luhn and A. J. Stuart, JHEP **1305**, 084 (2013)
  M. S. Boucenna, S. Morisi, E. Peinado, Y. Shimizu and J. W. F. Valle, Phys. Rev. D **86**, 073008 (2012)
- The interplay of flavour and CP must be carefully discussed, this will make up the remainder of the talk. Literature:
  M. -C. Chen, M. Fallbacher, K. T. Mahanthappa, M. Ratz and A. Trautner, arXiv:1402.0507
  F. Feruglio, C. Hagedorn and R. Ziegler, JHEP **1307**, 027 (2013)
  C. C. Nishi, Phys. Rev. D **88**, 033010 (2013)
  W. Grimus and M. N. Rebelo, Phys. Rept. **281**, 239 (1997)
  M. Holthausen, M. Lindner and M. A. Schmidt, JHEP **1304**, 122 (2013)

Generalised CP (3)

- For fields that transform as $\varphi_r \mapsto \rho_r(g)\varphi_r$ under G_{Flavour}, define generalised CP as

$$\varphi_r \mapsto X_r \left(\varphi_r^*(x^P)\right).$$

  X_R is a unitary matrix.

- Only gCP transformations that map each representation on itself make observables conserve CP (M. -C. Chen, M. Fallbacher, K. T. Mahanthappa, M. Ratz and A. Trautner, arXiv:1402.0507).
- The mass matrices will be constrained by

$$X_r^{e\dagger} M^e (M^e)^\dagger X_r^e = (M^e)^* (M^e)^T \text{ and } X_r^{\nu\,T} M^\nu X_r^\nu = (M^\nu)^*$$

  .

- If both flavour and gCP symmetries are present, they have to fulfill the consistency equation: $X_r \rho^*(g) X_r^\dagger = \rho_r(g')$.
- For faithful representations, this defines a bijective mapping on the group:

$$u_X(g) := \rho_r^{-1}(X_r \rho_r^*(g) X^\dagger)$$

  .

Brief Digression: Group automorphisms

- Inner automorphisms: A single group element h_u exists such that $\forall g : u(g) = h_u^{-1} g h_u$
- Outer automorphisms: All other automorphisms
- (u inner) $\Rightarrow$ (All group elements are mapped into their original class)
- (u outer) $\Rightarrow$ (u inner) $\Rightarrow$ (Not all group elements are mapped into their original class)
- (All group elements are mapped into their original class) $\Rightarrow$ (u inner)
- (Not all group elements are mapped into their original class) $\Rightarrow$ (u outer)
- This proves

$$(u \text{ inner}) \Leftrightarrow (\text{All group elements are mapped into their original class})$$

Group automorphisms and generalised CP: $X_r \in e^{i\alpha}G$ for real representations

- For $u_X(g) := \rho_r^{-1}(X_r \rho_r^*(g) X^\dagger)$, if $\rho_r(g)$ is real and $X_r \in e^{i\alpha}G$, u_X is an inner automorphism.
- Can there be a matrix $\tilde{X}_r$ that is not in $e^{i\alpha}G$ but where $u_{\tilde{X}}$ maps every group element into its original class?
- $u_{\tilde{X}}$ is inner $\Rightarrow \exists! h_u : u_{\tilde{X}}(g) = h_u^{-1} g h_u$.
- From this follows that $\rho_r(h_u)^{-1}\tilde{X}_r\rho_r(g) = \rho_r(g)\rho_r(h_u^{-1})\tilde{X}_r$, i.e. $\rho_r(h_u^{-1})\tilde{X}_r$ commutes with every group element.
- With Schur's Lemma follows $\tilde{X}_r = \lambda\rho_r(h_u)$ with $|\lambda| = 1$ which contradicts $X_r \notin e^{i\alpha}G$.
- For real representations this proves that inner automorphisms correspond to $X_r \in e^{i\alpha}G$.

Group automorphisms and gCP: $X_r \in e^{i\alpha} G$ for complex representations

- Assume a matrix w_r exists such that $\rho_r(g) \mapsto w_r^\dagger \rho_r(g)^* w_r$ is in the class of the inverse of g, $C(g^{-1})$.
- This can be seen as mapping g onto g^{-1} followed by an inner automorphism onto another element in $C(g^{-1})$.
- Are there $\tilde{X}_r \notin e^{i\alpha} G$ that map elements into a different class as g^{-1}? - No, c.f. previous page.
- Only gCP transformations that map into the class of the inverse make observables conserve CP (M. -C. Chen, M. Fallbacher, K. T. Mahanthappa, M. Ratz and A. Trautner, arXiv:1402.0507).
- $\Delta(6n^2)$ contains an element that can be w_r, namely $w_r = \rho_r(b)$.
- $\Rightarrow$ gCP $\left(\Delta(6n^2)\right) = e^{i\alpha} \Delta(6n^2)$.

Predictions for Majorana phases and $0\nu\beta\beta$ purely from flavour and CP symmetry (1)

- Residual flavour and residual gCP symmetries must still be consistent: For $K = \{1, c^{n/2}, abc^{\gamma}, abc^{\gamma+n/2}\}$, consistent gCP transformations are

$$X_r = \rho_r(e^{i\alpha} c^x d^{2x+2\gamma}), \rho_r(e^{i\alpha} abc^x d^{2x}) \text{ with } x = 0, \ldots, n-1$$

- $\Rightarrow$ Choice of residual flavour symmetry determines allowed residual gCP symmetries
- $\Rightarrow$ For all $\Delta(6n^2)$ groups enhanced by CP invariance the mixing matrix is fixed up to a discrete choice and always has the form

$$U_{\text{PMNS}}^{(+)/[(-)]} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\left(\frac{\pi\gamma}{n}\right) & \frac{e^{i(\varphi_1-\varphi_3)/2}}{\sqrt{3}} & [i]i\sqrt{\frac{2}{3}}\sin\left(\frac{\pi\gamma}{n}\right) \\ -\sqrt{\frac{2}{3}}\sin\left(\pi\left(\frac{1}{6}+\frac{\gamma}{n}\right)\right) & \frac{e^{i(\varphi_1-\varphi_3)/2}}{\sqrt{3}} & [i]i\sqrt{\frac{2}{3}}\cos\left(\pi\left(\frac{1}{6}+\frac{\gamma}{n}\right)\right) \\ \sqrt{\frac{2}{3}}\sin\left(\pi\left(\frac{1}{6}-\frac{\gamma}{n}\right)\right) & -\frac{e^{i(\varphi_1-\varphi_3)/2}}{\sqrt{3}} & [i]i\sqrt{\frac{2}{3}}\cos\left(\pi\left(\frac{1}{6}-\frac{\gamma}{n}\right)\right) \end{pmatrix}$$

where $\varphi_1 - \varphi_3 = -\frac{6\pi(\gamma+x)}{n}$ for $X = c^x d^{2x+2\gamma}, abc^x d^{2x}$

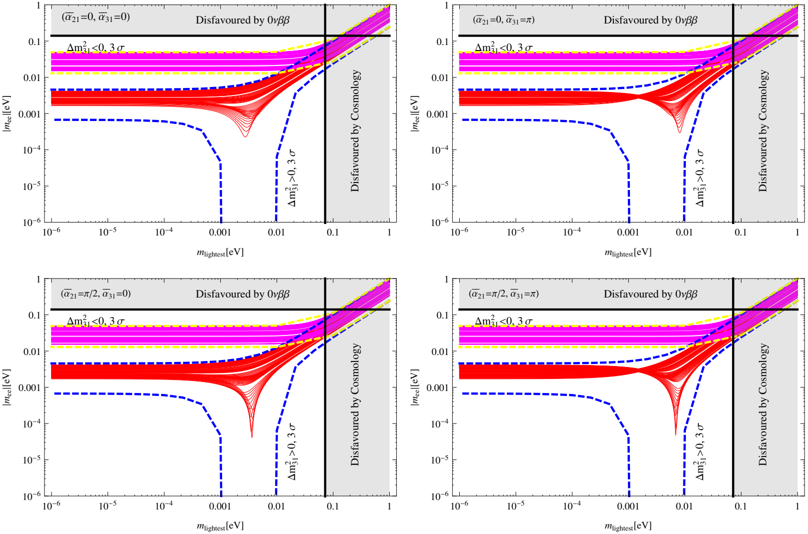
Predictions for Majorana phases and $0\nu\beta\beta$ purely from flavour and CP symmetry (2)

- The key observable for Majorana phases is neutrinoless double-beta decay.
- In this framework, the effective mass is given by

$$|m_{ee}| = |\frac{2}{3} m_1 \cos^2(\frac{\pi\gamma'}{n}) + \frac{1}{3} m_2 e^{i\alpha_{21}} + \frac{2}{3} m_3 \sin^2(\frac{\pi\gamma'}{n}) e^{i(\alpha_{31}-2\delta)}|$$

  with $m_1 = m_l$, $m_2 = \sqrt{m_l^2 + \Delta m_{21}^2}$, $m_3 = \sqrt{m_l^2 + \Delta m_{31}^2}$ for normal ordering and $m_1 = \sqrt{m_l^2 + \Delta m_{31}^2}$, $m_2 = \sqrt{m_l^2 + \Delta m_{21}^2 + \Delta m_{31}^2}$, $m_3 = m_l$ for inverted ordering, where m_l is the mass of the lightest neutrino and $\gamma' = \gamma \mod \frac{1}{6}$.
- There are 8 cases to distinguish for $\bar{\alpha}_{21} = \alpha_{21} + 6\pi\frac{\gamma+x}{n}$, $\bar{\alpha}_{31} = \alpha_{31} - 2\delta$.

$0\nu\beta\beta$ (1)



$0\nu\beta\beta$ (2)

- For inverted hierarchy there is no particular structure visible. Additionally, the predicted values for $|m_{ee}|$ are well within the reach of e.g. phase III of the GERDA experiment of $|m_{ee}^{\text{exp}}| \sim 0.02 \ldots 0.03$ eV.
- For normal ordering, it follows that for the values of γ/n and x/n considered there always is a lower limit on $|m_{ee}|$ which means that these parameters are accessible to future experiments.
- Further for normal ordering, in the very low m_{lightest} region, predicted values of $|m_{ee}|$ are closer to the upper end of the blue three sigma range.
- With the current data, no combination of $\bar{\alpha}_{21}$ and $\bar{\alpha}_{31}$ is favoured. Only for values of $|m_{ee}| \lesssim 0.0001$ eV and $m_{\text{lightest}} \lesssim 0.01 \ldots 0.001$ eV it would be possible to distinguish different values of $\bar{\alpha}_{21}$ and $\bar{\alpha}_{31}$.

Summary and Outlook

- We examine lepton mixing patterns in direct models with $\Delta(6n^2)$ groups and consistent generalised CP.
- For direct models, without corrections, $\Delta(6n^2)$ is the most (only?) promising class of flavour groups
- Further in direct flavour models, one can analyse $\Delta(6n^2)$ for all n simultaneously.
- This yields experimentally viable predictions for lepton mixing parameters and a sum rule.
- (Broken) invariance under consistent generalised CP transformations is the only framework that allows to predict Majorana phases purely from symmetry.
- We show, using a general arguement, that in presence of $\Delta(6n^2)$, physical CP transformations are $X_r = e^{i\alpha}\Delta(6n^2)$.
- Predictions for neutrinoless double-beta decay are accessible to experiments in the (near) future.