CP violation from discrete (flavor) symmetries

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based on NPB883 (2014) 267-305 (arXiv:1402.0507)
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FLASY 2014
18.6.14

Outline

Conditions for consistent physical CP transformations

Existence of CP transformations

- Two(three) types of groups

Toy example for predictive CPV from a finite group

Conclusion

Setup

- QFT with some sort of discrete (finite) symmetry G
  e.g. S_3, A_4, T_7, T', $\Delta(27)$, $\ldots$

- Most interesting: flavor model building
  - + Reduce # of flavor parameters
  - + Specific mixing patterns (TBM,...)
  - + Avoid Goldstones
  - (+ This talk: predictive CP violation)

But not limited to flavor! Other applications thinkable...

CP symmetries in these settings

Scalar field for example:

$$\boldsymbol{\phi}(x) \;=\; \int \mathrm{d}^3 p \, \frac{1}{2E_{\vec{p}}} \left[\boldsymbol{a}(\vec{p})\,\mathrm{e}^{-\mathrm{i}\,p\cdot x} + \boldsymbol{b}^\dagger(\vec{p})\,\mathrm{e}^{\mathrm{i}\,p\cdot x}\right]$$

Transformation under CP ? $(j^\mu \overset{\mathcal{C}}{\longmapsto} -j^\mu)$

$$(\mathcal{C}\,\mathcal{P})^{-1}\;\boldsymbol{a}\;(\vec{p})\,\mathcal{C}\,\mathcal{P} \;\propto\; \boldsymbol{b}(-\vec{p})$$
$$(\mathcal{C}\,\mathcal{P})^{-1}\;\boldsymbol{b}^\dagger(\vec{p})\,\mathcal{C}\,\mathcal{P} \;\propto\; \boldsymbol{a}^\dagger(-\vec{p})$$

Transformation of the fields

$$\mathcal{C}\,\mathcal{P} : \phi(x) \;\xrightarrow{\;?\,\boldsymbol{?}\,?\;}\; \ldots\;\phi'(\mathcal{P}x)$$

Physical CP transformations

What does a physical CP symmetry need to do?

Physical observable: Asymmetry $\Leftrightarrow$ Basis–invariants, e.g. J.

$$\varepsilon_{i\to f} \;=\; \frac{\left|\Gamma(i\to f)\right|^2 - \left|\Gamma(\bar{\imath}\to \overline{f})\right|^2}{\left|\Gamma(i\to f)\right|^2 + \left|\Gamma(\bar{\imath}\to \overline{f})\right|^2} \;\Leftrightarrow\; J \;=\; \det\left[M_u M_u^\dagger, M_d M_d^\dagger\right]$$

CP conservation:
need a map $M_{u/d} \to M_{u/d}^*$ to guarantee $\varepsilon, J \equiv 0$.
note: non–trivial map $M_{u/d} \to M_{u/d}$ *cannot* do this! (would just increase flavor symmetry...)
Therefore:

$$\mathscr{L} \;\supset\; c\;\mathcal{O}(x) \;+ c^*\;\mathcal{O}^\dagger(x)$$

i.e. $\mathcal{CP}$

$$\mathcal{CP} : \phi(x) \;\longmapsto\; \textbf{?} \times \phi^*(\mathcal{P}x)$$

CP symmetries in settings with G

Clearly: ϕ in representation $\boldsymbol{r} \implies \phi^*$ in representation $\boldsymbol{r}^*$

- in general: **inequivalent** representations: $\phi_{\boldsymbol{a}}$, $\phi^*_{\dot{\boldsymbol{a}}}$

$\Rightarrow$ CP has to act on the group from the "outside"

Character table:

$\boldsymbol{G}$	$\ldots$
.	
$\boldsymbol{r}$	$\ldots$
$\boldsymbol{r}^*$	$\ldots$
.	
.	

($\mathcal{CP}$: $\boldsymbol{r} \leftrightarrow \boldsymbol{r}^*$)

need matrix U (think about spinors...)

$$\phi_a \overset{\mathcal{CP}}{\longmapsto} U_{a\dot{a}}\,\phi^*_{\dot{a}}$$

(True for charge conjugation of *any* symmetry with complex irreps. Often $U_{a\dot{a}} = \delta_{a\dot{a}}$, so people forget...)

U is basis dependent!

Consistency with $G \iff U\,\rho_{\boldsymbol{r}^*}(g)\,U^\dagger \overset{!}{=} \rho_{\boldsymbol{r}}(u(g))$

[Holthausen, Lindner, Schmidt, 2013; Feruglio, Hagedorn, Ziegler, 2013]

- $\rho_{\boldsymbol{r}}(g)$: representation matrix for group element $g \in G$
- $u : g \mapsto u(g)$: **outer** automorphism

Note: There is **no distinction** "canonical CP" $\leftrightarrow$ "generalized CP", whether $U = \mathbb{1}$ or not (if possible) is *basis dependent*!

CP symmetries in settings with G

- CP transformations are **outer automorphisms** (auts) of G.

[Holthausen, Lindner, Schmidt, 2013]

(this is actually true for all symmetries, not only discrete symmetries)

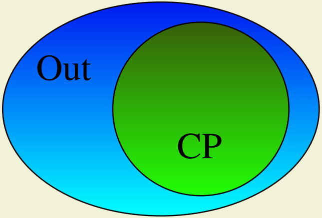
$$\mathrm{Out} \;:=\; \frac{\mathrm{Aut}}{\mathrm{Inn}}$$

Inn: reshuffling of $g \in G$ within conjugacy classes

Out: reshuffling of conjugacy classes *and* representations

- **But:** *not* all outer auts are CP transformations!

[Chen, Fallbacher, Mahanthappa, Ratz, AT, 2014]



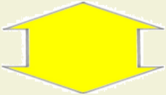
$$\mathrm{Out}: \boldsymbol{r} \;\mapsto\; \boldsymbol{r}'$$
$$\text{CP}: \boldsymbol{r} \;\mapsto\; \boldsymbol{r}^*$$

CP symmetries in settings with G

Given a model it is clear what is needed for CP :
Outer automorphism which maps *all* representations present to their c.c.

	G	$\dots$
	.	
$\mathcal{CP}$	$\boldsymbol{r}_1$	$\dots$
	$\boldsymbol{r}_1^*$	$\dots$
	.	
$\mathcal{CP}$	$\boldsymbol{r}_2$	$\dots$
	$\boldsymbol{r}_2^*$	$\dots$
	.	

But: Not all groups allow for such trafos. Fine print: often depends on representation content of the model.

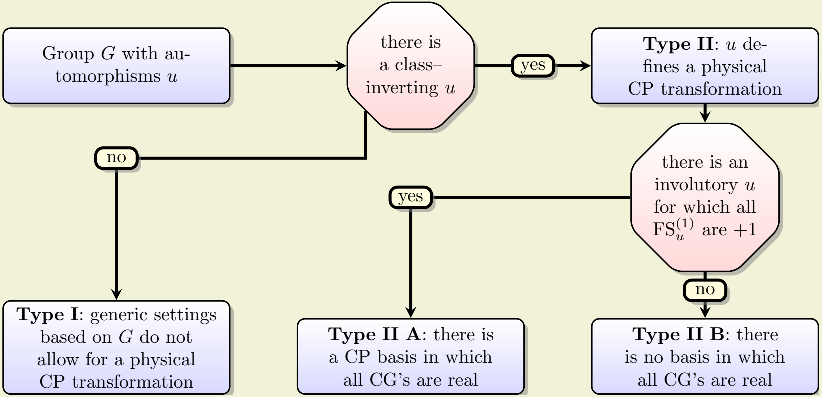


close relation to whether

$$\boldsymbol{r}_i \otimes \boldsymbol{r}_j \supset \boldsymbol{r}' \oplus \boldsymbol{r}'' \oplus \cdots$$

Clebsch–Gordan (CG) coefficients can be taken real.

CP symmetries in settings with G



(For details see [Chen, Fallbacher, Mahanthappa, Ratz, AT, 2014])

Mathematical tool to decide: Twisted Frobenius–Schur indicator FS_u(Backup slides)

Basically two types of groups

- Groups which **do not** allow for CP symmetry: **Type I**

Fine print: assuming sufficient # of irreps are there

G	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG id	$(20,3)$	$(21,1)$	$(27,3)$	$(27,4)$

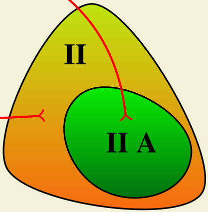
- Groups which **do** allow for CP symmetry: **Type II**

Among those: all groups which allow for real CG's: Type II **A**

G	S_3	A_4	T'	S_4	A_5
SG id	$(6,1)$	$(12,3)$	$(24,3)$	$(24,12)$	$(60,5)$

But also: CP trafo w/o real CG's: Type II **B**

G	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG id	$(72,41)$	$(144,120)$



Type II A groups: CP violation completely analogue to well known case: $\mathbf{SU}(\boldsymbol{N})$
Type II B groups: CP violation tied to certain operators

Toy example for CPV from Type I group

Type I group: take $G \equiv \Delta(27)$ and sufficient # of irreps.

[Such that any map $r_i \mapsto r_i^*$ is inconsistent w/ the group.]

$\Rightarrow$ Presence of G itself signals (explicit) CPV.

Toy model:

	X	Y	Z	Ψ		Σ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$		$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	0	$q_\Psi - q_\Sigma$	q_Ψ	$\neq$	q_Σ

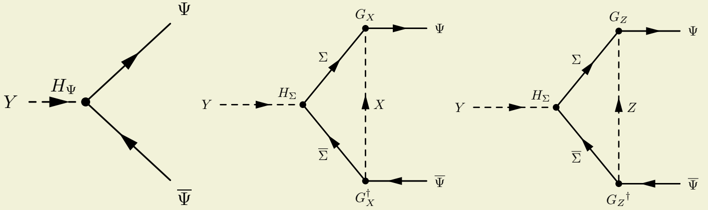
$$\mathscr{L}_{\mathrm{toy}} \supset \; + G_X^{ij}\, X\, \overline{\Psi}_i \Sigma_j + G_Z^{ij}\, Z\, \overline{\Psi}_i \Sigma_j + H_\Psi^{ij}\, Y\, \overline{\Psi}_i \Psi_j + H_\Sigma^{ij}\, Y\, \overline{\Sigma}_i \Sigma_j + \text{h.c.}\,.$$

w/ Yukawa couplings dictated by the symmetry:

$$G_X = g_X \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, G_Z = g_Z \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}, H_{\Psi/\Sigma} = h_{\Psi/\Sigma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

Here $g_X, g_Z, h_{\Psi/\Sigma} \in \mathbb{C}$ and $\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3}$.

Toy example: Decay asymmetry $Y \to \overline{\Psi}\Psi$



$$\varepsilon_{Y \to \overline{\Psi}\Psi} \propto |g_X|^2 \, \mathrm{Im}\,[I_X] \, \mathrm{Im}\,[\omega\, h_\Psi\, h_\Sigma^*] + |g_Z|^2 \, \mathrm{Im}\,[I_Z] \, \mathrm{Im}[\omega^2\, h_\Psi\, h_\Sigma^*] \neq 0 \,.$$

($I_X = I(M_X, M_Y)$, $I_Z = I(M_Z, M_Y)$ loop integral & phase space)

Indeed: CP violated! Crosschecks:

- ✓ invariant under rephasing of the fields
- ✓ basis independent
- ✓ cancellation between the two terms? Not RGE stable! (or $\boldsymbol{G} > \Delta(27)$)
- ✓ imposing [Holthausen et al., 2013] "generalized CP" which maps $\mathbf{3} \mapsto \mathbf{3}$ does not lead to CPC

Even better: SSB $\Rightarrow$ prediction of CP phases

Take this model and embed it in $\boldsymbol{G \equiv \Delta(27) \subset H}$

- where $\boldsymbol{H}$ is **Type II** (explicitly $H \equiv \Delta(27) \rtimes \mathbb{Z}_2 \cong \mathrm{SG}(54,5)$)
- X, Z combined to a doublet
- coupling relations $|g_X| = |g_Z|$, $h_\Psi = h_\Sigma$, $M_X = M_Z$

$\Rightarrow$ $\varepsilon_{Y \to \overline{\Psi}\Psi} = 0$, i.e. **CP conserved** at the level of $\boldsymbol{H}$

Now SSB: $\boldsymbol{H} \xrightarrow{\langle\phi\rangle} \boldsymbol{\Delta(27)}$

- coupling relations stay, *but*: mass splitting $M_X \neq M_Z$

$$\Rightarrow \varepsilon_{Y \to \overline{\Psi}\Psi} \propto |g_X|^2 \, |h_\Psi|^2 \, \mathrm{Im}\,[\boldsymbol{\omega}] \, (\mathrm{Im}\,[I_X] - \mathrm{Im}\,[I_Z])$$

- CP phase is predicted *from the flavor symmetry*!

Conclusion

- CP transformations are **outer automorphisms** of G
- *Not* all outer automorphisms are CP transformations
- Two types of discrete groups:
  - Type I: incompatible with CP ✗ Attention: some (model dependent) fine print!
  - Type II: compatible with CP ✓
- In generic model w/ Type I group: CP is violated *as a consequence* of the flavor symmetry
- Explicit example: CP phase is *predicted from the flavor symmetry*

Thank You!

Bibliography I

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Backup slides

Twisted Frobenius–Schur indicator

Criterion to decide: existence of a CP outer automorphism.
$\curvearrowright$ can be probed by computing the

"twisted Frobenius–Schur indicator" $\mathbf{FS}_{\boldsymbol{u}}$

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;:=\; \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g\,u(g))$$

($\chi_{\boldsymbol{r}_i(g)}$: Character)
[Chen, Fallbacher, Mahanthappa, Ratz, AT, 2014]

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \begin{cases} +1 \text{ or } -1 \quad \forall\, i, & \Rightarrow\; u \text{ is good for CP,} \\ \text{different from } \pm 1, & \Rightarrow\; u \text{ is no good for CP.} \end{cases}$$

In analogy to the Frobenius–Schur indicator

$\mathrm{FS}_{\not{u}}(\boldsymbol{r}_i) = +1, -1, 0$ for real / pseudo–real / complex irrep.