Leptonic Dipole operators in RS

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Outline

- Introduction
- Strategy and Formalism
- Phenomenology

in collaboration with M. Beneke, P. Dey arXiv:1209.5897 and P. Moch arXiv:1405.5385

Setup

Slice of AdS_5 in interval ($[1/k, 1/T]$ in conformal coordinates)

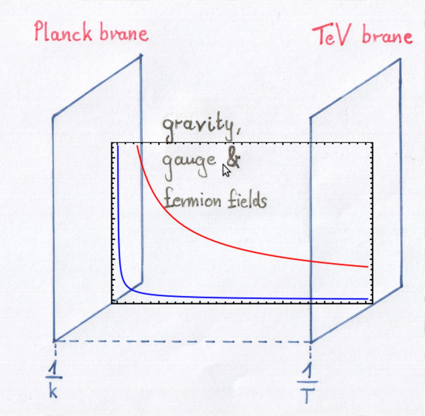
$$ds^2 = \left(\frac{1}{kz}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - d^2 z)$$

$$M_{Pl}^{4d\,2} \approx \frac{M_{Pl}^{5d\,3}}{k}$$

$$\varepsilon \equiv \frac{T}{k} \approx 10^{-16} \approx \frac{1 \text{ TeV}}{M_{Pl}^{4d}}$$

proper distance between branes (= boundaries):

$$1/k \times \ln(k/T)$$



- IR localized Higgs to address the gauge-gravity hierarchy [Randall, Sundrum 1999]
- geometric interpretation of flavour [Huber, Shafi 2000,2001]

Models & Particle Content

- minimal RS: only SM fields (promoted to 5D) + Higgs localised near IR branes
- custodially protected RS: extended gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times \mathbf{Z}_2$
  $\hookrightarrow$ custodial symmetry only broken by UV brane BC
  $\hookrightarrow$ extended fermion sector (but NO unique choice for fermion reps.)

$$\xi_{1L}^{il} = \begin{pmatrix} \chi_L^{\nu_i}(-,+)_1 & l_L^{\nu_i}(+,+)_0 \\ \chi_L^{l_i}(-,+)_0 & l_L^{l_i}(+,+)_{-1} \end{pmatrix}$$

$$\xi_{2R}^{il} = \nu_R(+,+)_0$$

$$\xi_{3R}^{il} = T_{3R}^i \otimes T_{4R}^i = \begin{pmatrix} \tilde{\lambda}_R^i(-,+)_1 \\ \tilde{N}_R^i(-,+)_0 \\ \tilde{L}_R^i(-,+)_{-1} \end{pmatrix} \otimes \begin{pmatrix} \lambda_R^i(-,+)_1 \\ N_R^i(-,+)_0 \\ E_R^i(+,+)_{-1} \end{pmatrix}$$

- models where the Higgs 'leaks' into the bulk
- …

Dipoles and Loops

- Exhaustive phenomenology of tree-level processes (electroweak, flavour)

  From direct production $M_{KK} \sim 2.7$TeV
  higher from EWPO without custodial protection [e.g. Agashe et al., 2003; Duling et al., 2009; Casagrande et al., 2010]
  from gluon FCNCs $\sim$ 20 TeV without extra flavour structure [e.g. Csaki, Falkowski, Weiler, 2008]
- Higgs production/decay [Casagrande et al., 2010; Azatov et al.,2010; Carena et al.,2012; Malm et al.,2013; Hahn et al.,2013]
- Dipoles and Penguin loops
  - we are after the coefficients of the operators

$$\boxed{\mathcal{C}_{ij}} \quad \boxed{\bar{f}_i \sigma^{\mu\nu} f_j \, F^{\mu\nu}}$$

  i,j flavour indices
  - Flavour-changing radiative transitions related to $\mu \to e\gamma$ [Agashe et al., 2006; Csaki et al., 2010], $b \to s\gamma$,$b \to sg$,[Gedelia, Isidori, Perez 2009; Blanke et al.,2012], and $c \to ug$ [Delaunay et al.,2012]
  - Flavour-preserving; (anomalous) dipole moments [$g-2$, electric dipole moments, . . .]

Strategy

- 5D Lagrangians are quite involved at first glance

distinct scale hierarchy $\underbrace{\overbrace{k \gg T}^{\text{5d scales}}}_{UV} \gg \underbrace{\overbrace{v \gg m_\ell}^{\text{SM scales}}}_{IR}$

matching scale μ

- strategy:

1. Step (in symmetric phase $\to$ no vev):
integrate out the “bulk” $\to$ match onto an $SU(2)_L \times U(1)_Y$ symmetric effective theory

$$\mathcal{L}_{RS} \to \mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{T^2} \sum_i C_i \mathcal{O}_i$$

[Buchmüller & Wyler]

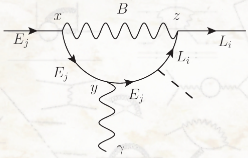
2. Step: change into the “broken” phase

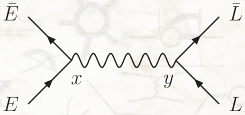
3. Step: compute the final dipole coefficient in the effective theory

After 1. Step: EFT before EWSB

- Operators that contribute in the lepton sector

$$\sum_i C_i \mathcal{O}_i \supset a_{B,ij}\, \bar{L}_i \Phi \sigma_{\mu\nu} E_j B^{\mu\nu} + a_{W,ij}\, \bar{L}_i \tau^a \Phi \sigma_{\mu\nu} E_j\, W^{a,\mu\nu} + \text{h.c.}$$
$$+\, b_{ij}\, (\bar{L}_i \gamma^\mu L_i)(\bar{E}_j \gamma_\mu E_j) + c_{1,i}\, (\bar{E}_i \gamma_\mu E_i)(\Phi^\dagger i D^\mu \Phi)$$
$$+\, c_{2,i}\, (\bar{L}_i \gamma_\mu L_i)(\Phi^\dagger i D^\mu \Phi) + c_{3,i}\, (\bar{L}_i \gamma^\mu \tau^a L_i)(\Phi^\dagger i \tau^a \overleftrightarrow{D}_\mu \Phi) + \ldots$$





- changing to the 'broken' phase

$$\Phi \to \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + iG) \end{pmatrix} \qquad E_i \to V_{ij} P_R \psi_j, \qquad L_i \to U_{ij} P_L \begin{pmatrix} \nu_j \\ \psi_j \end{pmatrix}$$

2. Step: EFT after EWSB

- Operators that contribute in the lepton sector

$$\sum_i C_i \mathcal{O}_i \supset a_{B,ij}\,\bar{L}_i \Phi \sigma_{\mu\nu} E_j B^{\mu\nu} + a_{W,ij}\,\bar{L}_i \tau^a \Phi \sigma_{\mu\nu} E_j\, W^{a,\mu\nu} + \text{h.c.}$$
$$+ b_{ij}\,(\bar{L}_i \gamma^\mu L_i)(\bar{E}_j \gamma_\mu E_j) + c_{1,i}\,(\bar{E}_i \gamma_\mu E_i)(\Phi^\dagger i D^\mu \Phi)$$
$$+ c_{2,i}\,(\bar{L}_i \gamma_\mu L_i)(\Phi^\dagger i D^\mu \Phi) + c_{3,i}\,(\bar{L}_i \gamma^\mu \tau^a L_i)(\Phi^\dagger i \tau^a \overleftrightarrow{D_\mu} \Phi) + \ldots$$

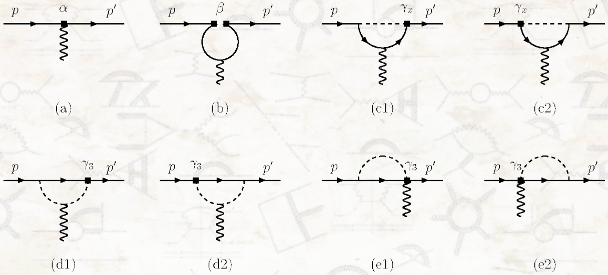
- changing to the 'broken' phase gives

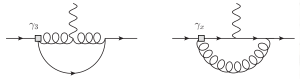
$$\sum_i C_i \mathcal{O}_i \to \frac{\alpha_{ij} + \alpha^\star_{ij}}{2} \frac{v}{\sqrt{2}}\, \bar{\psi}_i \sigma_{\mu\nu} \psi_j F^{\mu\nu} + \frac{\alpha_{ij} - \alpha^\star_{ij}}{2i} \frac{v}{\sqrt{2}}\, \bar{\psi}_i \sigma_{\mu\nu} i \gamma_5 \psi_j F^{\mu\nu}$$
$$+ \beta_{ijkl}\,(\bar{\psi}_i \gamma^\mu P_L \psi_j)(\bar{\psi}_k \gamma_\mu P_R \psi_l)$$
$$+ \gamma_{1,ij} \frac{v}{2}\,(\bar{\psi}_i P_L \gamma_\mu \psi_j)(i\partial^\mu H) + [\gamma_{2,ij} + \gamma_{3,ij}] \frac{v}{2}\,(\bar{\psi}_i P_R \gamma_\mu \psi_i)(i\partial^\mu H)$$
$$+ \gamma_{3,ij} \frac{v}{\sqrt{2}}\,(\bar{\psi}_i P_R \gamma^\mu \nu_i)(-i\partial_\mu \phi^-) + \gamma_{3,ij} \frac{v}{\sqrt{2}}\,(\bar{\psi}_i P_R \gamma^\mu \nu_i)(e A_\mu \phi^-)$$
$$+ \text{h.c. of previous line} + \ldots$$

the Greek Wilson coefficients are Latin ones dressed with flavour rotation matrices

3. Step: loops in the effective theory

$g-2$ & $\mu \to e\gamma$ correspond to the flavour conserving and violating part of





UV and IR divergences require regularisation

BUT finite due to $\frac{1}{\epsilon} \times \epsilon$

Scheme dependent $\to$ dependence must cancel with the dependence of the 5D loop in α

3. Step: loops in the effective theory

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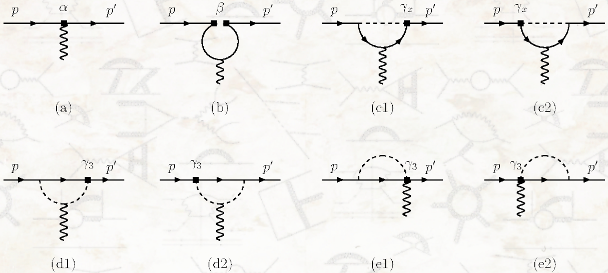
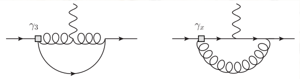




$$\Delta a_\mu = -\frac{4m_\mu^2}{T^2}\left(\frac{\mathrm{Re}[\alpha_{22}]}{y_\mu e} + \sum_k \frac{1}{16\pi}\frac{m_{l_k}}{m_\mu}\mathrm{Re}[\beta_{2kk2}] + \frac{1}{3(4\pi)^2}\mathrm{Re}\left[\gamma_2^{22} - \frac{3}{2}\gamma_1^{22} - \frac{3}{2}\gamma_3^{22} + \sin^2\Theta_W \sum_{i=1}^{3}\gamma_i^{22}\right]\right)$$

Back to step 1: 5D formalism

- work in a 5D QFT
  no KK sums; vertices and propagators are five-dimensional [Randall, Schwartz, 2001]
- zero-mode ($\sim$ SM fields) must be separated explicitly [Grossman, Neubert 1999]

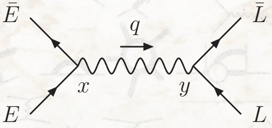
$$f_L^{(0)}(z) = \sqrt{\frac{1-2c_L}{1-\epsilon^{1-2c_L}}}\sqrt{T}(kz)^2(Tz)^{-c_L} \qquad g_E^{(0)}(z) = \sqrt{\frac{1+2c_E}{1-\epsilon^{1+2c_E}}}\sqrt{T}(kz)^2(Tz)^{c_E}$$

- use mixed coordinate-momentum representation for propagators in the unbroken theory [Randall, Schwartz, 2001]

$$\Delta^{\mu\nu}(p,x,y,\xi) = \Delta_\perp(p,x,y)\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + \frac{p^\mu p^\nu}{p^2}\Delta_\perp(p/\sqrt{\xi},x,y)$$

$$\Delta_\perp(p,x,y) = \Theta(x-y)\,\frac{ikxy(I_1(px)K_0(p/T) - K_1(px)I_0(p/T))(I_1(py)K_0(p/k) - K_1(py)I_0(p/k))}{I_0(p/T)K_0(p/k) - K_0(p/T)I_0(p/k)} + \{x \leftrightarrow y\}$$

Tree-level coefficients are 'for free'



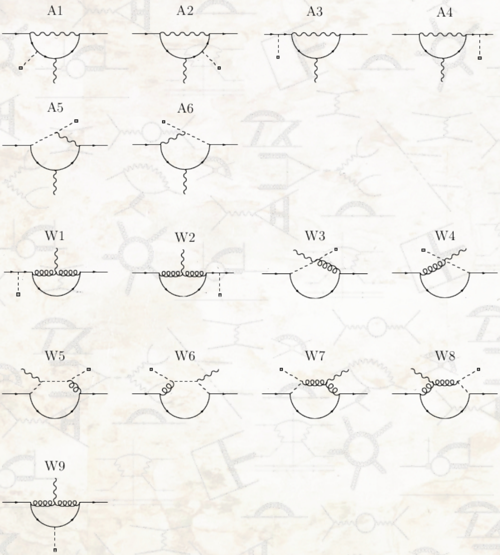
$$b_{ij} = -i(-g_5')^2 \frac{Y_L Y_E}{4} T^2 \int_{1/k}^{1/T} \mathrm{d}x\, \mathrm{d}y\, \frac{f_{L_i}^{(0)\,2}(x)}{(kx)^4} \frac{g_{E_j}^{(0)\,2}(y)}{(ky)^4} \Delta_\perp(q=0,x,y)$$

the hypercharge boson zero-momentum propagator is

$$\Delta_\perp(q,x,y) \overset{q\to 0}{=} \Theta(x-y)\, \frac{ik}{\ln\frac{k}{T}} \left(-\frac{1}{q^2} + \frac{1}{4}\left\{ \frac{1/T^2 - 1/k^2}{\ln\frac{k}{T}} - x^2 - y^2 + 2x^2\ln(xT) \right.\right.$$
$$\left.\left. + 2y^2 \ln(yT) + 2y^2 \ln\frac{k}{T} \right\} + \mathcal{O}(q^2) \right) + (x \leftrightarrow y),$$

all integrals are elementary
very similar to computation of $\Delta F = 2$ tree-level processes
$\hookrightarrow$ agrees with KK sum calculation [Casagrande et al. 2008]

Genuine 5D Loops

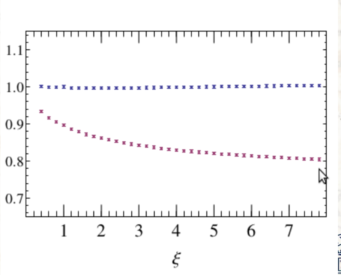


custodially prot. RS model:
15 topologies

evaluate à la expansion by regions
Beneke, Smirnov 1997

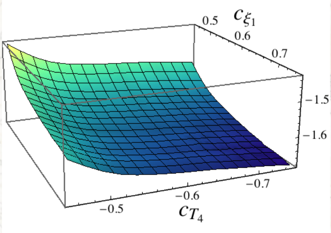
1PR diagram contribute to short distance coefficients

crucial for gauge invariance



Phenomenology and Results

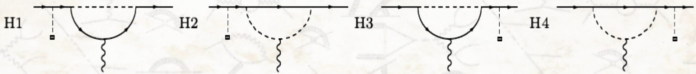
- a_{ij}^{gauge} relative to the mass matrix



$$a_{ij}^{gauge} = \text{Const} \times \text{Loop}(c_{L_i}, c_{E_j}) \cdot M_{ij}$$

$\hookrightarrow$ suppressed FCNCs
(in full analogy to GIM mechanism)

- Higgs contributions



a_{ij}^{Higgs} is technically less challenging, e.g.,

$$a_{ij}^{Higgs} = (iQ_\mu e) \times \frac{\mathbf{H}}{6} \times \frac{1}{16\pi^2}\frac{1}{T^2} \times f_{L_i}^{(0)}(1/T)[YY^\dagger Y]_{ij}\, g_{E_j}^{(0)}(1/T)\frac{T^3}{k^4} + \ldots$$

BUT result depends subtly on the Higgs localization [see also Carena 2012, Delaunay 2012]

Phenomenology and Results

- Flavour violation
  naive estimate: GIM mechanism suppresses gauge contributions to $\mu \to e\gamma$

$$\hookrightarrow \mathrm{Br}(\mu \to e\gamma) \approx \frac{3}{2}|a|^2 \frac{\alpha_{em}}{4\pi} \frac{\langle Y \rangle^4}{G_F^2 T^4} \frac{m_e}{m_\mu}, \quad a \approx \frac{\mathbf{H}}{6}$$

  MEG bound is violated unless $M_{KK} > 15\,\mathrm{TeV}$ for $\langle Y \rangle = 1$ and $\mathbf{H} = 1$

- Flavour preserving ($g_\mu - 2$ and d_{el})
  with above estimate $g_\mu - 2$ has the very simple form:

$$\Delta a_\mu = 26.7 \cdot 10^{-11} \frac{1\ \mathrm{TeV}^2}{T^2}$$

  (**independent** of all 5D parameters but T) compared to

$$a_\mu^{exp} - a_\mu^{the} = 287(63)(49) \times 10^{-11}$$

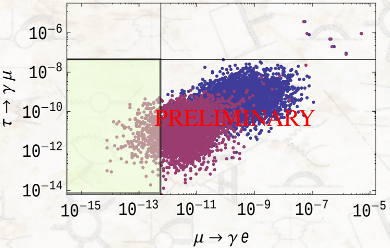
  and

$$d_{\mathrm{el}}^{gauge} < \mathrm{few} \times 10^{-29} \mathrm{e\ m} \frac{1\mathrm{TeV}^2}{T^2}$$

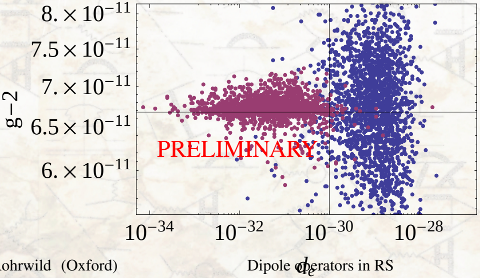
  (current bound 10^{-30}e m)

Phenomenology and Results

- Flavour violation [assuming ~ 5 TeV resonances and $\mathcal{O}(1)$ Yukawas]



- Flavour preserving ($g_\mu - 2$ and d_{el})



Summary

- complete 5D calculation of leptonic dipole operators
- results usually depend on the details of the Higgs treatment
- $g-2$ is model independent but numerically too small to be observable
- flavour-changing transitions are strongly dependent on the 5D parameters; very large effects possible
- correlations with other LFV processes (not dipole dominated)?