Radiative Generation of Quark Masses and Mixing Angles in the 2HDM

[arXiv:1403.2382]

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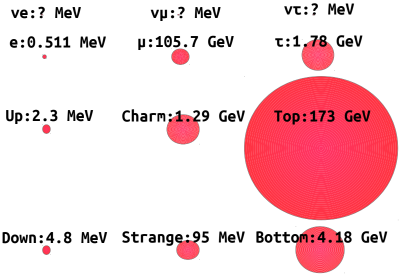
FLASY 2014, Brighton

Introduction

- **Motivation:** Hierarchy for masses and mixing angles in the Standard Model. $\Rightarrow$ **New Physics?**



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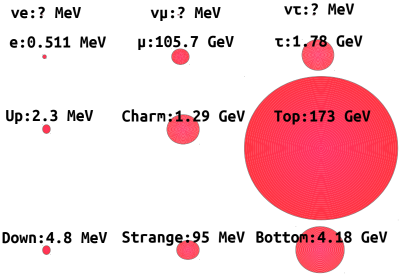

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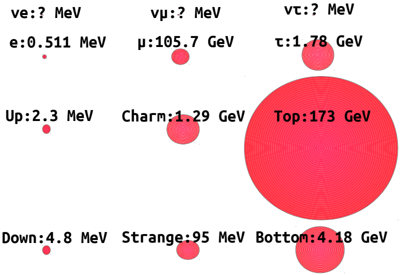

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- **Our goal:** Reproduce masses and mixing angles.
- **How?**: 2HDM (~~extra symmetries~~)

The Two-Higgs Doublet Model

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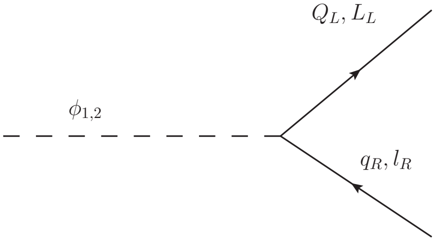

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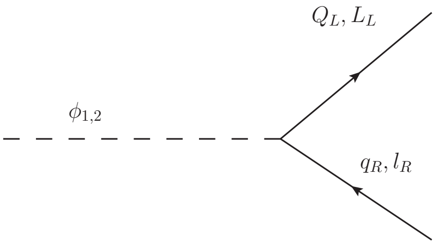

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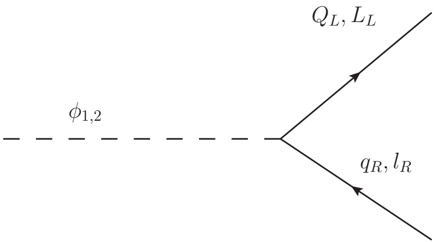

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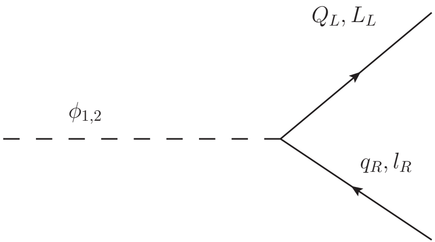

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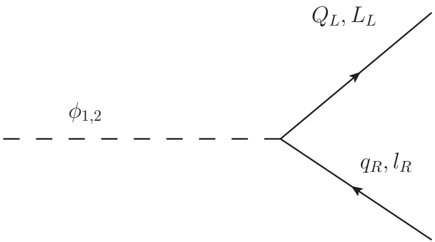

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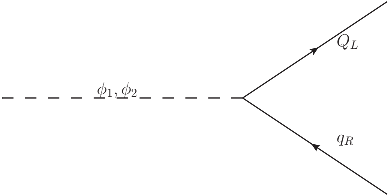
- Yukawa interaction (rank-1):



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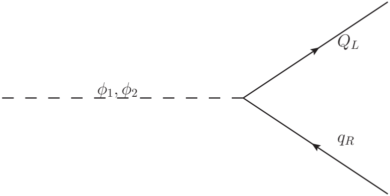

$$Y_u^{(1)}|_{\rm tree} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(1)} \end{pmatrix} \; , \quad Y_d^{(1)}|_{\rm tree} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon y_d^{(1)} \\ 0 & 0 & y_d^{(1)} \end{pmatrix}$$

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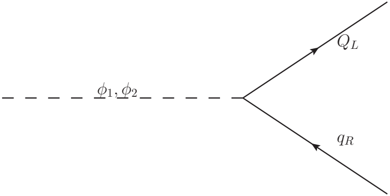

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- $Y_u^{(1)}|_{\text{tree}}$ & $Y_d^{(1)}|_{\text{tree}} \Rightarrow \begin{cases} m_t \\ m_b \end{cases}$ **@ tree level**

$$Y_u^{(2)}|_{\text{tree}} = U_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(2)} \end{pmatrix} U_R \ , \quad Y_d^{(2)}|_{\text{tree}} = D_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_d^{(2)} \end{pmatrix} D_R$$

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$$\begin{aligned} (U_L)_{31} &= e^{i\rho_{u_L}} \sin\theta_{u_L} \sin\omega_{u_L} \ , \\ (U_L)_{32} &= e^{i\xi_{u_L}} \sin\theta_{u_L} \cos\omega_{u_L} \ , \\ (U_L)_{33} &= \cos\theta_{u_L} \ , \end{aligned}$$

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- Neglect phases.
- Assume for simplicity $\mathrm{y_u^{(1)}}, \mathbf{y_u^{(2)}} >> \mathrm{y_d^{(1)}}, \mathbf{y_d^{(2)}}$

- 1-loop from β function:

$$Y_{u,d}^{(1)}|_{1-\mathrm{loop}} \simeq Y_{u,d}^{(1)}|_{\mathrm{tree}} + \frac{1}{16\pi^2}\beta_{u,d}^{(1)} \log \frac{\Lambda}{M_{\phi 2}}$$

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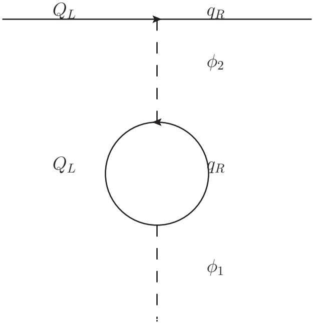
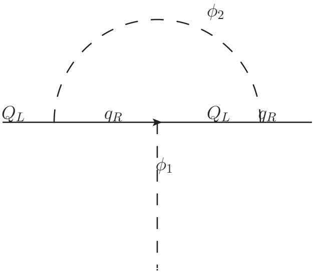
- 1-loop diagram (generate 2nd mass):





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Mass hierarchy for the quarks:

$$\frac{y_c}{y_t} \simeq \left(\frac{1}{16\pi^2}\log\frac{\Lambda}{M_H}\right)\frac{3}{4}(y_u^{(2)})^2 \times \text{mixing angles}$$

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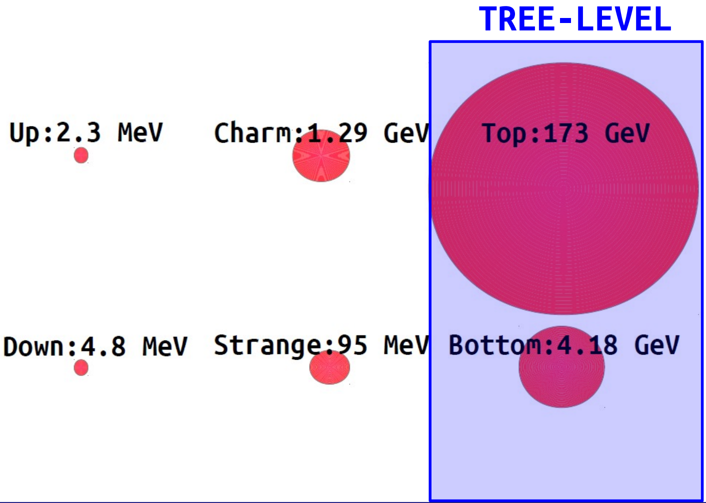
- First generation massless.

Quark Masses



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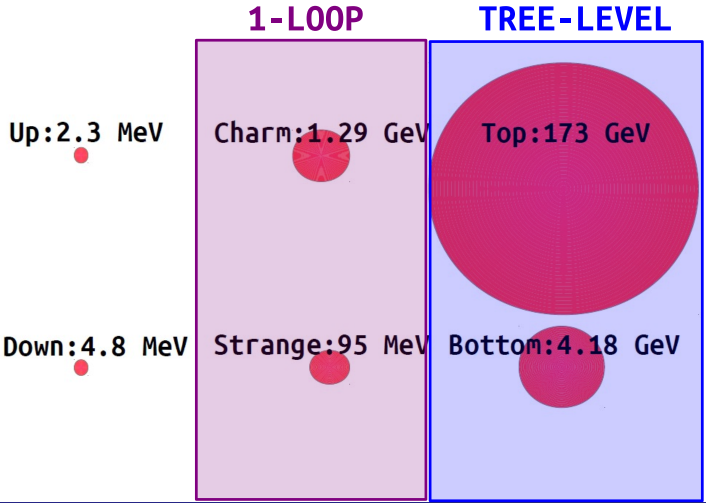

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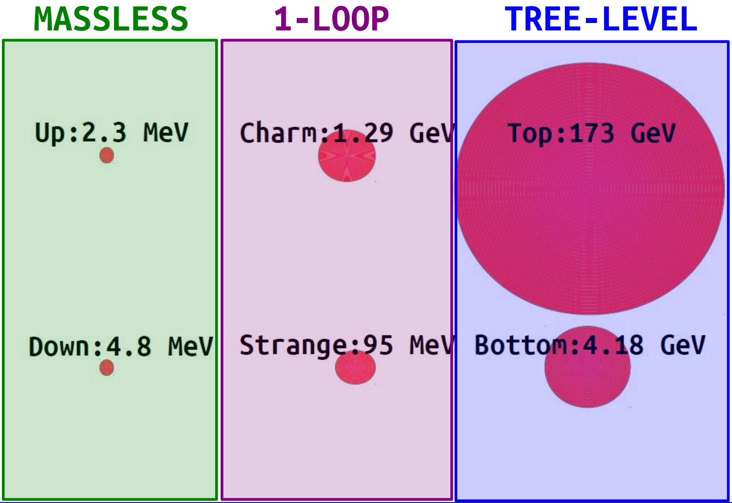

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- Cabibbo angle @ 0-order in perturbation theory:

$$V_{us} \simeq -V_{cd} \simeq \frac{3\sin\theta_{d_L}\cos\theta_{u_L}\sin(\omega_{d_L}-\omega_{u_L})}{N_d}$$

$$N_d = \left[9\sin^2\theta_{d_L}\cos^2\theta_{u_L} + 4\cos^2\theta_{d_L}\sin^2\theta_{u_L}\right.$$
$$\left.-3\sin 2\theta_{d_L}\sin 2\theta_{u_L}\cos(\omega_{d_L}-\omega_{u_L})\right]^{1/2}$$

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- Remaining off-diagonal terms @ 1st order in perturbation theory:

$$V_{ub} \simeq \left(\frac{1}{16\pi^2} \log \frac{\Lambda}{M_H}\right) \frac{3y_u^{(1)} y_u^{(2)} y_d^{(2)}}{y_d^{(1)}} \times \text{mixing angles}$$

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Tree-level

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1st Order Tree-level

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$$\Rightarrow \theta_{u_R} \approx 0.16, \ \theta_{d_R} \approx 1.06$$

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- Degeneracy for other parameters:
  - ✓ All masses and angles can be reproduced (example: $y_u^{(2)} \approx 1.04$, $y_d^{(2)} \approx 0.02$, $\theta_{d_L} \approx 0.61$, $\theta_{u_L} \approx 0.51$, $\omega_{d_L} - \omega_{u_L} \approx 0.10$)

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μ:105.7 GeV

ν_τ:? MeV

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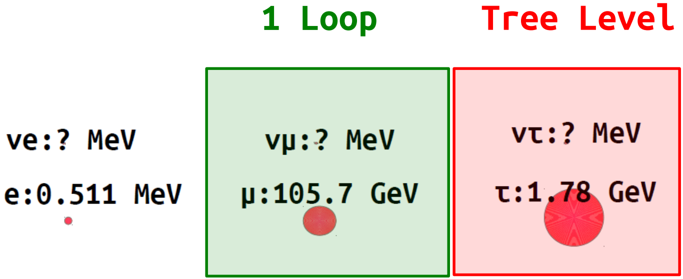

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Massless	1 Loop	Tree Level
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0-Order **Tree-Level**

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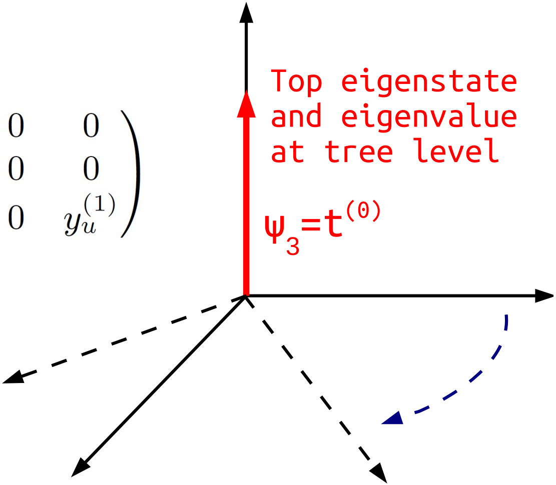

Backup slides

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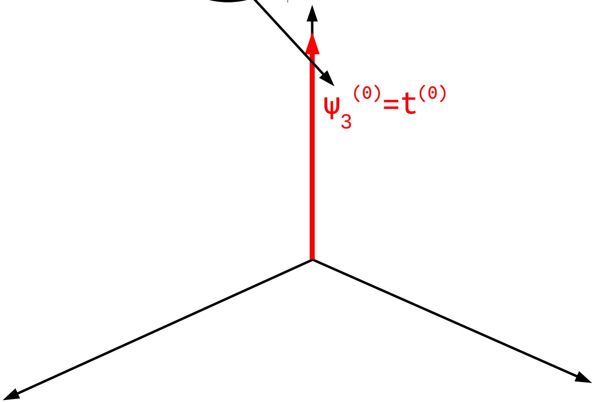

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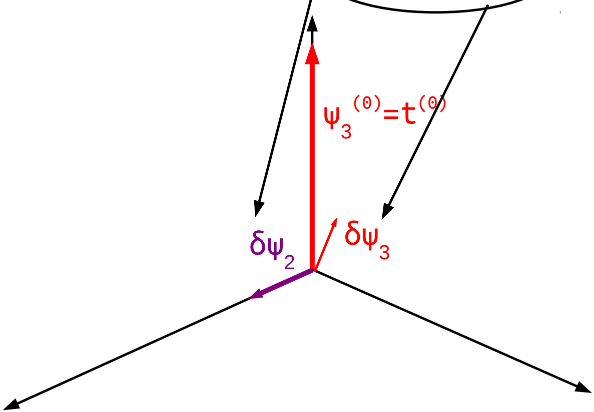

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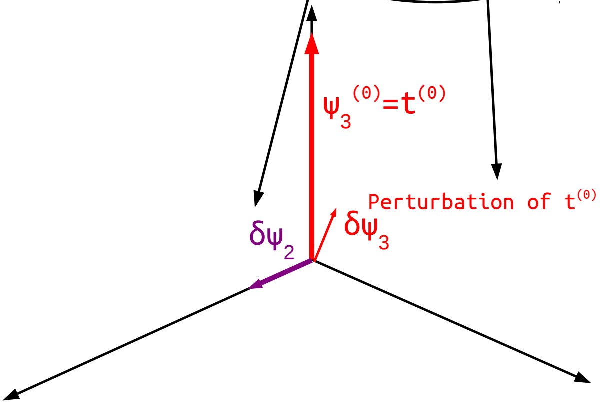

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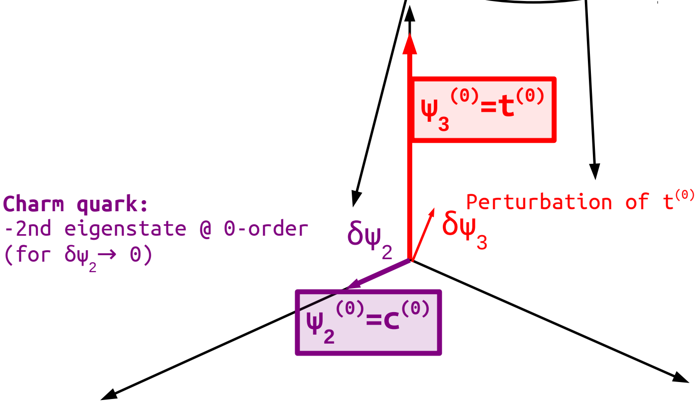

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- Hamiltonian $\mathsf{H} = \mathsf{H}^{(0)} + \lambda \mathsf{W}$
- $\mathsf{H}^{(0)}$ of rank-1 determines 1-eigenstate ($\left|t^{(0)}\right\rangle$)
- $\mathsf{W}^{(n)}$ is the perturbation in subspace $\perp \left|t^{(0)}\right\rangle$
- Eigenstates and eigenvalues of $\lambda \mathsf{W}^{(n)}$ are eigenstates (0-order) and eigenvalues (1st-order) of H from the subspace $\perp \left|t^{(0)}\right\rangle$.