

**Sizable D-term contribution  
as a signature of  
 $E_6 \times SU(2)_F \times U(1)_A$  SUSY GUT model**

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# 1. Introduction

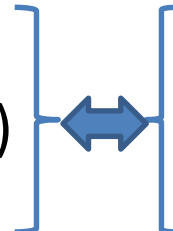
# Advantages of SUSY GUT

two unifications

- gauge interactions
- particles (especially matters)

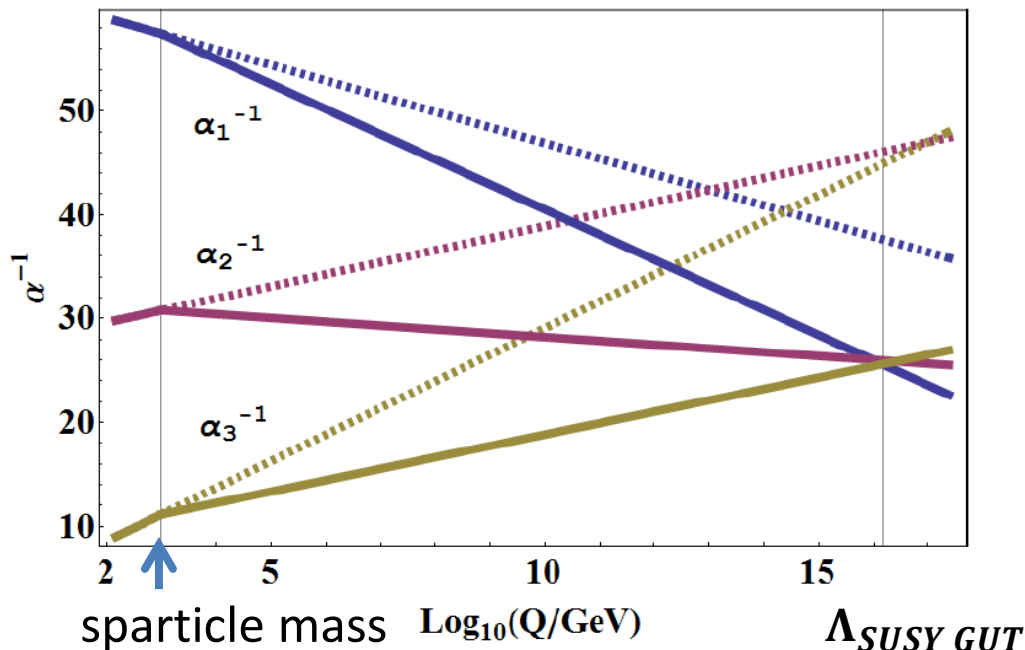
two supports

- gauge coupling unification
- quark and lepton masses and mixings



## ● unification of gauge interactions

SM gauge group  $G_{SM}$  is unified into grand unification group.



$$\Lambda_{SUSY\ GUT} \sim 2 \times 10^{16} \text{ GeV}$$

● unification of particles (especially matters)

$$SU(5) \rightarrow G_{SM} \equiv (SU(3)_C, SU(2)_L)_{U(1)_Y} \quad \bar{\mathbf{5}} \rightarrow d_R^c(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$$

$$\mathbf{10} \rightarrow q_L(\mathbf{3}, \mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1}, \mathbf{1})_1$$

**10** SM matter induces stronger hierarchies for Yukawa coupling than  $\bar{\mathbf{5}}$  SM matter.

$$\begin{pmatrix} \mathbf{10}_1 \\ \mathbf{10}_2 \\ \mathbf{10}_3 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda^3 \\ \lambda^2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \bar{\mathbf{5}}_3 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda^3 \\ \lambda^{2.5} \\ \lambda^2 \end{pmatrix} \quad \lambda \sim 0.22$$

$$Y_{d,e} \mathbf{10} \cdot \bar{\mathbf{5}} \cdot \bar{\mathbf{5}}_H$$

$$M_d = M_e^t = \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^{3.5} & \lambda^3 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H_d \rangle$$

$$\psi_{Li} M_{ij} \psi_{Rj}^c = (L_\psi^\dagger \psi_L)_i (L_\psi^T M R_\psi)_{ij} (R_\psi^\dagger \psi_R^c)_j$$

$$= \psi'_{Li} M_{diag\ ij} \psi'_{Rj}{}^c$$

➔ not only realistic quark and lepton mixings  $L_\psi, R_\psi$  : diagonalizing matrix

$$U_{CKM} = L_u^\dagger L_d \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad U_{MNS} = L_\nu^\dagger L_e \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

~ diagonalizing matrix for **10** SM matters      ~ diagonalizing matrix for  $\bar{\mathbf{5}}$  SM matters

but also realistic quark and lepton masses

# Model identification

$$\underline{\Lambda_{GUT} = 10^{13-17} \text{ GeV} \gg \Lambda_{LHC} = 10^{3-4} \text{ GeV}}$$

It is hard to identify models by accelerator directly.

Phenomenon to identify GUT model

- Nucleon decay
- D-term contribution to squark and slepton masses
- FCNC process



topic of this talk

In  $E_6 \times SU(2)_F \times U(1)_A$  SUSY GUT, we can expect

- sizable D-term contribution

as a signature of GUT model.

# 2. D-term contribution as a signature of GUT model

# D-term contribution

larger rank gauge group breaks to smaller rank one

D-term contribution to squark and slepton masses

$$\Delta \tilde{m}_\psi^2 = \sum_I Q_I(\psi) D_I$$

$D_I$  : squared gauge coupling  $\times$  D-term

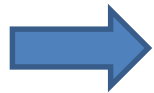
In  $E_6 \times SU(2)_F \times U(1)_A$  SUSY GUT there are four D-term contributions

- $E_6 \rightarrow SO(10) \times U(1)_{V'}$   $D_6$  : D-term of  $U(1)_{V'}$
- $SO(10) \rightarrow SU(5) \times U(1)_V$   $D_{10}$  : D-term of  $U(1)_V$
- $SU(2)_F$  breaking  $D_F$  : D-term of  $U(1)_F$
- $U(1)_A$  breaking  $D_A$  : D-term of  $U(1)_A$

$Q_I(\psi)$  :  $U(1)$  charge of the field  $\psi$

$U(1)_F$  : Cartan part of  $SU(2)_F$

rank of  $SO(10)$  and  $E_6$  gauge group  $>$  rank of SM gauge group



D-term contribution is useful to identify GUT model

e. g. Kawamura, Tanaka (1994)

$G_n$	Scalar Masses
$E_6$	$m_{\bar{d}}^2 = m_{\bar{t}}^2,$ $m_{\bar{u}}^2 = m_{\bar{q}}^2 = m_{\bar{e}}^2,$ $m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$

\*In usual GUT model (without flavour symmetry), D-term contribution is flavour blind.



In  $E_6 \times SU(2)_F \times U(1)_A$  SUSY GUT

D-term contribution is not flavour blind.

- $SU(2)_F$  flavour symmetry
- special unification of matters to realize realistic quark and lepton masses and mixings



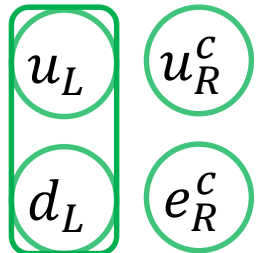
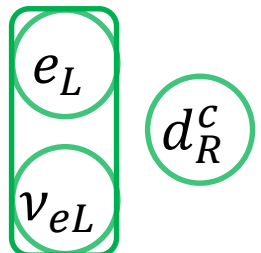

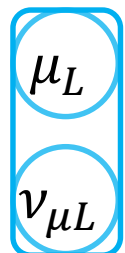

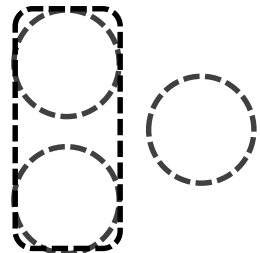

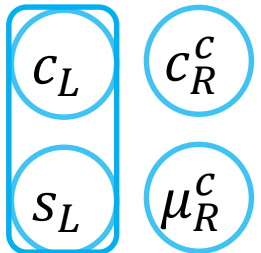
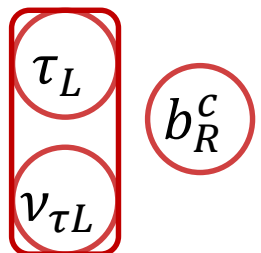

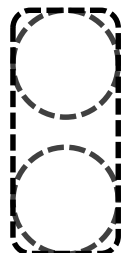
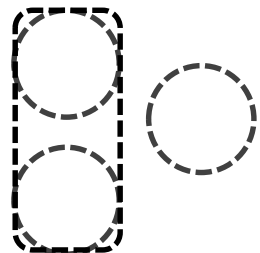

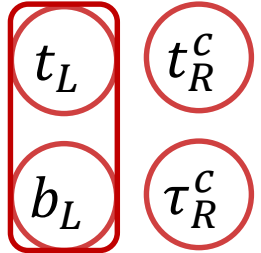
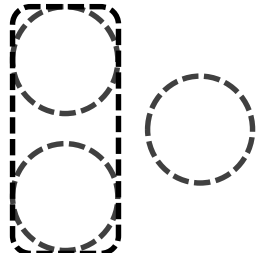

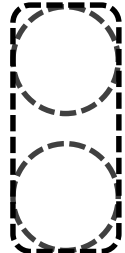
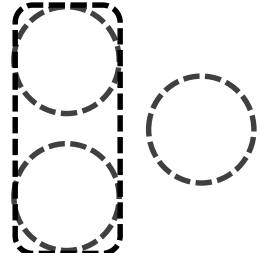

In  $SU(5)$  GUT model we assume that  $\mathbf{10}$  SM matter induces stronger hierarchies for Yukawa coupling than  $\bar{\mathbf{5}}$  SM matter.






$$E_6 \supset SO(10) \supset SU(5)$$

$$\mathbf{27} \rightarrow \mathbf{16} + \mathbf{10} + \mathbf{1}$$

$$\mathbf{16} \rightarrow \underbrace{q_L(\mathbf{3}, \mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1}, \mathbf{1})_1}_{\mathbf{10}} + \underbrace{d_R^c(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{\nu_R^c(\mathbf{1}, \mathbf{1})_0}_{\mathbf{1}}$$

$$\mathbf{10} \rightarrow \underbrace{D_R^c(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + L_L(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}'}} + \underbrace{\bar{D}_R^c(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}} + \bar{L}_L(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}}_{\mathbf{5}}$$

$E_6$	$27$					
$SO(10)$	$16$			$10$		$1$
$SU(5)$	$10$	$\bar{5}$	$1$	$\bar{5}$	$5$	$1$
$\Psi_1 (27)$					 	
$\Psi_2 (27)$						
$\Psi_3 (27)$						

 First generation matter  
  Second generation matter  
  Third generation matter  
 Right-handed neutrino  
  extra particle

# Features of $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT

Yukawa structure at GUT scale is restricted.

27 dimensional representation for matter :  $\Psi$

$\Psi_a (a = 1, 2) : SU(2)_F$  doublet for first- and second-generation matters

$\Psi_3 : SU(2)_F$  singlet for third-generation matters

- Realize realistic quark and lepton masses and mixings within restricted Yukawa structure


 9 real parameters and 2 CP phases

- Suppress flavour Changing Neutral Current processes

 modified universality for squark and slepton masses

$$\tilde{m}_{\frac{5}{5}}^2 \sim \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_0^2 \end{pmatrix} \quad \tilde{m}_{10}^2 \sim \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_3^2 \end{pmatrix}$$

$m_0 \gg m_3$  In this work we add **D-term contribution**.

- Solve SUSY CP problem (Chromo-EDM constraint) by spontaneous CP violation mechanism  real  $Y_u$

# Non flavour blind D-term contribution is dangerous.

← SUSY FCNC problem

resent LHC result



- No SUSY signal
- 126 GeV Higgs

Natural SUSY scenario

- heavy stop mass to realize 126 GeV Higgs  
(but near current experimental lower bound for naturalness)
- heavy squark and slepton mass to suppress SUSY FCNC processes → sizable D-term contribution

$$\tilde{m}_{\frac{5}{5}}^2 \sim \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_0^2 \end{pmatrix} \quad \tilde{m}_{10}^2 \sim \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_3^2 \end{pmatrix}$$
$$m_3 = O(1\text{TeV}) \quad \longrightarrow \quad m_0 = O(10\text{TeV})$$

$$\frac{m_0}{m_3} \lesssim 5$$

to realize positive stop mass  
Arkani-Hamed and Murayama (1997)

# squark and slepton masses including D-term contributions

$$\begin{aligned} \tilde{m}_{10}^2 &= (m_0^2 + D_6 + D_{10} + D_F + 4D_A)\mathbf{1}_{3\times 3} \\ &+ \begin{pmatrix} 0 & & \\ & -2D_F & \\ & & -D_F - \frac{5}{2}D_A + m_3^2 - m_0^2 \end{pmatrix} \\ &\equiv m_{10,0}^2\mathbf{1}_{3\times 3} + \begin{pmatrix} 0 & & \\ & \Delta m_{10,2}^2 & \\ & & \Delta m_{10,3}^2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \tilde{m}_{\bar{5}}^2 &= (m_0^2 + D_6 - 3D_{10} + D_F + 4D_A)\mathbf{1}_{3\times 3} \\ &+ \begin{pmatrix} 0 & & \\ & -3D_6 + 5D_{10} & \\ & & -2D_F \end{pmatrix} \\ &\equiv m_{\bar{5},0}^2\mathbf{1}_{3\times 3} + \begin{pmatrix} 0 & & \\ & \Delta m_{\bar{5},2}^2 & \\ & & \Delta m_{\bar{5},3}^2 \end{pmatrix} \\ &\Delta m_{10,2}^2 = \Delta m_{\bar{5},3}^2 \end{aligned}$$

## diagonalizing matrices

diagonalizing matrix for **10** matters

$$L_u \sim L_d \sim R_u \sim R_e \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

diagonalizing matrix for  $\bar{\mathbf{5}}$  matters

$$L_e \sim L_\nu \sim R_d \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$



$$(\delta_{ij}^\psi)_{\Gamma\Gamma} \equiv \frac{(U_{\psi\Gamma}^\dagger \tilde{m}_{\psi\Gamma}^2 U_{\psi\Gamma})_{ij}}{m_{\tilde{\psi}}^2} \quad (\Gamma = L, R) \quad U : \text{diagonalizing matrix}$$

mass insertion parameter

# Non flavour blind D-term contribution

flavour physics  size of D-term contribution

$K^0 - \bar{K}^0$  mixing ( $\varepsilon_K$  parameter)

$$\sqrt{|\text{Im}(\delta_{12}^d)_{LL}^2|} < 2.9 \times 10^{-3} \left( \frac{m_{\tilde{d}}}{500\text{GeV}} \right)$$

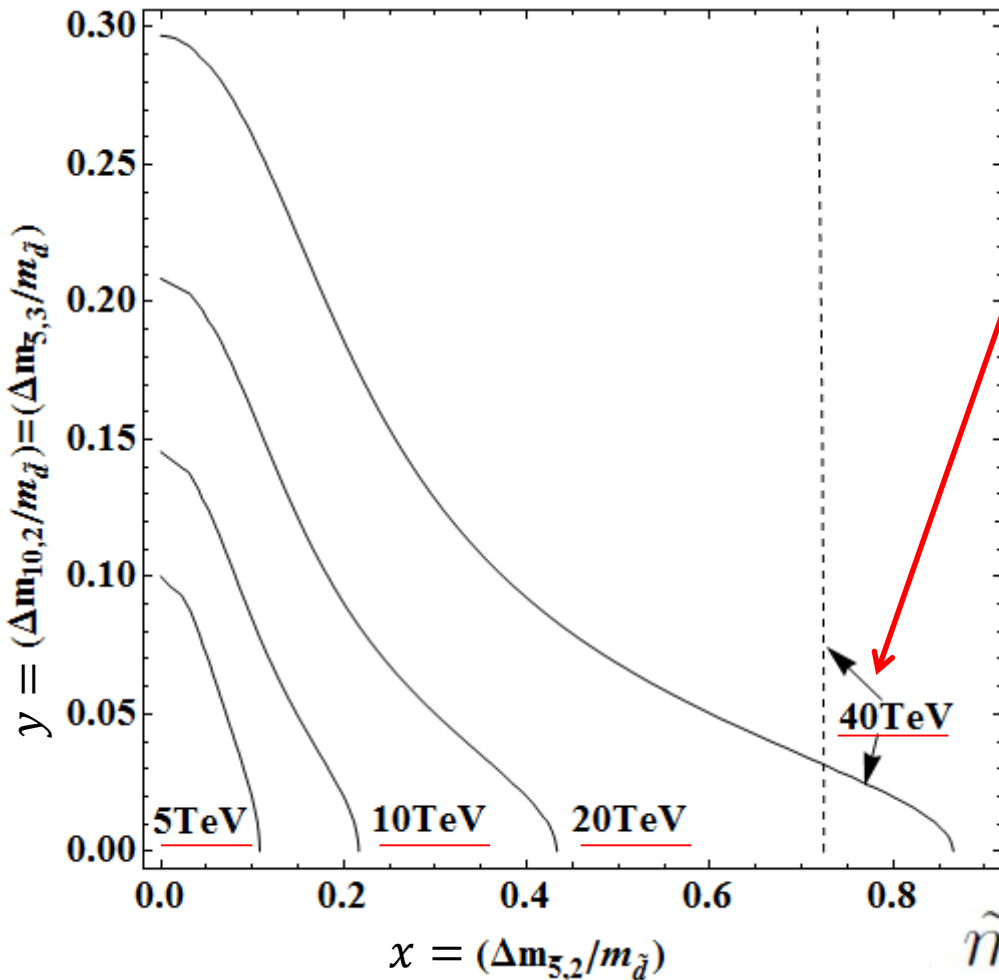
$$\sqrt{|\text{Im}(\delta_{12}^d)_{RR}^2|} < 2.9 \times 10^{-3} \left( \frac{m_{\tilde{d}}}{500\text{GeV}} \right)$$

$$\sqrt{|\text{Im}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}|} < 1.1 \times 10^{-4} \left( \frac{m_{\tilde{d}}}{500\text{GeV}} \right) \quad \text{Ciuchini et al. (1998)}$$

mass insertion parameter in this model

$$(\delta_{12}^d)_{LL} \sim \left( \frac{2}{3} + i\frac{4}{27} \right) \left( \lambda \frac{\Delta m_{10,2}^2}{m_{\tilde{d}}^2} + \lambda^5 \frac{\Delta m_{10,3}^2}{m_{\tilde{d}}^2} \right)$$

$$(\delta_{12}^d)_{RR} \sim \frac{2}{3}(1+i) \left( \lambda^{0.5} \frac{\Delta m_{5,2}^2}{m_{\tilde{d}}^2} + \lambda^{1.5} \frac{\Delta m_{5,3}^2}{m_{\tilde{d}}^2} \right) \quad \lambda \sim 0.22$$



average down type squark mass :  $m_{\tilde{d}}$

— :constraint from

$$\sqrt{|\text{Im}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}|}$$

..... :constraint from

$$\sqrt{|\text{Im}(\delta_{12}^d)_{RR}^2|}$$

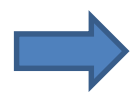
40TeV

Result 1

When  $m_{\tilde{d}} = 10 \text{ TeV}$

$$x \sim y \sim 0.1$$

= sizable D-term contribution



$$\tilde{m}_{\tilde{5}_3}^2 - \tilde{m}_{\tilde{5}_1}^2 = \tilde{m}_{\tilde{10}_2}^2 - \tilde{m}_{\tilde{10}_1}^2$$

signature of  $E_6 \times SU(2)_F \times U(1)_A$  SUSY GUT model

in future experiments (100 TeV proton collider or muon collider)

# Summary

Modified universality for squark and slepton masses is useful

$$\tilde{m}_{\frac{5}{3}}^2 \sim \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_0^2 \end{pmatrix} \quad \tilde{m}_{10}^2 \sim \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_3^2 \end{pmatrix}$$

to suppress Flavour Changing Neutral Current processes.

If

$$m_3 = O(1\text{TeV}) \quad \text{and} \quad m_0 = O(10\text{TeV})$$

it is possible to introduce sizable and flavour non blind D-term contributions.

As a result,

- D-term contribution to squark and slepton masses can be a signature of GUT model.



# Future work

- FCNC processes

B physics?

chargino contribution for  $b \rightarrow s + \gamma$ ?

- explicit calculation

$$(\delta_{12}^d)_{LL} \simeq \left\{ - \left( \frac{2}{3} R_{12}^{dL} + \frac{4}{27} I_{12}^{dL} e^{i(2\rho-\delta)} \right) \lambda \frac{\Delta m_{10,2}^2}{m_{\tilde{d}}^2} \right. \\ \left. - R_{23}^{dL} \left( -\frac{1}{3} R_{13}^{dL} + \frac{2}{3} R_{23}^{dL} R_{12}^{dL} + \frac{4}{27} R_{23}^{dL} I_{12}^{dL} e^{i(2\rho-\delta)} \right) \lambda^5 \frac{\Delta m_{10,3}^2}{m_{\tilde{d}}^2} \right\}$$

$R, I$  : combination of nine  $O(1)$  parameters

$$(\delta_{12}^d)_{LL} \sim \left( \frac{2}{3} + i \frac{4}{27} \right) \left( \lambda \frac{\Delta m_{10,2}^2}{m_{\tilde{d}}^2} + \lambda^5 \frac{\Delta m_{10,3}^2}{m_{\tilde{d}}^2} \right)$$

Back up

$$Y_u = \begin{pmatrix} 0 & \frac{1}{3}d_q\lambda^5 & 0 \\ -\frac{1}{3}d_q\lambda^5 & c\lambda^4 & b\lambda^2 \\ 0 & b\lambda^2 & a \end{pmatrix}, \quad (6)$$

$$Y_d = \begin{pmatrix} -\left(\frac{(bg-af)^2}{ac-b^2} + g^2\right) \frac{\beta_H}{a} e^{i(2\rho-\delta)} \lambda^6 & -\frac{bg-af}{ac-b^2} \frac{2}{3} d_5 \beta_H e^{i(\rho-\delta)} \lambda^{5.5} & \frac{1}{3} d_q \lambda^5 \\ \left(-\frac{d_q}{3} - \frac{bg-af}{ac-b^2} \frac{b^2 d_5}{g}\right) \lambda^5 & \left(f\beta_H e^{i(\rho-\delta)} - \frac{(\frac{2}{3}d_5)^2}{ac-b^2} \frac{ab}{g} e^{-i\rho}\right) \lambda^{4.5} & \frac{cg-bf}{g} \lambda^4 \\ -\frac{bg-af}{ac-b^2} \frac{a^2 d_5}{g} \lambda^3 & \left(g\beta_H e^{i(\rho-\delta)} - \frac{(\frac{2}{3}d_5)^2}{ac-b^2} \frac{a^2}{g} e^{-i\rho}\right) \lambda^{2.5} & \frac{bg-af}{g} \lambda^2 \end{pmatrix}, \quad (7)$$

$$Y_e = \begin{pmatrix} -\left(\frac{(bg-af)^2}{ac-b^2} + g^2\right) \frac{\beta_H}{a} e^{i(2\rho-\delta)} \lambda^6 & d_l \lambda^5 & 0 \\ 0 & f\beta_H e^{i(\rho-\delta)} \lambda^{4.5} & g\beta_H e^{i(\rho-\delta)} \lambda^{2.5} \\ -d_l \lambda^5 & \frac{cg-bf}{g} \lambda^4 & \frac{bg-af}{g} \lambda^2 \end{pmatrix}, \quad (8)$$

$$L_u \sim \begin{pmatrix} 1 & \frac{1}{3}\lambda & 0 \\ \frac{1}{3}\lambda & 1 & \lambda^2 \\ \frac{1}{3}\lambda^3 & \lambda^2 & 1 \end{pmatrix}, R_u \sim \begin{pmatrix} 1 & \frac{1}{3}\lambda & 0 \\ \frac{1}{3}\lambda & 1 & \lambda^2 \\ \frac{1}{3}\lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (20)$$

$$L_d \sim \begin{pmatrix} 1 & (\frac{2}{3} + i\frac{4}{27})\lambda & \frac{1}{3}\lambda^3 \\ (\frac{2}{3} + i\frac{4}{27})\lambda & 1 & \lambda^2 \\ (\frac{2}{3} + i\frac{4}{27})\lambda^3 & \lambda^2 & 1 \end{pmatrix}, R_d \sim \begin{pmatrix} 1 & \frac{2}{3}(1+i)\lambda^{0.5} & \frac{2}{3}\lambda \\ \frac{2}{3}(1+i)\lambda^{0.5} & 1 & (1+i)\lambda^{0.5} \\ \frac{2}{3}(1+i)\lambda & (1+i)\lambda^{0.5} & 1 \end{pmatrix}, \quad (21)$$

$$L_e \sim \begin{pmatrix} 1 & (1+i)\lambda^{0.5} & 0 \\ (1+i)\lambda^{0.5} & 1 & (1+i)\lambda^{0.5} \\ \lambda & (1+i)\lambda^{0.5} & 1 \end{pmatrix}, R_e \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (22)$$