Flavor symmetry and mass relations

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Signatures of flavor models

- Mixing Sum Rules
- Mass Sum Rules

} Neutral Sector [1]

[1] Talk given by Feruglio and Rodejohann.

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} Neutral Sector [1]

- Mass relations

} Charged Sector

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Signatures of flavor models

- **mixing sum rules**[2]. For instance,

$$s \approx r \cos\delta \tag{1}$$

where s and r represent deviations of the TBM in the following parametrization,

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s), \quad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)$$
$$\text{and} \quad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

[2] King JHEP0508:105,2005. For details see King et al. New J.Phys. 16 (2014) 045018 and references therein.

Signatures of flavor models

- **neutrino mass sume rules** [3].

$\Longrightarrow$ lower bounds for the effective mass $|m_{ee}|$ in $\nu 0\beta\beta$

For example,

$$3m_2 + 3m_3 = m_1 \quad \text{(NH)} \tag{2}$$

[3] For the first time, Altarelli and Meloni, J.Phys.G36:085005,2009. Barry and Rodejohann, Phys.Rev.D81:093002,2010. Dorame et al. , Nucl.Phys. B861 (2012) 259-270. King, Merle and Stuart, JHEP 1312 (2013) 005.

Neutrino mass sum rule, [4]

$$3m_2 + 3m_3 = m_1 \tag{3}$$

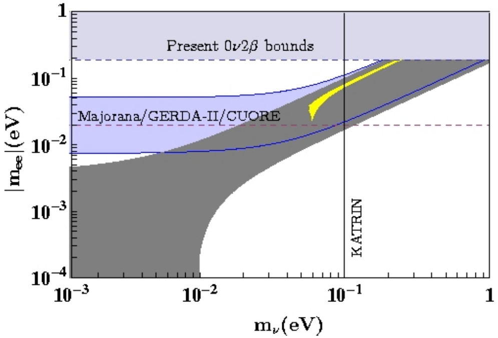

Figure: Effective $0\nu\beta\beta$ mass parameter $\langle|m_{ee}|\rangle$ as a function of the lightest neutrino mass. The yellow band corresponds to the model which predicts the sum mass rule $3m_2 + 3m_3 = m_1$ for the case of NH.

[4] The figure was taken from Dorame et al. , Nucl.Phys. B861 (2012) 259-270.

Signatures of flavor models

- **mass relation in a GUT-less framework**

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_s m_d}} \left(= \frac{m_t}{\sqrt{m_u m_c}} \right)^5. \tag{4}$$

[5] This part predicted by Wilczek and Zee in a model with SU(2) as flavor symmetry.

Mass relation in the charge sector[6]

- **mass relation in a GUT-less framework**

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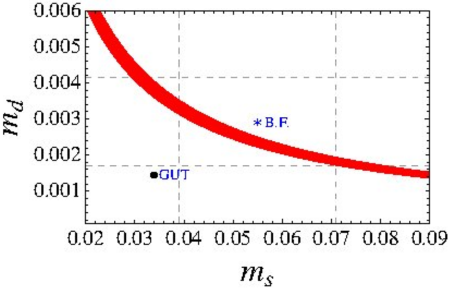

Figure: The shaded band gives our prediction for the down-strange quark masses at the M_z scale, vertical and horizontal lines are the 1σ experimental range.

[6] Morisi et al., Phys.Rev.D84:036003,2011. Morisi et. al., Phys.Rev. D88 (2013) 036001. King et. al., Phys.Lett. B 724 (2013) 68-72.

Mass relation between quarks and leptons[8]

Susy scenario and the matter content,

	$\hat{L}$	$\hat{E}^c$	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{H}^u$	$\hat{H}^d$
$SU(2)_L$	**2**	**1**	**2**	**1**	**1**	**2**	**2**
A_4	**3**	**3**	**3**	**3**	**3**	**3**	**3**

Table: Matter assignments of the model.

•The Yukawa Lagrangian is given by

$$L_{\text{Yuk}} = y^l_{ijk}\hat{L}_i\hat{H}^d_j\hat{E}^c_k + y^d_{ijk}\hat{Q}_i\hat{H}^d_j\hat{D}^c_k + y^u_{ijk}\hat{Q}_i\hat{H}^u_j\hat{U}^c_k\ , \tag{5}$$

•The neutrino masses are generated via the **dimension five operator** [7]

$$\mathcal{L}_{5d} = \frac{f_{ijlm}}{\Lambda}\hat{L}_i\hat{L}_j\hat{H}^u_j\hat{H}^u_m. \tag{6}$$

[7] The analysis only for the lepton sector in a non-susy A_4 model was done in S.Morisi and E. Peinado, Phys.Rev.D80:113011,2009

[8] Morisi et al., Phys.Rev.D84:036003,2011

How to obtain the mass relation

The mass matrices for down-type charged fermions are given by,

$$M_f = \begin{pmatrix} 0 & y_1^f v_3 & y_2^f v_2 \\ y_2^f v_3 & 0 & y_1^f v_1 \\ y_1^f v_2 & y_2^f v_1 & 0 \end{pmatrix} \quad (7)$$

where $f = \ell, d$ and v_i are the Higgs scalar vevs.

Mass relation between quarks and leptons[9]

The charged lepton mass matrix can be rewritten as,

$$M_f = \begin{pmatrix} 0 & a^f\alpha^f & b^f \\ b^f\alpha^f & 0 & a^f r^f \\ a^f & b^f r^f & 0 \end{pmatrix} \qquad (8)$$

where
$\langle H^{0u,d}\rangle \sim (1,0,0) \Rightarrow$ $\boxed{A_4 \text{ softly broken}}$ $\Rightarrow \langle H^{0u,d}\rangle = (v_1^{u,d}, \epsilon_2^{u,d}, \epsilon_3^{u,d})$
and $v_1^{u,d} >> \epsilon_{2,3}^{u,d}$.
Given that,

$$\boxed{\alpha^f = \frac{v_3}{v_2},\ r^f = \frac{v_1}{v_2}} \Longrightarrow \boxed{r^f \gg a^f}$$

[9] Morisi et al., Phys.Rev.D84:036003,2011

Mass relation between quarks and leptons

Using the invariants of $M_f M_f^\dagger$ one obtains,

$$\boxed{\frac{r^f}{\sqrt{\alpha^f}} \approx \frac{m_3^f}{\sqrt{m_1^f m_2^f}}.}$$

where,

$$r^\ell = r^d \quad \text{and} \quad \alpha^\ell = \alpha^d$$

and then

$$\boxed{\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_s m_d}}.}$$

This was obtained in a **GUT-less** framework.

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$$|V_{CKM}| \sim \begin{pmatrix} 1 & \lambda_C & 0 \\ \lambda_C & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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  - the RH up-type quarks transform as singlets under A_4[11].

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[11] King et. al., Phys.Lett. B 724 (2013) 68-72.

CKM and mass relation[12]

	L	l_R	Q	d_R	u_{R_1}	u_{R_2}	u_{R_3}	H	φ_u	φ_d	φ_ν	ξ_ν
A_4	3	3	3	3	1	$1''$	$1'$	1	3	3	3	1
Z_2^u	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$	$-$	$+$	$+$	$+$
Z_2^d	$+$	$-$	$+$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$+$
Z_3^ν	ω	ω^2	1	1	1	1	1	1	1	1	ω	ω

Table: Matter content of the model.

[12] King et. al., Phys.Lett. B 724 (2013) 68-72.

CKM and mass relation

- The Yukawa Lagrangian for the **charged sector** is made of **dimension-five operators**.

$$\begin{aligned}\mathcal{L} &= \frac{y^d_{\alpha\alpha'}}{M}(Q\, d_R)_\alpha H \varphi_{d_{\alpha'}} + \frac{y^l_{\alpha\alpha'}}{M}(L\, l_R)_\alpha H \varphi_{d_{\alpha'}} + \\ &+ \frac{y^u_\beta}{M}(Q\, \varphi_u)_\beta \tilde{H} u_{R\beta'} + H.c.,\end{aligned}$$

where $\tilde{H} = i\sigma_2 H^*$.

- The **neutrino sector** is given by **dimension-six operators**.

$$\mathcal{L} \supset \frac{y^\nu_\phi}{\Lambda^2} LLHH\phi_\nu + \frac{y^\nu_\xi}{\Lambda^2} LLHH\xi_\nu \tag{9}$$

CKM and mass relation

After EWSB,

$$M_f = \begin{pmatrix} 0 & y_1^f v_3 & y_2^f v_2 \\ y_2^f v_3 & 0 & y_1^f v_1 \\ y_1^f v_2 & y_2^f v_1 & 0 \end{pmatrix} \rightarrow \textbf{mass relation is PRESERVED.}$$

where $f = \ell, d$ and the mass matrix for the up-type quarks is given by,

$$M_u = \begin{pmatrix} v_1^u & 0 & 0 \\ 0 & v_2^u & 0 \\ 0 & 0 & v_3^u \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} y_1^u & 0 & 0 \\ 0 & y_{1''}^u & 0 \\ 0 & 0 & y_{1'}^u \end{pmatrix}, \quad (10)$$

where $v_3^u \gg v_{1,2}^u$.

CKM and mass relation

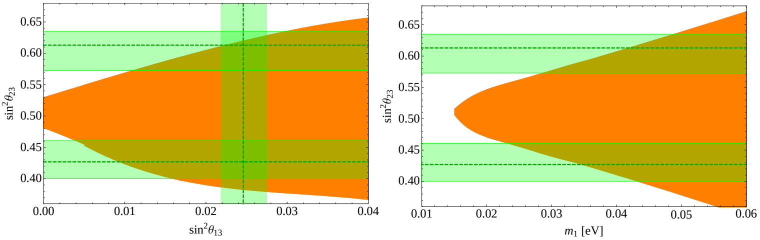
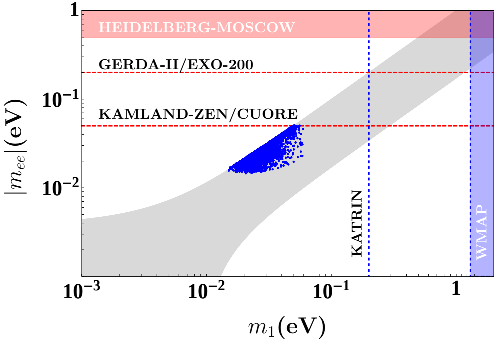


Figure: Correlations between the atmospheric angle, and the reactor angle (left panel) and the lightest neutrino mass (right panel). Straight bands are the allowed 1 σ bands of the oscillation angles in, Tortola et al. (2012).

CKM and mass relation



"Questions"

- **Can we obtain the mass relation we just mentioned from another group?**

[13] (Part of a work in progress)

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- **Can we obtain the mass relation we just mentioned from another group?**
- **Are there other mass relations in the charged sector?**[13]

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"Questions"

- **Can we obtain the mass relation we just mentioned from another group?**
  We can with T_7[14].

[14] Some works with T_7, Luhn et al. Phys.Lett. B652 (2007) 27-33, Cao et al. Phys.Rev.Lett. 106 (2011) 131801.

T_7 group[15]

T_7 is a $SU(3)$ subgroup with 21 elements. It contains three singlets ($\mathbf{1}_i$ with $i = 1, 2, 3$) and two triplets ($\mathbf{3}$ and $\bar{\mathbf{3}}$).
Multiplication rules:

- $\mathbf{3} \times \bar{\mathbf{3}} = \sum_i \mathbf{1}_i + \mathbf{3} + \bar{\mathbf{3}}$
- $\mathbf{3} \times \mathbf{3} = \mathbf{3} + \bar{\mathbf{3}} + \bar{\mathbf{3}}$
- $\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \bar{\mathbf{3}} + \mathbf{3} + \mathbf{3}$

[15] Ishimori et al.

The model

	$\overline{L}$	ℓ_R	N_R	N_{R_4}	$\overline{Q}$	d_R	u_{R_i}	H	φ_ν	φ_u	φ_d	ξ_ν
T_7	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_i$	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$
$\mathbb{Z}_7$	a^3	a^3	a^5	a^2	a^3	a^3	a^2	1	a^4	a^2	a^1	a^3

Table: Matter assignments of the model where $a^7 = 1$.

•The Yukawa Lagrangian for the charged sector is given by,

$$\mathcal{L} = \frac{Y^\ell}{M}\overline{L}\ell_R H_d + \frac{Y^d}{M}\overline{Q}d_R H_d + \frac{Y^u}{M}\overline{Q}u_R H_u + h.c. \tag{11}$$

where $H_d = H\varphi_d$, $H_u = \tilde{H}\varphi_u$ and $\tilde{H} = i\sigma_2 H^*$.

•The Lagrangian for the neutrino sector is given by,

$$\mathcal{L}_\nu = \frac{y^\nu}{\Lambda}\bar{L}N_R\tilde{H}_d + \frac{\zeta}{\Lambda}\bar{L}N_{R_4}H_u + \beta\bar{N}_R^c N_R\varphi_\nu + \zeta_2\bar{N}_{R_4}^c N_{R_4}\xi_\nu + h.c. \tag{12}$$

where, $\tilde{H}_d = \tilde{H}\overline{\varphi}_d$.

After EWSB,

$$M_f = \begin{pmatrix} 0 & y_1^f v_3 & y_2^f v_2 \\ y_2^f v_3 & 0 & y_1^f v_1 \\ y_1^f v_2 & y_2^f v_1 & 0 \end{pmatrix} \rightarrow \textbf{mass relation is PRESERVED.}$$

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where $v_3^u \gg v_{1,2}^u$.

•The neutrino masses come from the **Type-I seesaw**.

Phenomenology (PRELIMINARY RESULTS)[16]

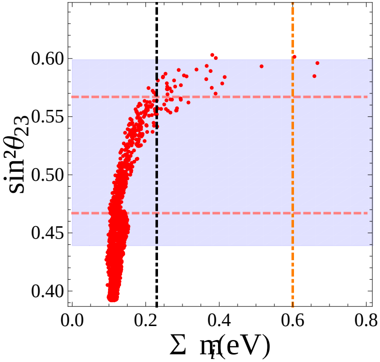

Figure: Correlation between the atmospheric angle and the sum of the neutrino masses. The black dashed vertical line is the cosmological constraint, the orange dot-dashed vertical line is the future sensitivity of KATRIN. The panel covers the 3σ range allowed for θ_{23}, Forero et al.2014.

[16] Work in progress with Morisi, Peinado and Valle.

Phenomenology (PRELIMINARY RESULTS)

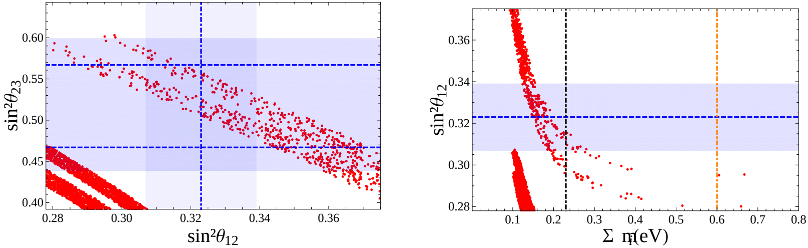

Figure: (Left panel). Correlation between the atmospheric and solar angle. (Right panel) Correlation between the solar angle and the sum of the neutrino masses. The panels cover the 3σ range allowed for θ_{12} and θ_{23} in Forero et al.2014.

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- Thank you.