Accurate renormalization group analyses in neutrino sector

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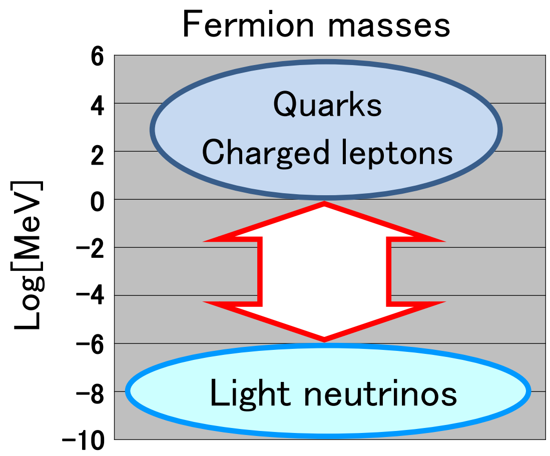
Introduction

- Light neutrinos are very different from other fermions
  - Smallness of masses
  - Structure of mixing angles

CKM matrix: $\theta_{12} = 13^\circ,\ \theta_{23} = 2.4^\circ,\ \theta_{13} = 0.2^\circ$

PMNS matrix: $\theta_{12} \simeq 33^\circ,\ \theta_{23} \simeq 41^\circ,\ \theta_{13} \simeq 8.8^\circ$

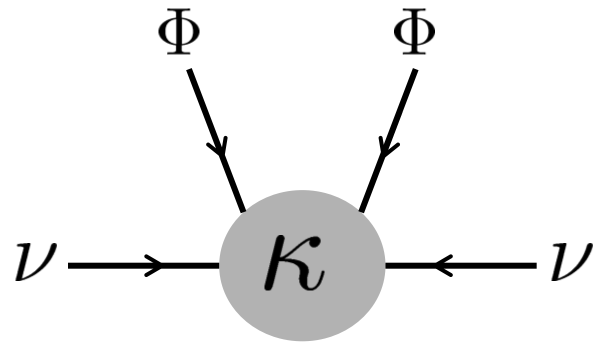


- The behavior may suggest high energy physics beyond the SM
  - Ex.) Seesaw mechanism, flavor symmetry, etc.

- In order to know the high energy behavior accurately, we consider decoupling effects of top quark and Higgs boson on the RGEs of light neutrino mass matrix

Light neutrino mass matrix

- Lepton mass terms



$$\mathcal{L} = -Y_E \overline{L} \Phi E_R - \frac{\kappa}{2} (\overline{L^C} \Phi)(L\Phi)$$

κ : coefficient of an effective dim. 5 operator

- Light neutrino mass matrix

$$M_\nu = \kappa v^2$$

v is a relevant Higgs vacuum expectation value:

$$v = \begin{cases} 174\,\mathrm{GeV} \text{ in the SM} \\ 174 \times \sin\beta\,\mathrm{GeV} \text{ in the MSSM} \end{cases}$$

Renormalization group equation for κ

- RGE for κ at one-loop level

$$16\pi^2 \frac{\mathrm{d}\kappa}{\mathrm{d}t} = C_E (Y_E^\dagger Y_E)^T \kappa + C_E \, \kappa \, (Y_E^\dagger Y_E) + \bar{\alpha} \, \kappa$$

$$\begin{cases} \bar{\alpha}_{\mathrm{SM}} = 2\,\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] - 3g_2^2 + \lambda \\ \bar{\alpha}_{\mathrm{MSSM}} = 6\,\mathrm{Tr}\left[Y_U^\dagger Y_U\right] - \frac{6}{5}g_1^2 - 6g_2^2 \end{cases}$$

where $t = \ln\mu$, and $C_E = -3/2\ (1)$ in the SM (MSSM)

S.Antusch, M.Drees, J.Kersten, M.Lindner, and M.Ratz (2001)

- Solving this equation, $M_\nu\ (= \kappa v^2)$ is written by

$$M_\nu(\Lambda) = R\,(\,I\, M_\nu(\Lambda_{\mathrm{EW}})\, I\,)$$

J.R.Ellis and S.Lola (1990)

High energy Low energy

$$I^{-1} \equiv \mathrm{Diag}\{\sqrt{I_e}, \sqrt{I_\mu}, \sqrt{I_\tau}\}, \quad I_\alpha \equiv \exp\left[-\frac{C_E}{8\pi^2}\int_{t_{\mathrm{EW}}}^{t_\Lambda} dt\, y_\alpha^2\right] \quad (\alpha = e, \mu, \tau)$$

Neutrino mass matrix with r and ϵ

- Then, neutrino mass matrix is written by

$$M_\nu(\Lambda) = \frac{R}{I_e}\begin{pmatrix} (M_\nu(\Lambda_{\rm EW}))_{ee} & (M_\nu(\Lambda_{\rm EW}))_{e\mu}\sqrt{\frac{I_e}{I_\mu}} & (M_\nu(\Lambda_{\rm EW}))_{e\tau}\sqrt{\frac{I_e}{I_\tau}} \\ (M_\nu(\Lambda_{\rm EW}))_{e\mu}\sqrt{\frac{I_e}{I_\mu}} & (M_\nu(\Lambda_{\rm EW}))_{\mu\mu}\frac{I_e}{I_\mu} & (M_\nu(\Lambda_{\rm EW}))_{\mu\tau}\sqrt{\frac{I_e}{I_\mu}\frac{I_e}{I_\tau}} \\ (M_\nu(\Lambda_{\rm EW}))_{e\tau}\sqrt{\frac{I_e}{I_\tau}} & (M_\nu(\Lambda_{\rm EW}))_{\mu\tau}\sqrt{\frac{I_e}{I_\mu}\frac{I_e}{I_\tau}} & (M_\nu(\Lambda_{\rm EW}))_{\tau\tau}\frac{I_e}{I_\tau} \end{pmatrix}$$

  - We define $r \equiv R/I_e$, $\epsilon_\tau \equiv \sqrt{I_e/I_\tau} - 1$ and $\epsilon_\mu \equiv \sqrt{I_e/I_\mu} - 1$
  - Since ϵ_μ is numerically almost equal to 0, we can neglect ϵ_μ

- Finally, neutrino mass matrix is approximately written by

$$M_\nu(\Lambda) \simeq r\begin{pmatrix} (M_\nu(\Lambda_{\rm EW}))_{ee} & (M_\nu(\Lambda_{\rm EW}))_{e\mu} & (M_\nu(\Lambda_{\rm EW}))_{e\tau}(1+\epsilon) \\ (M_\nu(\Lambda_{\rm EW}))_{e\mu} & (M_\nu(\Lambda_{\rm EW}))_{\mu\mu} & (M_\nu(\Lambda_{\rm EW}))_{\mu\tau}(1+\epsilon) \\ (M_\nu(\Lambda_{\rm EW}))_{e\tau}(1+\epsilon) & (M_\nu(\Lambda_{\rm EW}))_{\mu\tau}(1+\epsilon) & (M_\nu(\Lambda_{\rm EW}))_{\tau\tau}(1+\epsilon)^2 \end{pmatrix}$$

where we redefine $\epsilon \equiv \epsilon_\tau$

N.Haba and R.Takahashi (2013)

Neutrino mass matrix with r and ϵ

- r and ϵ are calculated by

$$r(\Lambda) = \frac{(M_\nu(\Lambda))_{ee}}{(M_\nu(\Lambda_{\rm EW}))_{ee}}\,, \qquad \epsilon(\Lambda) = \exp\left[\frac{1}{2}\frac{C_E}{8\pi^2}\int_{t_{\rm EW}}^{t_\Lambda} dt\,(y_\tau^2 - y_e^2)\right] - 1$$

- $M_\nu(\Lambda_{\rm EW})$ is given by PMNS matrix (assuming mass eigenvalues):

$$(M_\nu)_{\alpha\beta} = \sum_i U^*_{\alpha i} U^*_{\beta i} m_i$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{-i\frac{\phi_1}{2}} & 0 & 0 \\ 0 & e^{-i\frac{\phi_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Mass squared differences and mixing angles in the EW scale:

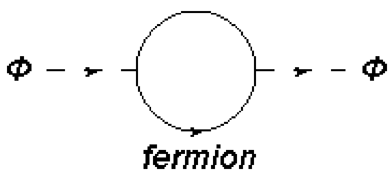
Δm^2_{21}	Δm^2_{31}	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$
$7.54\times10^{-5}{\rm eV}^2$	2.48×10^{-3} eV^2 (NH) 2.44×10^{-3} eV^2 (IH)	0.308	0.425 (NH) 0.437 (IH)	0.0234 (NH) 0.0239 (IH)

  - We assume $\Lambda_{\rm EW} = M_Z$

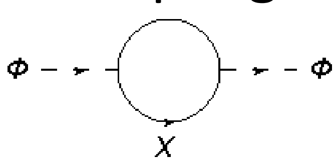
F.Capozzi, et al., arXiv:1312.2878.

Decoupling theorem

- The RGEs in the previous slide are effective for $m_t^{\rm pole} \leq \mu < M_a$
  cutoff scale
  Effective theory is valid in this region
  - For $\mu < m_t^{\rm pole}$ top quark is decoupled
  - For $\mu < m_h$ Higgs boson is also decoupled
  - :
- One of the quantum effects by fermions are shown by



- Decoupling theorem says that, for $\mu < m_{\rm x}$ (x is some fermion)



already does not contribute to quantum effects

(neglecting threshold corrections)

T.Appelquist and J.Carazzone (1975)

RGEs in the SM for $\mu \geq m_t^{\rm pole}$

$$
\begin{aligned}
16\pi^2\beta_\kappa &= -\frac{3}{2}(Y_E^\dagger Y_E)^T\,\kappa - \frac{3}{2}\,\kappa\,(Y_E^\dagger Y_E) + 2\,\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right]\,\kappa \\
&\quad -3g_2^2\,\kappa + \lambda\kappa\,, \\
16\pi^2\beta_{Y_U} &= Y_U\left\{\frac{3}{2}Y_U^\dagger Y_U - \frac{3}{2}\,Y_D^\dagger Y_D + \mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right]\right. \\
&\qquad \left. -\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right\}, \\
16\pi^2\beta_{Y_D} &= Y_D\left\{\frac{3}{2}Y_D^\dagger Y_D - \frac{3}{2}\,Y_U^\dagger Y_U + \mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right]\right. \\
&\qquad \left. -\frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right\}, \\
16\pi^2\beta_{Y_E} &= Y_E\left\{\frac{3}{2}Y_E^\dagger Y_E + \mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right\}, \\
16\pi^2\beta_\lambda &= 6\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{2}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4\right) \\
&\quad +4\,\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right]\lambda \\
&\quad -8\,\mathrm{Tr}\left[3Y_U^\dagger Y_U Y_U^\dagger Y_U + 3Y_D^\dagger Y_D Y_D^\dagger Y_D + Y_E^\dagger Y_E Y_E^\dagger Y_E\right]\,.
\end{aligned}
$$

RGEs in the SM for $m_h \leq \mu < m_t^{\rm pole}$

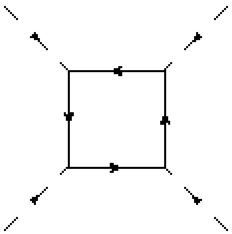
Diagrams with internal line of top quark do not contribute.

$$
\begin{aligned}
16\pi^2 \beta_\kappa &= -\frac{3}{2}(Y_E^\dagger Y_E)^T \kappa - \frac{3}{2}\kappa\,(Y_E^\dagger Y_E) + 2\left(\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] \underline{- 3y_t^2}\right)\kappa \\
&\quad -3g_2^2\,\kappa + \lambda\kappa\,, \\
16\pi^2 \beta_{Y_U \in \{y_u, y_c\}} &= Y_U\left\{\frac{3}{2}Y_U^\dagger Y_U - \frac{3}{2}Y_D^\dagger Y_D + \left(\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] \underline{- 3y_t^2}\right) - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right\}, \\
16\pi^2 \beta_{y_b} &= y_b\left\{\frac{3}{2}y_b^2 + \left(\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] \underline{- 3y_t^2}\right) - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right\}, \\
16\pi^2 \beta_{Y_D \in \{y_d, y_s\}} &= Y_D\left\{\frac{3}{2}Y_D^\dagger Y_D - \frac{3}{2}Y_U^\dagger Y_U + \left(\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] \underline{- 3y_t^2}\right) - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right\}, \\
16\pi^2 \beta_{Y_E} &= Y_E\left\{\frac{3}{2}Y_E^\dagger Y_E + \left(\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] \underline{- 3y_t^2}\right) - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right\}, \\
16\pi^2 \beta_\lambda &= 6\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{2}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4\right) \\
&\quad +4\left(\mathrm{Tr}\left[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] \underline{- 3y_t^2}\right)\lambda \\
&\quad -8\left(\mathrm{Tr}\left[3Y_U^\dagger Y_U Y_U^\dagger Y_U + 3Y_D^\dagger Y_D Y_D^\dagger Y_D + Y_E^\dagger Y_E Y_E^\dagger Y_E\right] \underline{- 3y_t^4}\right).
\end{aligned}
$$

RGEs in the SM for $M_Z \leq \mu < m_h$

Diagrams with internal line of Higgs boson also do not contribute.

Fermion box diagram



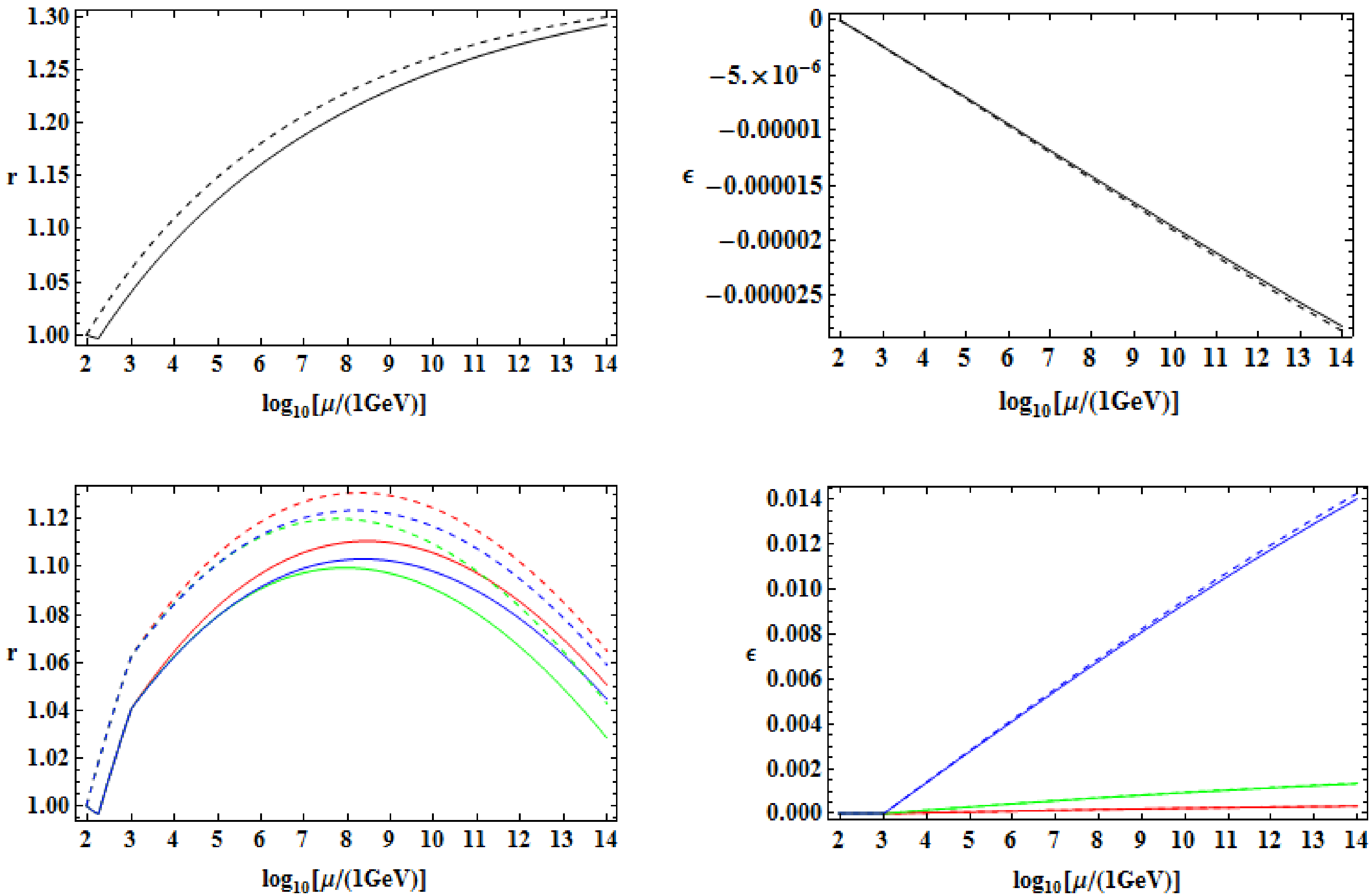
$$
\begin{aligned}
16\pi^2 \beta_\kappa &= -3 g_2^2\, \kappa\,, \\
16\pi^2 \beta_{y_t} &= 0\,, \\
16\pi^2 \beta_{Y_U \in \{y_u, y_c\}} &= Y_U \left(-\frac{2}{3} g_1^2 - 8\, g_3^2 \right), \\
16\pi^2 \beta_{Y_D} &= Y_D \left(\frac{1}{5} g_1^2 - 8\, g_3^2 \right), \\
16\pi^2 \beta_{Y_E} &= Y_E \left(-\frac{9}{5} g_1^2 \right), \\
16\pi^2 \beta_\lambda &= -8 \left(\mathrm{Tr} \left[3 Y_U^\dagger Y_U Y_U^\dagger Y_U + 3 Y_D^\dagger Y_D Y_D^\dagger Y_D + Y_E^\dagger Y_E Y_E^\dagger Y_E \right] - 3 y_t^4 \right) .
\end{aligned}
$$

Numerical results

Runnings of r and ϵ

SM

MSSM
(SUSY = 1TeV)

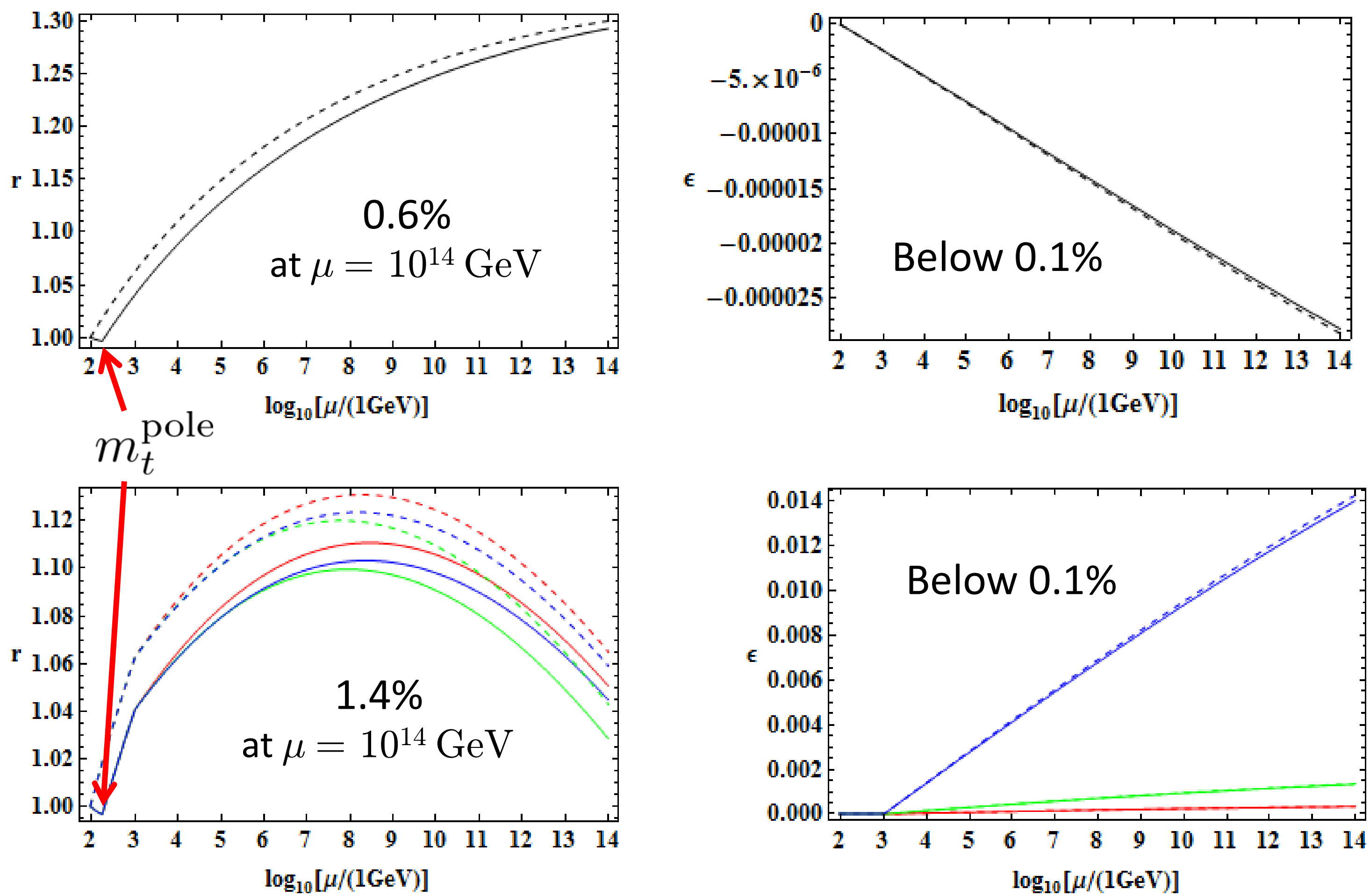


- Solid line: Including the decoupling effects
- Dashed line: Not including the decoupling effects
  - Red, Green, and Blue lines: $\tan\beta = 5$, 10, and 30

Runnings of r and ϵ

SM

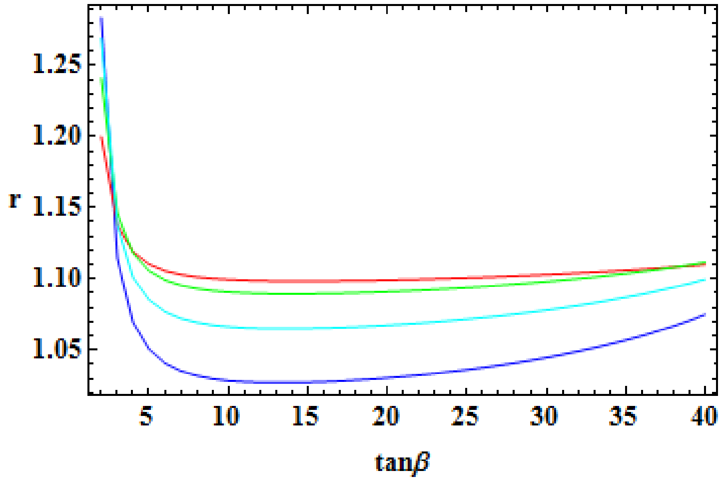
MSSM
(SUSY = 1TeV)

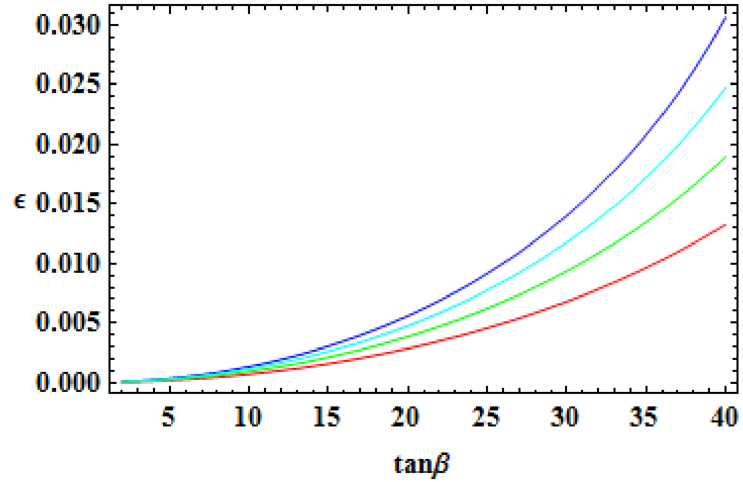


- The differences between including the decoupling effects or not are not negligible for r
- The main differences are occurred by top quark decoupling

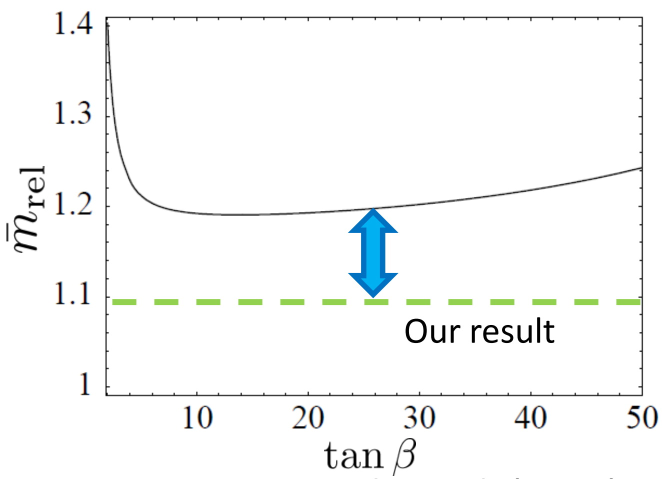
$\tan\beta$ dependence of r and ϵ

MSSM
(SUSY = 1TeV)





- These figures are results including the decoupling effects
  - Red, Green, Cyan and Blue lines: $\mu = 10^8$, 10^{10}, 10^{12}, and 10^{14} GeV



S.Antusch, et al. (2003)

【Comparison with previous work】

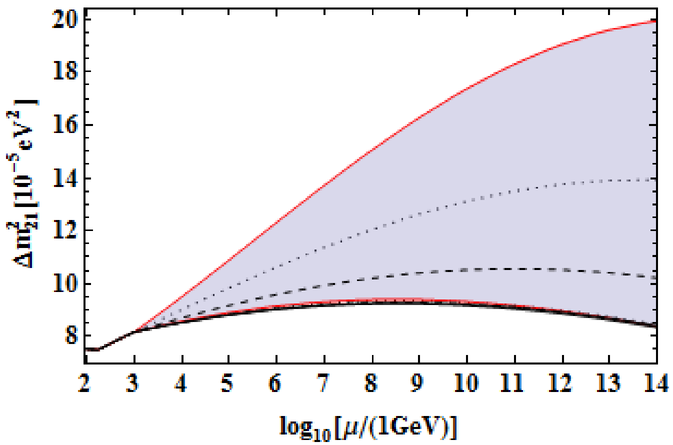
$$\bar{m} = \sqrt{m_1^2 + m_2^2 + m_3^2},\ \ \bar{m}_{\rm rel} \equiv \frac{\bar{m}(10^{10}\,\mathrm{GeV})}{\bar{m}(M_Z)}$$

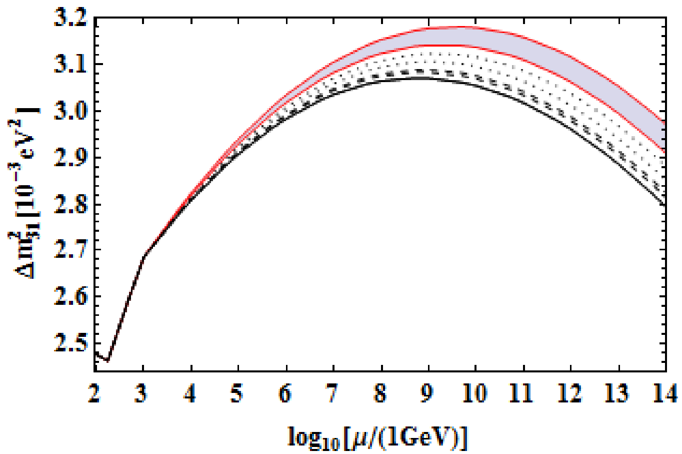
$\bar{m}_{\rm rel}$ corresponds to the green line (upper left fig.)

This value is important for leptogenesis.
The difference between our result and previous work is not negligible.

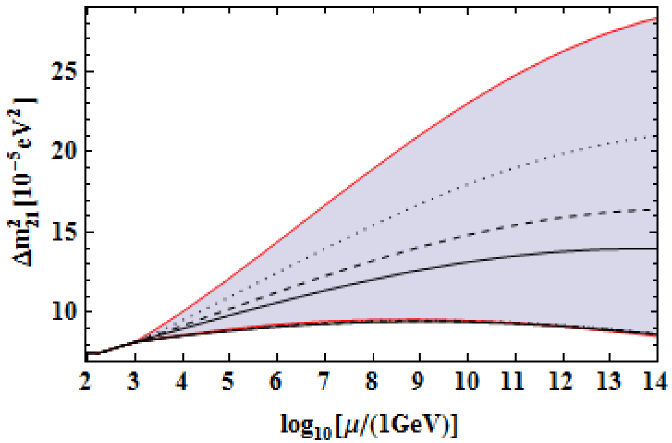
Runnings of mass squared differences

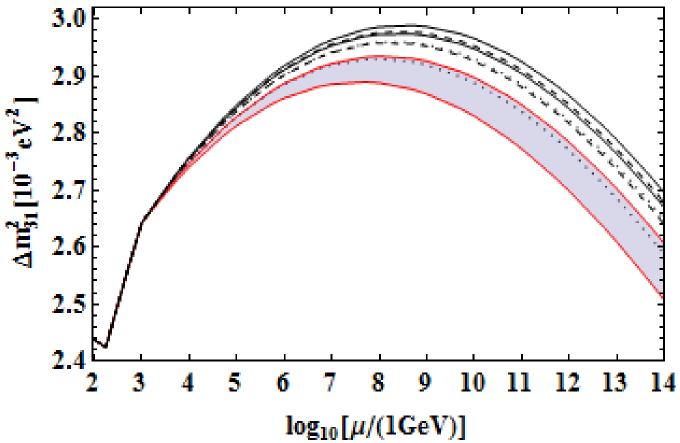
NH
(SUSY = 1TeV,
$\tan\beta$ = 30)





IH
(SUSY = 1TeV,
$\tan\beta$ = 30)

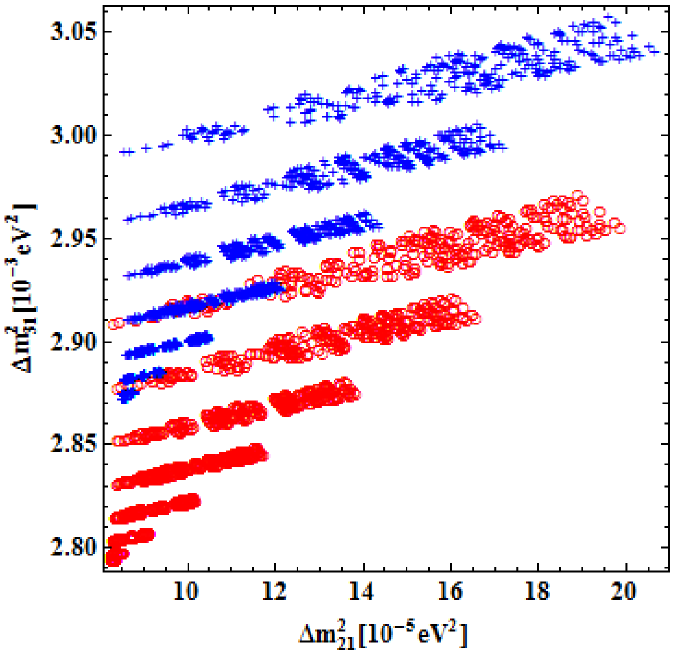




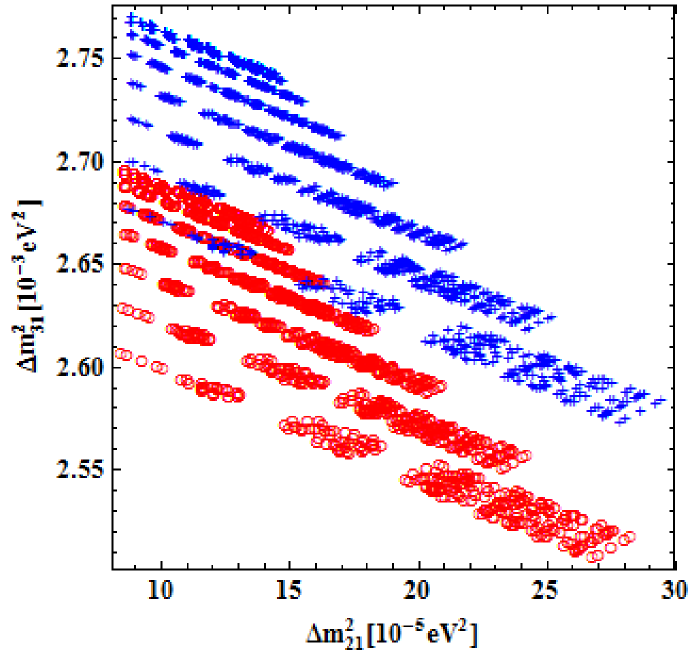
- These figures are results including the decoupling effects
- Solid, dashed, dotted, and red-solid lines: $m_{1(\text{or } 3)} = 0, 0.03, 0.05$, and $0.07\,\mathrm{eV}$
- Shaded regions can be taken according to CP-phases for $m_{1(\text{or } 3)} = 0.07\,\mathrm{eV}$

Mass squared differences at $\mu = 10^{14}\,\mathrm{GeV}$

NH (SUSY = 1TeV, $\tan\beta$ = 30)



IH (SUSY = 1TeV, $\tan\beta$ = 30)

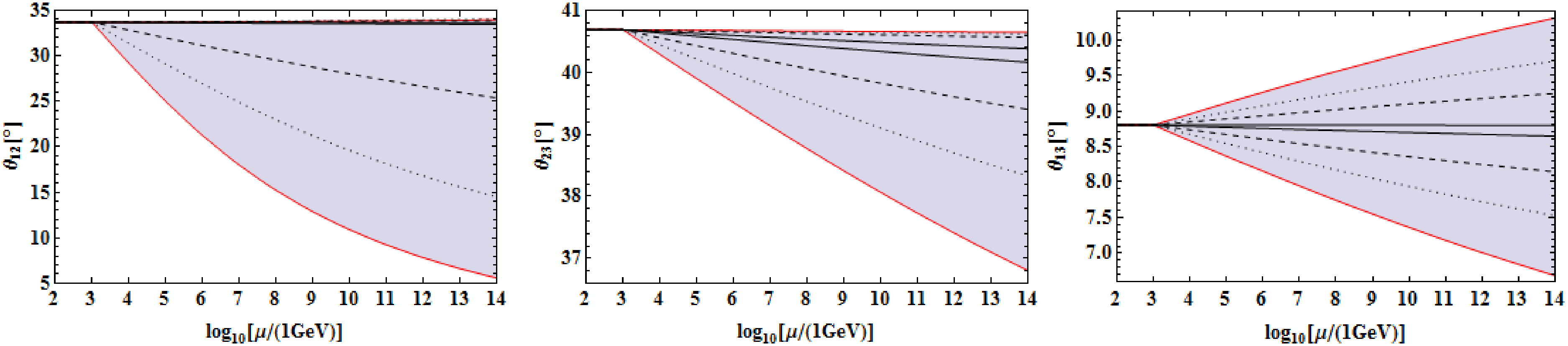


- ○(Red): Including the decoupling effects
- + (Blue): Not including the decoupling effects
  - ➢ The clusters correspond to $m_{1(\text{or }3)} = 0$, 0.01, 0.02, …, and $0.07\,\mathrm{eV}$ from the bottom (top) in the NH (IH)

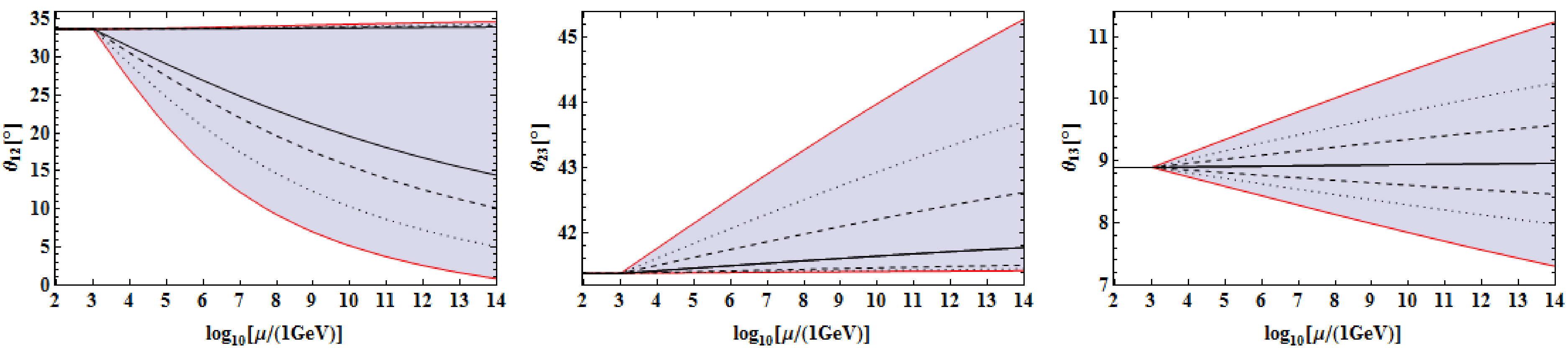
- The allowed parameters with the decoupling effects are about 3% lower than those without the decoupling effects

Runnings of mixing angles

NH (SUSY = 1TeV, $\tan\beta$ = 30)



IH (SUSY = 1TeV, $\tan\beta$ = 30)

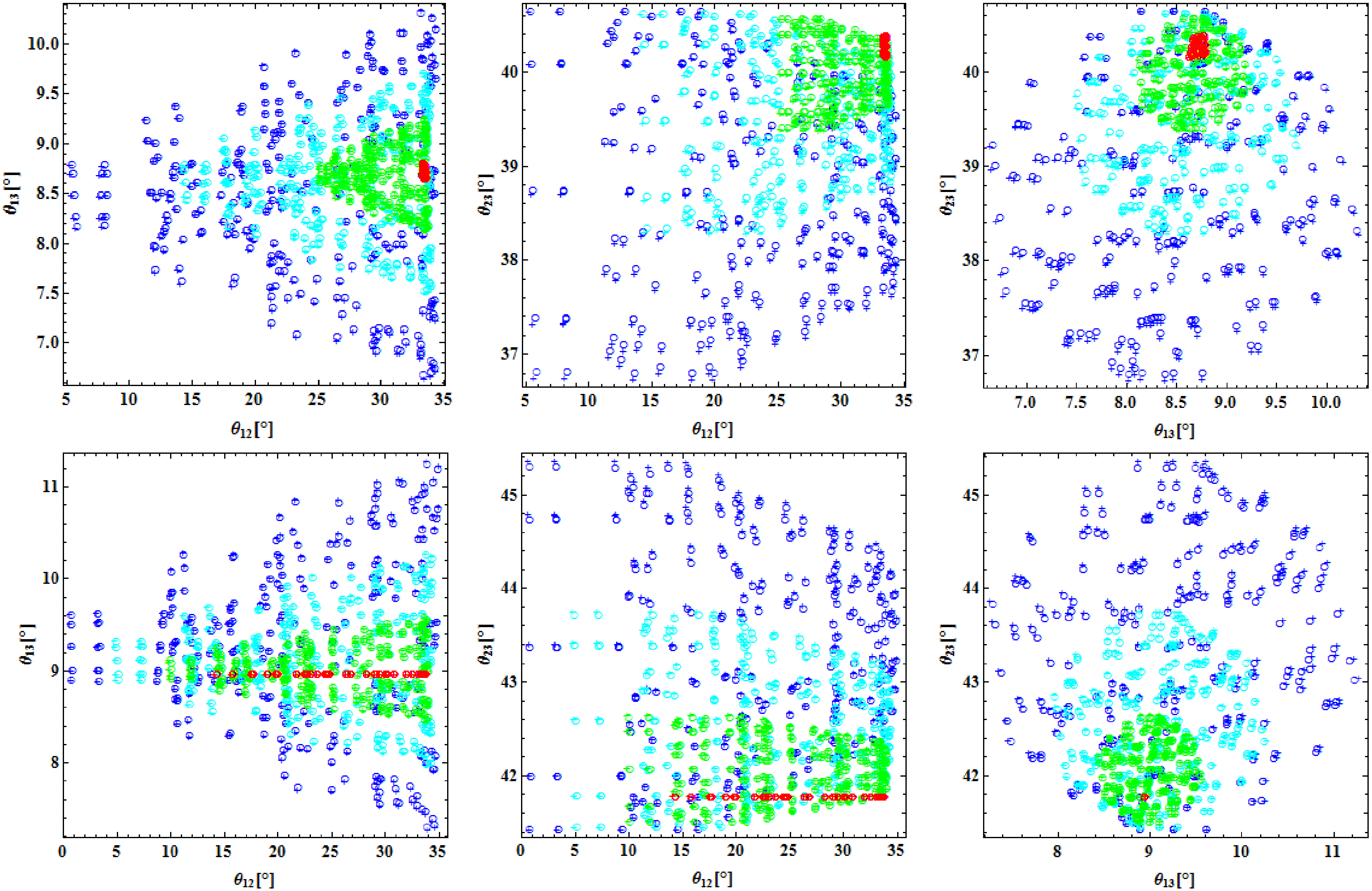


- These figures are the results with the decoupling effects
- Solid, dashed, dotted, and red-solid lines: $m_{1(\text{or }3)} = 0$, 0.03, 0.05, and $0.07\,\mathrm{eV}$
- Shaded regions can be taken according to CP-phases for $m_{1(\text{or }3)} = 0.07\,\mathrm{eV}$

mixing angles at $\mu = 10^{14}\,\mathrm{GeV}$

NH
(SUSY = 1TeV, $\tan\beta$ = 30)

IH
(SUSY = 1TeV, $\tan\beta$ = 30)



- ○: Including the decoupling effects
- + : Not including the decoupling effects
  - ➤ Red, Green, Cyan and Blue symbols: $m_{1(\text{or }3)} = 0$, 0.03, 0.05, and $0.07\,\mathrm{eV}$

There are almost no differences between “○” and “+”.

Summary

- We have considered the decoupling effects of top and Higgs
  - Usually these effects are ignored
- The effects of top quark decoupling are negligible for mixing angles but mass eigenvalues
  - When we build models in high energy scale, we should be careful about the difference
- Phenomenological aspects
  - $r \simeq \bar{m} = \sqrt{m_1^2 + m_2^2 + m_3^2}$ is a parameter which give a bound of right-handed neutrino mass in leptogenesis
  - Decay rate of 0νββ decay is proportional to $(M_\nu)_{ee} = r \times (M_\nu(M_Z))_{ee}$

Appendix

EFT in type-I seesaw mechanism

- The analysis only with Weinberg operator is not always possible
  - Type-I seesaw ($M_{N1} < M_{N2} < M_{N3}$) M. Lindner, et. al, JHEP 0503(2005)

① $\mu > M_{N3}$

$$(M_\nu)_{ij} = \sum_{k=1}^{3} \frac{(Y_\nu^T)_{ik}(Y_\nu)_{kj}}{(M_N)_k} v^2$$

N_3 decouple

② $M_{N2} < \mu < M_{N3}$

$$(M_\nu)_{ij} = \kappa_{ij} v^2 + \sum_{k=1}^{2} \frac{(Y_\nu^T)_{ik}(Y_\nu)_{kj}}{(M_N)_k} v^2$$

N_2 decouple

③ $M_{N1} < \mu < M_{N2}$

$$(M_\nu)_{ij} = \kappa_{ij} v^2 + \sum_{k=1}^{1} \frac{(Y_\nu^T)_{ik}(Y_\nu)_{kj}}{(M_N)_k} v^2$$

N_1 decouple

④ $\mu < M_{N1}$

$$(M_\nu)_{ij} = \kappa_{ij} v^2$$

When $\mu < M_{N1}$, we can analyze only with Weinberg operator

Merit of using r and ϵ

$$M_\nu(\Lambda) \simeq r \begin{pmatrix} (M_\nu(\Lambda_{\rm EW}))_{ee} & (M_\nu(\Lambda_{\rm EW}))_{e\mu} & (M_\nu(\Lambda_{\rm EW}))_{e\tau}\,(1+\epsilon) \\ (M_\nu(\Lambda_{\rm EW}))_{e\mu} & (M_\nu(\Lambda_{\rm EW}))_{\mu\mu} & (M_\nu(\Lambda_{\rm EW}))_{\mu\tau}\,(1+\epsilon) \\ (M_\nu(\Lambda_{\rm EW}))_{e\tau}\,(1+\epsilon) & (M_\nu(\Lambda_{\rm EW}))_{\mu\tau}\,(1+\epsilon) & (M_\nu(\Lambda_{\rm EW}))_{\tau\tau}\,(1+\epsilon)^2 \end{pmatrix}$$

- r and ϵ are calculated by

$$r(\Lambda) = \frac{(M_\nu(\Lambda))_{ee}}{(M_\nu(\Lambda_{\rm EW}))_{ee}} \quad \text{and} \quad \epsilon(\Lambda) = \exp\left[\frac{1}{2}\frac{C_E}{8\pi^2}\int_{t_{\rm EW}}^{t_\Lambda} dt\,(y_\tau^2 - y_e^2)\right] - 1$$

- We have to solve only 1 RGE for M_ν (κ)
- We can extract mass eigenvalues and mixing angles from M_ν at arbitrary energy scale
  - We usually solve 6 RGEs for them (3 mass eigenvalues + 3 mixing angles)
- It is easy to understand the light neutrino's behavior
  - Mass eigenvalues depend on both r and ϵ, but mainly depend on r
  - Mixing angles depend on only ϵ

RGEs for gauge couplings

$$16\pi^2 \frac{\mathrm{d}g_A}{\mathrm{d}t} \equiv 16\pi^2 \beta_{g_A} = b_A\, g_A^3$$

$$\Rightarrow \quad g_A^2(\Lambda) = \frac{g_A^2(\Lambda_{\mathrm{EW}})}{1 - \frac{b_A}{16\pi^2} g_A^2(\Lambda_{\mathrm{EW}}) \log\left(\frac{\Lambda}{\Lambda_{\mathrm{EW}}}\right)^2}$$

- SM

$$\begin{cases} b_1 = \frac{2}{3}\frac{3}{5}\left[\left(\frac{1}{6}\right)^2 6N_Q + \left(\frac{2}{3}\right)^2 3N_U + \left(\frac{1}{3}\right)^2 3N_D + \left(\frac{1}{2}\right)^2 2N_L + N_E\right] + \frac{1}{3}\frac{3}{5}\left(\frac{1}{2}\right)^2 2N_H \\ b_2 = -\frac{11}{3}2 + \frac{2}{3}\frac{1}{2}(3N_Q + N_L) + \frac{1}{3}\frac{1}{2}N_H \\ b_3 = -\frac{11}{3}3 + \frac{2}{3}\frac{1}{2}(2N_Q + N_U + N_D) \end{cases}$$

$$\downarrow \; N_Q = 3, N_U = 3, N_D = 3, N_L = 3, N_E = 3, N_H = 1$$

$$(b_1, b_2, b_3) = \left(\frac{41}{10}, -\frac{19}{6}, -7\right)$$

- For the MSSM, we can get $(b_1, b_2, b_3) = \left(\frac{33}{5}, 1, -3\right)$ in the same way.

RGEs in the MSSM

$$16\pi^2\beta_\kappa = (Y_E^\dagger Y_E)^T\,\kappa + \kappa\,(Y_E^\dagger Y_E) + 2\,\mathrm{Tr}\left[3Y_U^\dagger Y_U\right]\kappa - \frac{6}{5}g_1^2\,\kappa - 6g_2^2\,\kappa,$$

$$16\pi^2\beta_{Y_U} = Y_U\left\{3Y_U^\dagger Y_U + Y_D^\dagger Y_D + \mathrm{Tr}\left[3Y_U^\dagger Y_U\right] - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right\},$$

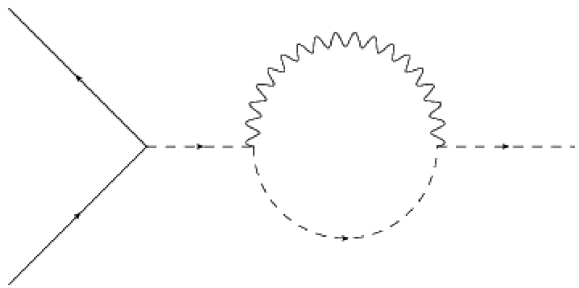
$$16\pi^2\beta_{Y_D} = Y_D\left\{3Y_D^\dagger Y_D + Y_U^\dagger Y_U + \mathrm{Tr}\left[3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}\,g_3^2\right\},$$

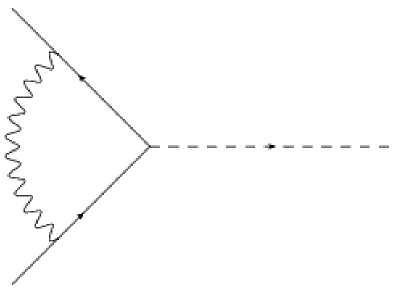
$$16\pi^2\beta_{Y_E} = Y_E\left\{3Y_E^\dagger Y_+ \mathrm{Tr}\left[3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right] - \frac{9}{5}g_1^2 - 3g_2^2\right\}.$$

S.Antusch, J.Kersten, M.Lindner, M.Ratz, and M.A.Schmidt (2005)

How to calculate the RGEs

- In Landau gauge, contributions of electroweak gauge bosons are calculated by the following two diagrams





The solid, dashed, and wavy lines show fermions, Higgs boson, and gauge bosons, respectively.

Boundary conditions

- To solve the RGEs, we take the boundary conditions for fermions and bosons as below:

$$
\begin{aligned}
m_u &= 2.3\,\mathrm{MeV}, \quad m_c = 1.28\,\mathrm{GeV}, \\
m_d &= 4.8\,\mathrm{MeV}, \quad m_s = 95\,\mathrm{MeV}, \quad m_b = 4.18\,\mathrm{GeV}, \\
m_e &= 0.511\,\mathrm{MeV}, \quad m_\mu = 106\,\mathrm{MeV}, \quad m_\tau = 1.78\,\mathrm{GeV}, \\
M_Z &= 91.2\,\mathrm{GeV}, \quad m_h = 126\,\mathrm{GeV}, \\
\alpha_{em}^{-1} &= 127.944\,, \quad \sin^2\theta_w = 0.23116\,, \quad \alpha_s \equiv g_3^2/(4\pi) = 0.1184\,,
\end{aligned}
$$

at $\mu = M_Z$, and $m_t = 160\,\mathrm{GeV}$ at $\mu = m_t^{pole} = 173\,\mathrm{GeV}$.

PDG data (2012)

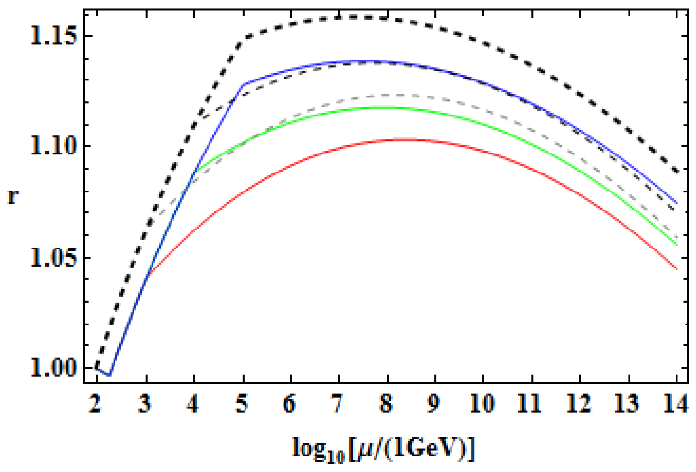
Matching conditions

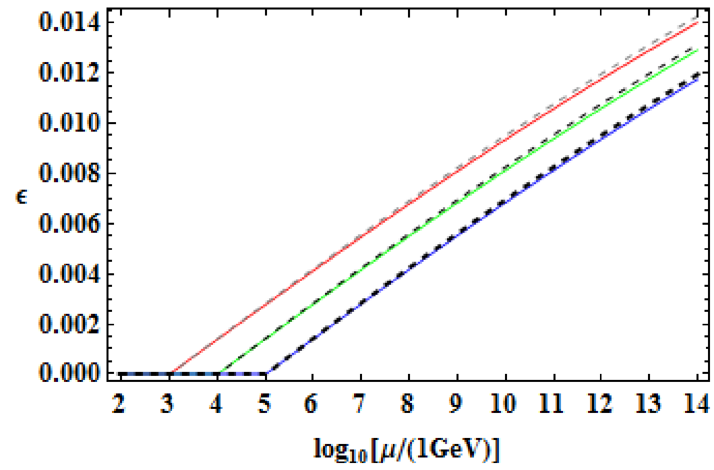
- We use the RGEs at one loop level, and matching conditions at tree level

- The RGEs of κ are continuously connected at the thresholds without the corrections

SUSY threshold dependence of r and ϵ

MSSM
$(\tan\beta = 30)$





- Solid line: Including the decoupling effects
- Dashed line: Not including the decoupling effects
  - Red, Green, and Blue lines: SUSY $= 1$, 10, and $100\,\text{TeV}$

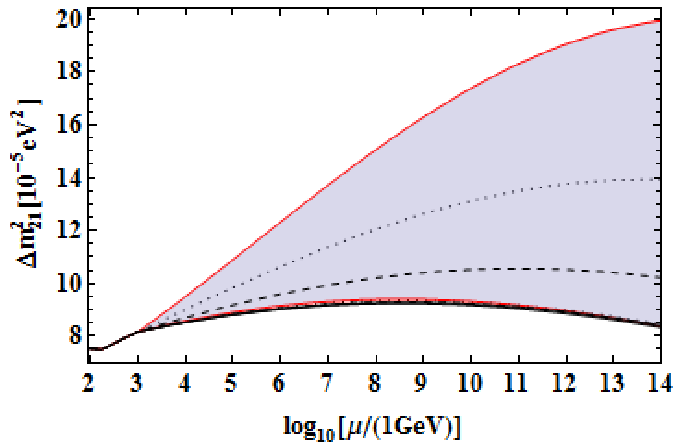
- The fundamental behavior is same as before

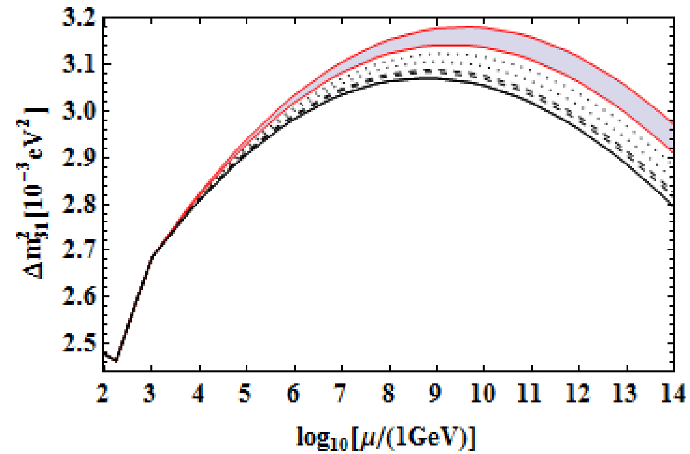
→ The differences between including the decoupling effects or not are almost independent of SUSY threshold.

- All results for r and ϵ are independent of mass spectrum of the light neutrinos and all CP-phases

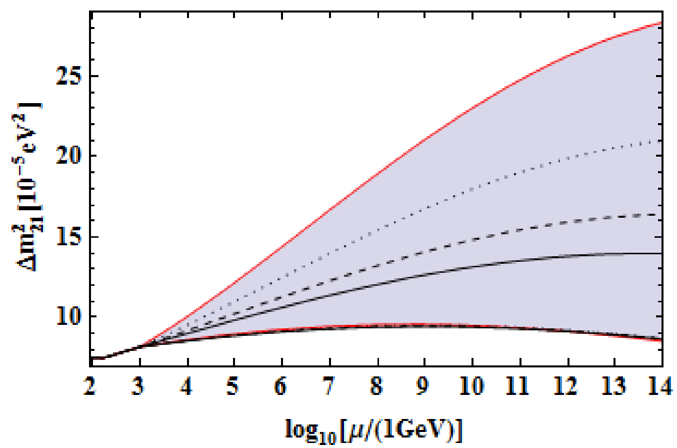
Runnings of mass squared differences

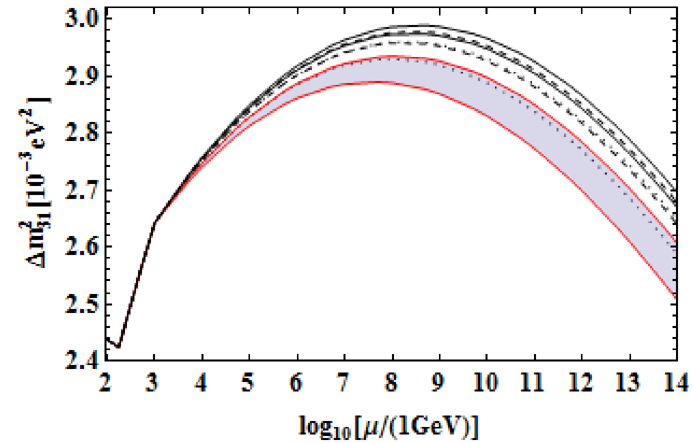
NH
(SUSY = 1TeV,
$\tan\beta$ = 30)





IH
(SUSY = 1TeV,
$\tan\beta$ = 30)

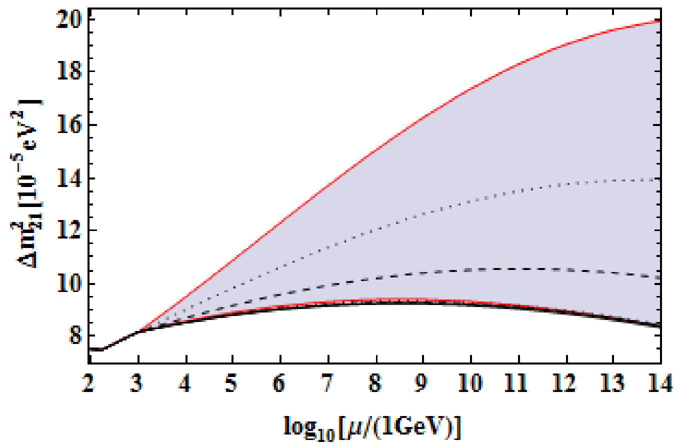


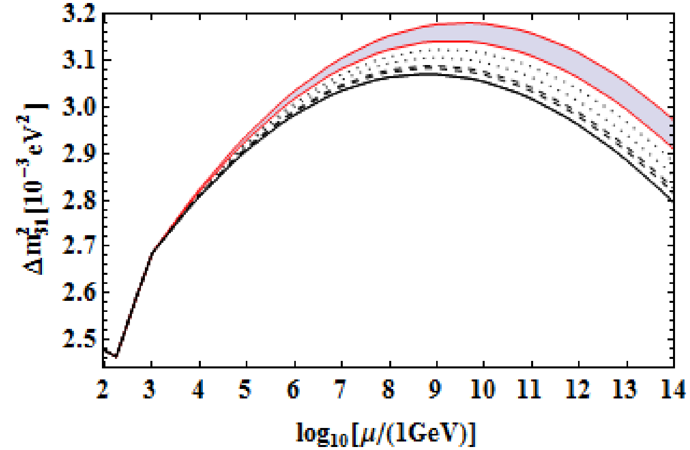


- These figures are results including the decoupling effects
- Solid, dashed, dotted, and red-solid lines: $m_{1(\text{or }3)} = 0$, 0.03, 0.05, and $0.07\,\mathrm{eV}$
- Shaded regions can be taken according to CP-phases for $m_{1(\text{or }3)} = 0.07\,\mathrm{eV}$

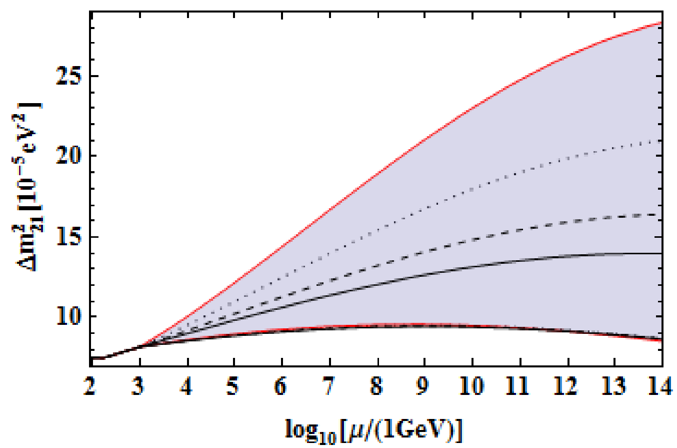
Runnings of mass squared differences

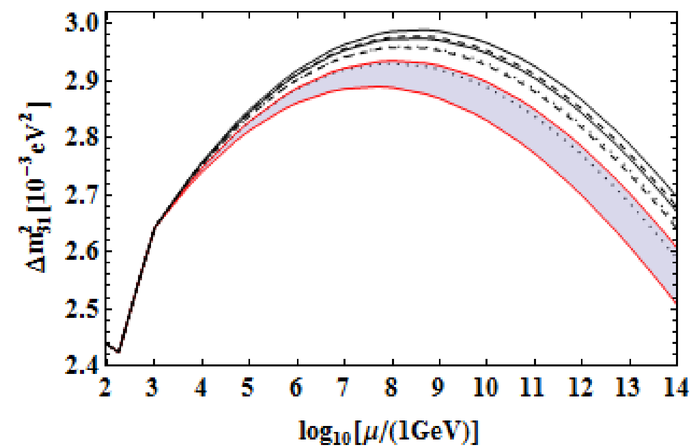
NH
(SUSY = 1TeV,
$\tan\beta$ = 30)





IH
(SUSY = 1TeV,
$\tan\beta$ = 30)





- As $m_{1(\text{or }3)}$ is large allowed region of Δm^2_{21} is much larger than that of Δm^2_{31}
  - For $m_{1(\text{or }3)} = O(0.01)\mathrm{eV}$, $\Delta m^2_{21} < m_1^2 < \Delta m^2_{31}$

→ Δm^2_{21} is sensitive to the quantum effect of m_1

Runnings of mass squared differences

- Combinations of CP-phases which give the upper and lower bounds

NH	Δm^2_{21}	Δm^2_{31}
Upper bound	$(0,\ \text{any},\ \pi)$ / $(0,\ \pi,\ \pi)$	$(0,\ \text{any},\ 0)$ / $(0,\ 0,\ 0)$
Lower bound	$(\pi,\ \text{any},\ 0)$ / $(\pi,\ \pi,\ 0)$	$(0,\ \text{any},\ \pi)$ / $(\pi,\ \pi,\ 0)$

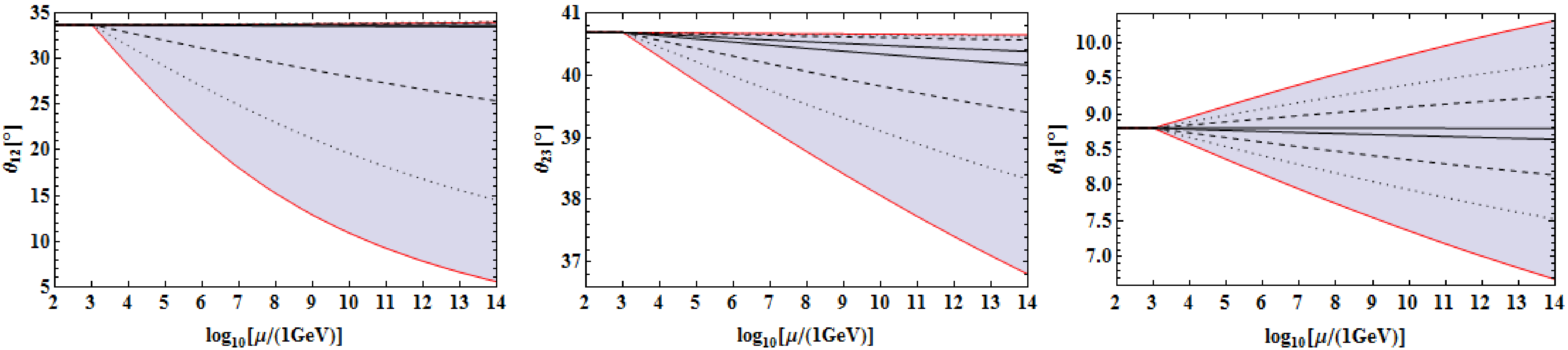
IH	Δm^2_{21}	Δm^2_{31}
Upper bound	$\delta = 0, \lvert\phi_1 - \phi_2\rvert = 0$ / $(0,\ 0,\ 0)$	$\delta = \pi, \lvert\phi_1 - \phi_2\rvert = \pi$ / $(\pi,\ 0,\ \pi)$
Lower bound	$\delta = \pi, \lvert\phi_1 - \phi_2\rvert = \pi$ / $(\pi,\ 0,\ \pi)$	$\delta = 0, \lvert\phi_1 - \phi_2\rvert = 0$ / $(0,\ \pi,\ \pi)$

- The values in the table are $(\delta,\ \phi_1,\ \phi_2)$
- The former and latter combinations correspond to $m_{1(\text{or }3)} = 0\,\text{eV}$ and nonzero $m_{1(\text{or }3)}$

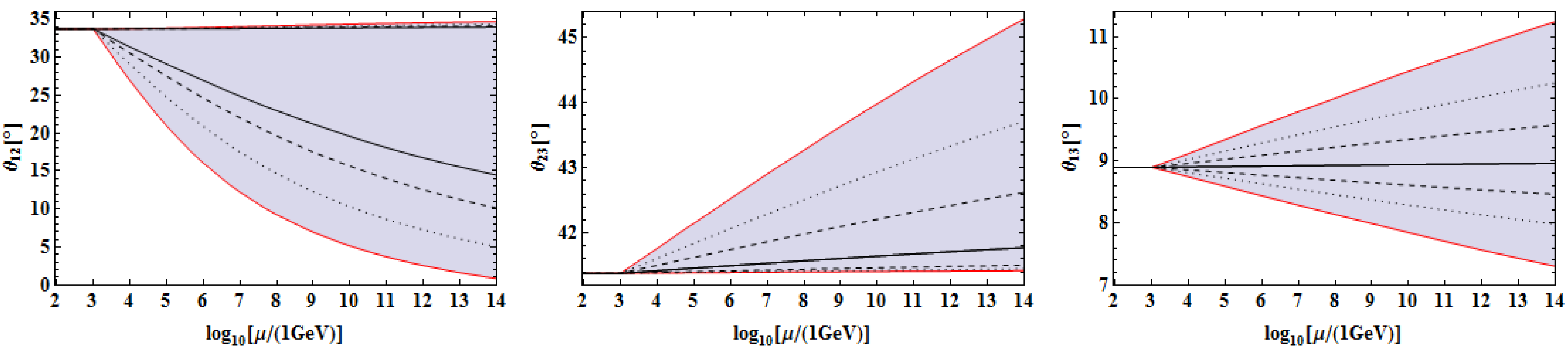
- For $m_1 = 0\text{eV}$ in NH, both Δm^2_{21} and Δm^2_{31} are independent of ϕ_1
- For $m_3 = 0\text{eV}$ in IH, they are independent of $|\phi_1 - \phi_2|$
- The upper and lower parts of the allowed regions (except Δm^2_{31} in IH) are taken by $\delta = 0$ and π, respectively
- For Δm^2_{31} in IH, they are taken by $\delta = \pi$ and 0, respectively

Runnings of mixing angles

NH (SUSY = 1TeV, $\tan\beta$ = 30)



IH (SUSY = 1TeV, $\tan\beta$ = 30)



- These figures are the results with the decoupling effects
- Solid, dashed, dotted, and red-solid lines: $m_{1(\text{or }3)} = 0$, 0.03, 0.05, and $0.07\,\mathrm{eV}$
- Shaded regions can be taken according to CP-phases for $m_{1(\text{or }3)} = 0.07\,\mathrm{eV}$

Runnings of mixing angles

- Combinations of CP-phases which give the upper and lower bounds

NH	θ_{12}	θ_{23}	θ_{13}
Upper bound	depend on m_1	$(0,\ \text{any},\ \pi)$ / $(0,\ \pi,\ \pi)$	$(\pi,\ \text{any},\ 0)$ / $(\pi,\ \pi,\ 0)$
Lower bound	$(\pi,\ \text{any},\ 0)$ / $(\pi,\ \pi,\ \pi)$	$(0,\ \text{any},\ 0)$ / $(0,\ 0,\ 0)$	$(0,\ \text{any},\ 0)$ / $(\pi,\ 0,\ \pi)$

IH	θ_{12}	θ_{23}	θ_{13}
Upper bound	depend on m_3	- / $(\pi,\ 0,\ 0)$	- / $(\pi,\ 0,\ \pi)$
Lower bound	$\delta = \pi, \lvert\phi_1 - \phi_2\rvert = 0$ / $(\pi,\ 0,\ 0)$	- / $(\pi,\ \pi,\ \pi)$	- / $(\pi,\ \pi,\ 0)$

m_1 (or m_3)	$0\,\text{eV}$	$0.03\,\text{eV}$	$0.05\,\text{eV}$	$0.07\,\text{eV}$	
Upper bound of θ_{12}	$(0,\ \text{any},\ 0)$	$(0,\ \pi,\ 0)$	$(\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2})$ or $(\frac{3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2})$	$(\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2})$ or $(\frac{3\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2})$	NH
	$\delta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, $\lvert\phi_1 - \phi_2\rvert = \pi$	$(\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2})$ or $(\frac{3\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2})$	$(\frac{\pi}{2}, 0, \pi)$ or $(\frac{3\pi}{2}, 0, \pi)$	$(\frac{\pi}{2}, 0, \pi)$ or $(\frac{3\pi}{2}, 0, \pi)$	IH

- For $m_1 = 0\text{eV}$ in NH, all mixing angles are independent of ϕ_1
- For $m_3 = 0\text{eV}$ in IH, θ_{12} is independent of $|\phi_1 - \phi_2|$, and θ_{23} and θ_{13} are almost independent of all CP-phases

	θ_{12} $\lvert\phi_1 - \phi_2\rvert$	θ_{23} (ϕ_1, ϕ_2)	θ_{13} $(\lvert\delta - \phi_1\rvert, \lvert\delta - \phi_2\rvert)$
Upper region	π	(π, π)	$(0, \pi)$
		$(0, 0)$	$(\pi, 0)$
Lower region	0	$(0, 0)$	$(\pi, 0)$
		(π, π)	$(0, \pi)$