FLASY 2014

Fourth workshop on flavour symmetries

University of Sussex, 17-21 June 2014

Quark Yukawa pattern from spontaneous breaking of $\mathrm{SU}(3)^3$

Enrico Nardi

INFN – Laboratori Nazionali di Frascati, Italy

The SM fermions gauge invariant kinetic term:

$$\sum_{f=Q,\ell,u,d,e} \bar{\Psi}_f \not{D}_f \Psi_f$$

Only five $\not{D}_f$ for 15 fermions.

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Formally: $\mathcal{G} = U(3)^5$ invariance

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No multiplet structure in the spectrum: $\Rightarrow SSB$

Restricting to quarks and the broken subgroup $SU(3)^3$

$$\mathcal{G}_{\mathcal{F}} = SU(3)_Q \times SU(3)_u \times SU(3)_d$$

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Symmt. breaking ansatz: Interpret the SM *explicit* breaking as *spontaneous*, driven by a set of scalar "Yukawa fields" :

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[1] E. Nardi, Phys.Rev. **D84**, 036008 (2011); [2] J. R. Espinosa, C. S. Fong, E. Nardi, JHEP **1302**, 137 (2013);

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Scalar field invariants and T,A,D parametrization

Singular value decomposition for the non-Abelian fields:

$$Y_u = \mathcal{V}_u^\dagger\, \chi_u\, \mathcal{U}_u\,, \qquad\qquad Y_d = \mathcal{V}_d^\dagger\, \chi_d\, \mathcal{U}_d\,.$$

$\mathcal{V}, \mathcal{U}$ unitary field matrices, $\chi = \mathrm{diag}\,(u_1,\, u_2,\, u_3)$; $u_i \geq 0$.

$\mathcal{G}_\mathcal{F}$ transformations: $Y \to V_Q Y_q V_q^\dagger$, $\quad YY^\dagger \to V_Q (YY^\dagger) V_Q^\dagger$

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$SU(N)$ invariants: Renormalizable $\quad$ Non-ren $D > 4$

$T = \mathrm{Tr}(YY^\dagger) = \sum_i u_i^2$; $\qquad (T^2)$

$A = \mathrm{Tr}\left[\mathrm{Adj}\,(YY^\dagger)\right] = \frac{1}{2}\sum_{i\neq j} u_i^2 u_j^2$

$\mathcal{D} = \mathrm{Det}(Y) = e^{i\delta}\prod_i u_i \equiv e^{i\delta} D$; $(\mathcal{D}^*)$

$T_m^n = T\left[(YY^\dagger)^m\right]^n$

$A_m^n = A\left[(YY^\dagger)^m\right]^n$

$\mathcal{D}_m^n = \mathcal{D}\left[Y^n Y^{\dagger m}\right]$

$\delta = \mathrm{Arg\,Det}\,(\mathcal{V}^\dagger\mathcal{U})$. $\quad$ Therefore: $\mathcal{L}(Y) = \mathcal{L}[T(\chi), A(\chi), \mathcal{D}(\chi)]$

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$\delta = \mathrm{Arg\,Det}\left(\mathcal{V}^\dagger \mathcal{U}\right)$. Therefore: $\mathcal{L}(Y) = \mathcal{L}[T(\chi), A(\chi), \mathcal{D}(\chi)]$

(Characteristic eqn.: $\mathcal{P}(\xi) = \det\left(\xi I - YY^\dagger\right) = \xi^3 - T\,\xi^2 + A\,\xi - D^2 = 0$)

Scalar potential and classification of the vacua

$$V = \frac{1}{\Lambda^4}\hat{V} = +\lambda\left[T - \frac{m^2}{2\lambda}\right]^2 + \lambda_A A + \underbrace{\tilde{\mu}\,\mathcal{D} + \tilde{\mu}^*\,\mathcal{D}^\dagger}_{2\,\mu\,\cos\tilde{\delta}\cdot D}$$

$$\langle T\rangle = \frac{m^2}{2\lambda}; \quad \begin{cases} \max \mathrm{A} : \langle\chi\rangle_s = (u,u,u) \\ A = 0 : \langle\chi\rangle_h = (0,0,u) \end{cases}; \quad \begin{cases} \max D : \langle\chi\rangle_s = (u,u,u) \\ D = 0 : \langle\chi\rangle' = (0,u',u') \end{cases}$$

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(1): $\lambda_A < 0$: $\Rightarrow$ $A_{\max}$, $D_{\max}$, $\langle\tilde{\delta}\rangle = \pi$, $\langle\chi\rangle_s$ $\underline{SU(3)\times SU(3)\to SU(3)}$

(2): $\lambda_A > 0$: $\Rightarrow$ $\begin{cases} \frac{\mu^2}{m^2} > \mathcal{F}(\frac{\lambda_A}{\lambda}) : \; D_{\max}, \; \langle\tilde{\delta}\rangle = \pi, \; \langle\chi\rangle_s \\ \frac{\mu^2}{m^2} < \mathcal{F}(\frac{\lambda_A}{\lambda}) : \; \boxed{A = D = 0} \; (\langle\delta\rangle = ?), \; \langle\chi\rangle_h \end{cases}$

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V admits hierarchical vacua $\langle\chi\rangle_h = (0,0,u)$! $\left[SU(3)\times SU(3)\to SU(2)\times SU(2)\times U(1)\right]$

Can the vanishing entries be lifted $(0,0,1) \to (\epsilon', \epsilon, 1)$?

Ref.[1]: $V \to V^{eff} = V_0 + V_1$; if $V_1 \supset \alpha \cdot A \log A; \quad \beta \cdot D \log D$ then:

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Georgi & Pais theorem (PRD16 (1977) 3520): *A reduction of the tree level vacuum symmetry via loop corrections can only occur if there are additional (non-NGB) massless scalars in the tree approximation.*

Intuitively: $\mathcal{G}_{\mathcal{F}}(8+8) \to H_h(3+3+1)$: 9 broken generators (NGB) + 9 massive.
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Examples of theories with additional massless scalars:
$V_{CW} = \lambda \phi^4$ (all states are massless at tree level)
$V_{\lambda_A, \mu_D = 0} = (T - v_T^2)^2$ accidental $SO(18)$ broken to $SO(17)$: 17 NGB, 1 massive

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CONCLUSION: $\mathcal{G}_\mathcal{F} \to H_\epsilon$ breaking should occur already at the tree level!
[$V(Y)$ potential is too simple. We need additional scalar reps.]

[Previous theorems only apply for the irreducible $SU(3)\times SU(3)$ representation Y]

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However, with only two "directions" Y_u and Y_d in $SU(3)_Q$ flavour space there is just one relative "angle". The potential $V(Y_u, Y_d)$ is minimized for $\chi_{u,d}$ *alignment* ($\lambda_{ud} < 0$) or *anti-alignment* ($\lambda_{ud} > 0$). All mixings then vanish, and $V_{CKM} \propto I$

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CONCLUSION: We need at least four "directions" in $SU(3)_Q$ flavour space to get three relative "angles". [We need additional scalar reps.]

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Simplest choice: fundamental reps. $Z_{Q_{1,2}} = (\mathbf{3},\mathbf{1},\mathbf{1})$, $Z_u = (\mathbf{1},\mathbf{3},\mathbf{1})$, $Z_d = (\mathbf{1},\mathbf{1},\mathbf{3})$

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– Flavour irrelevant: carry larger symmetries: $T_{\sim[SO(18):\langle\chi\rangle_h \to \langle\chi\rangle_s]}$, $|Z|^2_{\sim[SO(6)]}$

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2. Divide $V(Y_q,Z) = V_\mathcal{I} + V_\mathcal{AR} + V_\mathcal{A}$ and study $V_\mathcal{AR}$ and

$$V_\mathcal{A} \supset \left(\mu_q \mathcal{D}_q + \nu_{iq} Z_{Qi}^\dagger Y_q Z_q\right) + H.c.$$

If $\mu_q, \nu_{iq} < v_q = \langle T\rangle$ strong hierarchies can arise dinamically [with no hierarchical parameters].

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No type of perturbative effect can further break the little groups H left unbroken at tree level. Mixings require at least two other $SU(3)_Q$ fields.

Ref [3] [C.S.Fong, EN] program: Search for $V(Y_{u,d},\{Z\})$ that can break at the tree level $\mathcal{G}_\mathcal{F} = SU(3)_Q \times SU(3)_u \times SU(3)_d$ generating hierarchies, mixings, and $\not{CP}$.

Simplest choice: fundamental reps. $Z_{Q_{1,2}} = (\mathbf{3},\mathbf{1},\mathbf{1})$, $Z_u = (\mathbf{1},\mathbf{3},\mathbf{1})$, $Z_d = (\mathbf{1},\mathbf{1},\mathbf{3})$

1. Classify the dynamical properties of the invariants w. respect to minimization:

– Flavour irrelevant: carry larger symmetries: $T_{\sim[SO(18):\langle\chi\rangle_h \to \langle\chi\rangle_s]}$, $|Z|^2_{\sim[SO(6)]}$
– Attractive/repulsive: Hermitian monomials: $\alpha|YZ|^2$: $\alpha < 0 (> 0)$ Y-Z (anti)alignment,
– Always attractive: non-Hermitian monomials: $Z_Q^\dagger Y_u Z_u + \mathrm{H.c.} = 2|Z_Q^\dagger Y_u Z_u| \cos\phi$

2. Divide $V(Y_q, Z) = V_\mathcal{I} + V_{\mathcal{AR}} + V_\mathcal{A}$ and study $V_{\mathcal{AR}}$ and

$$V_\mathcal{A} \supset \left(\mu_q \mathcal{D}_q + \nu_{iq} Z_{Qi}^\dagger Y_q Z_q\right) + H.c.$$

If $\mu_q, \nu_{iq} < v_q = \langle T \rangle$ strong hierarchies can arise dinamically [with no hierarchical parameters].

3. CP-violation: $V_\mathcal{A}$ contains four physical complex phases. At the minimum, they induce one $\not{CP}$ phase in $\langle \mathcal{V} \rangle = V_{CKM}$.

Conclusions

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6. [The MFV hypothesis can be automatically realized.]

One numerical example

With these inputs:

$$\mu_q = \nu_{1q} = \nu_{2q} = v/10, \qquad m_{12}^2 = 0.15\, v^2,$$
$$\gamma_{ud} = 0.81, \qquad \eta_{12} = 0.1, \qquad \lambda_{12} = 1.27,$$
$$\phi_{\gamma_{ud}} = 0.98\pi, \qquad \phi_{\eta_{12}} = 0.92\pi, \qquad \phi_{\nu_{2q}} = 0.95\pi.$$

and all other parameters set to 1 (or to -1), we obtain:

$$\begin{aligned}
|\hat{Y}_u| &= v\, \mathrm{diag}\,(0.0003, 0.009, 1.4)\,, \\
|\hat{Y}_d| &= v\, \mathrm{diag}\,(0.0007, 0.02, 1.2)\,, \\
K &= V_{CKM} = \begin{pmatrix} 0.974 & 0.223 & 0.027 \\ 0.224 & 0.974 & 0.042 \\ 0.017 & 0.046 & 0.999 \end{pmatrix}, \\
J &= \mathrm{Im}\left(K_{jk} K_{lm} K_{jm}^* K_{kl}^*\right) = 2.9 \times 10^{-5}.
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