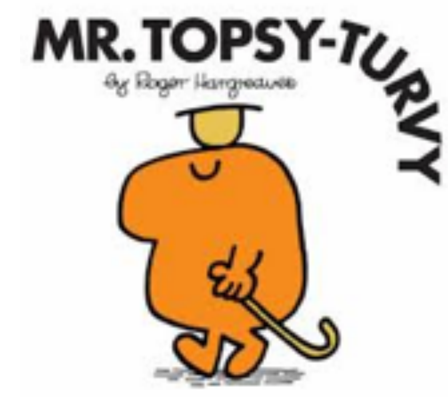
Resonances gone topsy turvy in b→ sll – the charm of QCD or new physics?



Lyon and RZ 1406.0566

CP3 Origins
Cosmology & Particle Physics

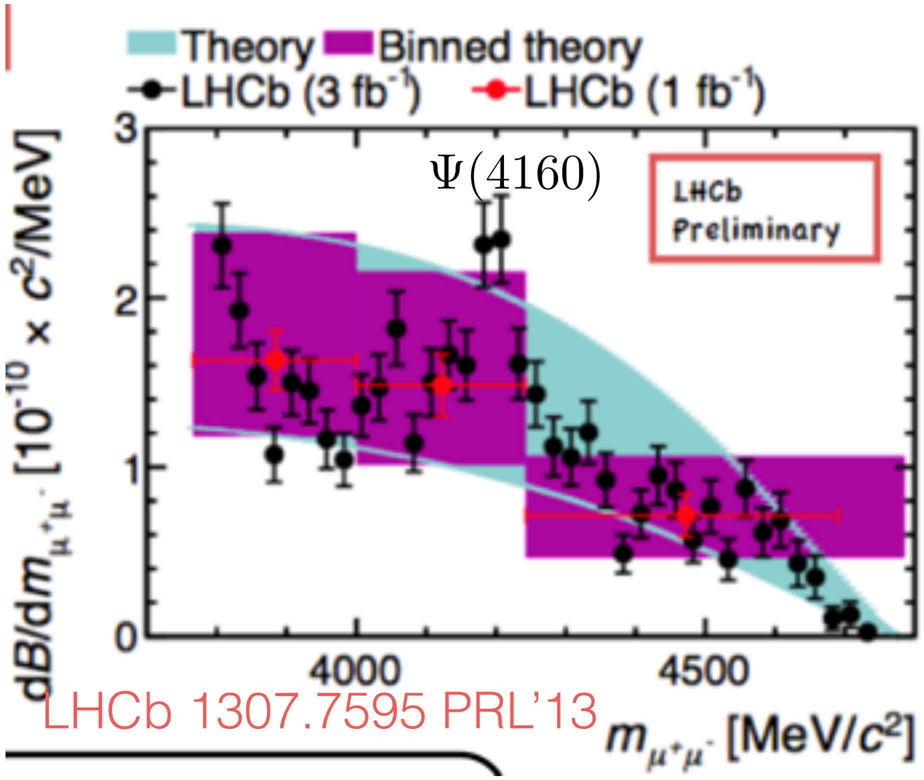
CENTRE FOR THEORETICAL PHYSICS · Higgs

Roman Zwicky
Edinburgh University

20 June 2014 — FLASY'14

Prologue

- LHCb EPS'13 surprise: pronounced open-charm resonances in B→Kll



LHCb 1307.7595 PRL'13

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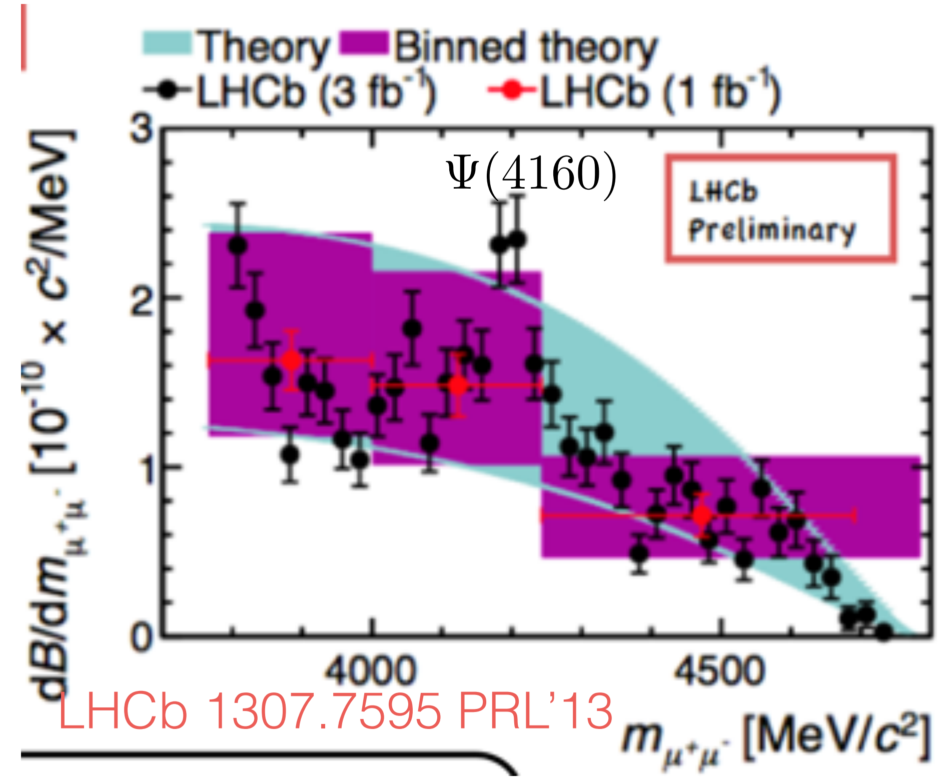

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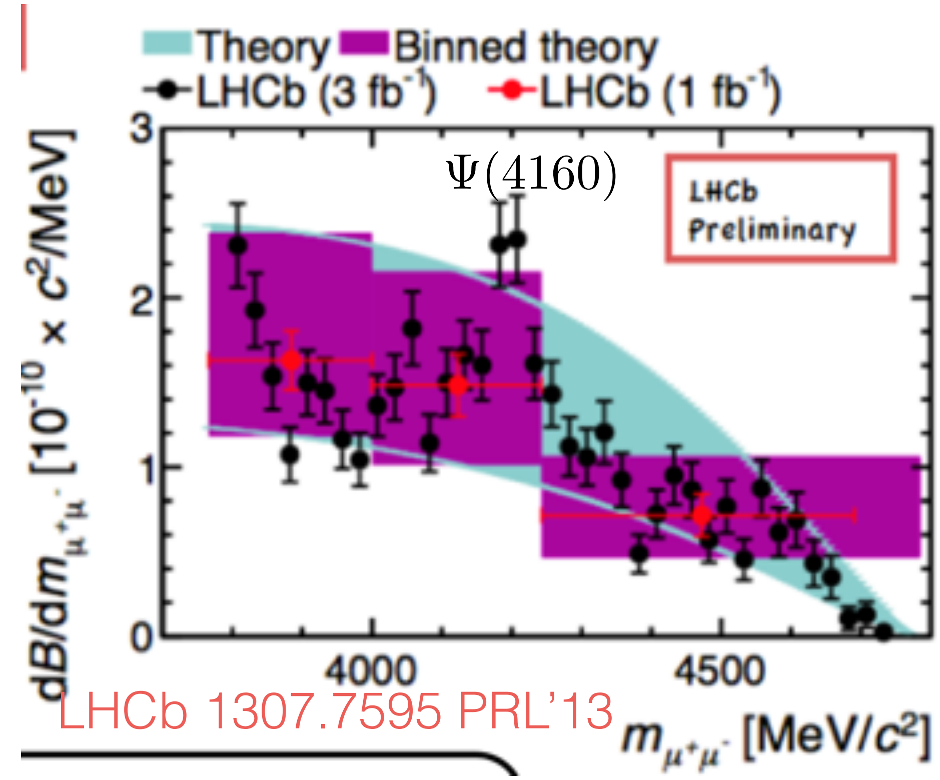

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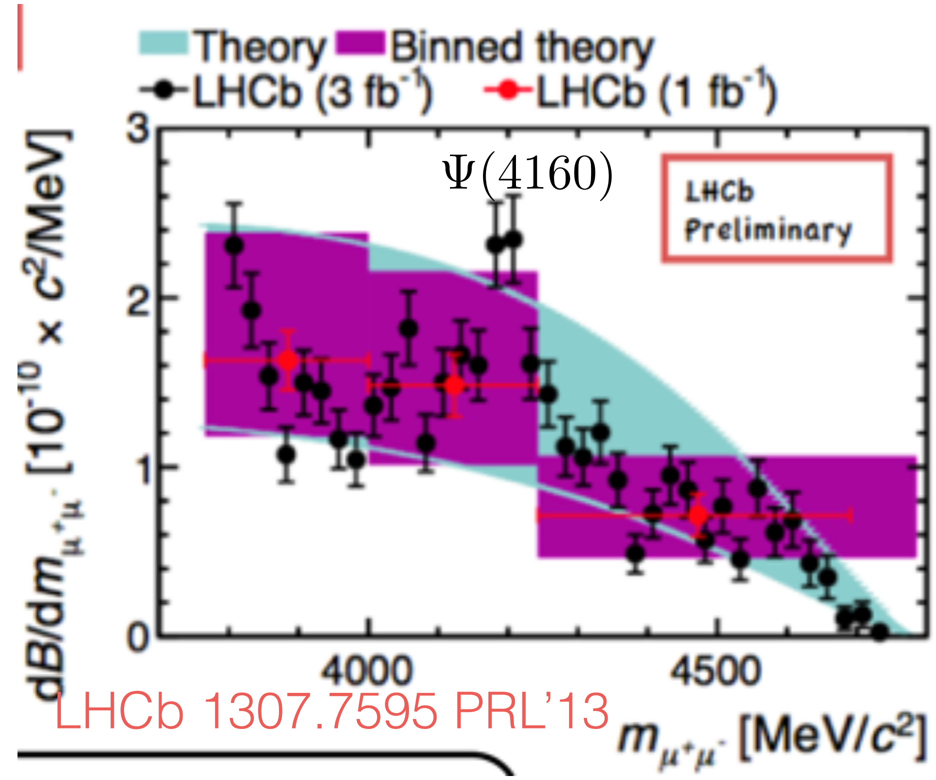

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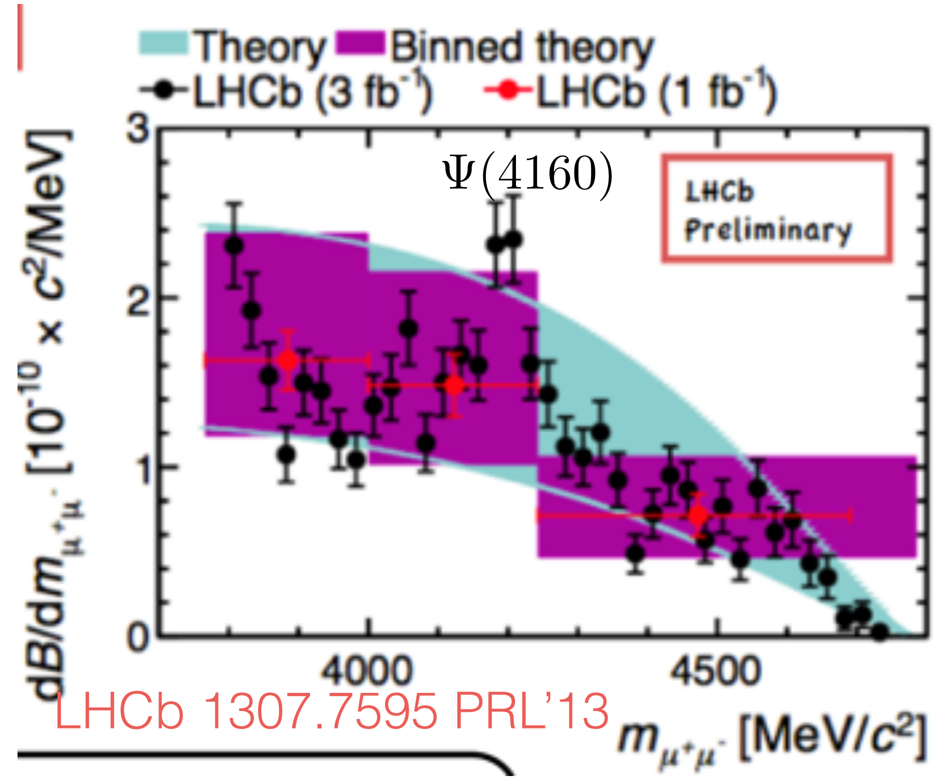

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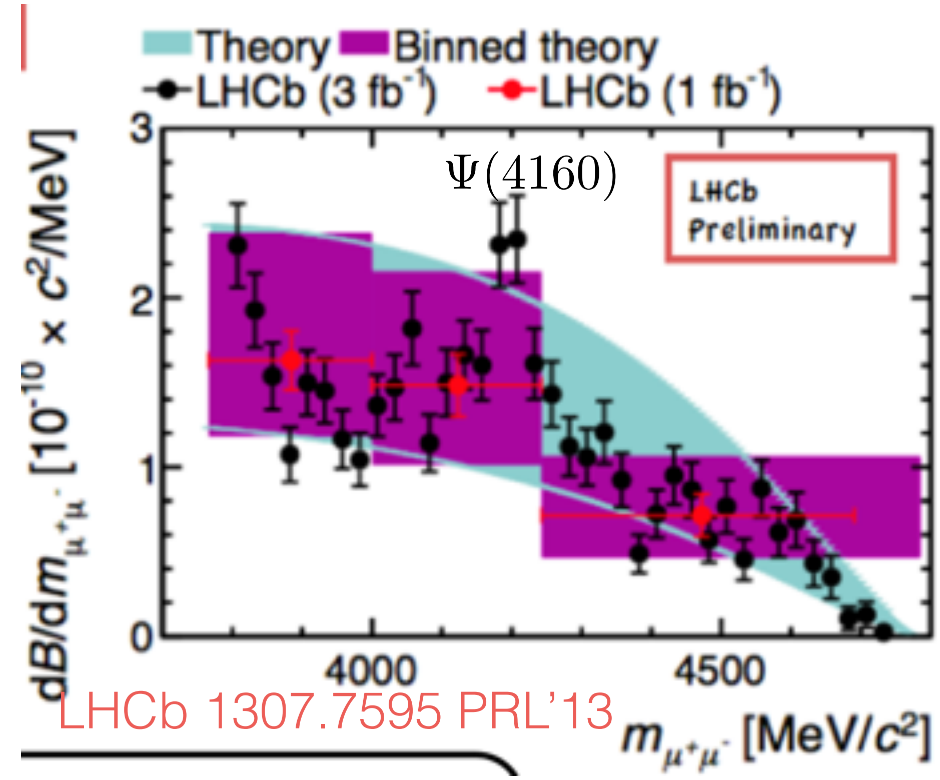

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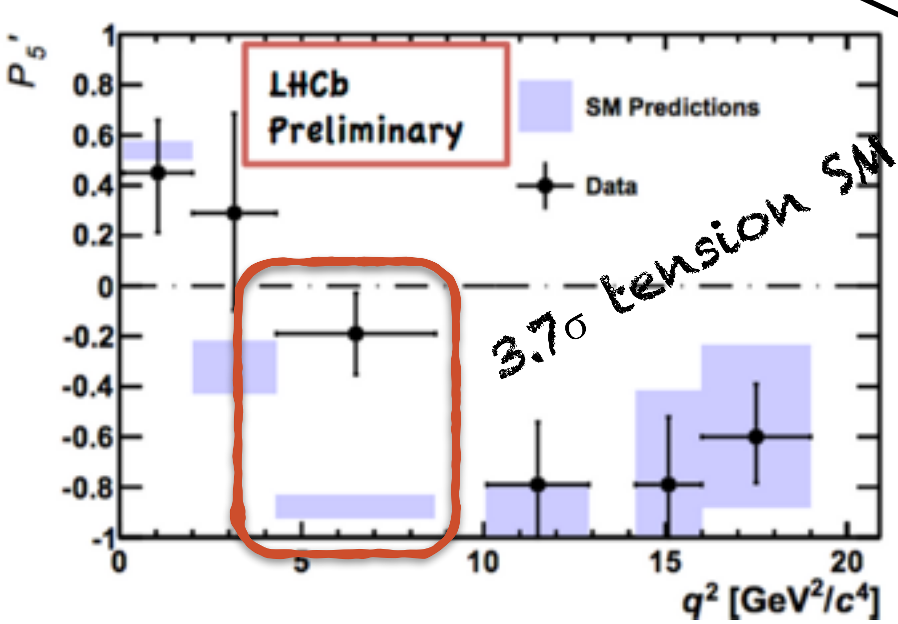
— Headlines —

Effect right sign and size to explain 2013-anomalies



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Overview

- A. Introduction $B \to K^{(*)}ll$ and resonances in factorisation
- B. (charm) vacuum polarisation from BESII-data
- C. combined fits LHCb- and BESII-data
- D. assessment non-factorisable corrections be? (**duality**)
- E. relation to 2013-anomalies
- F. discussion & conclusion

A. Basics of $B \to K^{(*)} ll$ for this talk

- framework effective Hamiltonian:

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operator
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Amplitude

$$\mathcal{A} = \langle V\ell\ell|H_{\text{eff}}|B\rangle = \sum_i C_i(m_b)\langle V\ell\ell|O_i(m_b)|B\rangle$$

non-perturbative

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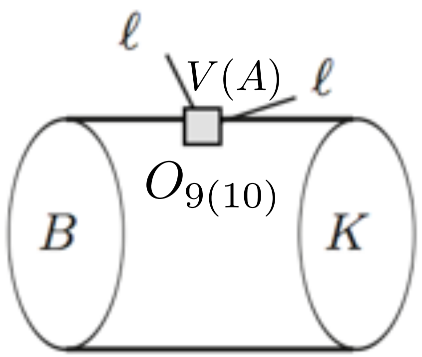
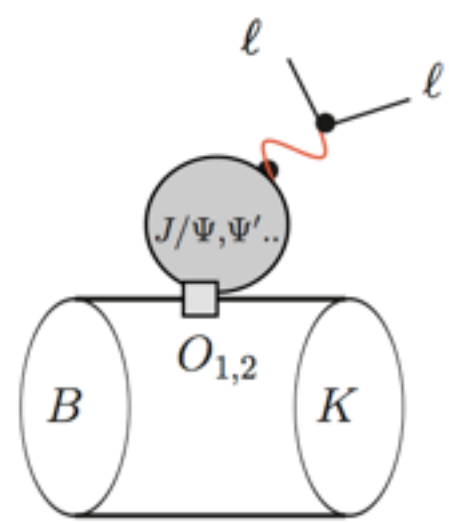


short distance



factorisation
fully described
vacuum polarisation

long distance

electroweak penguin (also O_7..)

4-quark operators (also $O_{3..6}$)

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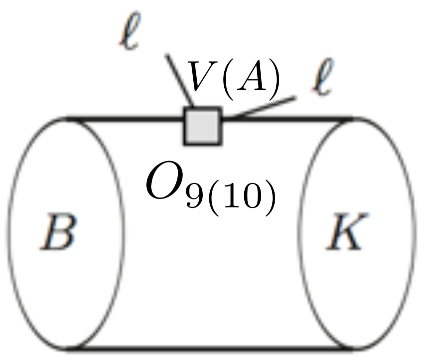

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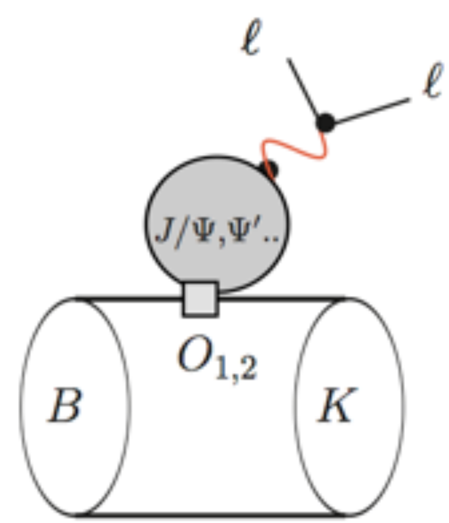

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electroweak penguin (also O_7..) **4-quark operators (also $O_{3..6}$)**

- chirality flipped operators (right-handed currents):

$$O' = O|_{s_L \to s_R} \quad \Leftrightarrow \quad \text{V-A} \to \text{V+A}$$

- one of the main dramas:

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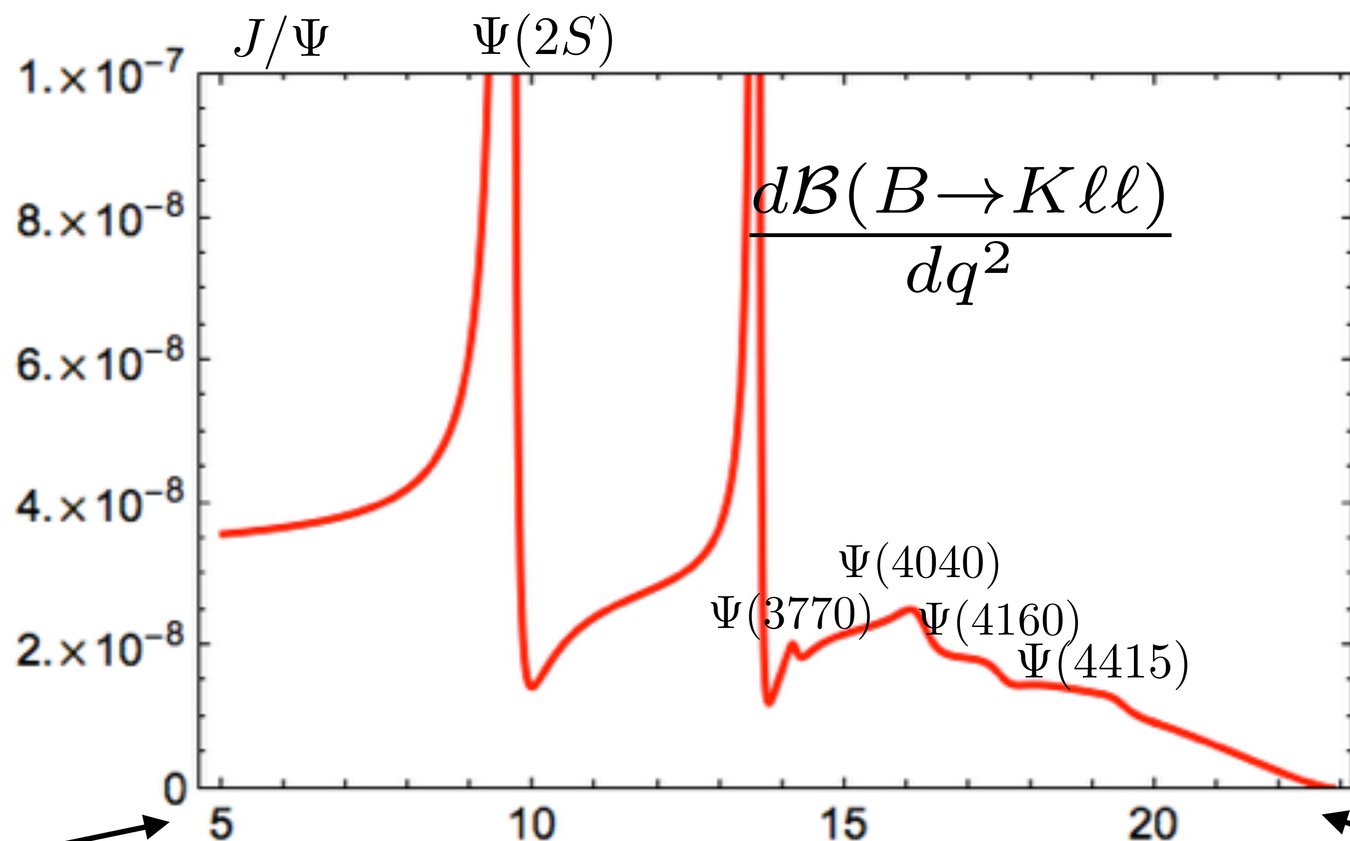
- partly reveal themselves in the q²-spectrum *(lepton-pair mom. squared)*

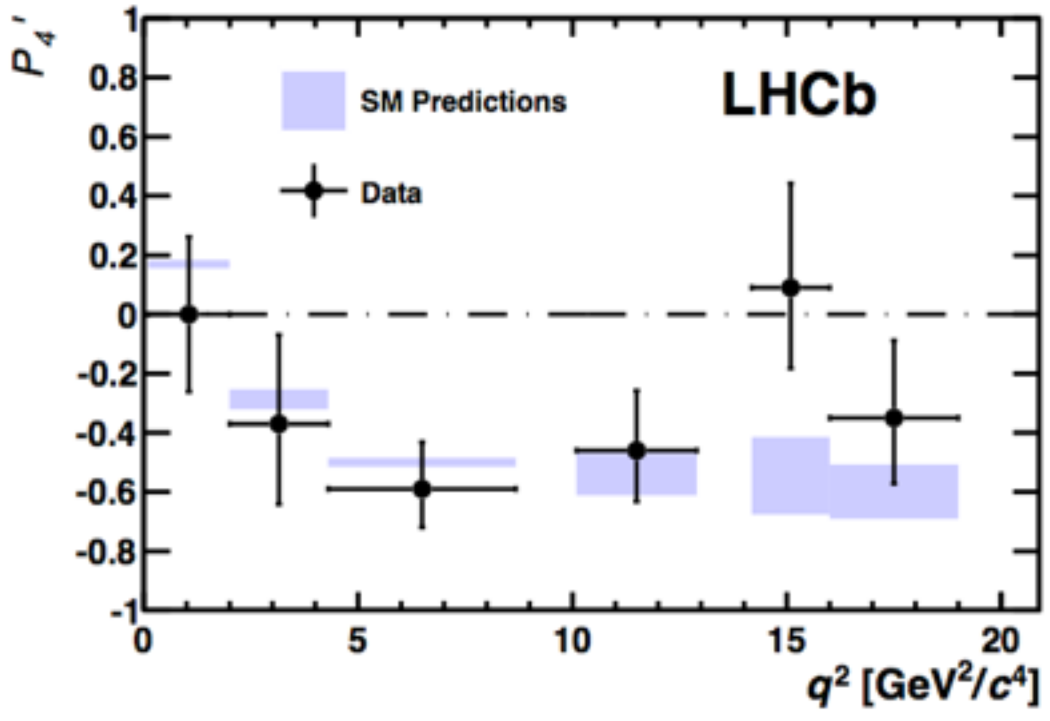


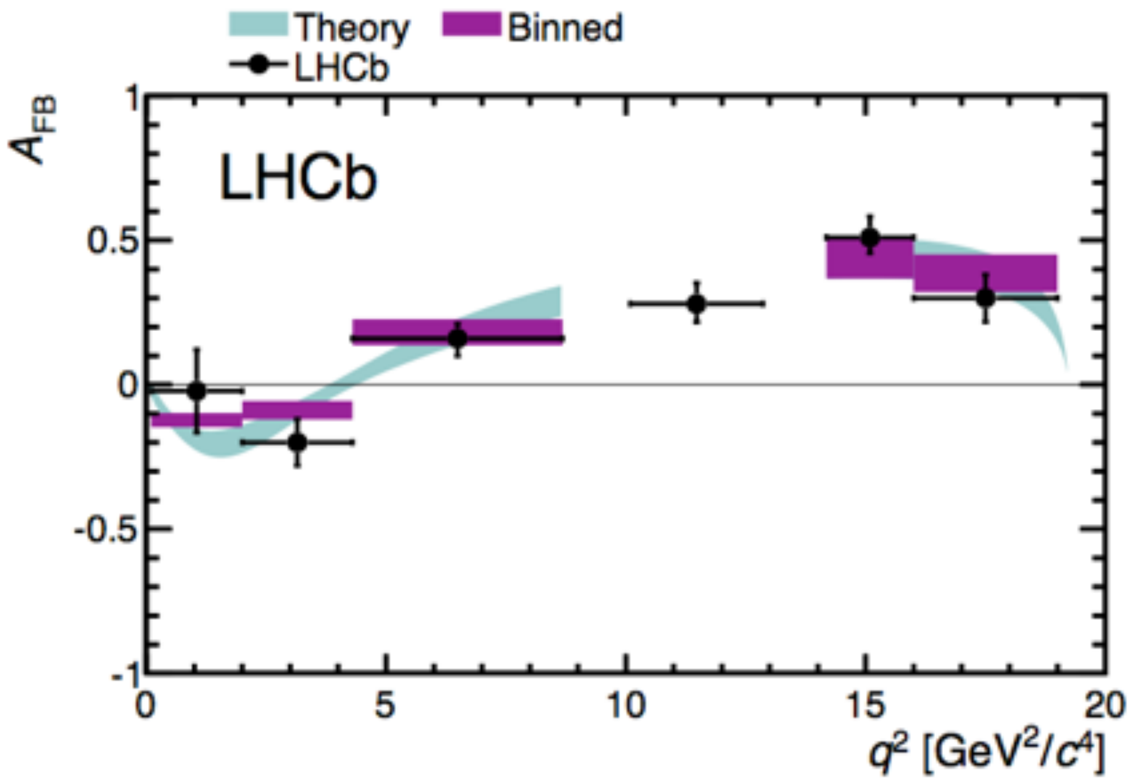
$O_{7,9}^2$-dominates O_2-$O_{7,9}$-interference	narrow resonances $(O_2)^2$-effect	O_9^2-dominates O_2-O_9-interference

K **fast**:
light-cone methods
LCSR, QCDF/SCET

K **slow**:
- **high-q² "OPE"**
-**endpoint relations**

after all it really works (angular observables)....





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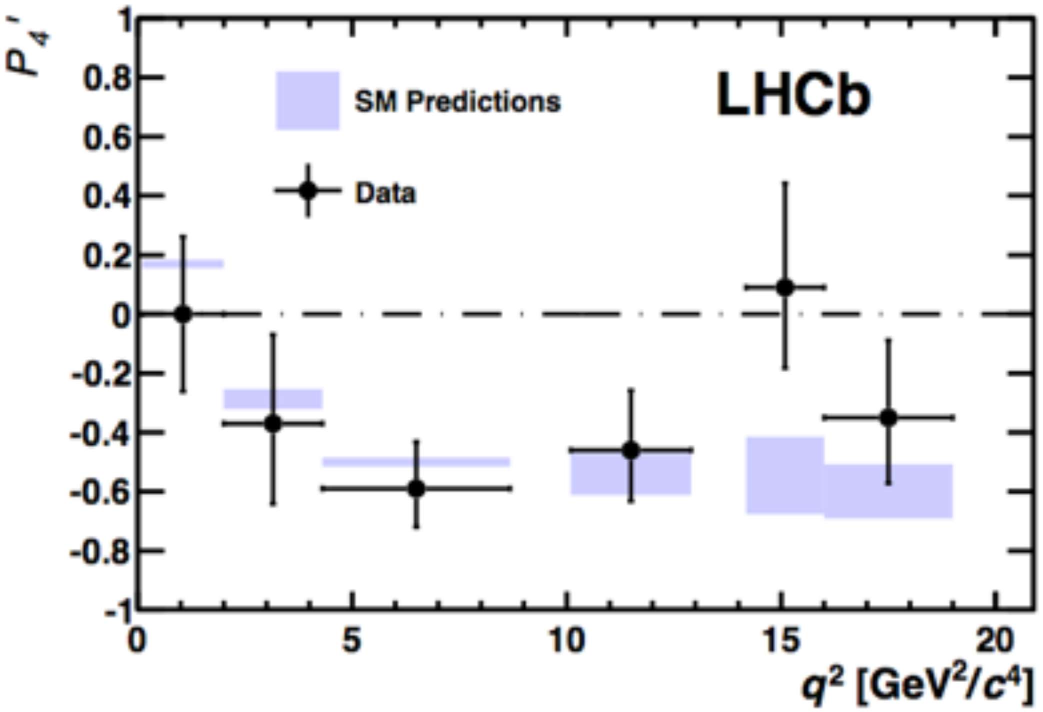
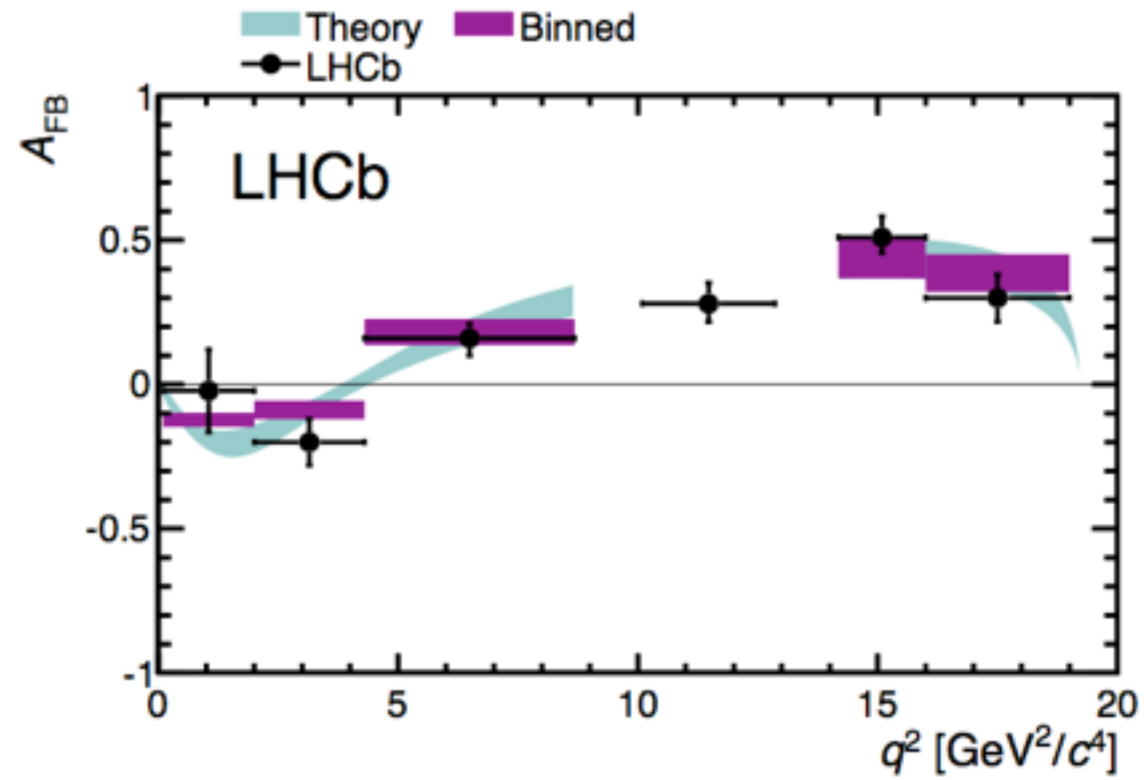
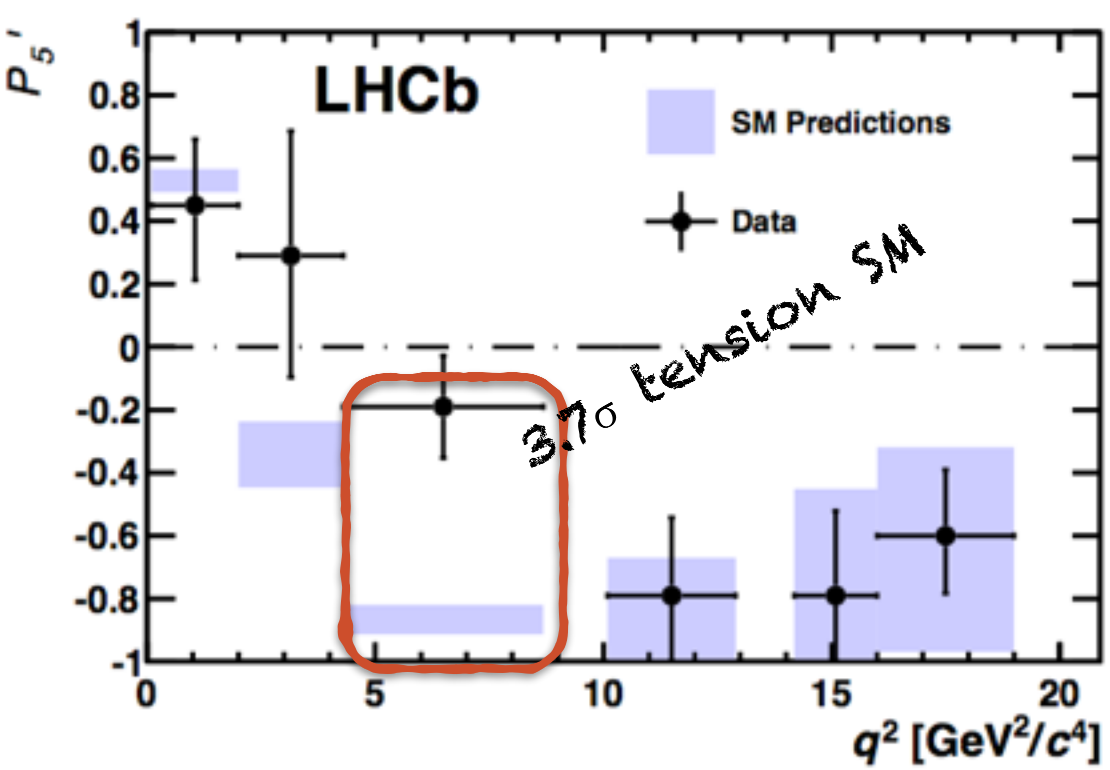




not quite everywhere (2013-anomalies) ..



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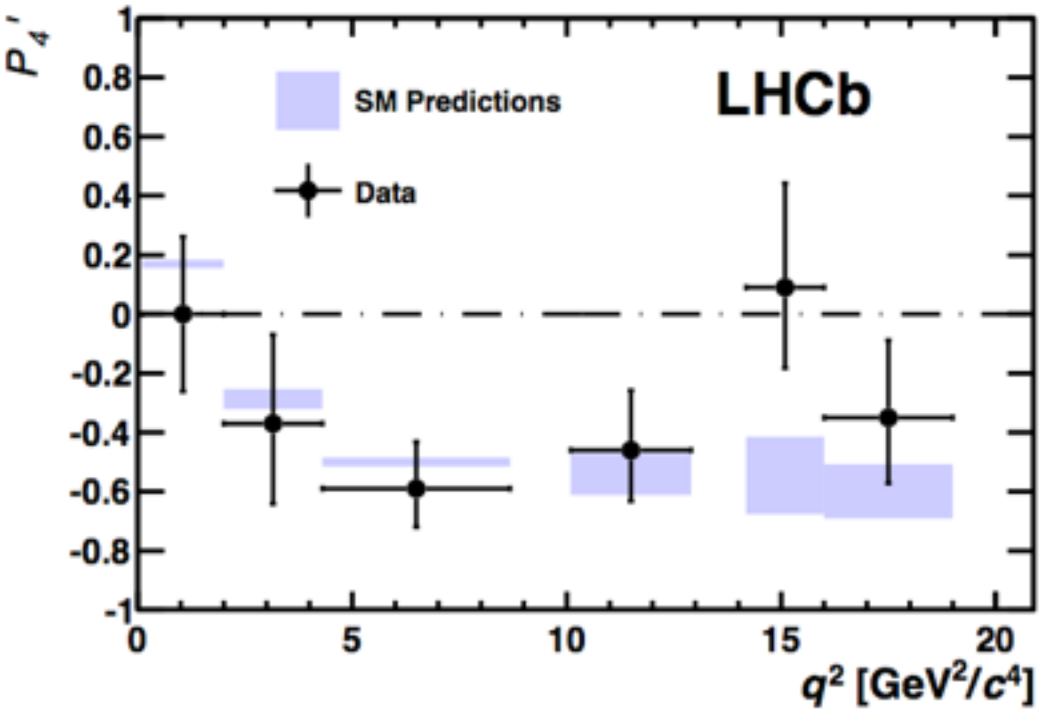

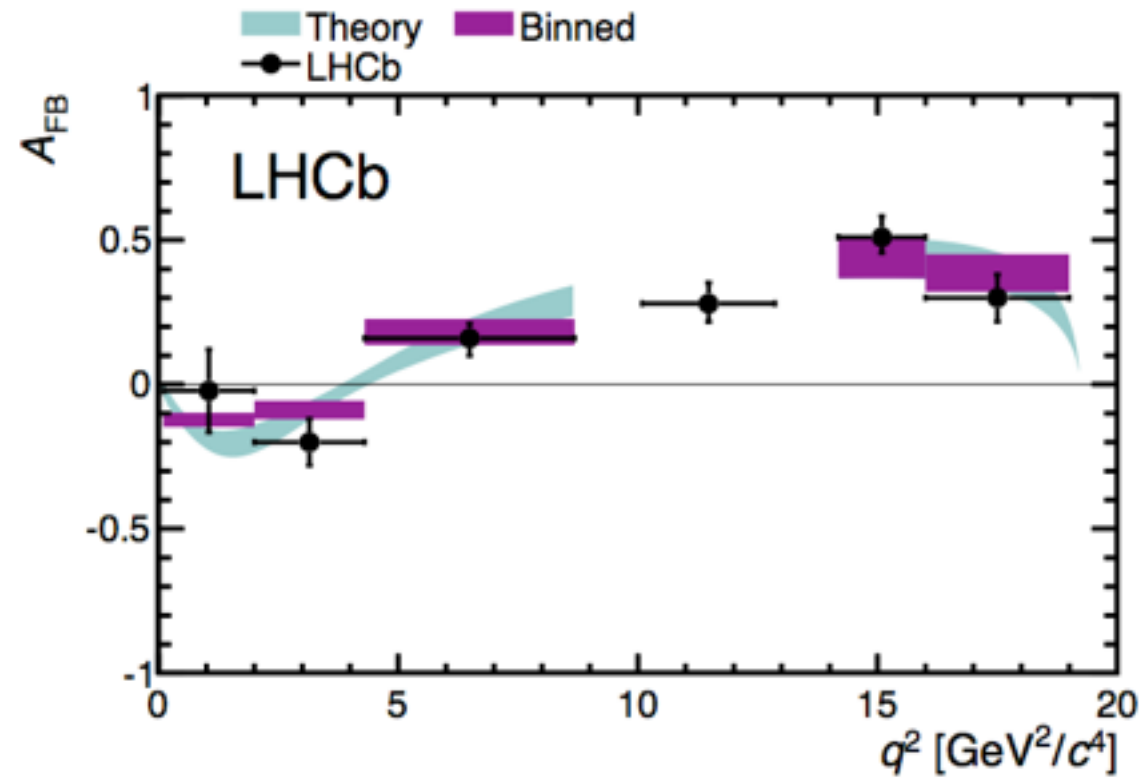

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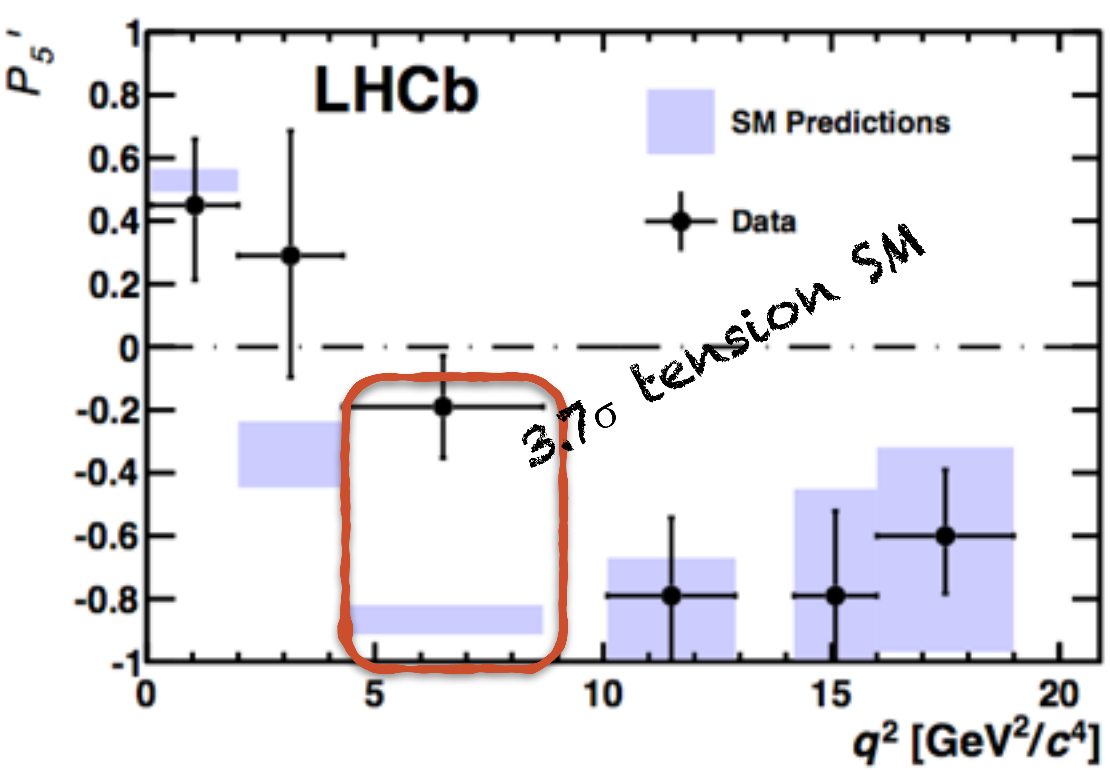
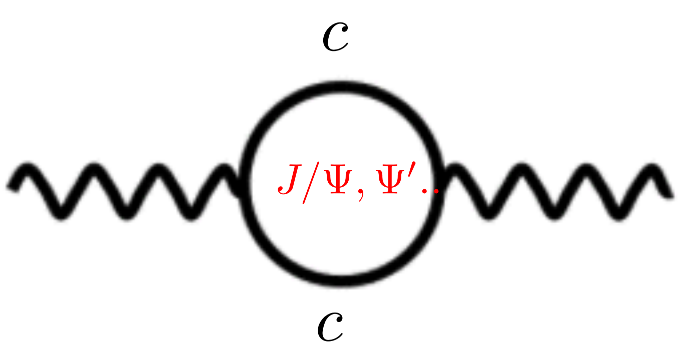


- what's charm got to with it? let's see ….

B. charm vacuum polarisation



- fully non-perturbative from BESII-data; as for (g-2)
- fully describes factorisation (later beyond)

Charm vacuum polarisation from BESII-data

- **Kallen-Lehmann-representation** follows (first principle dispersion relation)

$$\text{vac.pol.} \equiv h_c(q^2) = h_c(s_0) + \frac{q^2 - s_0}{2\pi i} P \int_{s_{J/\Psi}}^{\infty} \frac{dt}{t - s_0} \frac{\text{Disc}[h_c](t)}{t - q^2 - i0} \,,$$

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pQCD “ok”

$$\text{Disc}[h_c](s) = \frac{2\pi i}{3} \frac{\sigma(e^+e^- \to \text{c-hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \,, \quad \text{celebrated R-function}$$

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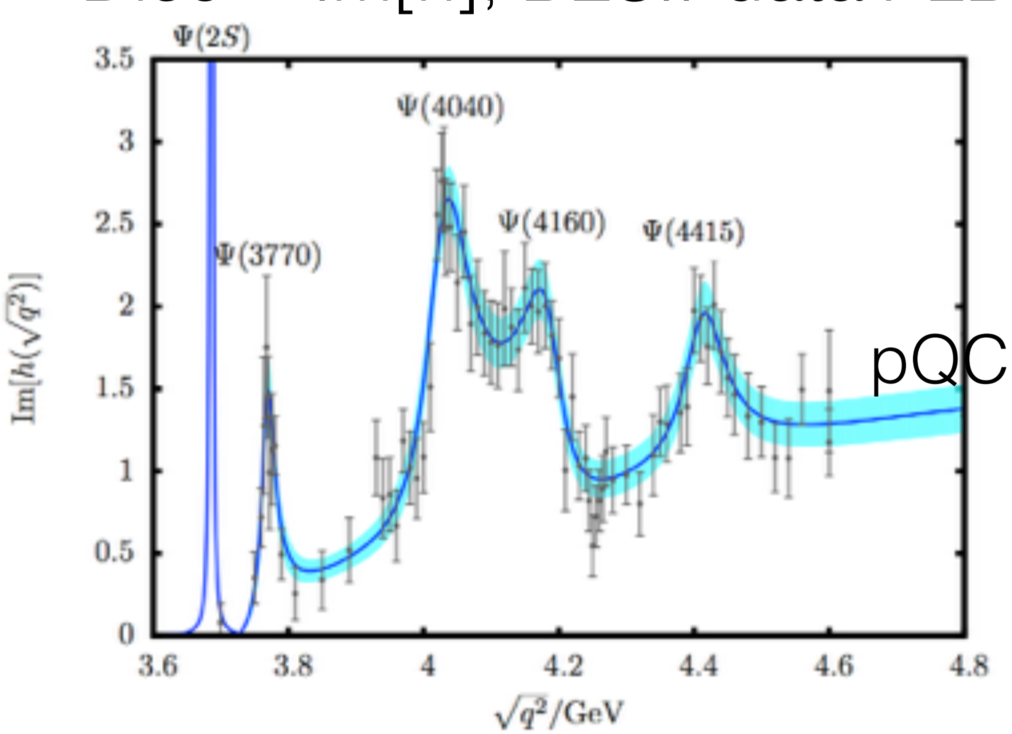
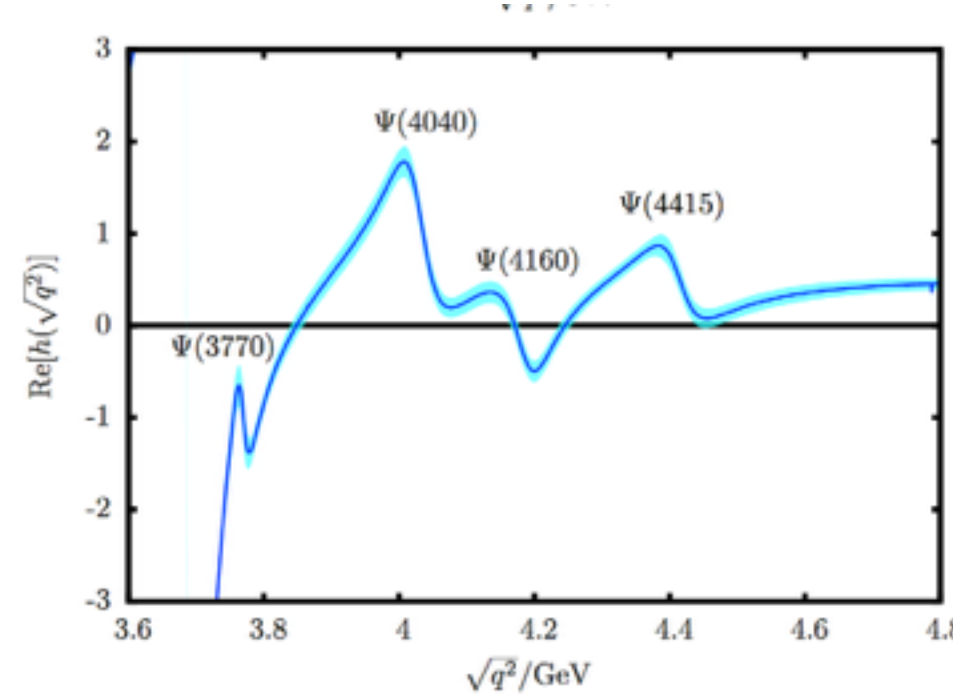
Disc ~ Im[h]; BESII-data'PLB08



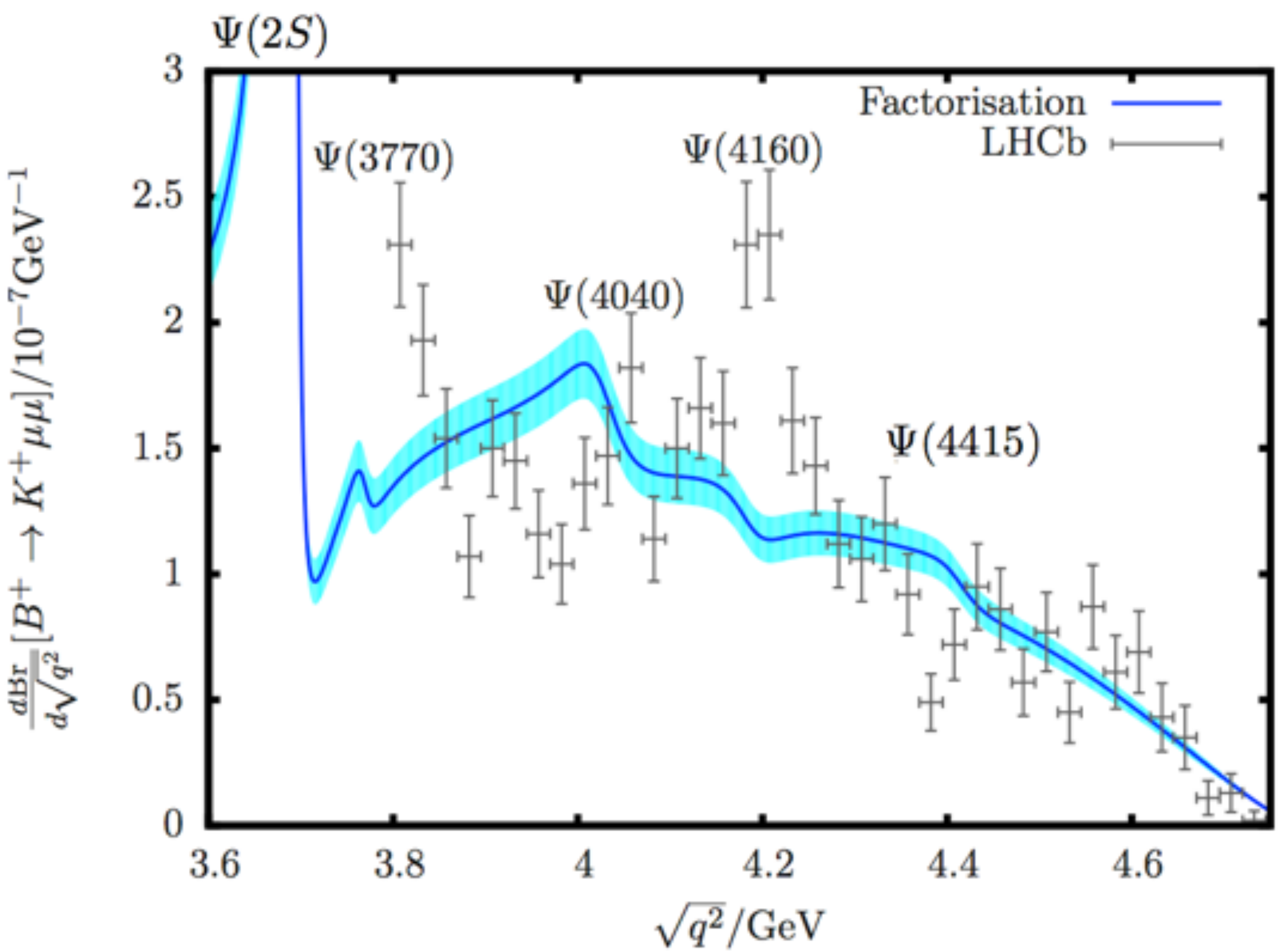
pQCD "ok"

our χ^2/dof = 1.015

Re[h] dispersion relation above



Factorisation (BESII-data) applied to B→Kll at high q^2



"oh dear!" does not work at all (topsy turvy)!
much worse than "expected" (later)

phase 1:

- phenomenological assessment through combined fits to LHCb- and BESII-data

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phase 2:

- assessment of non-factorisable corrections — discussion of duality

C. Combined fits to LHCb- and BESII-data

- Masses and width of resonances are same in both data 4 fits:

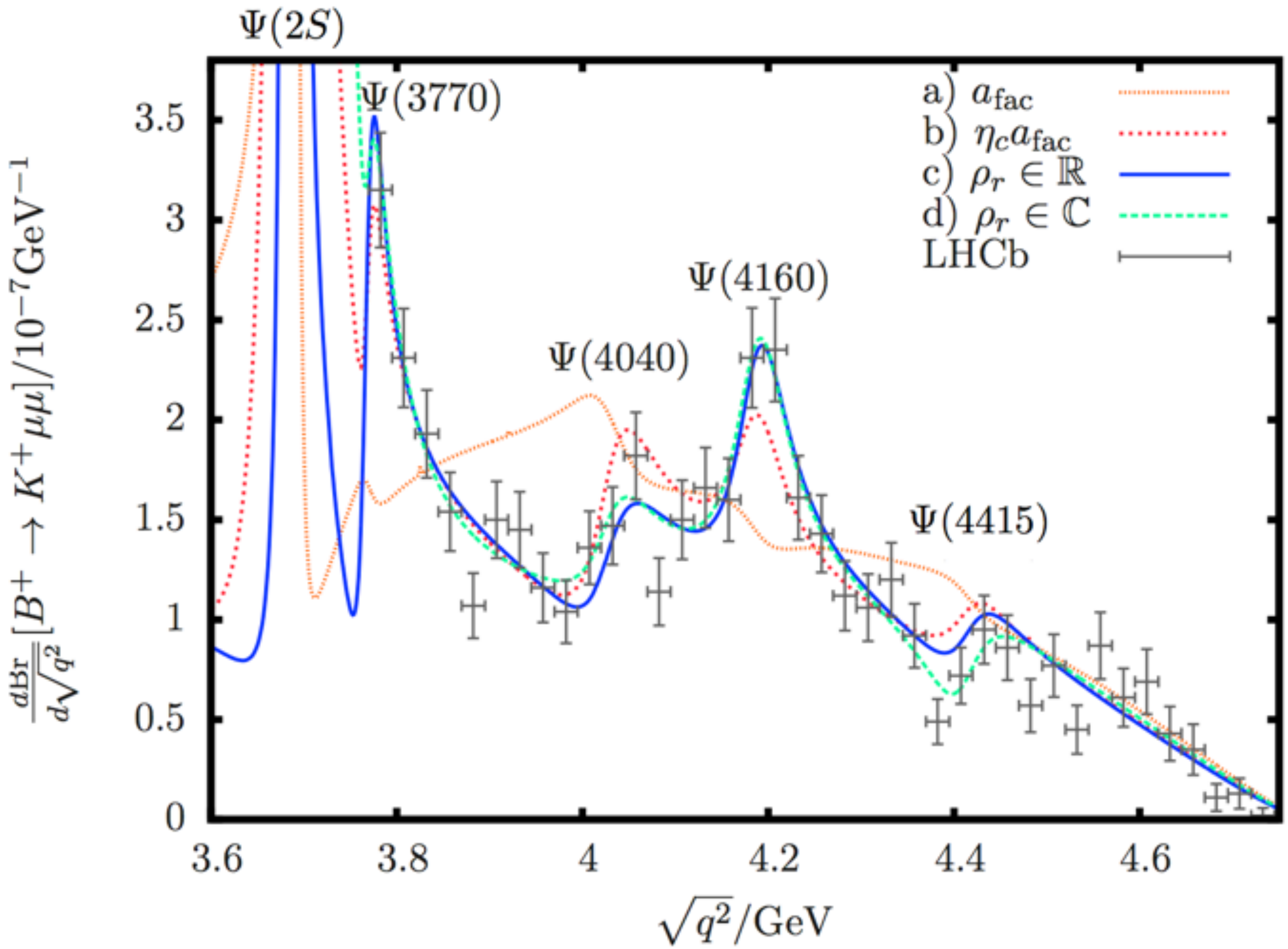
a) fit normalisation $\eta_{\mathcal{B}}$

b) fit BESII-prefactor η_c and $\eta_{\mathcal{B}}$

c) fit residues $\rho_r \in \mathbb{R}$ of LHCb-resonances (allow for non-factorisable effects)

d) fit residues $\rho_r \in \mathbb{C}$ of LHCb-resonances

results ....



Fit	$\eta_{\mathcal{B}}$	η_c	$\rho_{\Psi(2S)}$	$\rho_{\Psi(3370)}$	$\rho_{\Psi(4040)}$	$\rho_{\Psi(4160)}$	$\rho_{\Psi(4415)}$	χ^2/d.o.f.	d.o.f.	pts
$a)$	0.98	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	3.59	99	117
$b)$	1.08	-2.55	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	1.334	98	117
$c)$	0.81	$\equiv 1$	-1.3	-7.2	-1.9	-4.6	-3.0	1.169	94	117
$d)$	1.06	$\equiv 1$	3.8-5.1i $6.4e^{-i53.3^\circ}$	-0.1-2.3i $2.0e^{-i92^\circ}$	-0.5-1.2i $1.3e^{-i111^\circ}$	-3.0-3.1i $4.3e^{-i135^\circ}$	-4.5+2.3i $5.1e^{i153^\circ}$	1.124	89	117

conclusions of phase 1:

- factorisation scaled by a factor -2.5 (fit b) works surprisingly well
- this corresponds to a correction of -3.5 with regard to 1.0
  $\Rightarrow$ factorisation fails by 350% !
- keep it simple — fits c,d) refinements for later (duality)

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question:

- can QCD explain this? ⇒ phase 2:

D. how large are non-fac. corrections

- from QCD alone not chance to resolve locally in q^2
- at high q^2: q^2 is a large scale can integrate out charm quarks so-called high-q^2 “OPE” Grinstein,Pirjol’04 Beylich,Buchalla,Feldmann’11

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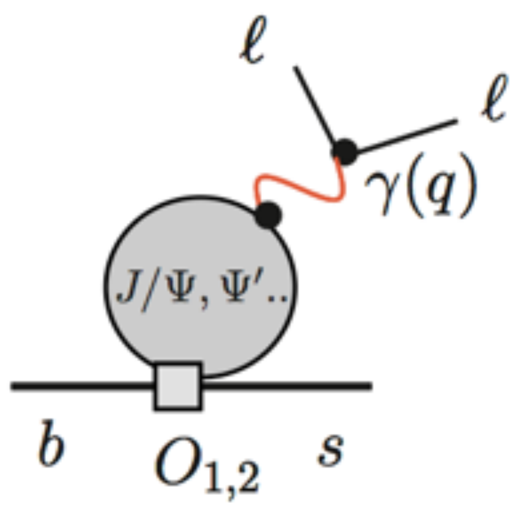
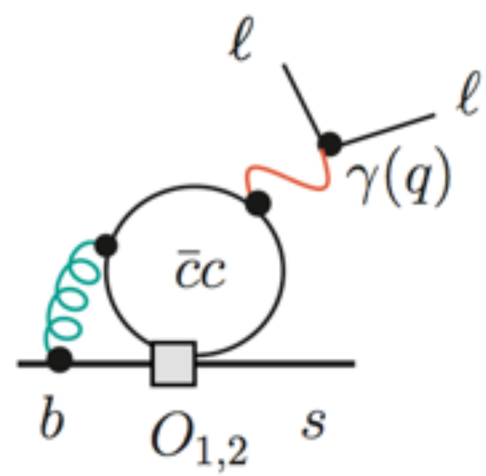
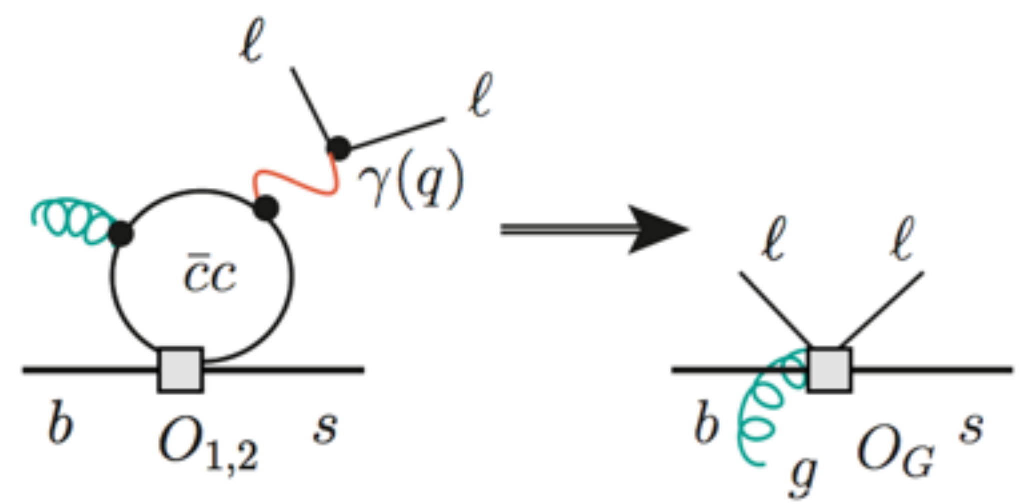


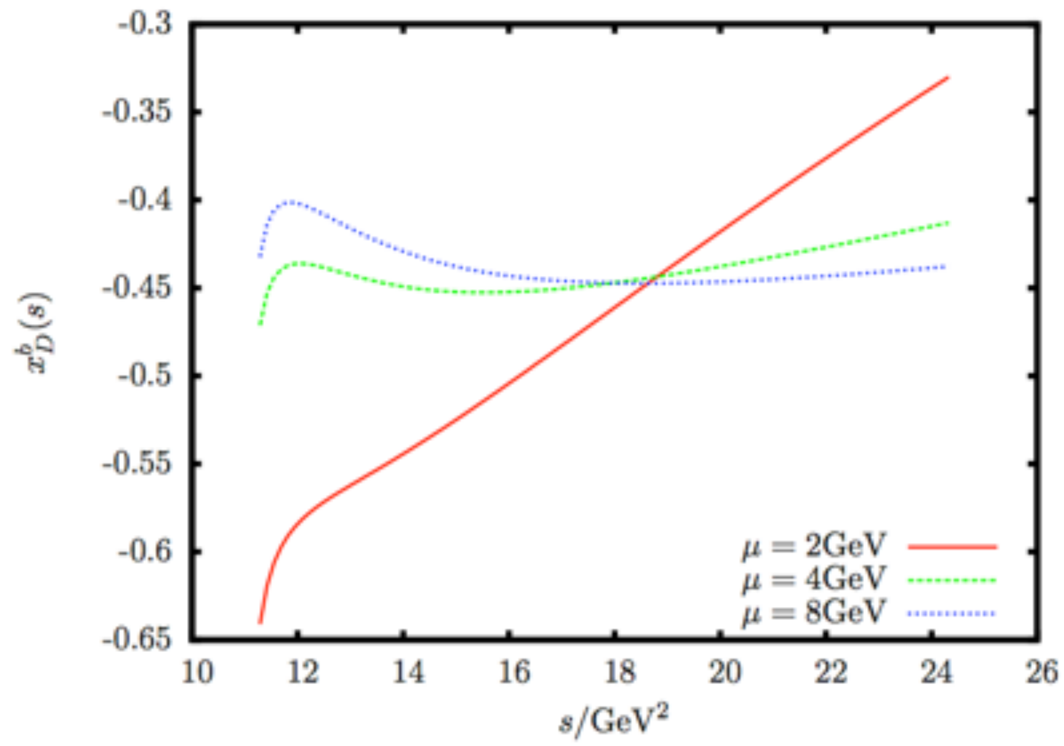




factorisation (BESII)	dim-3 vertex-corrections	dim-5 spectator & soft gluon
Lyon RZ'14	Hurth, Isidori, Ghinculov, Yao'03 Greub, Pilipp, Schupach'08	Beylich,Buchalla,Feldmann'11
100% in our units	roughly -50% throughout q^2-domain N.B. large due to color-enhancement (not repeated higher orders)	small O(2%) QCDF consistent dim. suppression

first conclusions of phase 2:

- -50%-correction is nowhere near -350%



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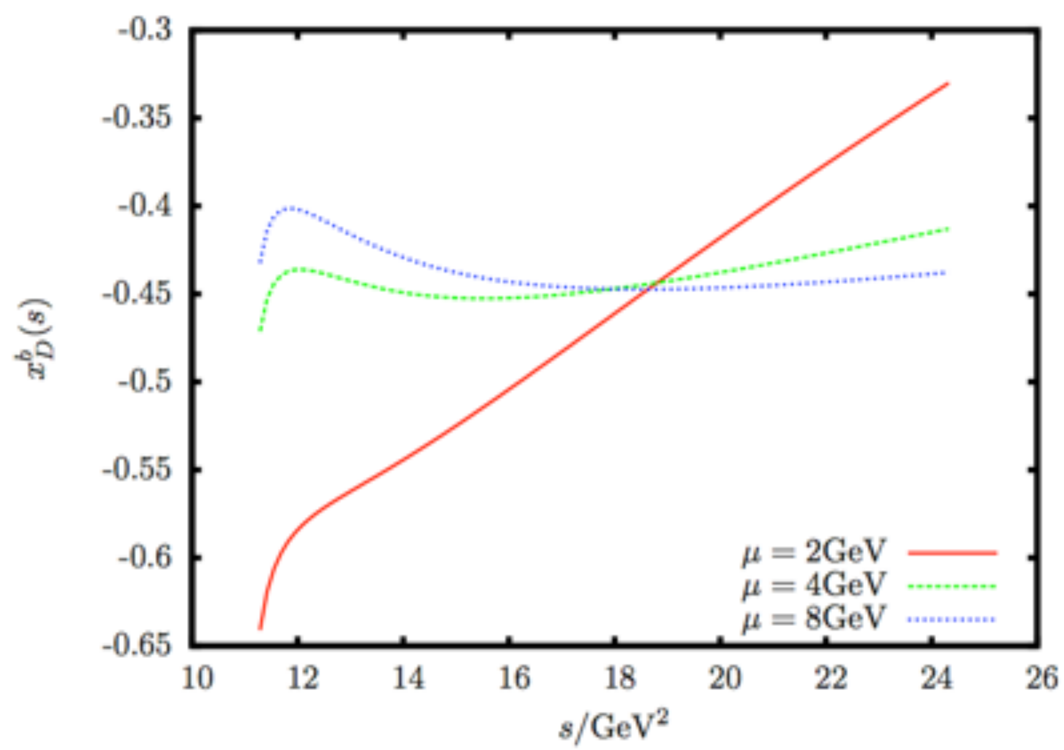

- can we trust partonic QCD?
  no not locally ⇒ quark-hadron duality …

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$$H^{V,X}(s) = H^{V,X}(s_0) + \frac{(s-s_0)}{2\pi i} \int_{s_{J/\Psi}}^{\infty} \frac{dt}{t-s_0} \frac{\mathrm{Disc}[H^{V,X}](t)}{t-s-i0} \;, \quad X \in \{\mathrm{fac}, \mathrm{cor}\} \;.$$

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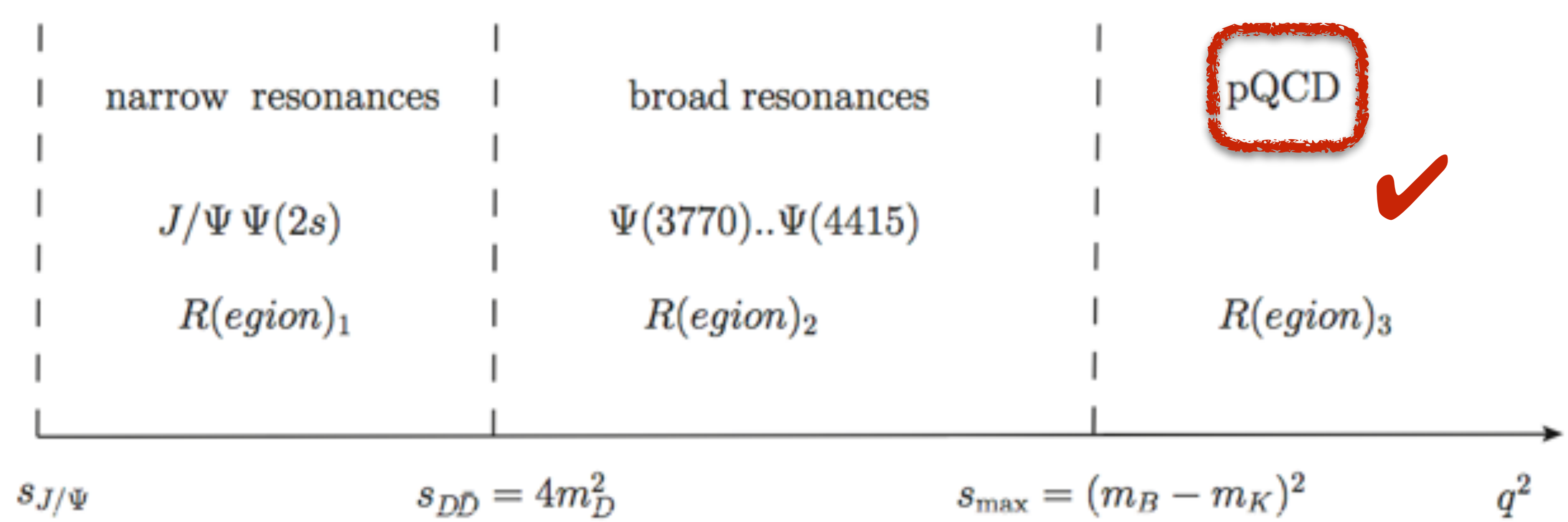

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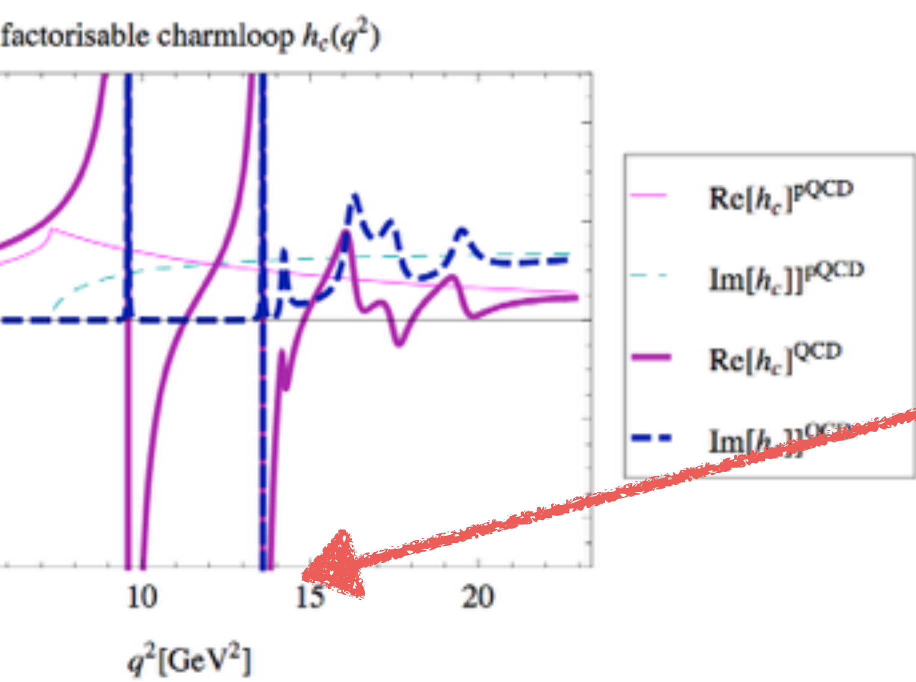

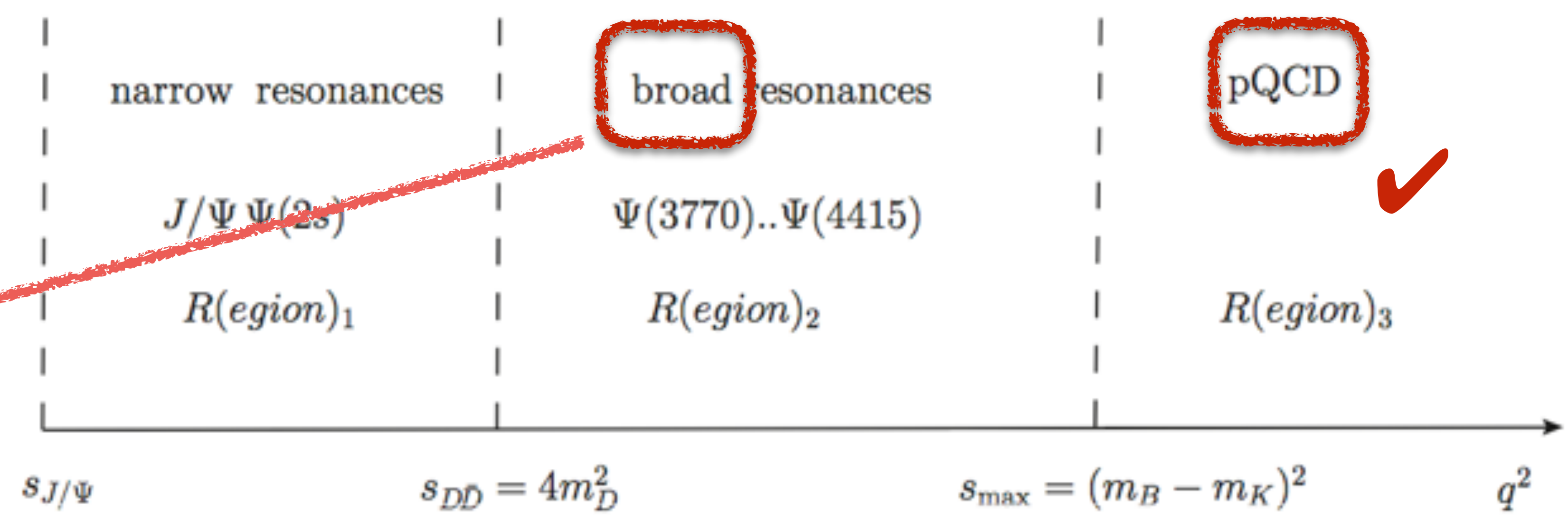

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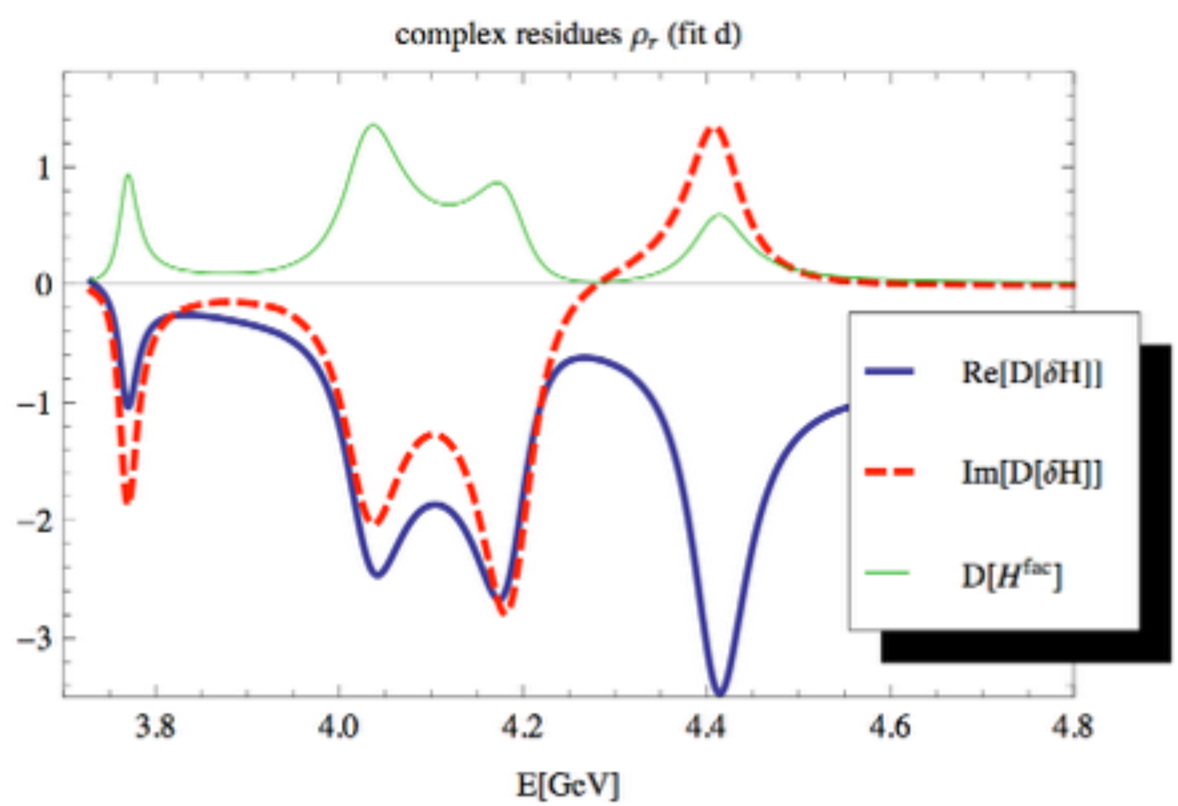

- fit d) effect of cancellations 20%
  instead of 350% its 280%
  ⇒ that's not it!

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- how to **improve**:

  1) measure residues (phases) of all resonances in $B \to (\Psi \to ll)K^{(*)}$
  2) perform fits to various fine binned observables (more robust results)
  3) if 1) is successful $\Rightarrow$ spectral information to reconstruct

  charm amplitude fully non-perturbative from DR (fit subtraction constant)

remains to be seen whether 3) is realistic

E. possible impact on low q^2

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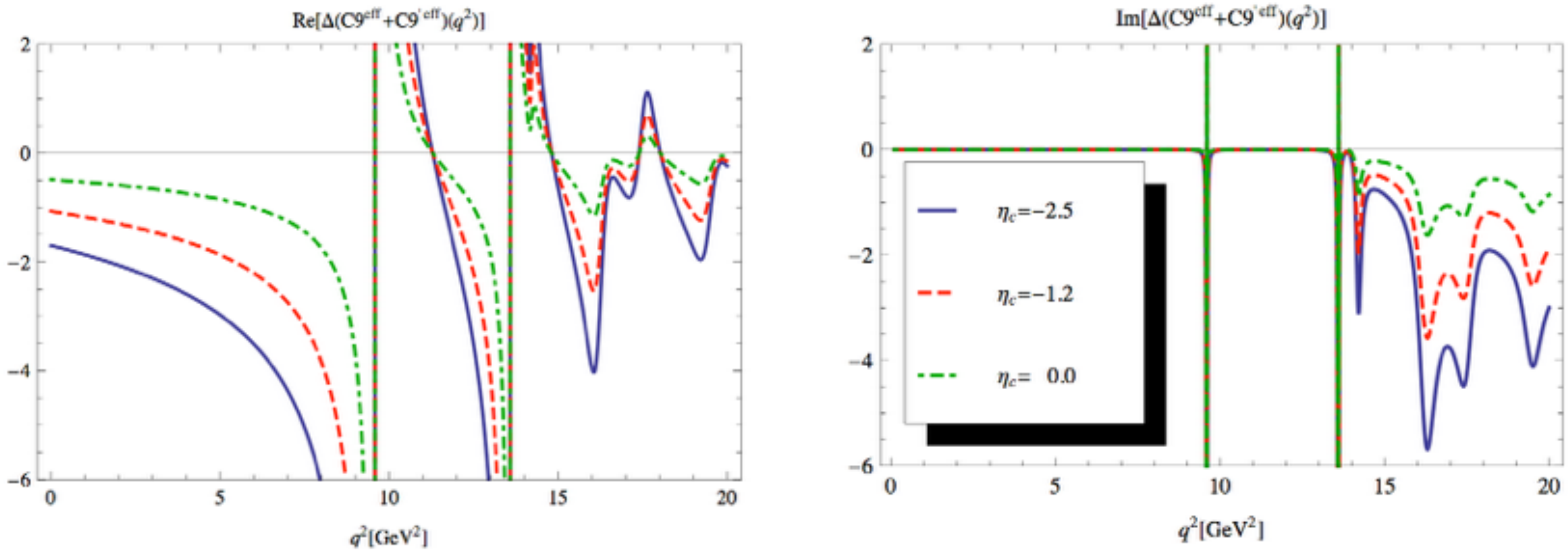
- fit b) scaling of factorisable part (at least close) 4D field theory
  and describes data astonishingly well.
  > shall take this as a model to assess the size of the effect

effect on $C_{9+} = C_9 + C_9'$ ....



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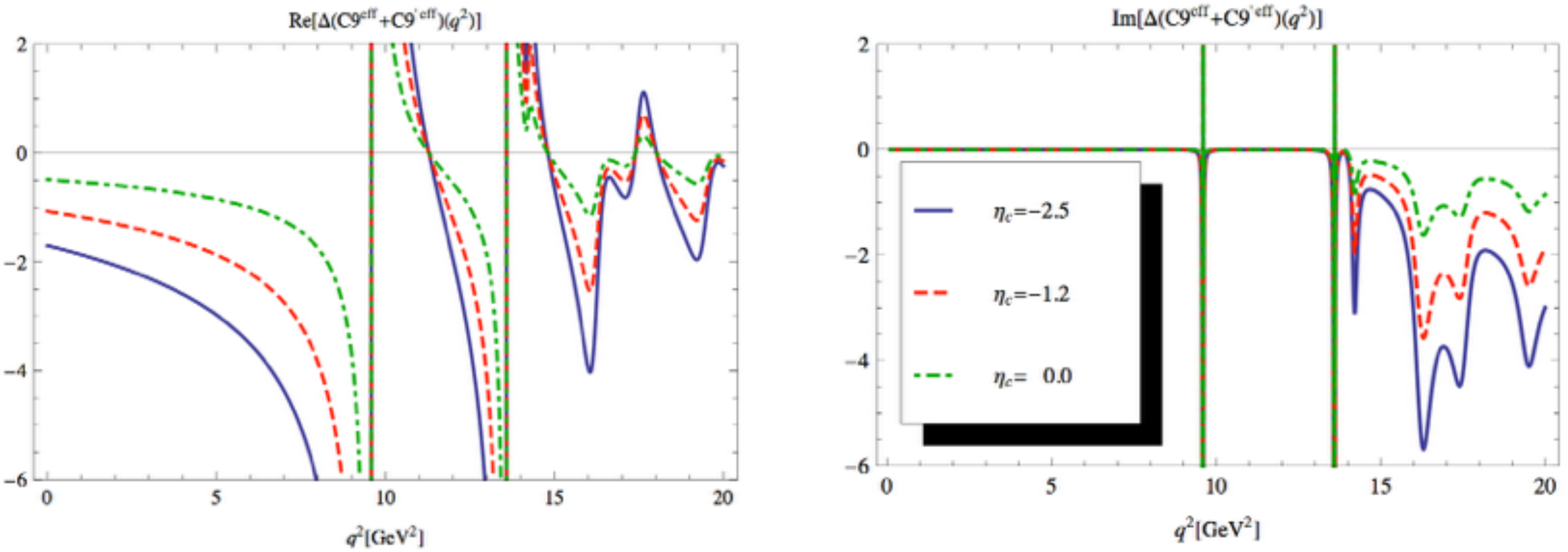

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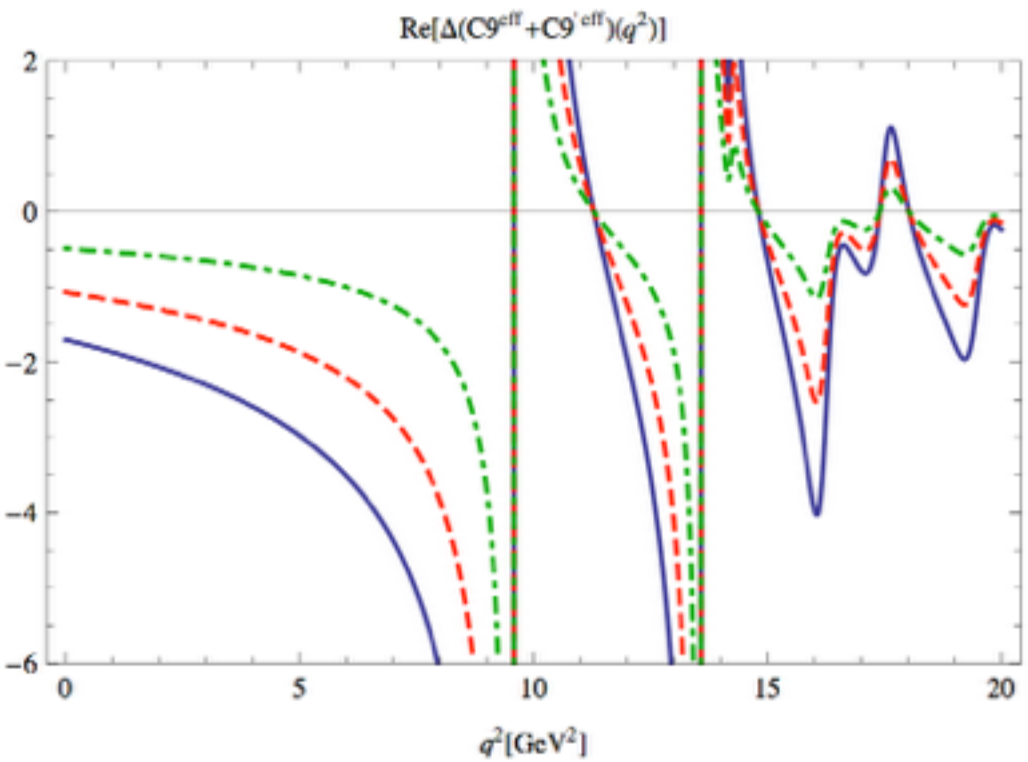

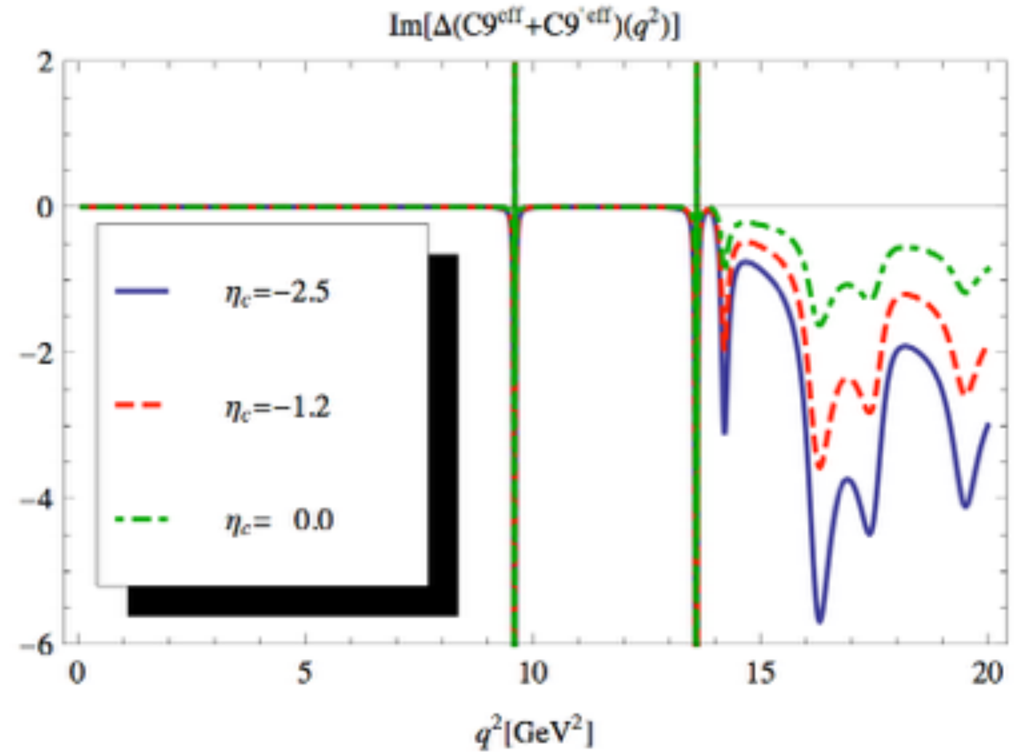

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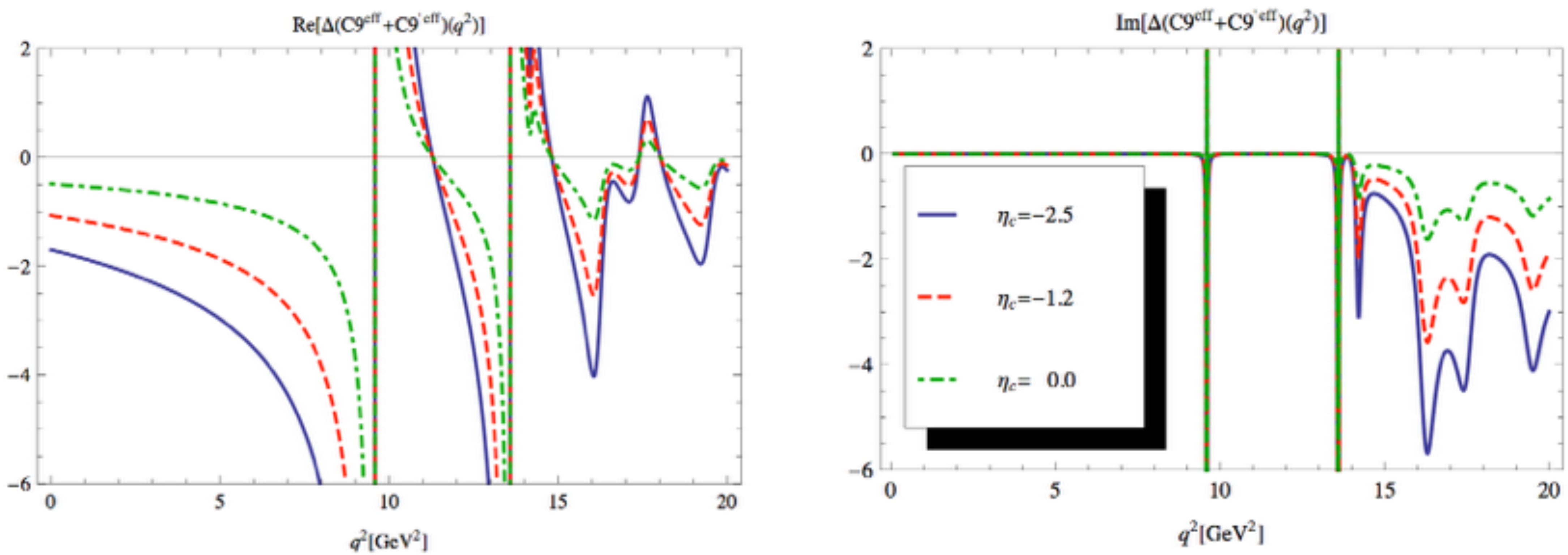

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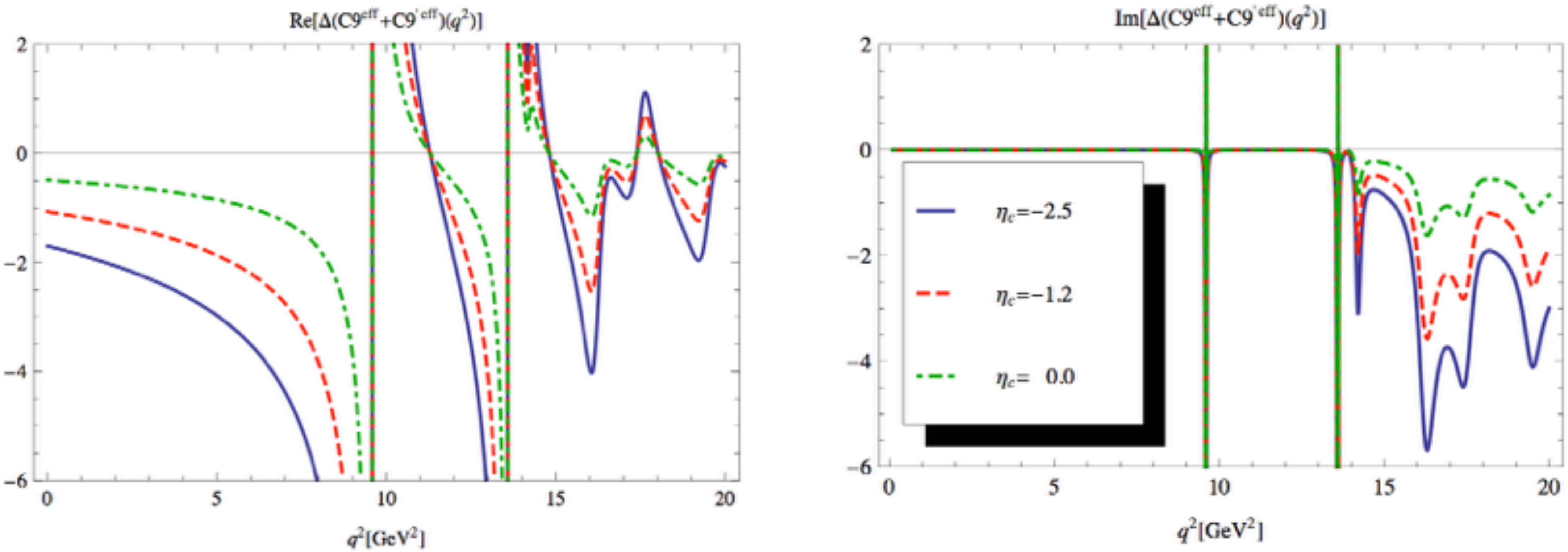

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- hence charm effect good omen but on top more pronounced towards charm resonances and this is what is needed to account for LHCb-results

choose three scenarios

Observable	q^2-bin	LHCb	SM	$\eta_c =$ -1.25(1, 1)	-2.5(0, 1)	-2.5(1, 0)
$\langle P_2\rangle$	[1.00, 6.00]	$0.33^{+0.11}_{-0.12}$	0.0085	0.16	-0.013	0.33
$\langle P_2\rangle$	[2.00, 4.30]	$0.50^{+0.00}_{-0.07}$	0.15	0.25	0.067	0.39
$\langle P_2\rangle$	[4.30, 8.68]	$-0.25^{+0.07}_{-0.08}$	-0.44	-0.05	-0.23	0.29
$\langle P_2\rangle$	[14.18, 16.00]	$-0.50^{+0.03}_{-0.00}$	-0.42	-0.39	-0.36	-0.36
$\langle P_2\rangle$	[16.00, 19.00]	$-0.32^{+0.08}_{-0.08}$	-0.34	-0.31	-0.25	-0.25
$\langle P'_4\rangle$	[1.00, 6.00]	$0.58^{+0.32}_{-0.36}$	0.57	0.66	0.80	0.64
$\langle P'_4\rangle$	[2.00, 4.30]	$0.74^{+0.54}_{-0.60}$	0.61	0.69	0.82	0.67
$\langle P'_4\rangle$	[4.30, 8.68]	$1.18^{+0.26}_{-0.32}$	1.0	1.0	1.2	0.98
$\langle P'_4\rangle$	[14.18, 16.00]	$-0.18^{+0.54}_{-0.70}$	1.2	1.2	1.2	1.2
$\langle P'_4\rangle$	[16.00, 19.00]	$0.70^{+0.44}_{-0.52}$	1.3	1.3	1.3	1.3
$\langle P'_5\rangle$	[1.00, 6.00]	$0.21^{+0.20}_{-0.21}$	-0.44	-0.15	-0.33	0.17
$\langle P'_5\rangle$	[2.00, 4.30]	$0.29^{+0.40}_{-0.39}$	-0.47	-0.17	-0.36	0.13
$\langle P'_5\rangle$	[4.30, 8.68]	$-0.19^{+0.16}_{-0.16}$	-0.88	-0.31	-0.44	0.26
$\langle P'_5\rangle$	[14.18, 16.00]	$-0.79^{+0.27}_{-0.22}$	-0.70	-0.66	-0.59	-0.61
$\langle P'_5\rangle$	[16.00, 19.00]	$-0.60^{+0.21}_{-0.18}$	-0.53	-0.49	-0.39	-0.38
$\langle A_{\rm FB}\rangle$	[1.00, 6.00]	$0.17^{+0.06}_{-0.06}$	0.0026	0.054	-0.0033	0.14
$\langle A_{\rm FB}\rangle$	[2.00, 4.30]	$0.20^{+0.08}_{-0.08}$	0.034	0.069	0.014	0.15
$\langle A_{\rm FB}\rangle$	[4.30, 8.68]	$-0.16^{+0.05}_{-0.06}$	-0.21	-0.025	-0.098	0.19
$\langle A_{\rm FB}\rangle$	[14.18, 16.00]	$-0.51^{+0.05}_{-0.07}$	-0.43	-0.40	-0.36	-0.37
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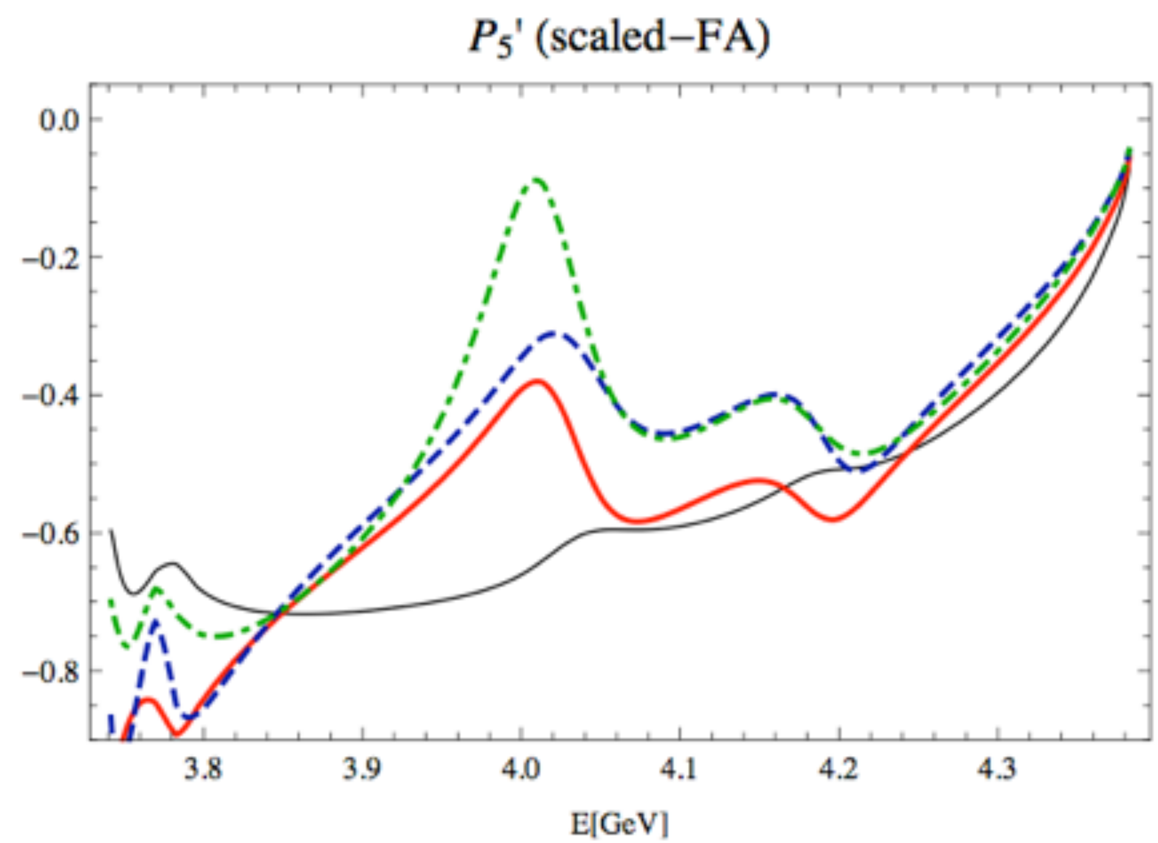
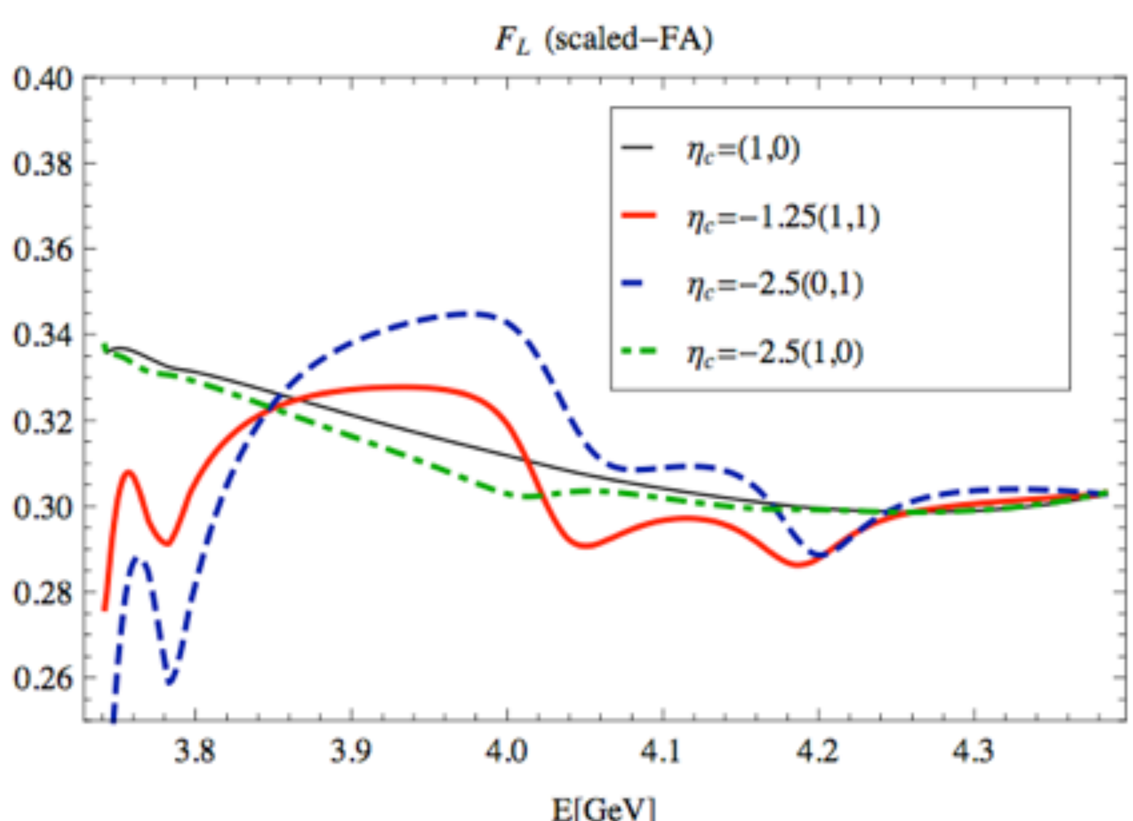
- inspection tells us that mix between scenario (i) and (iii) best for data …

what fine binning can do B→K*ll angular observables



noticeable effects



moderate effects
(at least when universal)

F. discussion and conclusions

- large effects in broad charm resonances
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new physics in form $\mathcal{O}_{\Gamma_1\Gamma_2} = \bar{c}\Gamma_1 c \bar{b}\Gamma_2 s$

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- of course there can still be ΔC_9 short distance new physics
  (power corrections cannot explain central values ….)

what to do ....

- high q^2: measure C_9 and C_9'-part of effects as well as polarisation non-universality (from non-factorisable effects)

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- need to address question of strong phases in $B \to \Psi K^{(*)}$
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- low q^2: makes sense to do SM predictions well below J/Ψ-resonances exciting times — close collaboration between theorists and experimentalists seems the way to go (at least to me) *exciting times – close collaboration between theorists and experimentalists seems the way to go (at least to me)*

Final words:
"charm (can) explain(s) $B \to K^* ll$-anomalies but charm doesn't explain itself"

BACKUP SLIDES

B→charmonium $K^{(*)}$ et al

- B→Ψ$K^{(*)}$ has got notorious reputation (also with factorisation)
- the setting we have with duality interval is on much safer grounds
  but what we have found is that not only factorisation is not very precise
  but that the sign of factorisation is the wrong one (indirect analysis)
- it seems the problem in b->css physics has become **much worse**
  with this new analysis
- motivates reinvestigation of b->css physics in general

Fit BESII-data (more details)

r	m_r[GeV]	Γ_r[MeV]	$^{2s+1}L_J$
J/Ψ	3.097	0.0934(21)	$^3S_1$
$\Psi(2S)$	3.686	0.337(13)	$^3S_1$
$\Psi(3370)$	3.771	23.3	$^3D_1$
$\Psi(4040)$	4.039	76.2	$^3S_1$
$\Psi(4160)$	4.192	73.5	$^3D_1$
$\Psi(4415)$	4.415	78.5	$^3S_1$

$$R_{\rm fit}(s) = R_{\rm res}(s) + R_{\rm con}(s)$$

$$R_{\rm con}(s) = R_{uds} + (1-z)(\Delta R_c + za)\ , \quad \Delta R_c \equiv R_{udsc} - R_{uds}\ ;$$

$$R_{\rm res}(s) = \frac{9}{\alpha^2}\sum_f |\sum_r T^{r\to f}(s)|^2$$

phase at production of resonance r

$$T^{r\to f}(s) = \frac{m_r\sqrt{\Gamma^{r\to e^+e^-}\Gamma^{r\to f}(s)}}{s - m_r^2 + i m_r \Gamma_r(s)} e^{i\delta_r}\ .$$

- Breit-Wigner ansatz with energy dependent width and interference effects