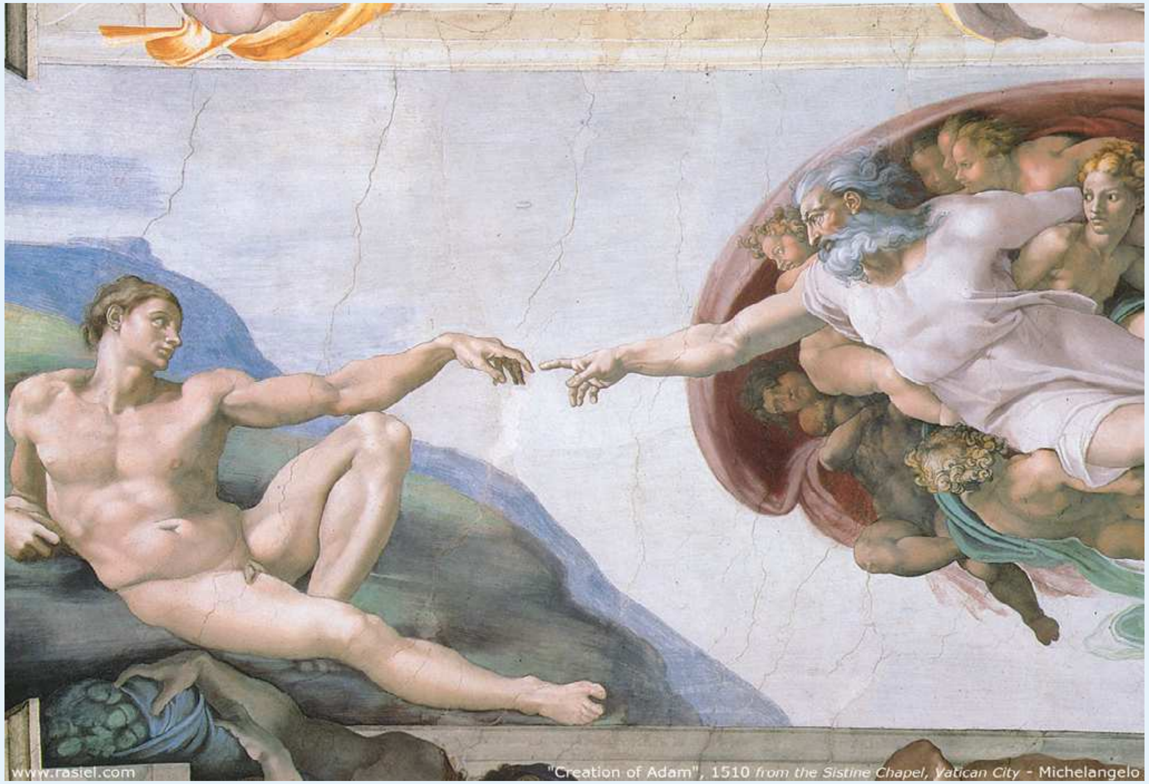
B-Mixing



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Outline

- Mixing in the standard model
  - Introduction
  - Mass difference
  - Decay rate difference and the HQE
- Mixing beyond the standard model
  - New physics in M_{12}
  - New physics in $\Delta\Gamma_d$
- Higher precision for M_{12} and Γ_{12}
- Conclusion

Introduction

Mixing is a common effect in particle physics!
(interaction eigenstate $\neq$ mass eigenstate)

- “Unification” of electromagnetic and weak interaction
- Neutrino oscillations
- Quark mixing via the CKM matrix
- Mixing of neutral mesons - macroscopic quantum effect!

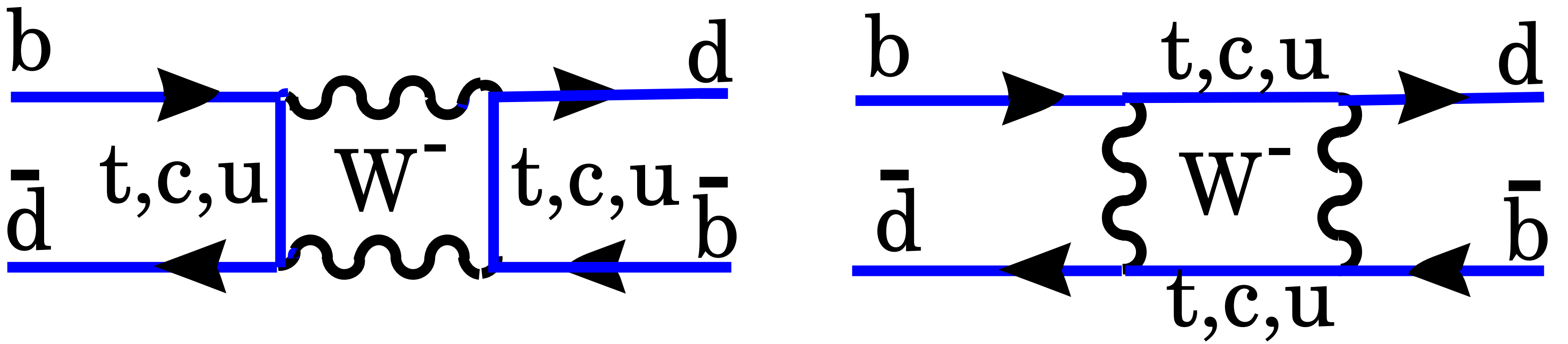
It was observed in

- K^0-system: 1950s (see text books, regeneration...)
- B_d-system: 1986 ΔM_d; ??? $\Delta\Gamma_d$
- B_s-system: 2006 ΔM_s; 2012 $\Delta\Gamma_s$
- D^0-system: 2007, 2012 $\Delta M_D, \ \Delta\Gamma_D$

Strongly suppressed in the SM (higher order in weak interaction)
New physics effects might be of comparable size

?Is QCD under control?

Introduction



$|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \text{arg}(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference:** $\Delta M := M_H - M_L \approx 2|M_{12}|$ **(off-shell)**
  $|M_{12}|$: heavy internal particles: t, SUSY, ...

- **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos\phi$ **(on-shell)**
  $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!

- **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to Xl\nu$ (*semi-leptonic*)

$$a_{sl} \quad \equiv \quad a_{fs} = \frac{\Gamma(\overline{B}_q(t) \to f) - \Gamma(B_q(t) \to \overline{f})}{\Gamma(\overline{B}_q(t) \to f) + \Gamma(B_q(t) \to \overline{f})} = \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin\phi$$

Mass difference ΔM

Calculating the box diagram with an internal top-quark yields

$$\boxed{M_{12,q} \quad = \quad \frac{G_F^2}{12\pi^2}(V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B}$$

- 1 loop calculation $S_0(x_t = m_t^2/M_W^2)$ **Inami, Lim, ’81**
- 2-loop perturbative QCD corrections $\hat{\eta}_B$ **Buras, Jamin, Weisz, ’90**
- Hadronic matrix element: $\frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q} = \langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B_q \rangle$

$$f_{B_s} \quad = \quad \left\{ \begin{array}{lll} 235 \pm 9 & 2+1+1 & \textbf{1311.2837: ETM '13} \\ 233 \pm 5 & 2+1 & \textbf{1311.0276: RBC/UKQCD '13} \\ 224 \pm 5 & 2+1+1 & \textbf{1302.2644: HPQCD '13} \\ 228 \pm 10 & 2+1 & \textbf{1202.4914: HPQCD '12} \\ 242.0 \pm 5.1 \pm 8.0 & 2+1 & \textbf{1112.3051: FNAL/MILC '11} \\ 225.0 \pm 2.9 \pm 2.9 & 2+1 & \textbf{1110.4510: HPQCD '11} \end{array} \right.$$

$$\Rightarrow f_{B_s} \quad = \quad 235 \pm 17 \quad ... \quad 224 \pm 5 \quad ?!?!?$$

$$B_{B_s} \quad = \quad 1.33 \pm 0.06 \quad \textbf{HPQCD '09} \quad 1.32 \pm 0.05 \quad \textbf{ETM '13}$$

Important bounds on the unitarity triangle and new physics

ΔM and $\Delta\Gamma$

- **Mass difference:** One Operator Product Expansion (OPE)

Theory **A.L., Nierste 1102.4274** vs. Experiment : **HFAG 14**

Theory	Experiment
$\Delta M_d = 0.543 \pm 0.091\ \text{ps}^{-1}$	$\Delta M_d = 0.510 \pm 0.003\ \text{ps}^{-1}$
$\Delta M_s = 17.30 \pm 2.6\ \text{ps}^{-1}$	$\Delta M_s = 17.761 \pm 0.022\ \text{ps}^{-1}$

  - Perfect agreement, still room for NP
  - Important bounds on the unitarity triangle and NP
  - **Dominant uncertainty = Lattice**

- **Decay rate difference:** Second OPE = Heavy Quark Expansion (HQE)

$$\Gamma_{12} = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^5 \left(\Gamma_5^{(0)} + \ldots\right) + \ldots$$

’96: Beneke, Buchalla; ’98: Beneke, Buchalla, Greub, A.L., Nierste;
’03: Beneke, Buchalla, A.L., Nierste; ’03: Ciuchini, Franco, Lubicz, Mescia, Tarantino;
’06; ’11: A.L., Nierste; ’07 Badin, Gabianni,Petrov

The Heavy Quark Expansion

HQE might be questionable - relies on quark hadron duality

Energy release is small ⇒ naive dim. estimate: series might not converge

- Mid 90's: Missing Charm puzzle $n_c^{\text{Exp.}} < n_c^{\text{SM}}$, semi leptonic branching ratio
- Mid 90's: Λ_b lifetime is too short, i.e. $\tau(\Lambda_b) \ll \tau(B_d) = 1.519$ ps
- before 2003: $\tau_{B_s}/\tau_{B_d} \approx 0.94 \neq 1$
- 2010/2011: dimuon asymmetry too large

Theory arguments for HQE

⇒ calculate corrections in all possible "directions", to test convergence

⇒ test reliability of HQE via lifetimes (no NP effects expected)

The Heavy Quark Expansion

(Almost) all discrepancies disappeared:

- ’12: $n_c^{2011\mathrm{PDG}} = 1.20 \pm 0.06$ vs. $n_c^{\mathrm{SM}} = 1.23 \pm 0.08$ **Krinner, A.L., Rauh 1305.5390**
- HFAG ’03 $\tau_{\Lambda_b} = 1.229 \pm 0.080$ ps^{-1} $\longrightarrow$ HFAG ’14 $\tau_{\Lambda_b} = 1.451 \pm 0.024$ ps^{-1}

  Shift by 2.8σ!
- **HFAG 2014:** $\tau_{B_s}/\tau_{B_d} = 0.995 \pm 0.006$
- 2010/2011: dimuon asymmetry too large — Test Γ_{12} with $\Delta\Gamma_s$!

Theory arguments for HQE

$\Rightarrow$ calculate corrections in all possible “directions”, to test convergence

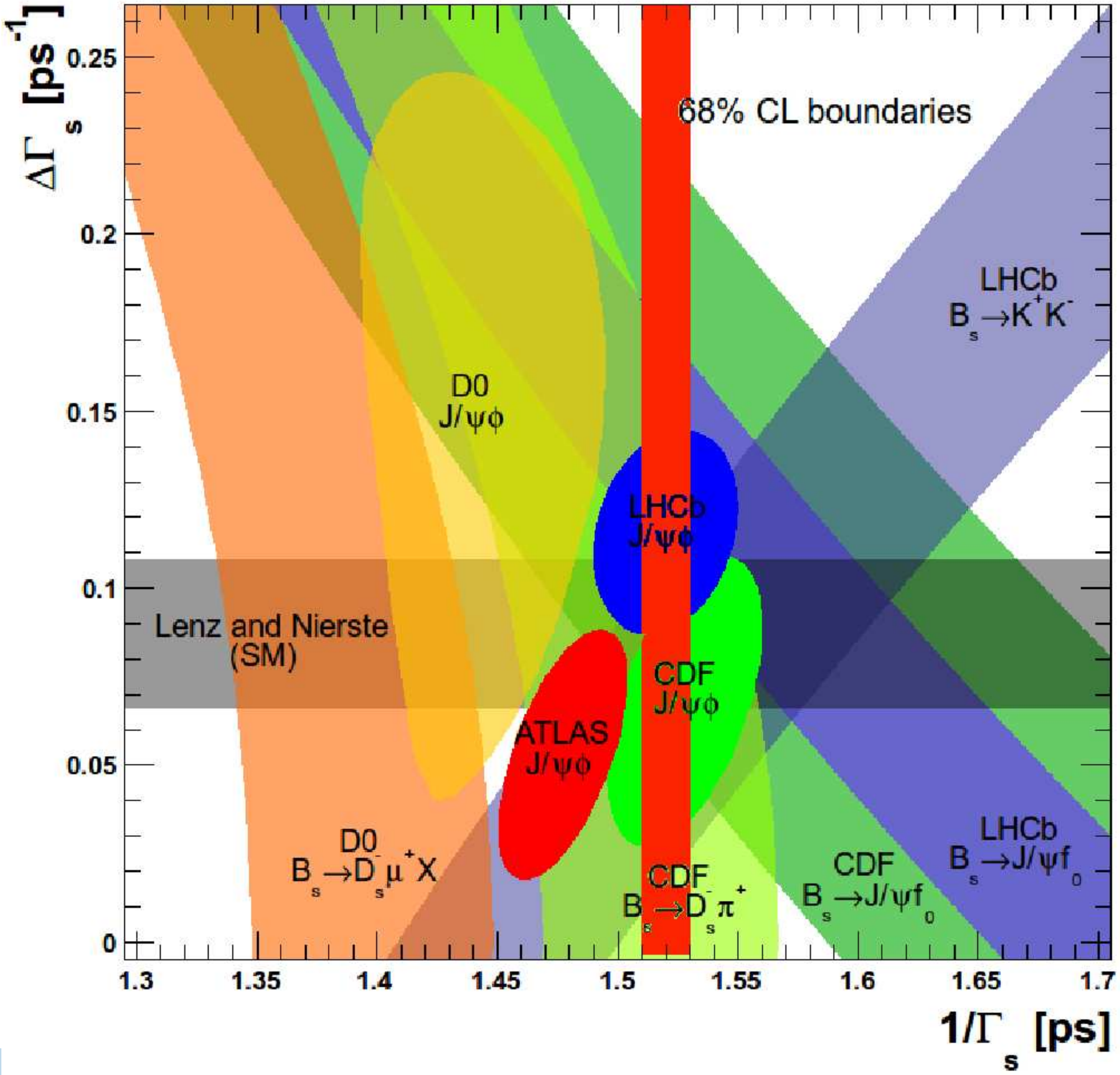
$$\boxed{\begin{aligned} \Delta\Gamma_s &= \Delta\Gamma_s^0 \left(1 + \delta^{\mathrm{Lattice}} + \delta^{\mathrm{QCD}} + \delta^{\mathrm{HQE}}\right) \\ &= 0.142\ \mathrm{ps}^{-1} \left(1 - 0.14 - 0.06 - 0.19\right) \end{aligned}}$$

$\Rightarrow$ looks ok!

$\Rightarrow$ test reliability of HQE via lifetimes (no NP effects expected)
$\Rightarrow \tau(B^+)/\tau(B_d)$ experiment and theory agree within hadronic uncertainties

Dominant uncertainties: NLO-QCD + Lattice

Finally $\Delta\Gamma_s$ is measured!



Thanks to Roger Jones

Finally $\Delta\Gamma_s$ is measured!

Finally $\Delta\Gamma_s$ is measured! E.g. from $B_s \to J/\psi\phi$

LHCb Moriond 2012, 2013; ATLAS; CDF; DO

$$\begin{aligned} \Delta\Gamma_s^{\rm Exp} &= (0.091 \pm 0.008)\,\text{ps}^{-1} \\ \Delta\Gamma_s^{\rm SM} &= (0.087 \pm 0.021)\,\text{ps}^{-1} \end{aligned}$$

HFAG 2014

A.L.,Nierste 1102.4274

Cancellation of non-perturbative uncertainties in ratios

$$\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm Exp} / \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm SM} = 1.02 \pm 0.09 \pm 0.19$$

Dominant uncertainty = NNLO-QCD + Lattice

Test of our theoretical Understanding

Most important lesson?: HQE works also for Γ_{12}!

- HQE works for the decay $b \to c\bar{c}s$
- Energy release $M_{B_s} - 2M_{D_s} \approx 1.4$ GeV (momentum release: 3.5 GeV)
- Violation quark hadron duality: Theoreticians were fighting for 35 years

How precise does it work? 20%? 10%?

Still more accurate data needed!

LHCb, ATLAS, CMS?, TeVatron, Super-Belle

1. Apply HQE also to $b \to c\bar{c}s$ transitions
2. Apply HQE to quantities that are sensitive to NP
3. Apply HQE also to quantities in the charm system?

Search for New Physics in B-mixing

HQE works! SM predictions: **A.L., U. Nierste, 1102.4274; A.L. 1108.1218**

$$
\begin{aligned}
a_{fs}^s &= (1.9 \pm 0.3) \cdot 10^{-5} & \phi_s &= 0.22^\circ \pm 0.06^\circ \\
a_{fs}^d &= -(4.1 \pm 0.6) \cdot 10^{-4} & \phi_d &= -4.3^\circ \pm 1.4^\circ \\
A_{sl}^b = 0.406 a_{sl}^s + 0.594 a_{sl}^d &= (-2.3 \pm 0.4) \cdot 10^{-4} \\
\left|\frac{\Delta\Gamma_d}{\Gamma_d}\right| &= (4.2 \pm 0.8) \cdot 10^{-3}
\end{aligned}
$$

Older experimental bounds:

$$\phi_s = -51.6^\circ \pm 12^\circ$$ **(A.L., Nierste, CKMfitter, 1008.1593)**

$$\left|\frac{\Delta\Gamma_d}{\Gamma_d}\right| = (1 \pm 10) \cdot 10^{-3}$$ **(HFAG 14)**

$$A_{sl}^b = -(7.87 \pm 1.72 \pm 0.93) \cdot 10^{-3}$$ **(D0,1106.6308)**

$A_{sl}^b(Exp.)/A_{sl}^b(Theory) = \mathbf{34}$ $3.9 - \sigma$**-effect**

Search for New Physics in B-Mixing

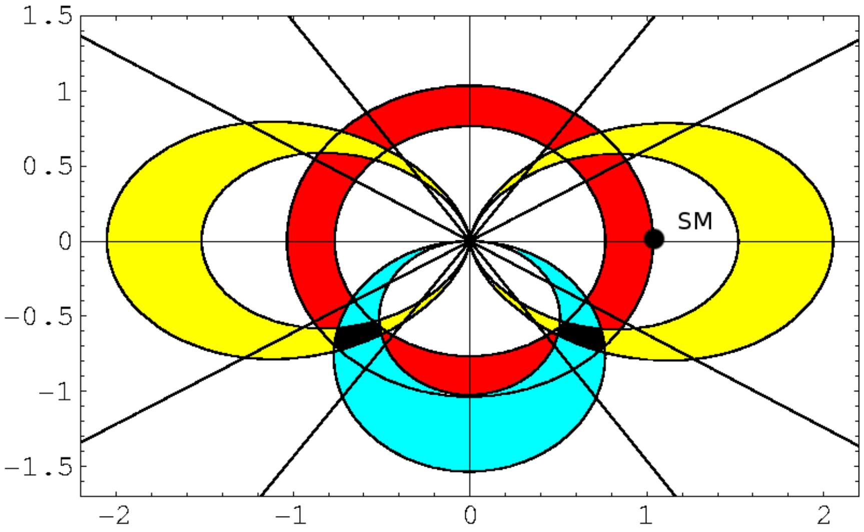
Model independent analysis: **A.L., Nierste, ’06**

$$\Gamma_{12,s} = \Gamma_{12,s}^{\rm SM}\,, \qquad M_{12,s} = M_{12,s}^{\rm SM}\cdot \Delta_s\,; \qquad \Delta_s = |\Delta_s| e^{i\phi_s^\Delta}$$

$$\begin{aligned}
\Delta M_s &= 2|M_{12,s}^{\rm SM}|\cdot|\Delta_s| \\
\Delta\Gamma_s &= 2|\Gamma_{12,s}|\cdot\cos\left(\phi_s^{\rm SM}+\phi_s^\Delta\right) \\
\frac{\Delta\Gamma_s}{\Delta M_s} &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\rm SM}|}\cdot\frac{\cos\left(\phi_s^{\rm SM}+\phi_s^\Delta\right)}{|\Delta_s|} \\
a_{fs}^s &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\rm SM}|}\cdot\frac{\sin\left(\phi_s^{\rm SM}+\phi_s^\Delta\right)}{|\Delta_s|}
\end{aligned}$$

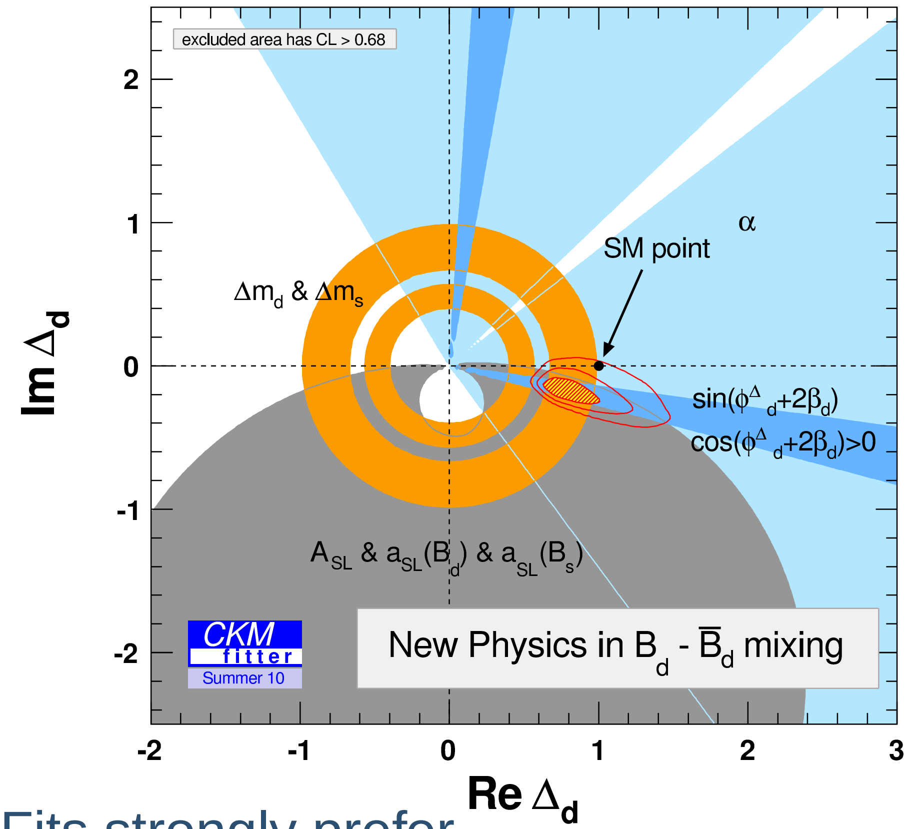
$$\sin(\phi_s^{\rm SM}) \approx 1/240$$

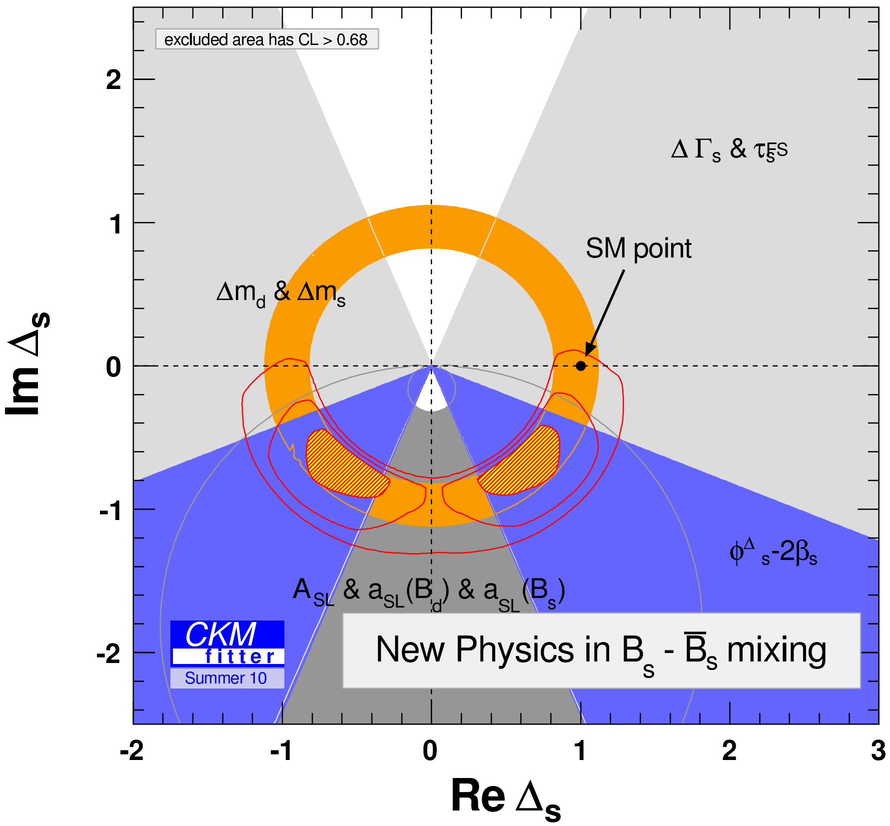
For $|\Delta_s| = 0.9$ and $\phi_s^\Delta = -\pi/4$ one gets the following bounds in the **complex Δ-plane**:



Search for New Physics in B-Mixing

Combine all data before summer 2010 and **neglect penguins**
fit of Δ_d and Δ_s **A.L.. Nierste. CKMfitter 1008.1593**





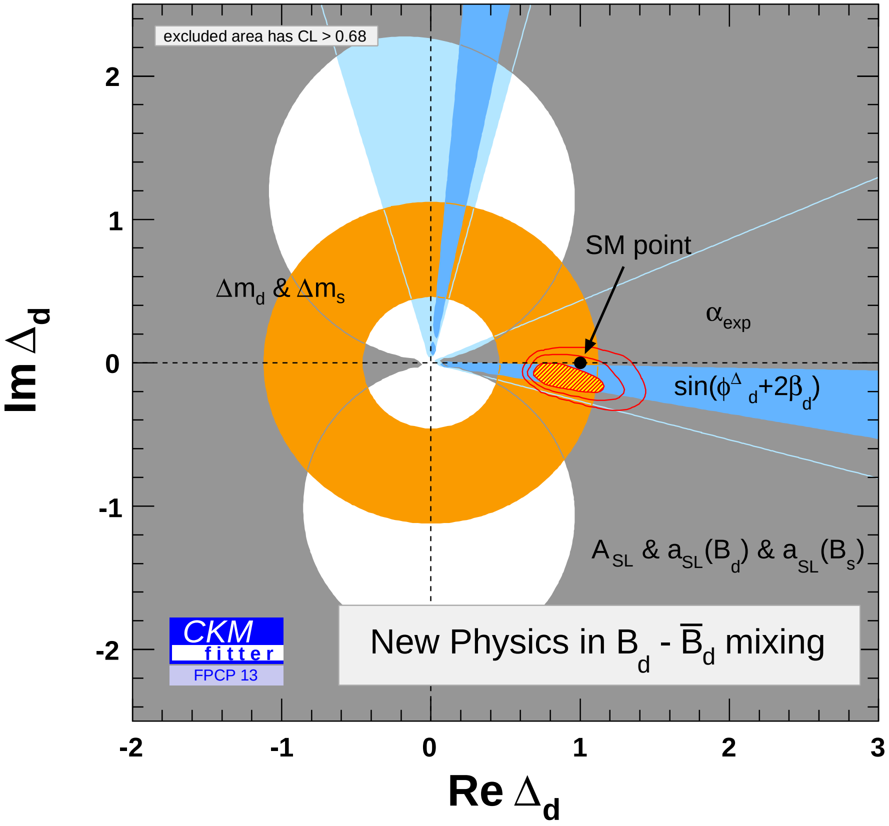
Fits strongly prefer

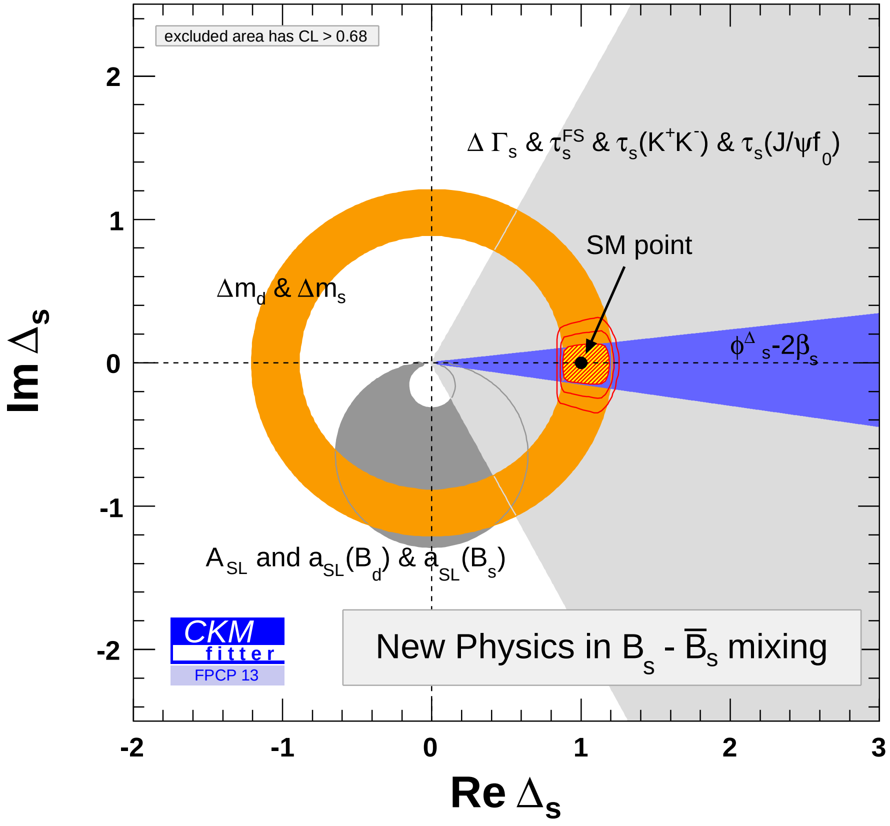
- **large new physics effects in the B_s-system**
- **some new physics effects in the B_d-system**

Search for New Physics in B-Mixing

Combine all data till FPCP 2013 and neglect penguins

fit of Δ_d and Δ_s; update of A.L., Nierste, CKMfitter 1203.0238v2





- **SM seems to be perfect**
- **Still quite some room for NP**

Search for NP in B-Mixing: A_{sl}^b?

$$A_{sl}^b \approx \frac{1}{2}\frac{|\Gamma_{12,d}|}{|M_{12,d}^{\rm SM}|}\cdot\frac{\sin\left(\phi_d^{\rm SM}+\phi_d^{\Delta}\right)}{|\Delta_d|}+\frac{1}{2}\frac{|\Gamma_{12,s}|}{|M_{12,s}^{\rm SM}|}\cdot\frac{\sin\left(\phi_s^{\rm SM}+\phi_s^{\Delta}\right)}{|\Delta_s|}$$

BUT: The experimental number is larger than “possible”! A.L. 1205.1444, 1106.3200

1. Huge (= several 100 $\%$) duality violations in Γ_{12}^s? $\rightarrow$ NO! see $\Delta\Gamma_s$
2. Huge NP in Γ_{12}? $\rightarrow$ NO! this also affects observables like $\tau_{B_s}/\tau_{B_d}, n_c, ...$
   But still some sizable NP possible - investigate e.g. n_c **Bobeth, Haisch 1109.1826**
3. Look at experimental side
   - Statistical fluctuation - **D0 update 1310.0447**
   - Cross-check via individual asymmetries - **LHCb, D0, BaBar**
     $\Rightarrow$ **consistent with SM, but not yet in conflict with** A_{sl}^b
   - Some systematics neglected - **Borissov, Hoeneisen 1303.0175**
     Discrepancy still more than 3σ - **also dependence on** $\Delta\Gamma_d$

$\Rightarrow$ A_{sl}^b **points towards effects in** a_{sl}^d, a_{sl}^s **and** $\Delta\Gamma_d$ **- look also somewhere else**

Search for NP in B-Mixing: A_{sl}^b?

- New measurements for the individual semi leptonic CP asymmetries

$$\begin{aligned} a_{sl}^s &= -0.06 \pm 0.50 \pm 0.36\% \\ a_{sl}^s &= -1.12 \pm 0.74 \pm 0.17\% \\ a_{sl}^d &= 0.68 \pm 0.45 \pm 0.14\% \\ a_{sl}^d &= 0.06 \pm 0.17^{+0.38}_{-0.32}\% \end{aligned}$$

LHCb 1308.1048
D0 1207.1769
D0 1208.5813
BaBar 1305.1575

All numbers are consistent with the SM
(no confirmation of large new physics effects)
but also consistent with the value of the dimuon asymmetry

more data urgently needed

- New interpretation of the dimuon asymmetry **Borissov, Hoeneisen 1303.0175**

$$A_{sl}^b = C_d a_{sl}^d + C_s a_{sl}^s + C_\Gamma \frac{\Delta\Gamma_d}{\Gamma_d}$$

There is still sizable space for NP in $\Delta\Gamma_d$

New physics in $\Delta\Gamma_d$

- $\Delta\Gamma_s$ cannot be enhanced dramatically by new physics - **Bobeth, Haisch 2011**
- $\Delta\Gamma_d$ could in principle be enhanced dramatically - **Bobeth, Haisch, A.L., Pecjak, Tetlalmatzi-Xolocotzi 2014**

Comparison

- $\Delta\Gamma_s$ **dominated by** $b \to c\bar{c}s$: $B(b \to c\bar{c}s) = (23.7 \pm 1.3)\%$ **Krinner, A.L., Rauh 2013**
- $\Delta\Gamma_d$ **dominated by** $b \to c\bar{c}d$: $B(b \to c\bar{c}d) = (1.31 \pm 0.07)\%$ **Krinner, A.L., Rauh 2013**
- $\Delta\Gamma_s$ is completely dominated by $b \to c\bar{c}s$, $\Delta\Gamma_d$ has also sizable contributions from $b \to c\bar{u}d$ and $b \to u\bar{u}d$, which **cancel** to some extent

Enhancement

- Violations of CKM duality
- New $bd\tau\tau$ operators
- New physics in current-current operators Q_1 and Q_2

Search for enhanced $b \to d, s\tau\tau$ transitions I

A class of (almost) invisible decays

- $b \to s\tau\tau$ can enhance $\Delta\Gamma_s$ and a_{sl}^s. It is constrained by
  - $B_s \to \tau\tau < 2.7\%$ **indirect** from $\tau(B_s)/\tau(B_d)$
  - $B \to X_s\tau\tau < 2.7\%$ **indirect** from $\tau(B_s)/\tau(B_d)$
  - $B^+ \to K^+\tau\tau < 3.3 \cdot 10^{-3}$ **direct** from **BaBar 2010**

$\Rightarrow$ Enhancement of up to **35**$\%$ in $\Delta\Gamma_s$ possible ($\approx$ hadronic uncertainties)

$\Rightarrow$ **Improve bounds on $b \to s\tau\tau$!**

Bobeth, Haisch 2011

Γ_{12}^s is dominated by the CKM favoured decay $b \to c\bar{c}s$, a huge effect would be seen everywhere - Γ_{12}^d looks more promising

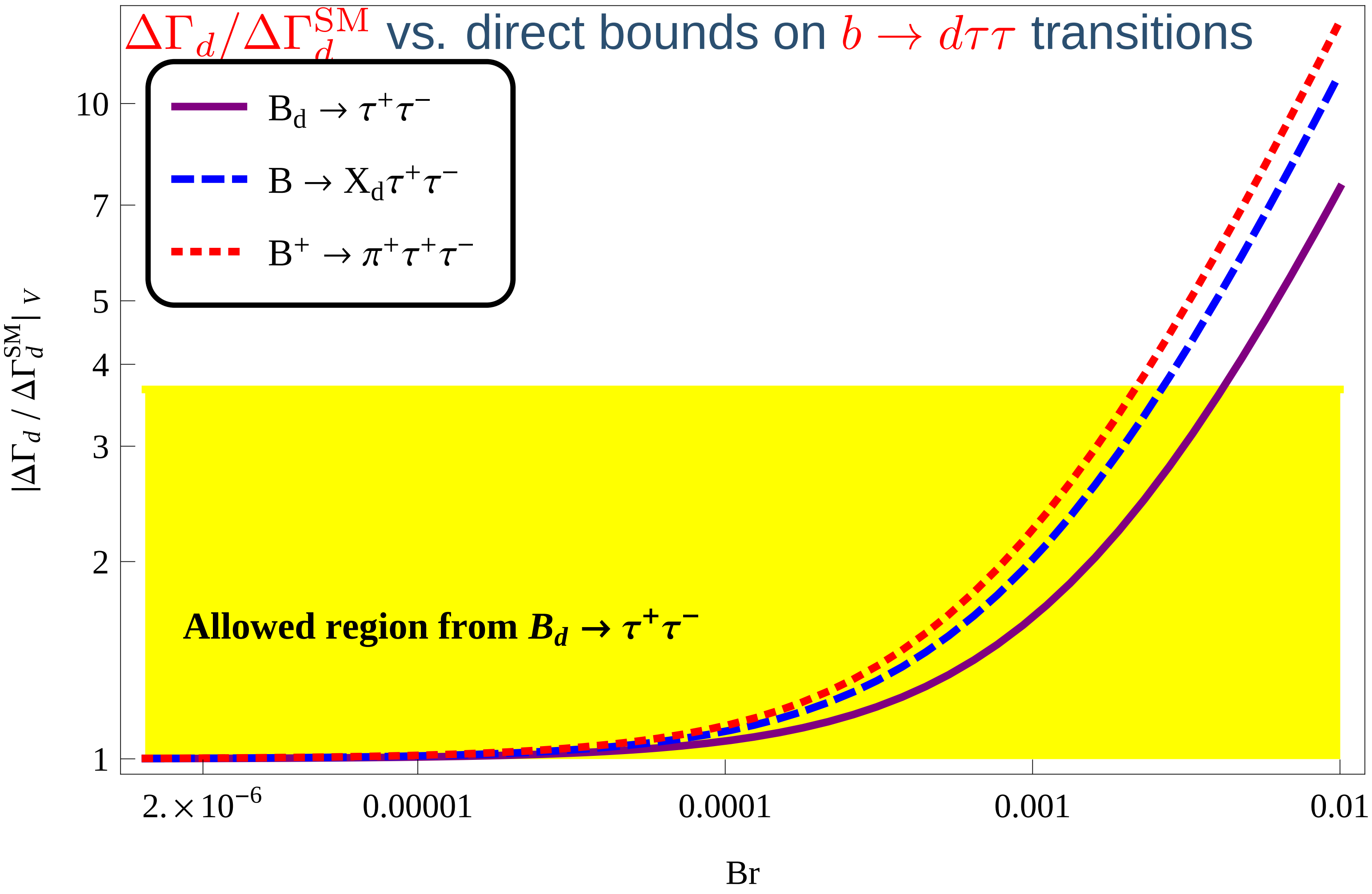
- $b \to d\tau\tau$ can enhance $\Delta\Gamma_d$ and a_{sl}^d. It is constrained by
  - $B_d \to \tau\tau < 4.1 \cdot 10^{-3}$ **direct** from **BaBar 2006**
  - $B \to X_d\tau\tau < 2.7\%$ **indirect from** $\tau(B_s)/\tau(B_d)$
  - $B^+ \to \pi^+\tau\tau < 2.7\%$ **indirect from** $\tau(B_s)/\tau(B_d)$

$\Rightarrow$ Enhancement of up to **270**$\%$ in $\Delta\Gamma_d$ possible

This might solve the dimuon asymmetry! $\Rightarrow$ Improve bounds on $b \to d\tau\tau$!

Bobeth, Haisch, AL, Pecjak, Tetlalmatzi-Xolocotzi, 2014

Search for enhanced $b \to d, s\tau\tau$ transitions II

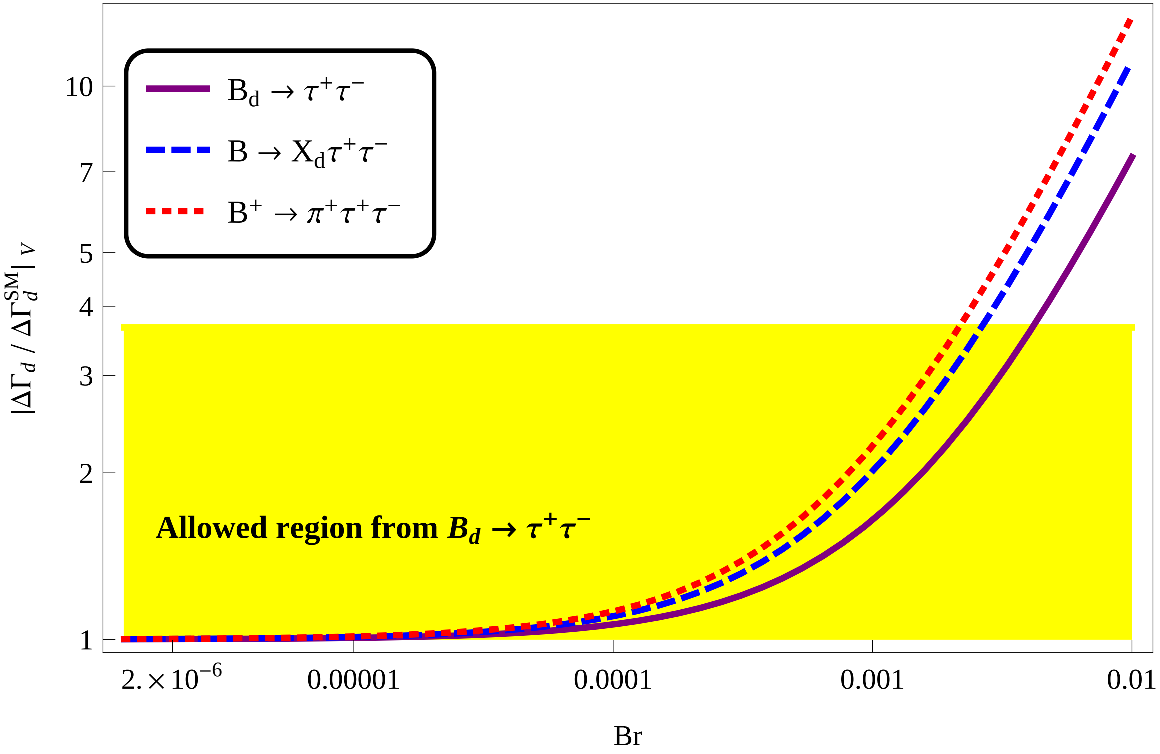


Bobeth, Haisch, AL, Pecjak, Tetlalmatzi-Xolocotz, 2014

New physics in $\Delta\Gamma_d$

New physics contributions to the current-current operators Q_1 and Q_2

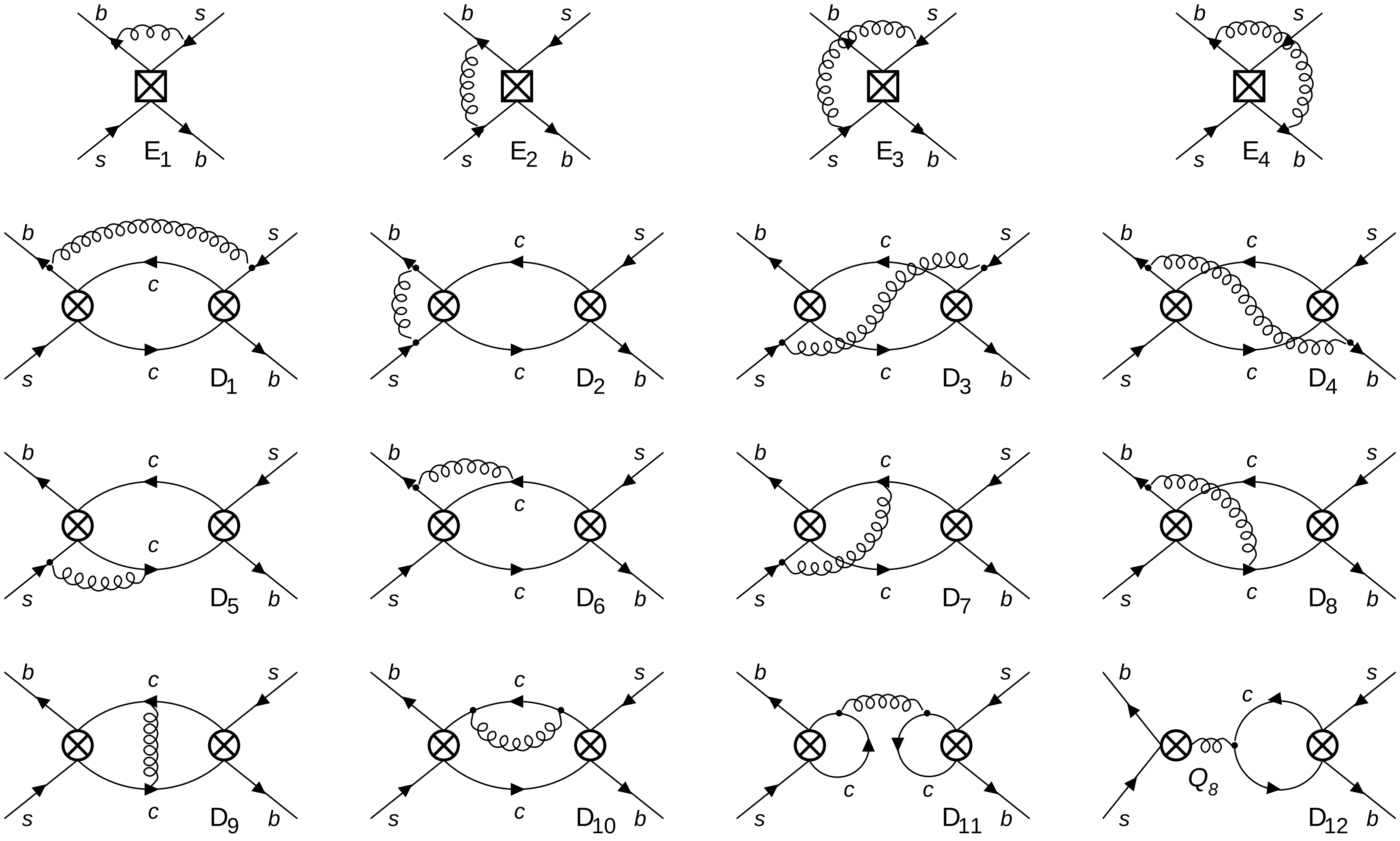
The decays $b \to c\bar{c}d, c\bar{u}d, u\bar{c}d, u\bar{u}d$ can get different new physics contributions to the Wilson coefficients (the SM-one is universal)



Constraints from $B \to \pi\pi, \rho\pi, \rho\rho, D^*\pi$, $B \to X_d\gamma$, $\sin 2\beta$ still allow enhancements of $\Delta\Gamma_d$ by **more than a factor of five**

Theory Prediction for $\Delta\Gamma_s$

Calculating the following diagrams



Theory Prediction for $\Delta\Gamma_s$

one gets Wilson coefficients of the following operators

$$\begin{aligned}
Q &= (\bar{b}_i s_i)_{V-A} \cdot (\bar{b}_j s_j)_{V-A} \\
\tilde{Q}_s &= (\bar{b}_i s_j)_{S-P} \cdot (\bar{b}_i s_j)_{S-P} \\
\langle \bar{B}_s | Q | B_s \rangle &= \frac{8}{3} f_{B_S}^2 M_{B_S}^2 B \\
\langle \bar{B}_s | \tilde{Q}_S | B_s \rangle &= \frac{1}{3} f_{B_S}^2 M_{B_S}^2 \tilde{B}_S' = \frac{1}{3} f_{B_S}^2 M_{B_S}^2 \frac{M_{B_S}^2}{(\bar{m}_b + \bar{m}_s)^2} \tilde{B}_S
\end{aligned}$$

f_{B_s}, B and $\tilde{B}_S$ have to be determined non-perturbatively!

Theory Prediction for $\Delta\Gamma_s$

Expanding also in the small s momenta one get contributions of dimension 7

$$
\begin{aligned}
R_0 &= Q_s + \tilde{Q}_S + \frac{1}{2} Q \\
R_1 &= \frac{m_s}{m_b} (\bar{b}_i s_i)_{S-P} (\bar{b}_j s_j)_{S+P} \\
R_2 &= \frac{1}{m_b^2} (\bar{b}_i \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho s_i)(\bar{b}_j \gamma_\mu (1-\gamma_5) s_j) \\
R_3 &= \frac{1}{m_b^2} (\bar{b}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho s_i)(\bar{b}_j (1-\gamma_5) s_j) \\
\tilde{R}_i &= \tilde{R}_i(R_j)
\end{aligned}
$$

There exist no non-perturbative determinations of these operators
A first estimate with QCD sum rules was made by **Mannel, Pecjak, Pivovarov**
Current estimates rely on vacuum insertion approximation

Theory Prediction for $\Delta\Gamma_s$

$\Delta\Gamma_s^{\mathrm{SM}}$	2011	2006
Central Value	$0.087\,\mathrm{ps}^{-1}$	$0.096\,\mathrm{ps}^{-1}$
$\delta(\mathcal{B}_{\widetilde{R}_2})$	17.2%	15.7%
$\delta(f_{B_s})$	13.2%	33.4%
$\delta(\mu)$	7.8%	13.7%
$\delta(\widetilde{\mathcal{B}}_{S,B_s})$	4.8%	3.1%
$\delta(\mathcal{B}_{R_0})$	3.4%	3.0%
$\delta(V_{cb})$	3.4%	4.9%
$\delta(\mathcal{B}_{B_s})$	2.7%	6.6%
...	....	...
$\sum \delta$	24.5%	40.5%

- **Additional Bag parameters at dimension 6 and 7 for** Γ_{12}
- α_s/m_b **corrections for** Γ_{12}
- α_s^2 **corrections for** Γ_{12} **first step: Asatrian, Hovhannisyan, Yeghiazaryan, arXiv:1210.7939**

What did we learn?

- Test of our theoretical Understanding
  - SM and CKM work perfectly
  - HQE work also perfectly

	HQE	HFAG 2014	Ref.
$\frac{\Delta\Gamma_s}{\Delta M_s}$	$0.0050 \cdot (1 \pm 0.19)$	$0.0051 \cdot (1 \pm 0.09)$	A.L., Nierste1102.4274
$\frac{\tau(\Lambda_b)}{\tau(B_d)}$	0.935 ± 0.054	0.955 ± 0.009	A.L., 1405.3601

No space for sizable duality violations

- Search for NP
  - No huge effects seen, but still some sizable space left
    $\Delta\Gamma_d, B \to \tau\tau, ...$ seem to be promising candidates for further searches

- **Life becomes harder: higher precision in experiment and theory needed**
  - **Non-perturbative parameters - lattice - corrent limitation of progress in HQE**
  - **Higher order perturbative corrections**
  - **Experimentally more difficult observables**
  - **Alternative non-perturbative methods (LCSR,...)**

Coming UK Flavour Events

- July 10th - July 11th: **50 Years of CP violation**
  London
- July 14th - July 18th: **BEAUTY 2014**
  Edinburgh
- July 21st - July 26th: **BEACH 2014**
  Birmingham
- xx.xx.2015: **Heavy Flavour 2015**
  Distillery in Scotland?

More info: “Workshops” on IPPP webpage