



FLASY'14, Brighton, June 21 2014



# Abelian Higgs models

Michal Malinský

Institute of Particle and Nuclear Physics, Charles University in Prague





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# Witten's loop in the flipped $SU(5)$ UT

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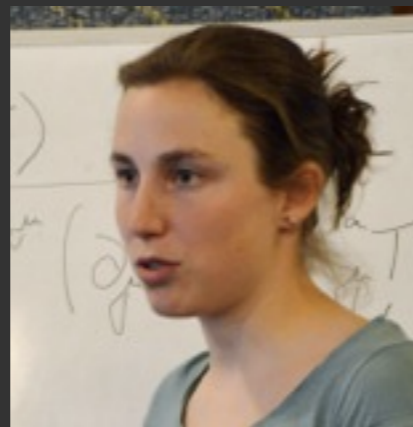


# Witten's loop in the flipped $SU(5)$ UT

Michal Malinský

Institute of Particle and Nuclear Physics, Charles University in Prague

in collaboration with



Helena Šediváková  
(IPNP Prague)



Carolina Arbelaez Rodriguez  
(IST Lisbon/AHEP Valencia)

## **Loop-induced (RH) Majorana neutrino masses in a non-SUSY unification**

C.Arbelaez Rodriguez, H. Kolečová, MM, PRD89, 055003 (2014)

# “The Higgs mass suffers from large radiative corrections!”

The standard (wrong) argument:

$$\Gamma_{hh} \propto \frac{h}{\text{-----}} \frac{h}{p^2 - m_H^2}$$

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$$\begin{aligned}
 \Gamma_{hh} \propto & \text{---} h \text{---} & p^2 - m_H^2 \\
 + & \text{---} h \text{---} \text{---} S \text{---} \text{---} h \text{---} & \frac{1}{16\pi^2} M_S^2 \left( C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right) \\
 + & \text{---} h \text{---} \text{---} S \text{---} \text{---} h \text{---} & \frac{1}{16\pi^2} v^2 \left( C_{UV} - \int_0^1 dx \log \frac{M_S^2 - x(1-x)p^2}{\mu^2} \right) \\
 + & \dots
 \end{aligned}$$

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 + & \dots &
 \end{aligned}$$

For large  $M_S^2$  the root shifts enormously...

$$m_H^2 \rightarrow m_H^2 + \frac{1}{16\pi^2} M_S^2 \left( 1 - \log \frac{M_S^2}{\mu^2} \right) + \dots$$

so the tree-level mass must be carefully readjusted order by order...




The “hierarchy problem”



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$$\Gamma_{hh} \propto$$

	$p^2 - m_H^2$
$+$ 	$\frac{1}{16\pi^2} M_S^2 \left( C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)$
$+$ 	$\frac{1}{16\pi^2} v^2 \left( C_{UV} - \int_0^1 dx \log \frac{M_S^2 - x(1-x)p^2}{\mu^2} \right)$



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 \Gamma_{hh} \propto & \quad h \text{---} h && p^2 - m_H^2 \\
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 & + \quad h \text{---} \textcircled{S} \text{---} h && \frac{1}{16\pi^2} v^2 \left( C_{UV} - \int_0^1 dx \log \frac{M_S^2 - x(1-x)p^2}{\mu^2} \right) \\
 \text{SUSY:} & + \quad h \text{---} \textcircled{\tilde{S}} \text{---} h && -\frac{1}{16\pi^2} M_S^2 \left( C_{UV} - 1 - \int_0^1 dx \log \frac{M_S^2 - x(1-x)p^2}{\mu^2} \right) + p^2 \text{ terms} \\
 & + \dots &&
 \end{aligned}$$

The shift of the root is small even for large  $M_S^2$

“The hierarchy is stabilized by SUSY near  $M_Z$ ”

# “The Higgs mass suffers from large radiative corrections!”

The devil is in detail:

$$\Gamma_{hh} \propto \begin{array}{c} h \qquad \qquad \qquad h \\ \text{-----} \bullet \text{-----} \\ p^2 - m_H^2 \end{array} + \begin{array}{c} h \qquad \qquad \qquad h \\ \text{-----} \circ \text{-----} \\ \text{S} \end{array} + \dots$$
$$\frac{1}{16\pi^2} M_S^2 \left( C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)$$

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$$\frac{1}{16\pi^2} M_S^2 \left( C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)$$

The root is **not the physical mass ! Mind the one-point function**

Perturbation theory contrived unless one-point function vanishes

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$$\Gamma_h \propto \text{---} \overset{h}{\bullet} \text{---} \times + \text{---} \overset{h}{\times} \text{---} \times$$

$$m_H^2 v \qquad \lambda v^3$$

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$$m_H^2 v \qquad \lambda v^3 \qquad \frac{v}{16\pi^2} M_S^2 \left( C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)$$

No need to be sorry about loops...

In the **true quantum vacuum** the polynomial  $M_S^2$ -dependence drops out.

The poor Higgs is in the same shape like anybody else in the SM...

**“Higgs anti-discrimination act”**

Full one-loop effective potential level approach: MM, EPJ C73 (2013) 2415

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In the **true quantum vacuum** the polynomial  $M_S^2$ -dependence drops out.

The poor Higgs is in the same shape like anybody else in the SM...

**“Higgs anti-discrimination act”**

Who cares? Do you mind getting rid of the UV divergences?

**Correlations** among observables ( = **physics**) **perfectly stable!**

Full one-loop effective potential level approach: MM, EPJ C73 (2013) 2415



## **Loop-induced (RH) Majorana neutrino masses in a non-SUSY unification**

C.Arbelaez Rodriguez, H. Kolečová, MM, PRD89, 055003 (2014)

# The Witten's loop

## NEUTRINO MASSES IN THE MINIMAL O(10) THEORY <sup>☆</sup>

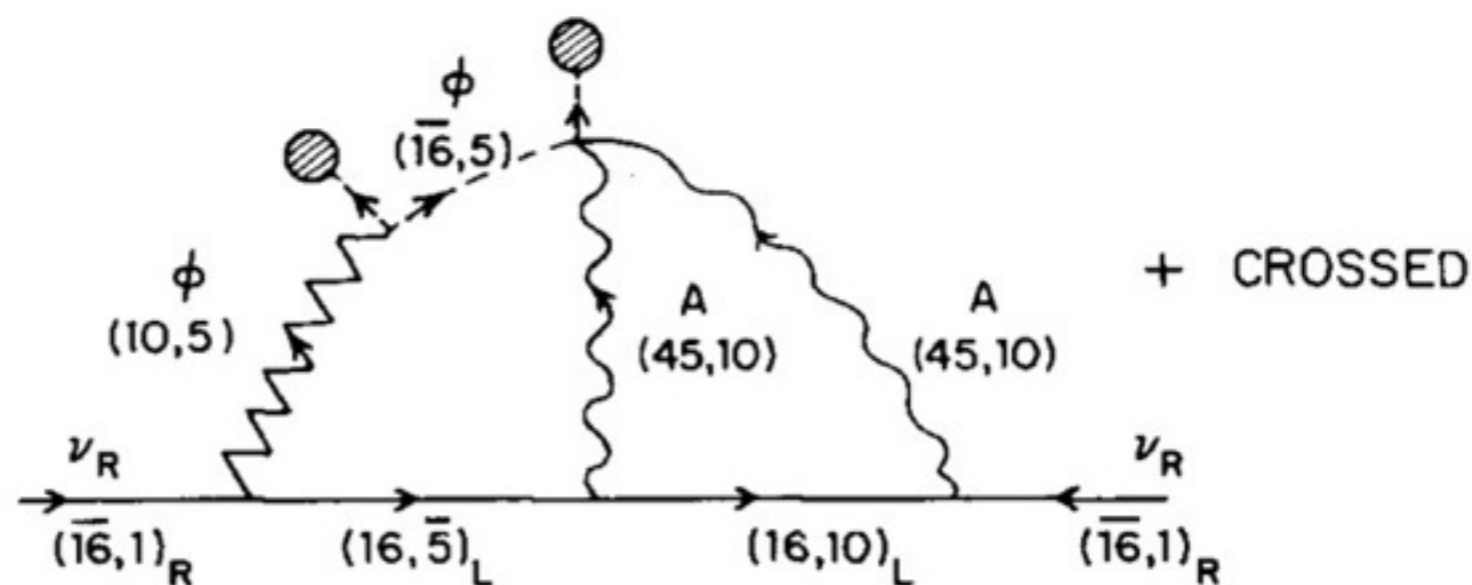
Phys. Lett. B91 (1980) 81

Edward WITTEN <sup>1</sup>

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Received 6 December 1979

Neutrino masses are discussed in the context of the O(10) grand unified theory. In the “minimal” form of this theory, with minimal Higgs and fermion content, the right-handed neutrinos acquire masses at the two loop level. The left-handed neutrino masses are correspondingly *larger* by a factor roughly  $(\alpha/\pi)^{-2}$  than they would be if the right-handed neutrino could acquire mass at the tree level. In the simplest form of this theory, the neutrino mass matrix is proportional to the up quark mass matrix, and the neutrino mixing angles equal the usual Cabibbo angles. The neutrino masses will be roughly in the range  $10^{0\pm 2}$  eV depending on the strength of O(10) symmetry breaking, and on certain unknown ratios of masses and couplings of superheavy particles.



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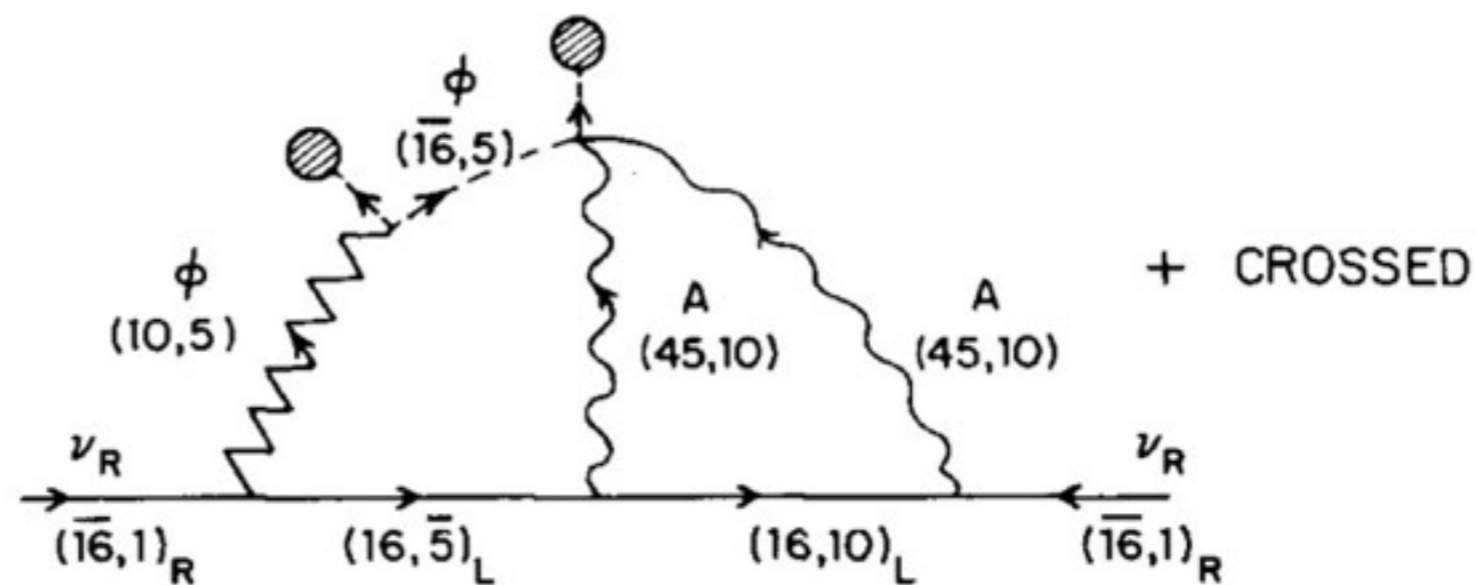
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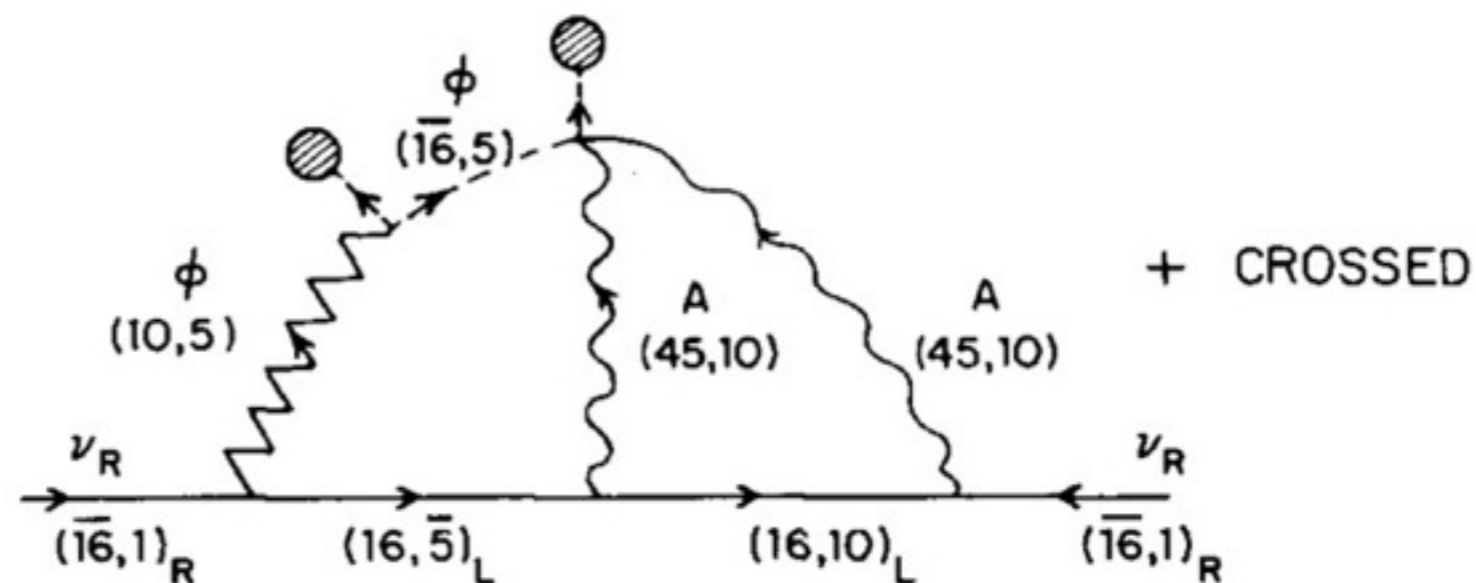
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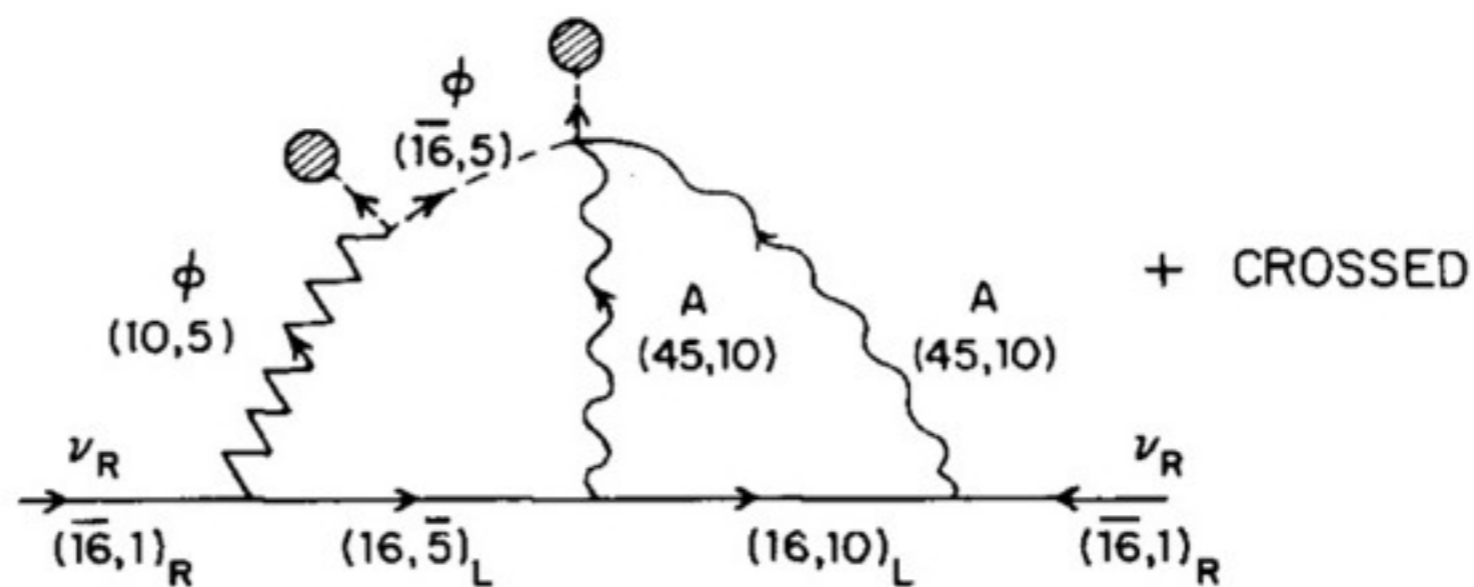
# The Witten's loop

- the structure is unique:

need to mimic  $126_H$  (5-index tensor) with fields coupled to  $16_M$

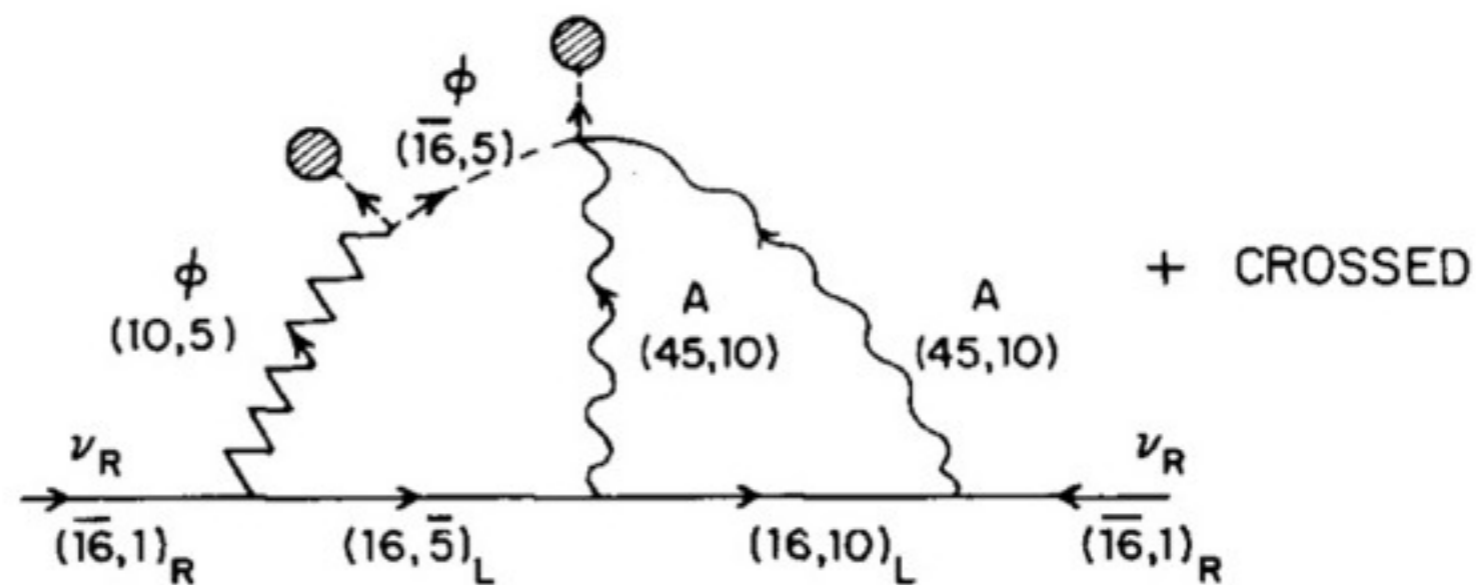
these are  $10_H$  and  $45_G$  with 1 and 2 indices, respectively

$$10_H \times 45_G \times 45_G$$



# The Witten's loop

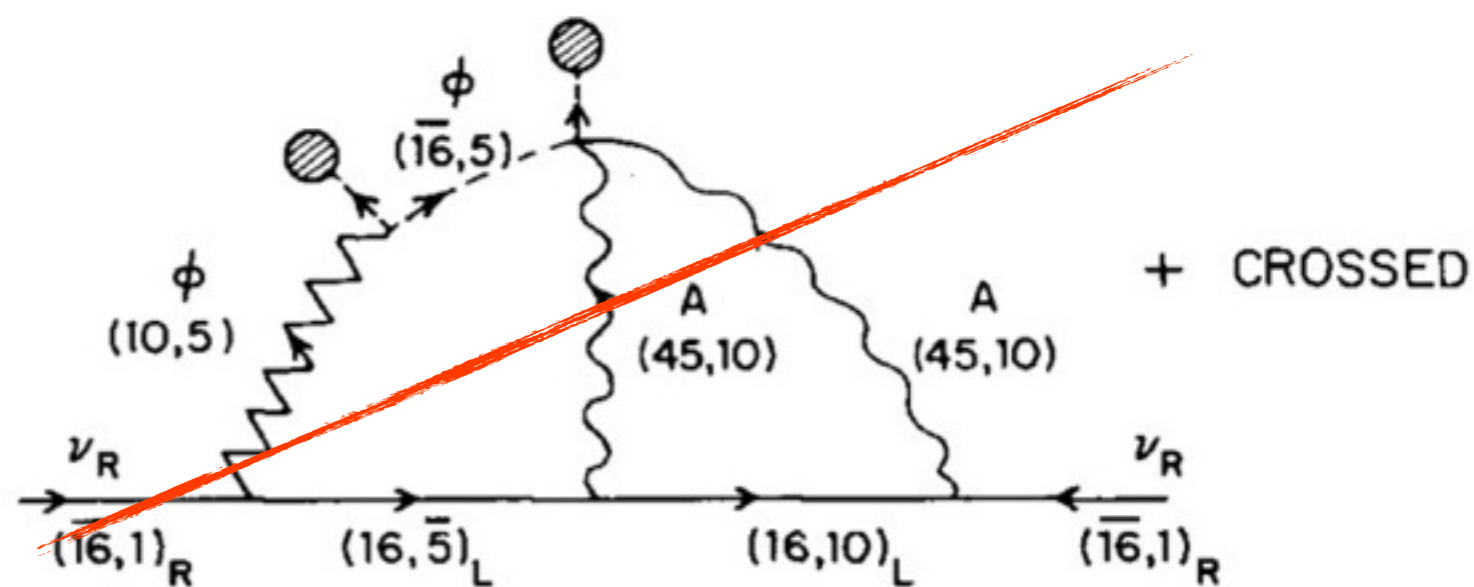
Not many real applications though!



# The Witten's loop

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1980: Not much known about the scales => **not needed**





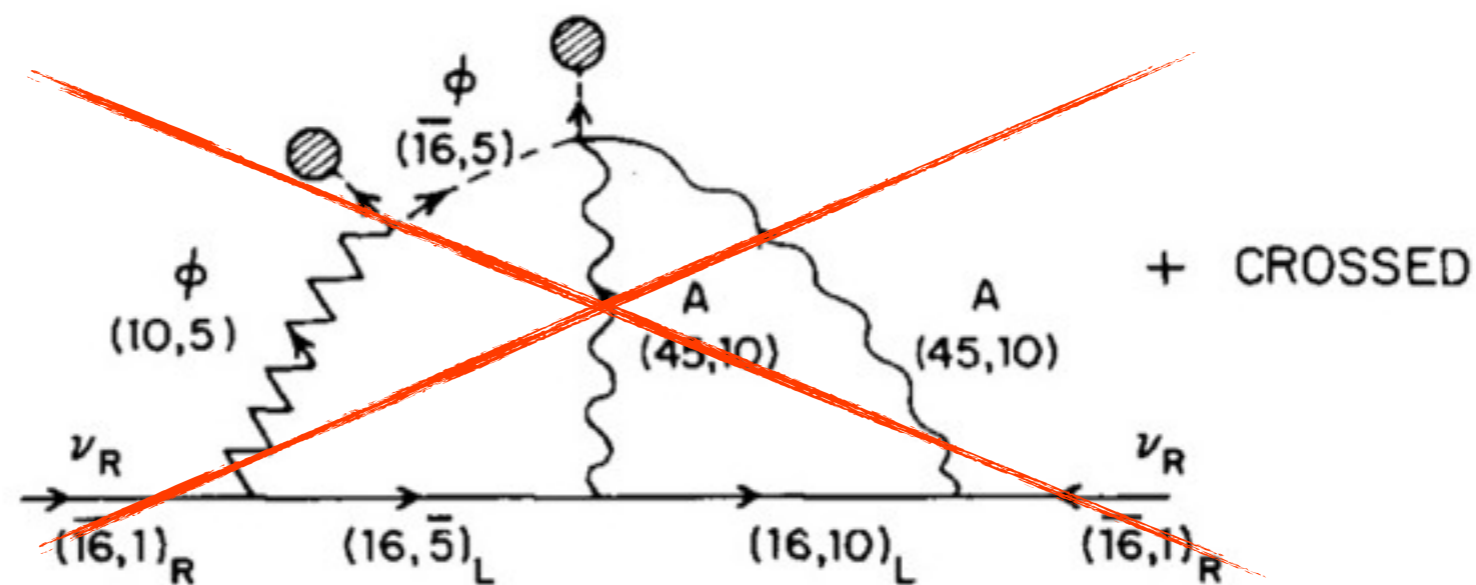
# The Witten's loop

Not many real applications though!

1980: Not much known about the scales => **not needed**

mid 1980's: one-step unification failure

TeV-scale SUSY “came to rescue” => **no point**

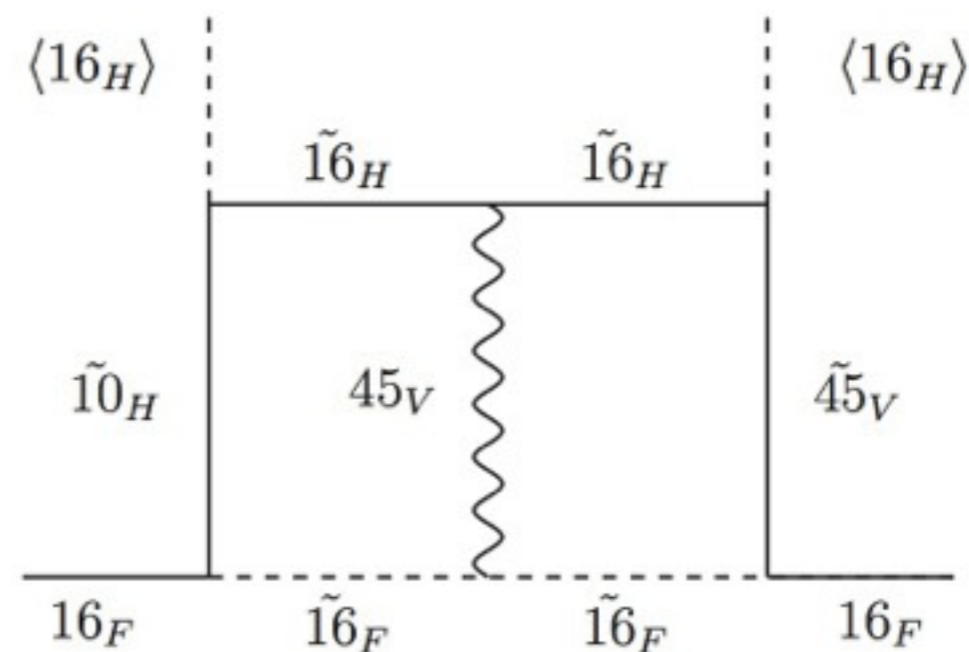
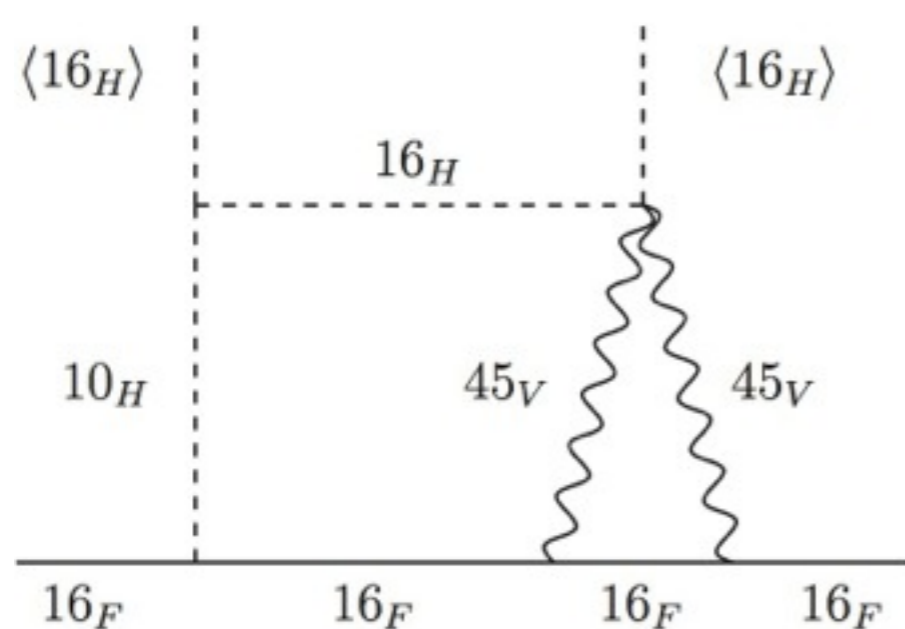


# The Witten's loop

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**Extremely split SUSY:** SUSY scalars at the GUT scale  
one-step unification  
loops not killed by SUSY

Bajc, Senjanovic, Phys. Lett. B610 (2005) 80



# Towards the core of the Witten's loop

**Other solutions to the “loop / scale / unification” issue?**

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non-SUSY GUTs do also unify

however, this is not one-step which lowers the critical VEV

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however, this is not one-step which lowers the critical VEV

**need more freedom in running than that in  $SO(10)$**

**retain testability**

# Towards the core of the Witten's loop



## Witten's loop in the flipped $SU(5)$ unification

C.Arbelaez Rodriguez, H. Kolečová, MM, PRD89, 055003 (2014)



# Flipped SU(5) basics

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$$SO(10) \supset SU(5) \times U(1)_Z$$

$$16_M \ni (10, +1)_M \oplus (\bar{5}, -3)_M \oplus (1, +5)_M$$

2 possible hypercharge assignments:

Standard:  $Y = T_{24}$        $u^c, Q, e^c$        $d^c, L$        $\nu^c$

Flipped:  $Y = \frac{1}{5}(Z - T_{24})$        $d^c, Q, \nu^c$        $u^c, L$        $e^c$

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Symmetry breaking:

$$16_H \ni (10, +1)_H \quad SU(5) \times U(1) \text{ to the SM}$$

$$10_H \ni (5, -2)_H \quad \text{SM to the QCD} \times \text{QED}$$

Gauge sector:  $45_G \ni (24, 0)_G \oplus (1, 0)_G$

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Gauge sector:  $45_G \ni (24, 0)_G \oplus (1, 0)_G \ni (3, 2, -\frac{1}{6})_G + h.c.$

$X', Y'$

# Flipped SU(5) basics

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- unification is not “Grand”, just  $SU(3) \times SU(2)$

VEV of  $(10,+1) =$  Witten’s VEV = mass of the  $X',Y'$  bosons

running is well under control ( $F_{\mu\nu}\langle H\rangle F^{\mu\nu}/M_{Pl}$  absent)

- monopoles absent  $\pi_2(SU(5) \otimes U(1)/SU(3) \otimes SU(2) \otimes U(1))$   
 $= \pi_2(SU(5)/SU(3) \otimes SU(2))$   
 $= \pi_1(SU(3) \otimes SU(2)) = 0$

# Flipped SU(5) basics

**Flavour structure:** rather different from the “standard” SU(5)

$$\mathcal{L} \ni Y_{10} 10_M 10_M 5_H + Y_{\bar{5}} 10_M \bar{5}_M 5_H^* + Y_1 \bar{5}_M 1_M 5_H + h.c.$$

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$$M_u = M_\nu^{D^T} \propto Y_{\bar{5}}$$

$$M_d = M_d^T \propto Y_{10} \quad M_e \propto Y_1 \text{ arbitrary}$$

+ down-quark - charged lepton degeneracy gone



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**Flavour structure:** rather different from the “standard” SU(5)

$$\mathcal{L} \ni Y_{10} 10_M 10_M 5_H + Y_{\bar{5}} 10_M \bar{5}_M 5_H^* + Y_1 \bar{5}_M 1_M 5_H + h.c.$$

$$\begin{aligned} M_u &= M_\nu^{D^T} \propto Y_{\bar{5}} & M_\nu^M &= 0 \\ M_d &= M_d^T \propto Y_{10} & M_e &\propto Y_1 \text{ arbitrary} \end{aligned}$$

- + down-quark - charged lepton degeneracy gone
- neutrinos massive, but heavy & Dirac

$50_H$  usually included to generate RH neutrino masses @ tree level...

# The strategy

We use the Witten's loop to generate

RH Majorana masses in the flipped SU(5) unification

C.Arbelaez Rodriguez, H. Kolečová, MM, PRD89, 055003 (2014)

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We use the Witten's loop to generate

RH Majorana masses in the flipped SU(5) unification

&

**use lepton flavour to learn about p-decay**


or vice versa

C.Arbelaez Rodriguez, H. Kolečová, MM, PRD89, 055003 (2014)

# Proton decay in SU(5)

Standard SU(5)

$(3, 2, -5/6)_G$

$$\overline{u^c} Q \overline{e^c} Q,^{(I)} \overline{u^c} Q \overline{d^c} L^{(II)}$$


Nath, Fileviez-Perez, Phys.Rept.441

Dorsner, Fileviez-Perez, PLB605

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$$\overline{u^c} Q \overline{e^c} Q,^{(I)} \overline{u^c} Q \overline{d^c} L^{(II)}$$

I: 
$$\left[ (U_{u^c}^\dagger U_u)_{11} (U_{e^c}^\dagger U_d)_{ab} + (U_{u^c}^\dagger U_d)_{1b} (U_{e^c}^\dagger U_u)_{a1} \right] \overline{u_{L(1)}^c} \gamma_\mu u_{L(1)} \overline{e_{L(a)}^c} \gamma_\mu d_{Lb}$$

neutral meson

II: 
$$\left[ (U_{u^c}^\dagger U_u)_{11} (U_{d^c}^\dagger U_e)_{ab} \right] \overline{u_{L(1)}^c} \gamma_\mu u_{L(1)} \overline{d_{L(a)}^c} \gamma_\mu e_{Lb} - \left[ (U_{u^c}^\dagger U_d)_{1a} (U_{d^c}^\dagger U_\nu)_{bc} \right] \overline{u_{L(1)}^c} \gamma_\mu d_{L(a)} \overline{d_{L(b)}^c} \gamma_\mu \nu_{Lc}$$

neutral meson charged meson

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neutral meson

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neutral meson charged meson

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = F_1 |(V_{CKM})_{11}|^2 + F_2 |(V_{CKM})_{12}|^2 C$$


$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) = F_3 |(V_{CKM})_{11}|^2 C$$

SU(5) may be tested by looking at the charged meson final states

# Proton decay in the flipped SU(5)

Flipped SU(5)

$(3, 2, 1/6)_G$

$$\overline{d^c} Q \overline{u^c} L^{(III)}, \overline{d^c} Q \overline{\nu^c} Q^{(IV)}$$


Nath, Fileviez-Perez, Phys.Rept.441

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$$\overline{d^c} Q \overline{u^c} L^{(III)}, \overline{d^c} Q \overline{\nu^c} Q^{(IV)}$$

$$\text{III: } - \left[ (U_{d^c}^\dagger U_u)_{a1} (U_{u^c}^\dagger U_e)_{1b} \right] \overline{u_{L(1)}^c} \gamma_\mu u_{L(1)} \overline{d_{L(a)}^c} \gamma_\mu e_{Lb} + \left[ (U_{d^c}^\dagger U_d)_{ba} (U_{u^c}^\dagger U_\nu)_{1c} \right] \overline{u_{L(1)}^c} \gamma_\mu d_{L(a)} \overline{d_{L(b)}^c} \gamma_\mu \nu_{Lc}$$

neutral meson

charged meson

$$\text{IV: } \left[ (U_{d^c}^\dagger U_u)_{a1} (U_{\nu^c}^\dagger U_d)_{bc} + (U_{d^c}^\dagger U_d)_{ac} (U_{\nu^c}^\dagger U_u)_{b1} \right] \overline{d_{L(a)}^c} \gamma_\mu u_{L(1)} \overline{\nu_{L(b)}^c} \gamma_\mu d_{Lc}$$

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$$\text{IV: } \left[ (U_{dc}^\dagger U_u)_{a1} (U_{\nu c}^\dagger U_d)_{bc} + (U_{dc}^\dagger U_d)_{ac} (U_{\nu c}^\dagger U_u)_{b1} \right] \overline{d_{L(a)}^c} \gamma_\mu u_{L(1)} \overline{\nu_{L(b)}^c} \gamma_\mu d_{Lc} \quad \text{charged meson}$$

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = 0$$

$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) = \tilde{F}_1 C$$

$$\frac{\Gamma(p \rightarrow K^0 \ell^+)}{\Gamma(p \rightarrow \pi^0 \ell^+)} = \frac{\tilde{F}_2 |(V_{CKM})_{12}|^2}{\tilde{F}_1 |(V_{CKM})_{11}|^2}$$

There is a clear feature in the charged mesons and in the **ratio** of the neutral...

# Proton decay in the flipped SU(5)

The central formula:

$$\Gamma(p \rightarrow \pi^0 \ell_{\alpha}^{+}) = \frac{1}{2} \Gamma(p \rightarrow \pi^{+} \bar{\nu}) |(V_{CKM})_{11}|^2 |(V_{PMNS} U_{\nu})_{\alpha 1}|^2$$

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If we knew  $U_\nu$  we would have **prediction** for ALL BNV channels!!!

$$\Gamma(p \rightarrow \pi^0 \ell_\alpha^+)$$

$$\Gamma(p \rightarrow \pi^+ \bar{\nu})$$

$$\Gamma(n \rightarrow \pi^- \ell_\alpha^+)$$

$$\Gamma(n \rightarrow \pi^0 \bar{\nu})$$

$$\Gamma(p \rightarrow K^0 \ell_\alpha^+)$$

$$\Gamma(p \rightarrow K^+ \bar{\nu})$$

$$\Gamma(n \rightarrow K^- \ell_\alpha^+)$$

$$\Gamma(n \rightarrow K^0 \bar{\nu})$$

$$\Gamma(p \rightarrow \eta \ell_\alpha^+)$$

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$$\Gamma(n \rightarrow \pi^- \ell_\alpha^+) \quad \Gamma(n \rightarrow \pi^0 \bar{\nu})$$

$$\Gamma(p \rightarrow K^0 \ell_\alpha^+)$$

$$\Gamma(p \rightarrow K^+ \bar{\nu})$$

$$\Gamma(n \rightarrow K^- \ell_\alpha^+) \quad \Gamma(n \rightarrow K^0 \bar{\nu})$$

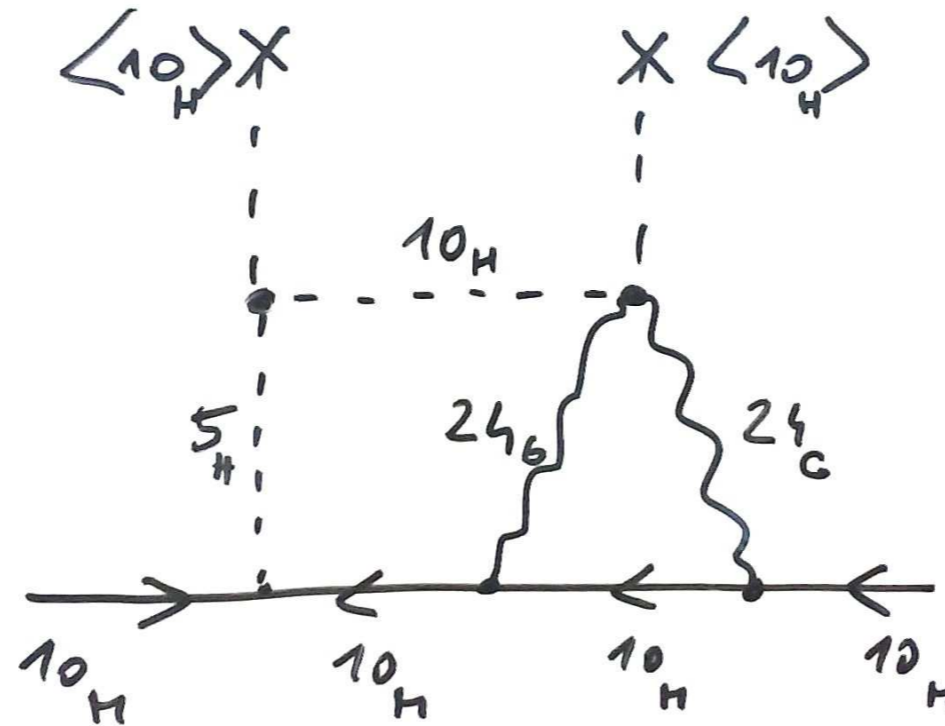
$$\Gamma(p \rightarrow \eta \ell_\alpha^+)$$

$$\Gamma(n \rightarrow \eta \bar{\nu})$$

**There are constraints on  $U_\nu$  in the Witten's loop scenario !!!**

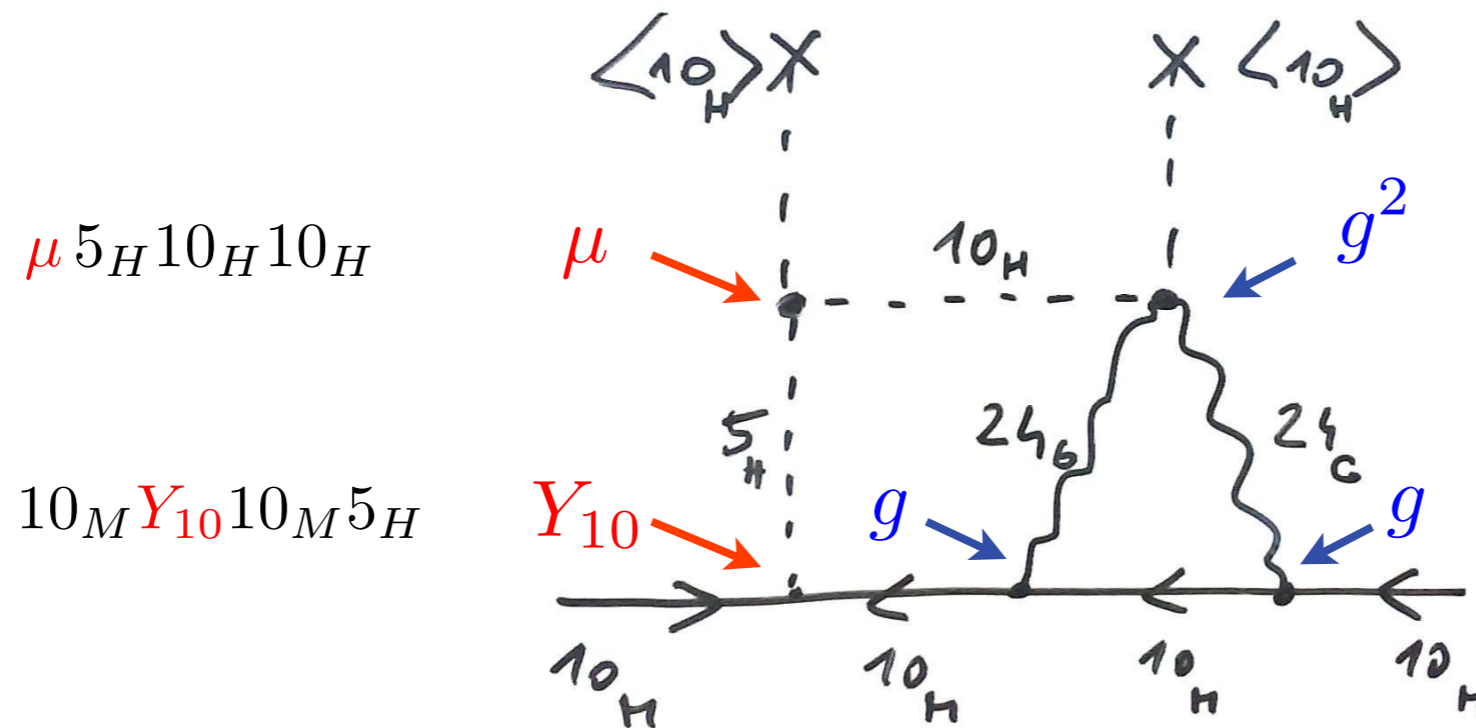
Nothing like that if the RH neutrino mass is due to  $50_H$  ...

# Witten's loop anatomy in the flipped SU(5)



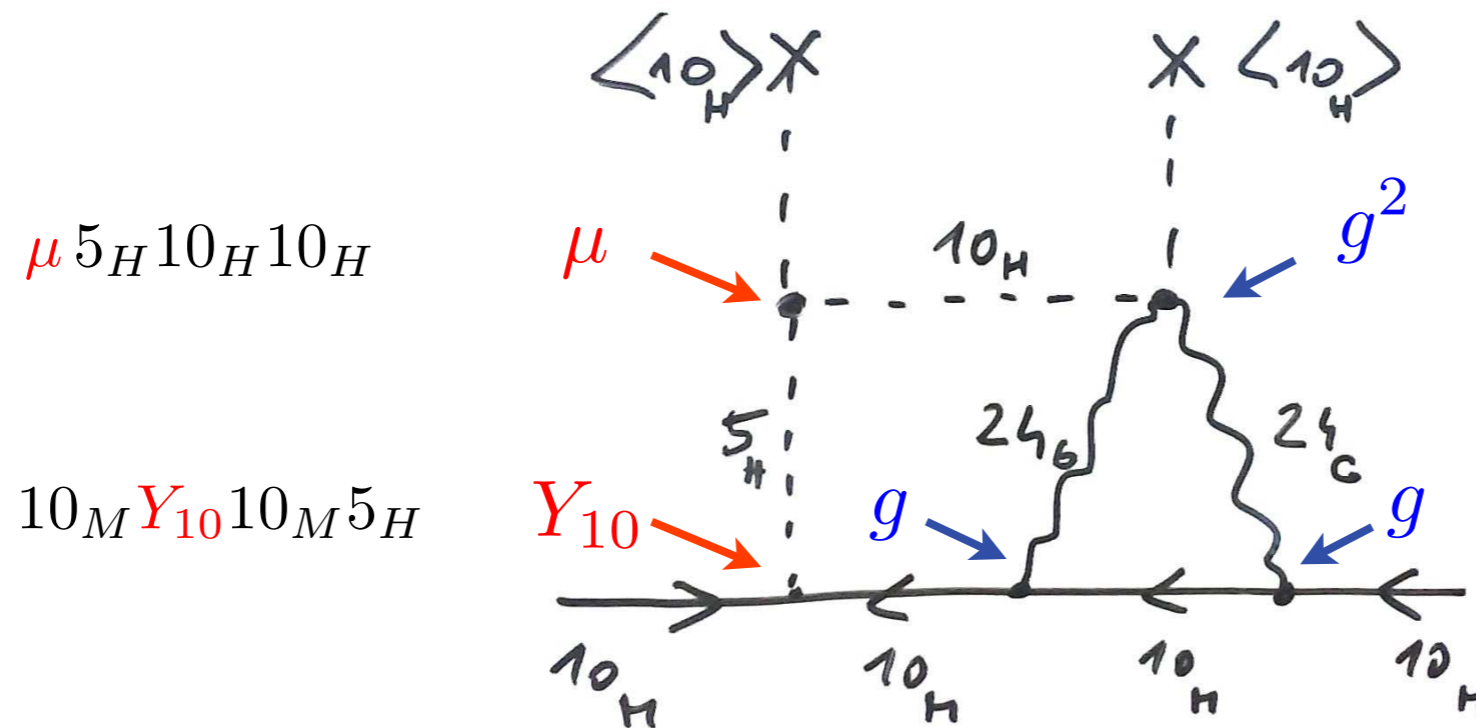
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# Witten's loop anatomy in the flipped SU(5)



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# Witten's loop anatomy in the flipped SU(5)



$$M_\nu^M = \left( \frac{1}{16\pi^2} \right)^2 g^4 Y_{10} \mu \frac{\langle 10_H \rangle^2}{M_G^2} K(\dots)$$

$K(\dots)$  is an  $\mathcal{O}(1)$  factor depending on the details of the heavy spectrum

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# Witten's loop anatomy in the flipped SU(5)

The  $U_\nu$  matrix kicks in via seesaw (using  $M_u = M_\nu^{D^T}$ ):

$$D_U U_\nu^\dagger D_\nu^{-1} (m_1, \dots) U_\nu^* D_U = \left( \frac{1}{16\pi^2} \right)^2 g^4 Y_{10} \mu \frac{\langle 10_H \rangle^2}{M_G^2} K(\dots)$$

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# Witten's loop in the perturbative & stable regime

$Y_{10}$  and  $\mu$  are constrained from **perturbativity** + **SM vacuum stability**

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LHS:  $\sim \begin{pmatrix} 10^4 & 0 & 0 \\ 0 & 5 \times 10^8 & 0 \\ 0 & 0 & 5 \times 10^{13} \end{pmatrix} \text{ GeV}$  for  $U_\nu = 1$  and  $m_1 = 8 \times 10^{-2}$  eV  
(assuming normal hierarchy)

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Beware of  $U_\nu$  as  $D_\nu^{-1}$  itself looks like  $\begin{pmatrix} 10^{10-\infty} & 0 & 0 \\ 0 & 10^{10-11} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix} \text{ GeV}^{-1} !!!$

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**Only some  $U_\nu$ 's can provide a consistent flavour fit...**

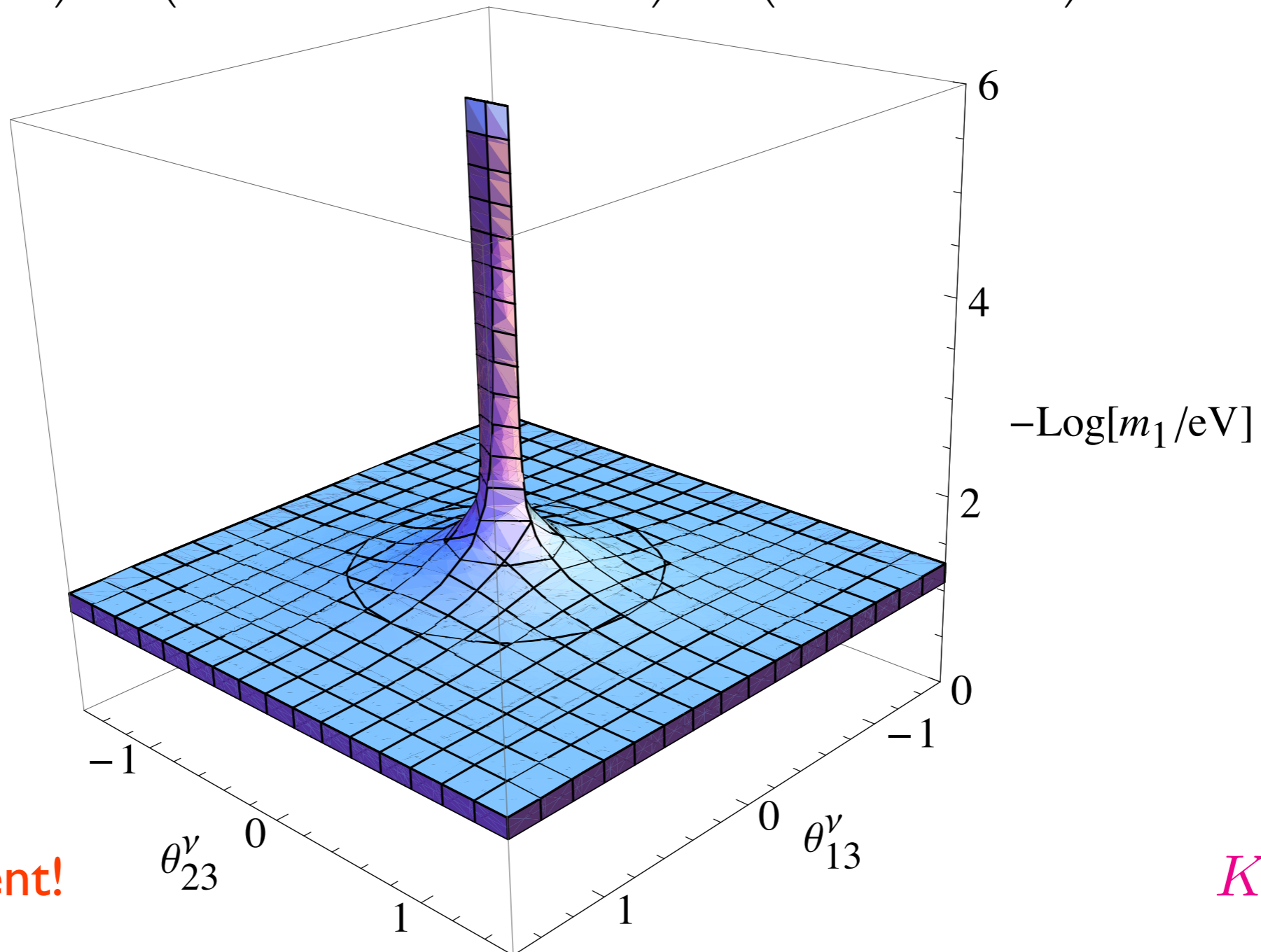
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# Only some $U_\nu$ 's allowed!

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_\nu^\dagger \begin{pmatrix} 10^{10-\infty} & 0 & 0 \\ 0 & 10^{10-11} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix} U_\nu^* \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \leq 10^{14} \text{GeV}^2 K$$

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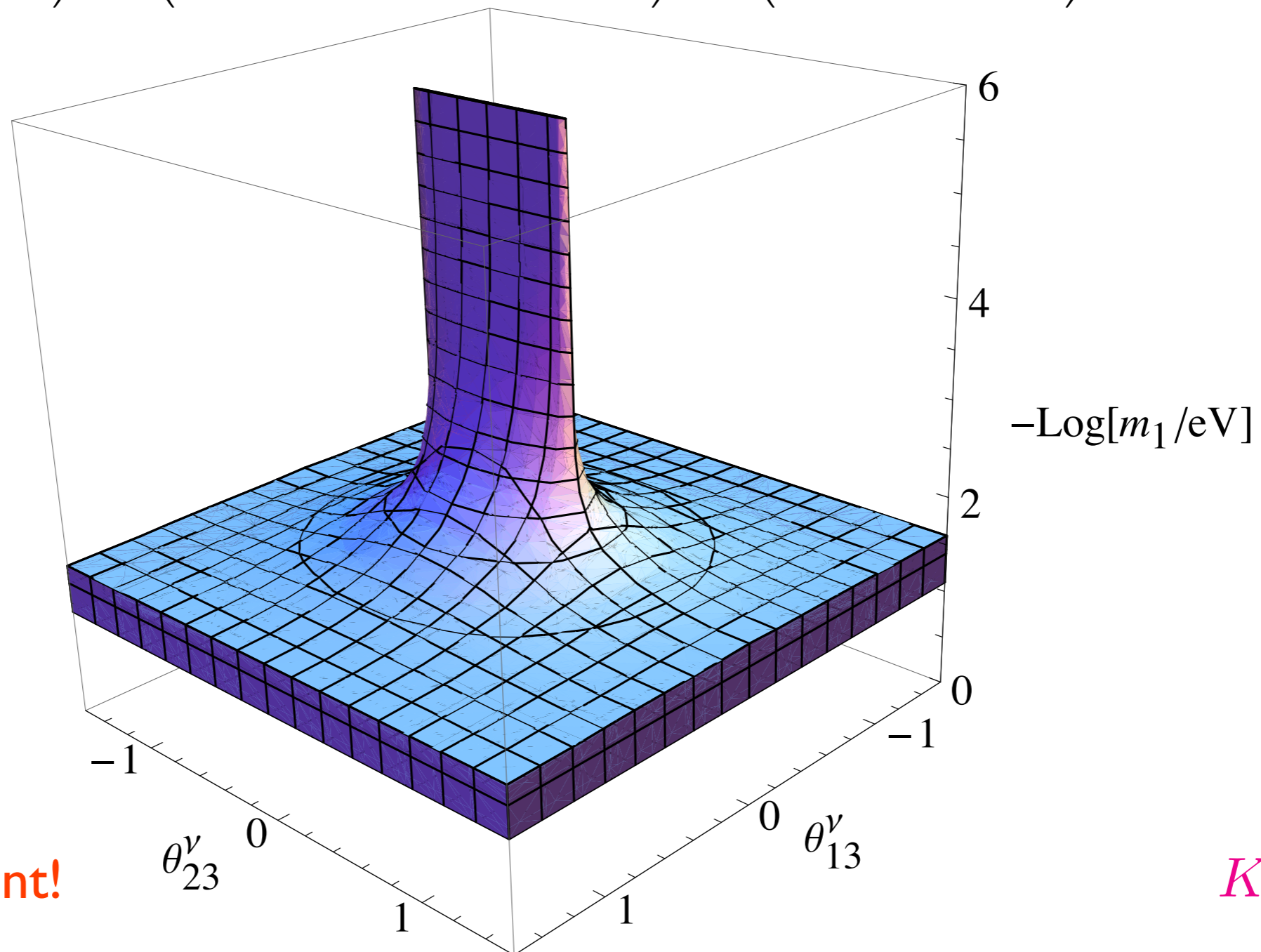
$\theta_{12}$ -independent!

$K = 1$



# Only some $U_\nu$ 's allowed!

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_\nu^\dagger \begin{pmatrix} 10^{10-\infty} & 0 & 0 \\ 0 & 10^{10-11} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix} U_\nu^* \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \leq 10^{14} \text{GeV}^2 K$$

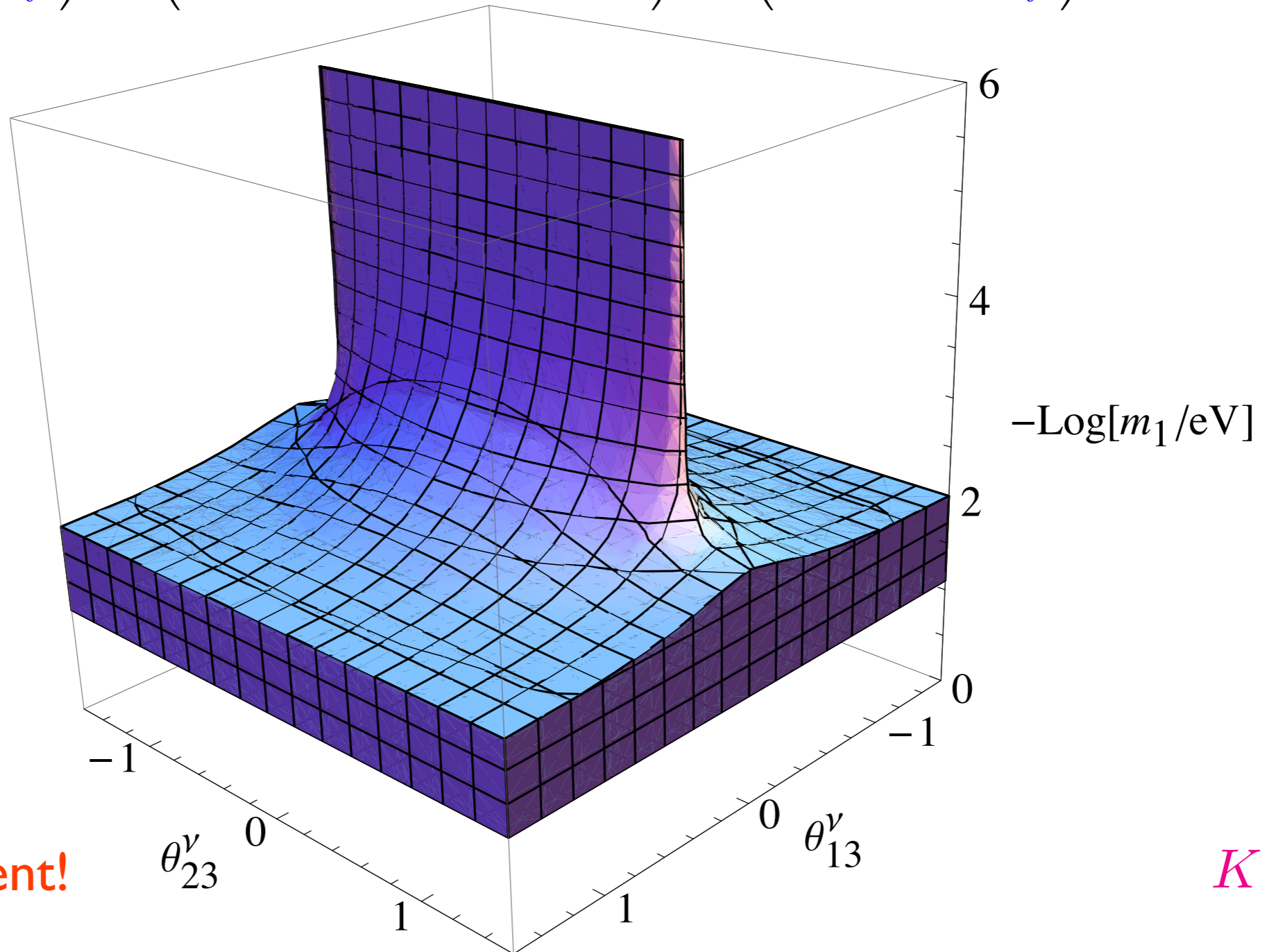


$\theta_{12}$ -independent!

$K = 2$

# Only some $U_\nu$ 's allowed!

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_\nu^\dagger \begin{pmatrix} 10^{10-\infty} & 0 & 0 \\ 0 & 10^{10-11} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix} U_\nu^* \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \leq 10^{14} \text{GeV}^2 K$$

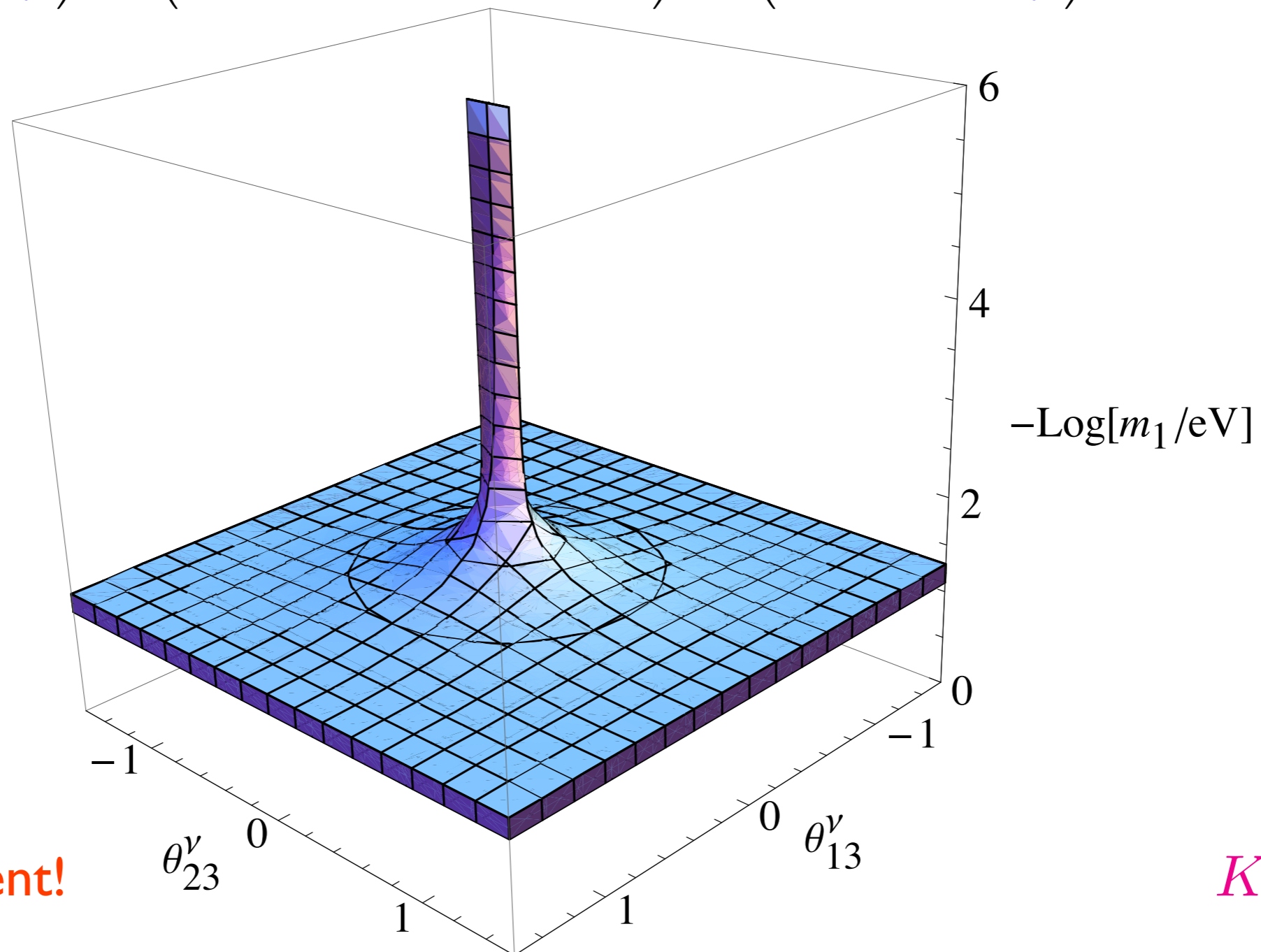


$\theta_{12}$ -independent!

$K = 5$

# Only some $U_\nu$ 's allowed!

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_\nu^\dagger \begin{pmatrix} 10^{10-\infty} & 0 & 0 \\ 0 & 10^{10-11} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix} U_\nu^* \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \leq 10^{14} \text{GeV}^2 K$$

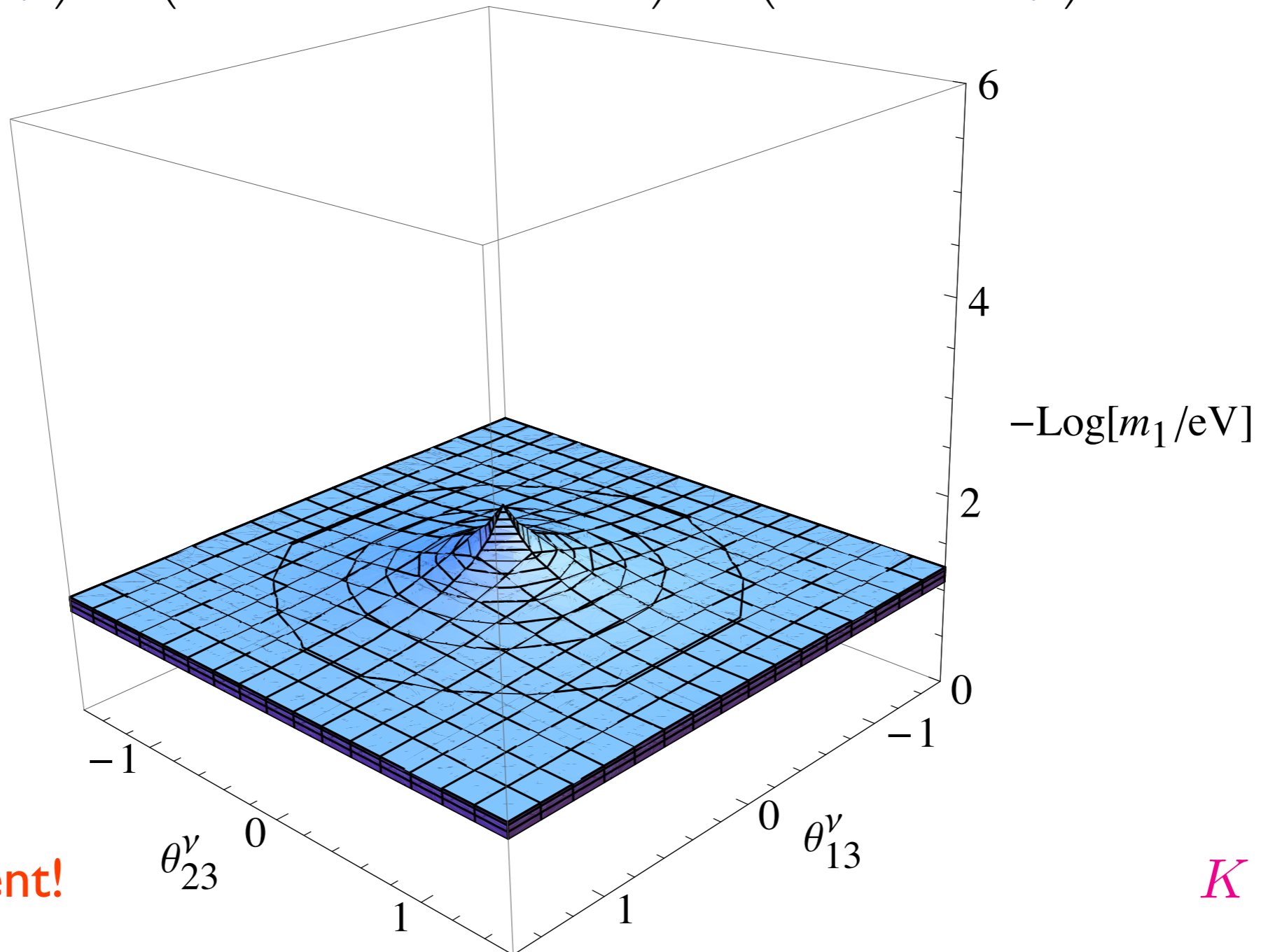


$\theta_{12}$ -independent!

$K = 1$

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$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_\nu^\dagger \begin{pmatrix} 10^{10-\infty} & 0 & 0 \\ 0 & 10^{10-11} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix} U_\nu^* \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \leq 10^{14} \text{GeV}^2 K$$



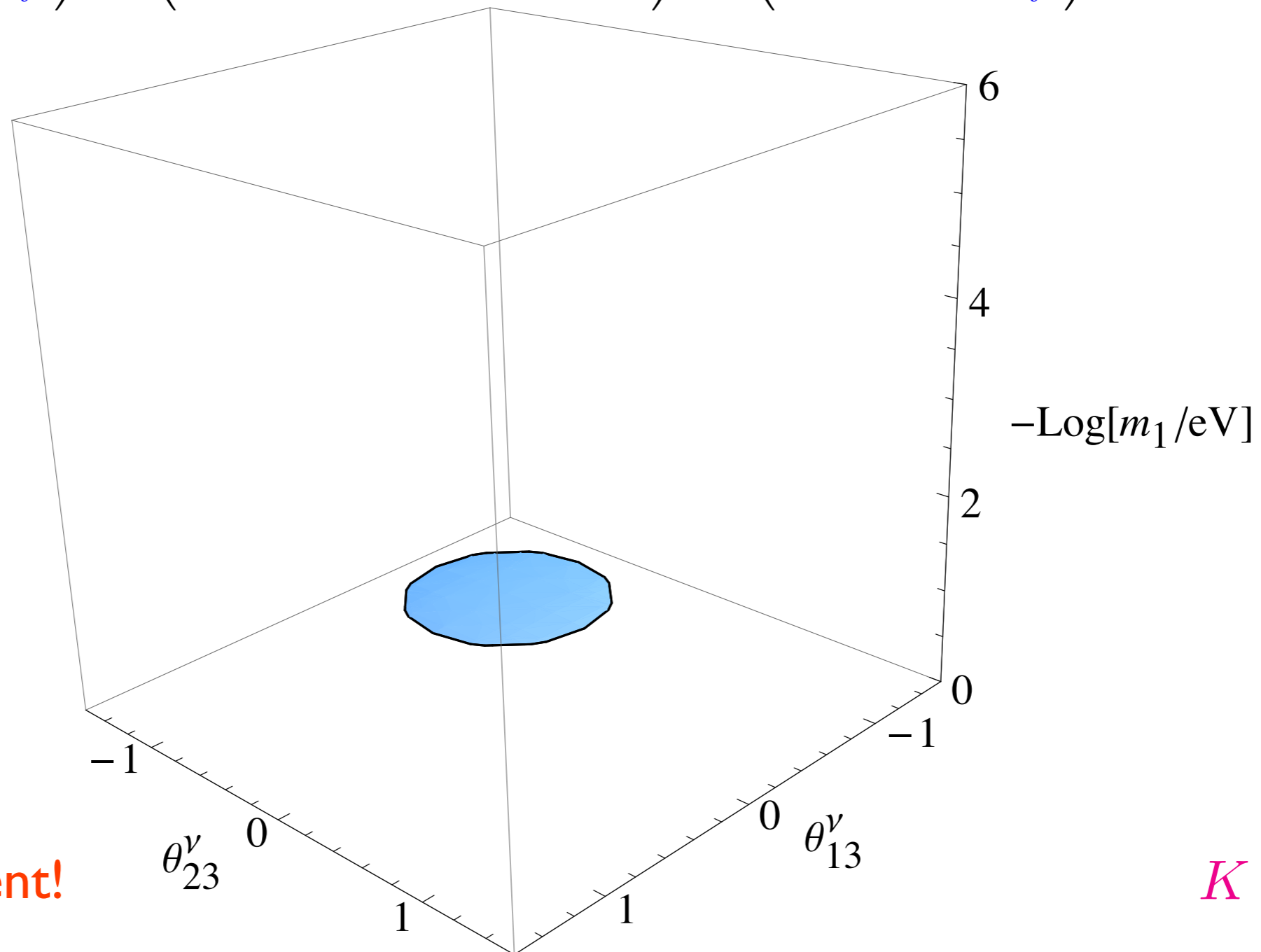
$\theta_{12}$ -independent!

$K = 0.9$



# Only some $U_\nu$ 's allowed!

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_\nu^\dagger \begin{pmatrix} 10^{10-\infty} & 0 & 0 \\ 0 & 10^{10-11} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix} U_\nu^* \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \leq 10^{14} \text{GeV}^2 K$$



$\theta_{12}$ -independent!

$K = 0.5$

# Proton decay to neutral mesons+charged leptons

To be superimposed over the observable(s) of interest ...

$$\Gamma(p \rightarrow \pi^0 e^+) + \Gamma(p \rightarrow \pi^0 \mu^+)$$

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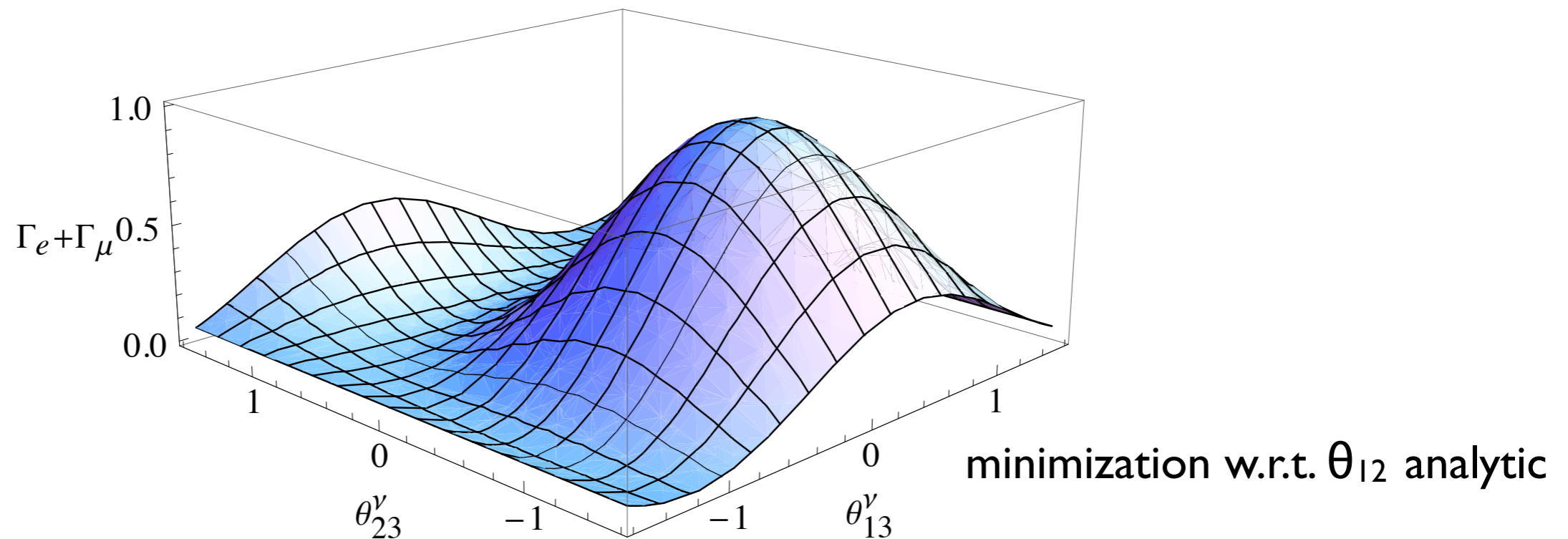
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the minima of the sum of the BNV matrix elements

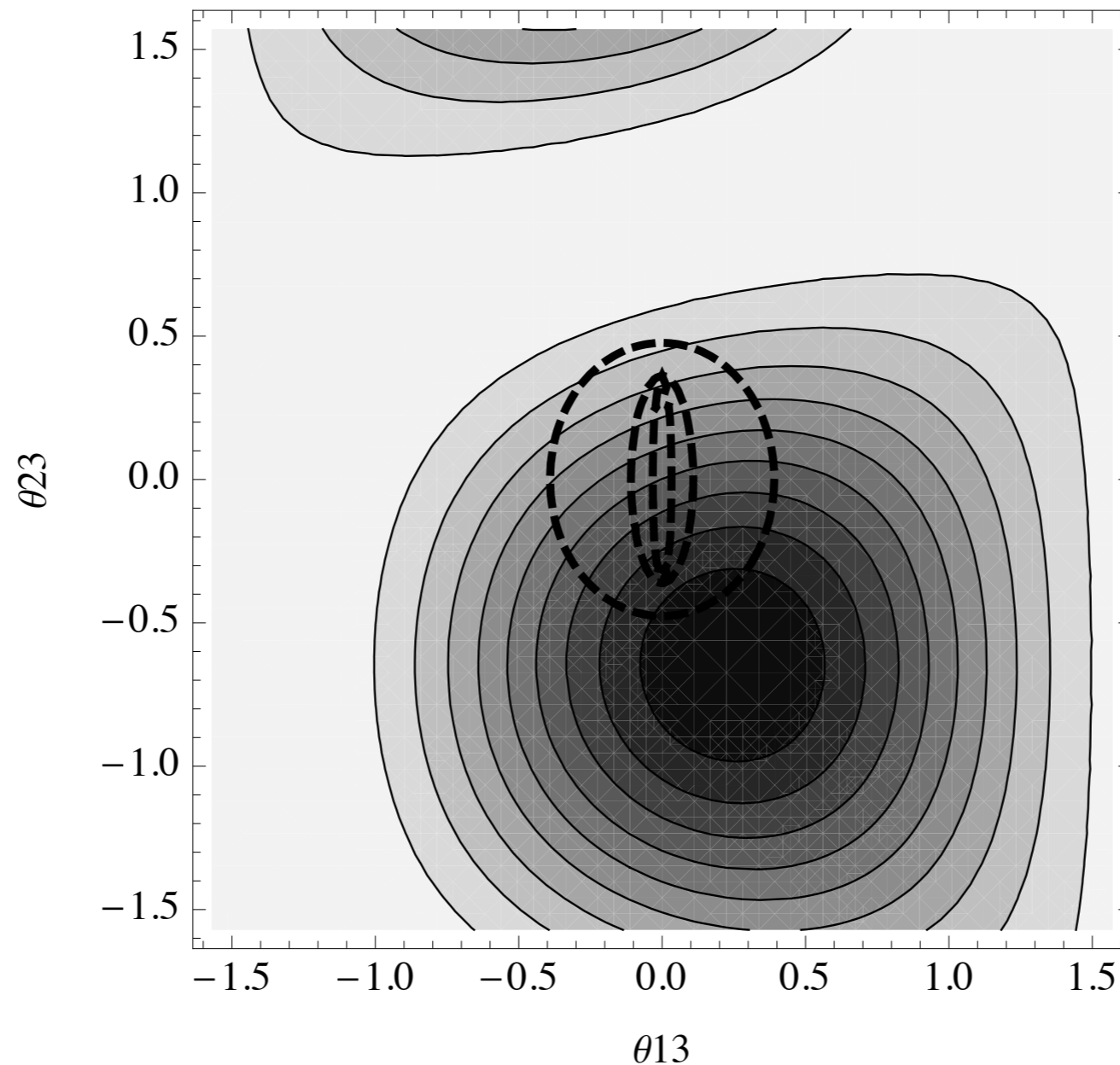
$$\Gamma(p \rightarrow \pi^0 \ell_\alpha^+) \propto |(V_{PMNS} U_\nu)_{\alpha 1}|^2$$



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# Proton decay to neutral mesons+charged leptons

Superimposing the two:  $\Gamma(p \rightarrow \pi^0 e^+) + \Gamma(p \rightarrow \pi^0 \mu^+)$  in the perturbative mode



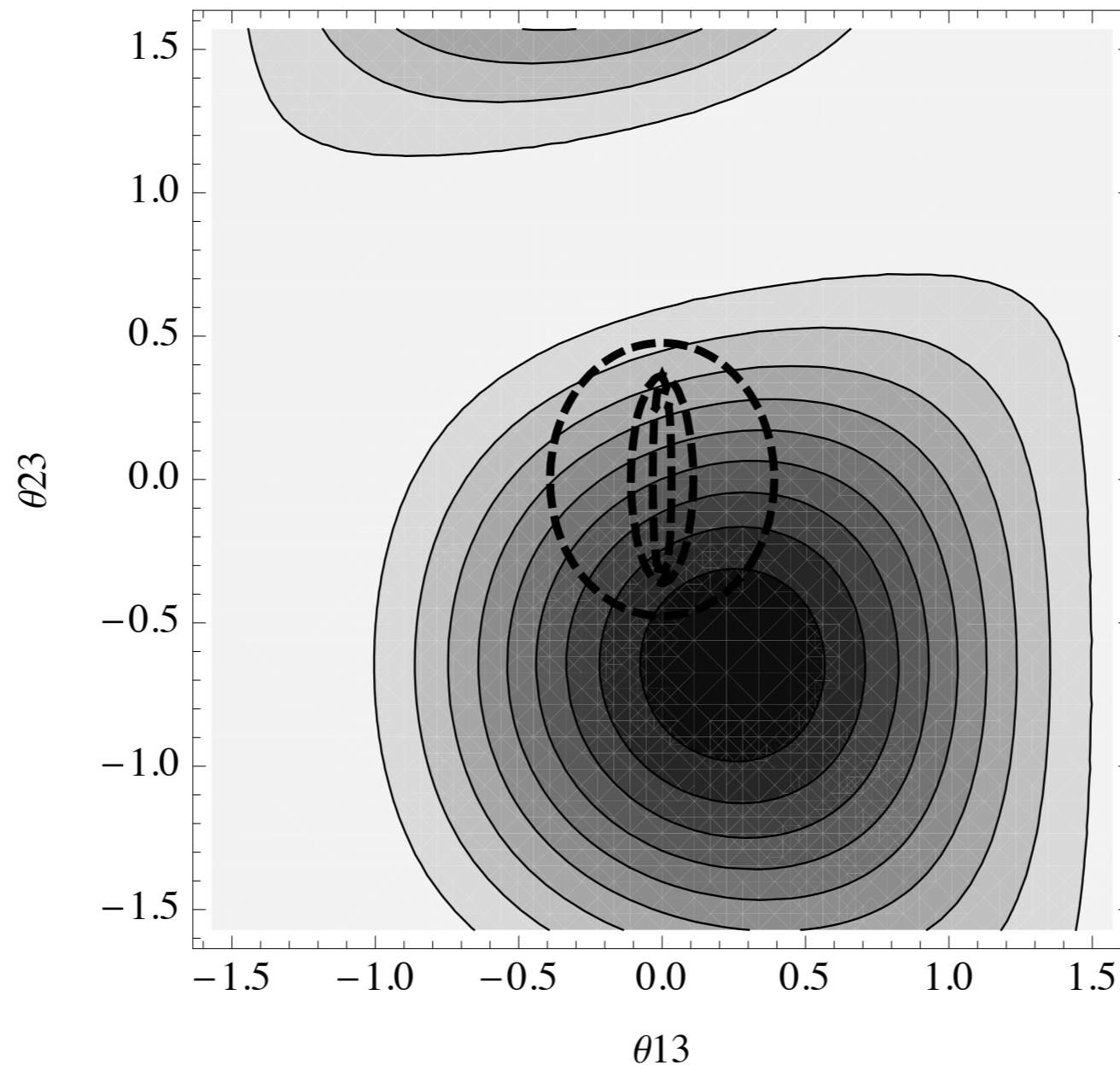
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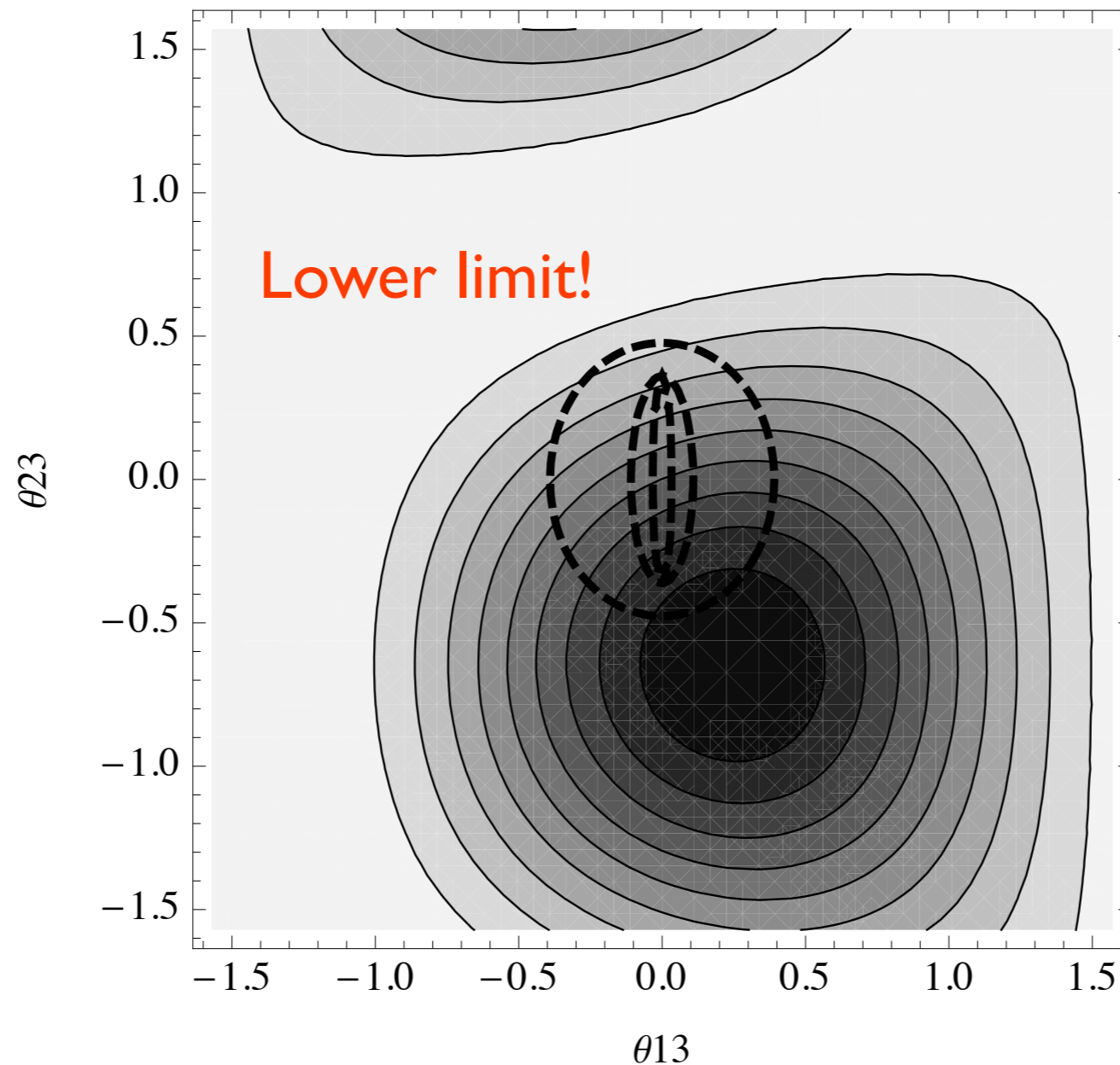
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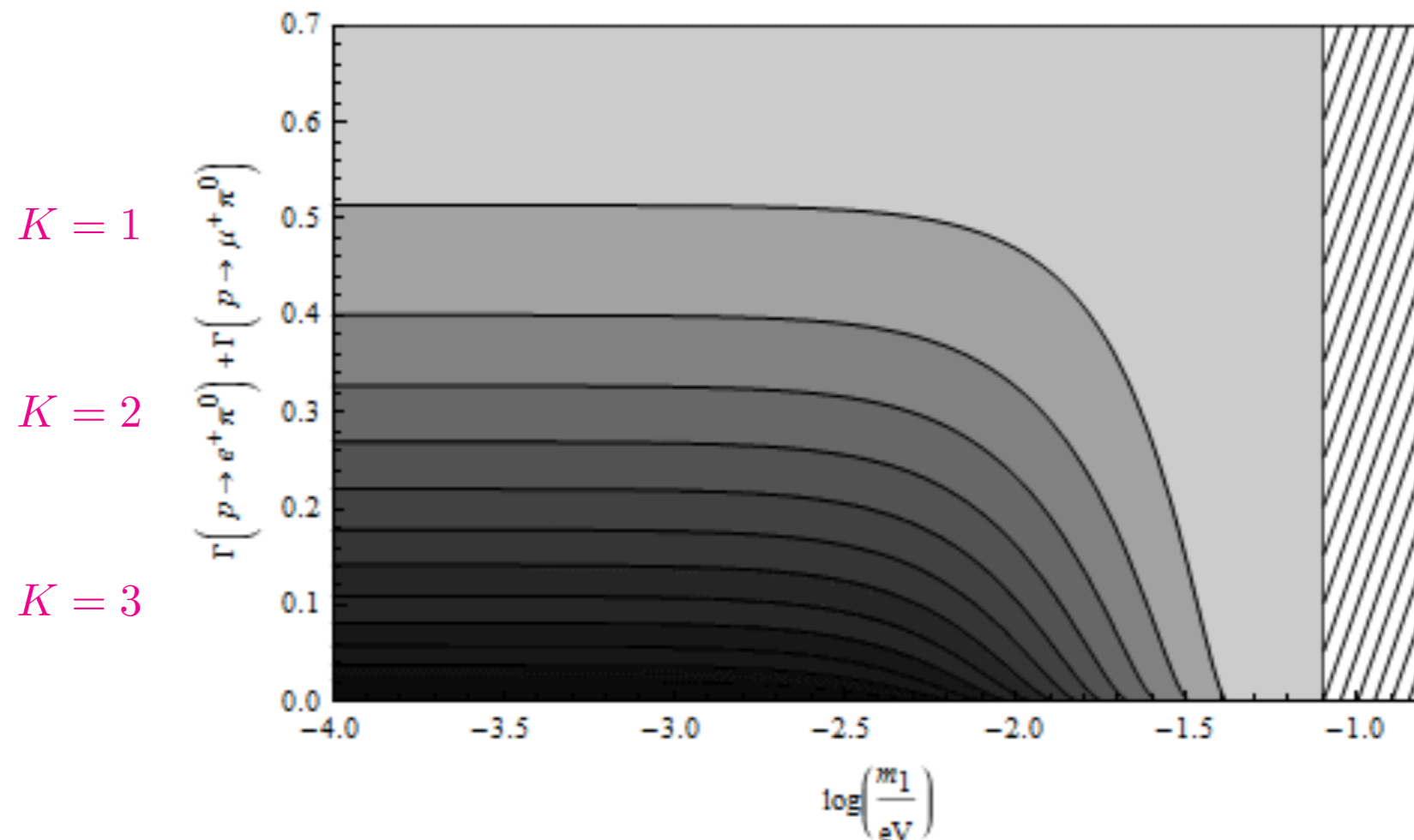
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# Proton decay to neutral mesons+charged leptons

Impossible to have both  $\Gamma(p \rightarrow \pi^0 e^+)$  and  $\Gamma(p \rightarrow \pi^0 \mu^+)$  suppressed!



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# Conclusions / outlook

Flavour structure of the LFV/LNV operators is great fun.

**Do not forget about the flavour structure of the d=6 BNV!**

Thanks for your kind attention!