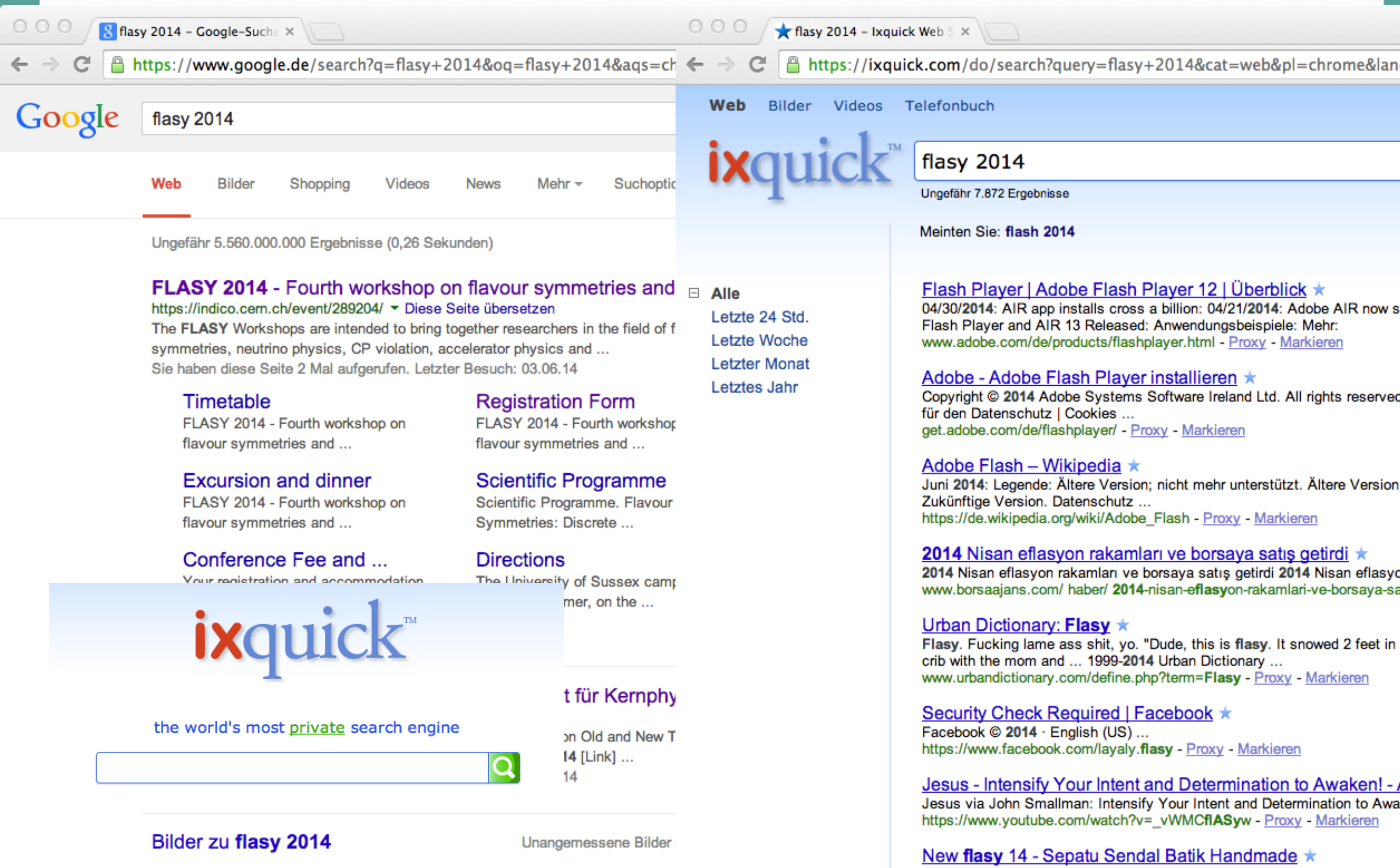
CP in Models with Discrete Flavour Symmetries

based on JHEP 1304 (2013) 122 [arXiv:1211.6953] with Manfred Lindner (HD) and Michael A. Schmidt (Melbourne)

FLASY 2014, Brighton

Martin Holthausen
Max-Planck-Institut für Kernphysik
Heidelberg

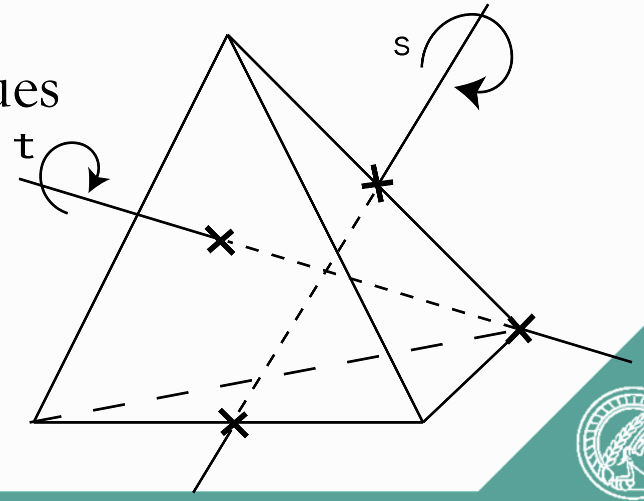
A new search engine...



Gives new insights...

CP Violation and Flavour

- CP Violation so far only observed in flavour sector
  - CP violation in lepton sector within exp. reach
- flavour symmetries are one possible explanation of the flavour puzzle
  - non-abelian discrete symmetries are motivated by close to maximal atmospheric mixing
- Goal of this talk: Clear up some issues surrounding the compatablility of CP and discrete flavour symmetries



Motivation

	E	T	T^2	S
$\underline{\mathbf{1}}_{\mathbf{1}}$	1	1	1	1
$\underline{\mathbf{1}}_{\mathbf{2}}$	1	ω	ω^2	1
$\underline{\mathbf{1}}_{\mathbf{3}}$	1	ω^2	ω	1
$\underline{\mathbf{3}}$	3	0	0	-1

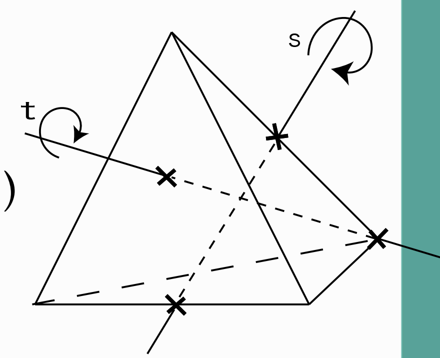
consider the group A_4:

$$A_4 = \langle S, T | S^2 = T^3 = (ST)^3 = E \rangle$$

consider a triplet $\chi \sim \underline{\mathbf{3}}$ transforming as

$$\omega = e^{i\frac{2\pi}{3}}$$

$$\rho_{\underline{\mathbf{3}}_{\mathbf{1}}}(S) = S_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho_{\underline{\mathbf{3}}_{\mathbf{1}}}(T) = T_3 \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$



and a non-trivial singlet $\xi \sim \underline{\mathbf{1}}_{\mathbf{3}}$ ($\rho_{\underline{\mathbf{1}}_{\mathbf{3}}}(S) = 1 \quad \rho_{\underline{\mathbf{1}}_{\mathbf{3}}}(T) = \omega^2$)

under the CP transformation $\chi \to \chi^* \quad \xi \to \xi^*$

the A4 invariant

$$I = \xi \left(\chi_1\chi_1 + \omega^2\chi_2\chi_2 + \omega\chi_3\chi_3\right) \sim \underline{\mathbf{1}}_{\mathbf{1}}$$

is mapped to sth. not invariant:

$$CP[I] = \xi^* \left(\chi_1^*\chi_1^* + \omega^2\chi_2^*\chi_2^* + \omega\chi_3^*\chi_3^*\right) \sim \underline{\mathbf{1}}_{\mathbf{2}}$$

- CP extends the group A4 and forbids this invariant??
- Is it possible to impose CP without forbidding wanted couplings?

How to define CP consistently

- Consider the vector made up out of all real(R), pseudo-real (P) and complex (C) representations of a given model

$$\phi = \left(\varphi_R, \quad \varphi_P, \quad \varphi_P^*, \quad \varphi_C, \quad \varphi_C^* \right)^T$$

- under the group G it transforms as $\phi \xrightarrow{G} \rho(g)\phi, \qquad g \in G.$
- the (reducible) representation $\rho : G \to U(N)$ is assumed to be faithful and complex
  - if not faithful then real symmetry group of theory is $G/\ker\rho$
  - ρ is homomorphism: ρ (a*b)= ρ (a) ρ (b)
- definition implies the existence of matrix W

$$\phi^* = W\phi \text{ or } \rho(g) = W\rho(g)^* W^{-1}$$

$$P : \varphi(t, \vec{x}) \to \varphi(t, -\vec{x})$$

$$C : \varphi(t, \vec{x}) \to \varphi^*(t, \vec{x})$$

$$CP : \varphi(t, \vec{x}) \to \varphi^*(t, -\vec{x})$$

- here only Lorentz-scalars, generalization straightforward

How to define CP consistently

- A generalized CP (gCP) acts upon the vector

[Bernabeu, Branco, Gronau 86]

$$\phi \xrightarrow{CP} U\phi^*$$

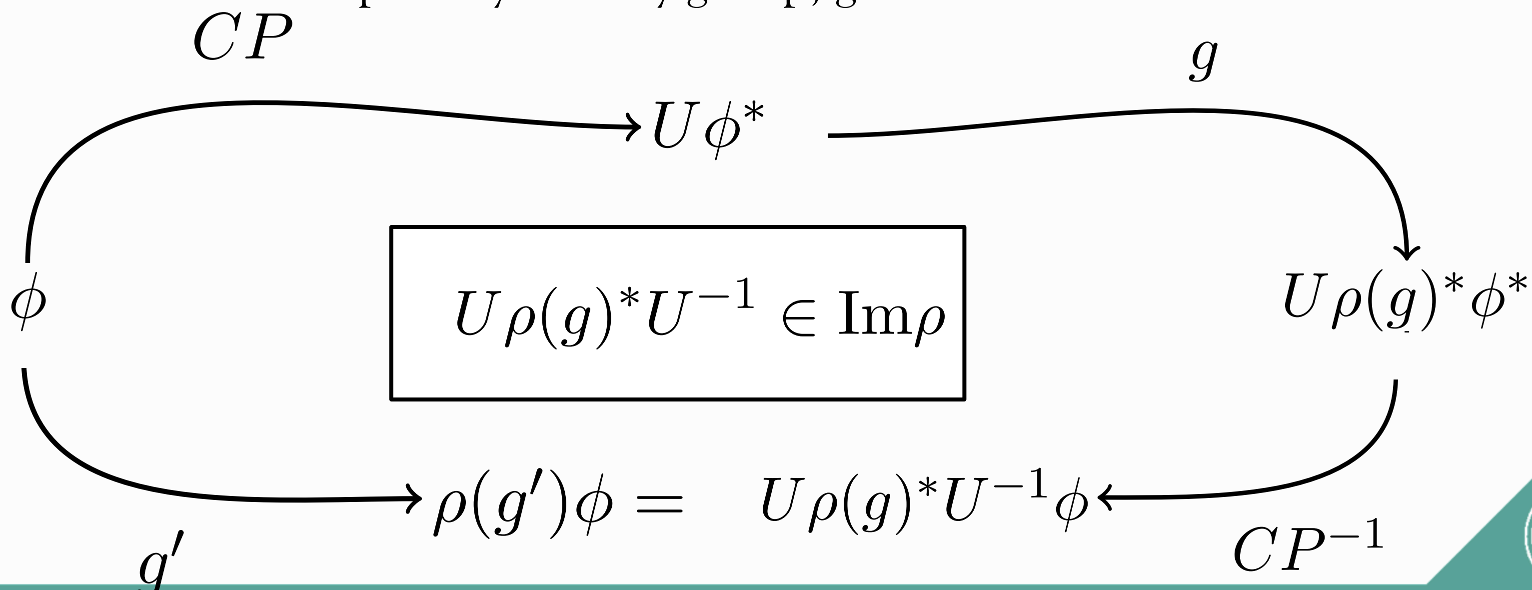
where U is unitary, to leave the kinetic term invariant.

for gauge groups this has been investigated by [Grimus, Rebelo 95]

CONSISTENCY CONDITION:

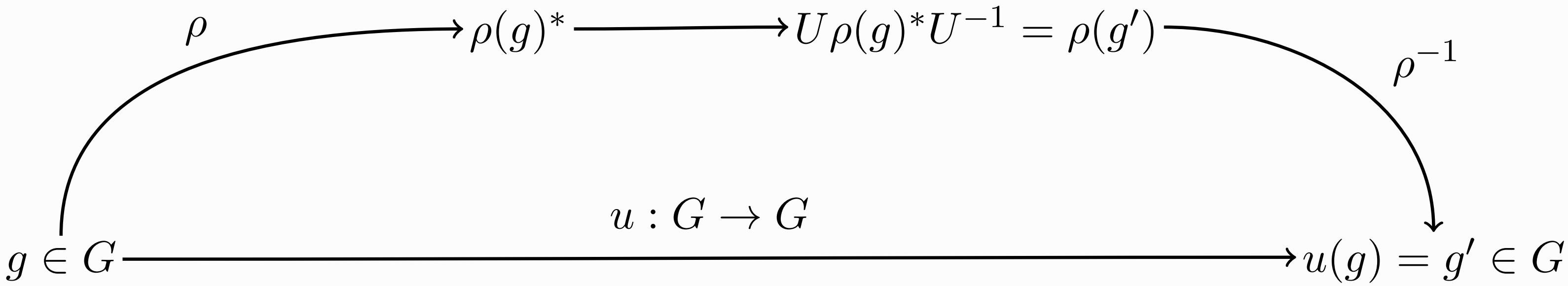
- If G is the complete symmetry group, gCP has to close in G:

[s.a Feruglio, Hagedorn, Ziegler 13]



gCP and the automorphism group

- The consistency condition $U\rho(g)^*U^{-1} \in \mathrm{Im}\rho$ defines an automorphism



$$U\rho(g)^*U^{-1} = \rho(u(g))$$

- the matrcies {U} furnish a representation of the automorphism group

$$\begin{aligned}\rho\left((a \circ b)(g)\right) &= \rho(a(b(g))) = U(a)\rho(b(g))^*U(a)^{-1} \\ &= U(a)W\rho(b(g))W^{-1}U(a)^{-1} \\ &= U(a)WU(b)\rho(g)^*U(b)^{-1}W^{-1}U^{-1}(a)\end{aligned}$$

$$U(a \circ b) = U(a)WU(b)$$

remember $\rho(g) = W\rho(g)^*W^{-1}$

neutral: $U(id) = W$

inverse: $U(u^{-1}) = WU^{-1}(u)W^{-1}$

gCP and the automorphism group

Inverse Direction: : Each automorphism u of G may be represented by such a matrix U.

$$U\rho(g)^*U^{-1} = \rho(u(g))$$

Proof:

- Construct group extended by automorphism u (u^n=id)

$$G' = G \rtimes_\theta Z_n \qquad \theta : \{0, ..., n-1\} \to Aut(G) \qquad \theta(1) = u$$

$$(g_1, z_1) \star (g_2, z_2) = (g_1\theta_{z_1}(g_2), z_1 + z_2)$$

- u acts as conjugation within this group

$$(E, 1) \star (g, 0) \star (E, 1)^{-1} = (u(g), 0)$$

- Consider representation $\rho' : G' \to U(M)$ induced via $\rho'(g, 0) = \rho(g)$

- automorphism u is represented by matrix

$$U(u) = \rho'((E, 1))W$$

$$\begin{aligned}\rho(u(g)) &= \rho'(u(g), 0)\\ &= \rho'((E, 1) \star (g, 0) \star (E, 1)^{-1})\\ &= \rho'((E, 1))\rho'((g, 0))\rho'((E, 1))^{-1}\\ &= \rho'((E, 1))W\rho(g)^*W^{-1}\rho'((E, 1))^{-1}\end{aligned}$$

Outer automorphism group

- if U is solution of $U\rho(g)^* U^{-1} = \rho(u(g))$ then so is $\rho(g')U$
  - corresponds to performing a gCP transformation followed by a group transformation described by ρ (g)
  - The group transformation corresponds to an inner homomorphism, which does not pose any new restrictions
- therefore interesting gCP transformations correspond to

$$\mathrm{Out}(G) \equiv \mathrm{Aut}(G)/\mathrm{Inn}(G)$$

where $\mathrm{Inn}(G) = \{u \in \mathrm{Aut}(G) | u(g) = AgA^{-1} \text{for some } A \in G\}$

- aside: continuous groups

$\mathrm{Out}(G) = E, Z_2$ except for $\mathrm{Out}(\mathrm{SO}(8)) = S_3$

- outer automorphism groups of small groups can be more involved: $\mathrm{Out}(\Delta(27)) \cong \mathrm{GL}(2,3)$

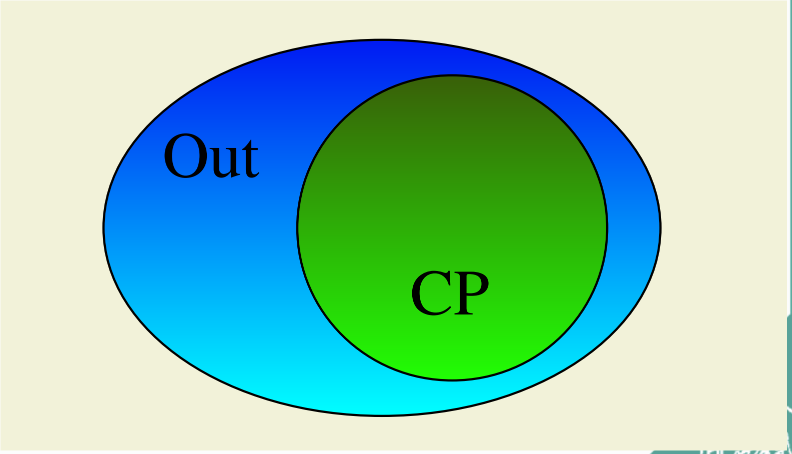
Physical CP Violation

see talk by A. Trautner

- not all gCP transformations correspond to physical CP violation [Nishi 13, Chen et al. 14]
- additional requirement: the square of a gCP transformation is a symmetry transformation

$$\rho\left(u^2(g)\right) = UU^*\rho(g)^*(U(a)U^*)^{-1} = \rho(g')$$

- CP has to map irr r to c.c. irr r*
  - U block diagonal [Chen et al. 14]
  - u has to be class-inverting
- „CP-like“ trafos useful
  - to construct U(u)
  - can be used to predict phases



t.f. talk by Trautner

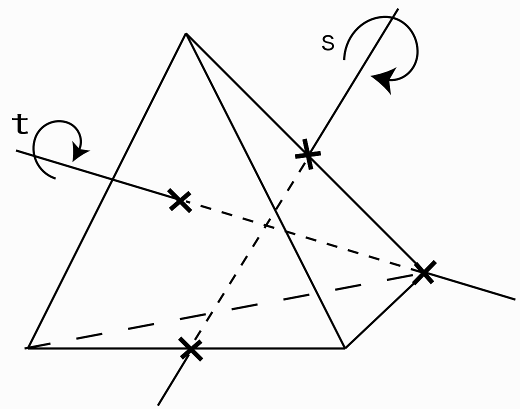
CP in A_4

outer automorphism group is Z_2, generated by

$$u : (S, T) \rightarrow (S, T^2).$$

	E	T	T^2	S
$\underline{\mathbf{1}}_1$	1	1	1	1
$\underline{\mathbf{1}}_2$	1	ω	ω^2	1
$\underline{\mathbf{1}}_3$	1	ω^2	ω	1
$\underline{\mathbf{3}}$	3	0	0	-1

$$A_4 = \langle S, T | S^2 = T^3 = (ST)^3 = E \rangle$$



outer automorphisms interchange representations and conjugacy classes

CP in A_4

on 3-dim representation

$$\rho_{\underline{\mathbf{3}}_{\mathbf{1}}}(S) = S_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \rho_{\underline{\mathbf{3}}_{\mathbf{1}}}(T) = T_3 \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

the consistency condition $U_3 \rho(T)^* U_3^{-1} = \rho(T^2)$

can be easily seen to require a 2-3 interchange: $U = U_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

- this can be easily read off, for more complicated setups it might not be so easy
- sketch of formalism described above:
  - construct group extended by outer automorphism, here S_4
  - extra group element gives matrix U

CP in A_4

- the ‚CP transformation' that is trivial with regard to A_4 runs into trouble if one considers a non-trivial singlet $\xi \sim \underline{\mathbf{1}}_{\mathbf{3}}$ in addition to the triplet $\chi \sim \underline{\mathbf{3}}$

- if one would use $\chi \to \chi^*$ and $\xi \to \xi^*$ one finds that the invariant is mapped to sth. non-invariant

$$\underline{\mathbf{1}}_{\mathbf{1}} \sim (\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}}\, \xi \to (\chi^*\chi^*)_{\underline{\mathbf{1}}_{\mathbf{2}}}\, \xi^* \sim \underline{\mathbf{1}}_{\mathbf{2}}$$

with $(\phi\phi)_{\underline{\mathbf{1}}_{\mathbf{2}}} = \frac{1}{\sqrt{3}}\left(\phi_1\phi_1 + \omega^2\phi_2\phi_2 + \omega\phi_3\phi_3\right)$

- this can be readily understood if one looks at how this ‚CP transformation' $\phi \to U\phi^*$ acts upon $\phi = (\xi, \xi^*, \chi)^T$
  - naive CP corresponds to $U=1_5$
  - A_4 does not close under this CP:

$$U\rho(T)^*U^{-1} = \rho(T)^* \notin \rho(G)$$

$$\rho(T) = \mathrm{diag}(\omega, \omega^2, T_3)$$
$$\rho(S) = \mathrm{diag}(1, 1, S_3)$$

- the real flavour group is larger, this has to be considered when constructing Lagrangian

often overlooked in literature [Toorop et. al. 2011, Ferreira, Lavoura 2011,....]

Geometric CP violation in $\Delta(27)$

$$\Delta(27) = \langle A, B | A^3 = B^3 = (AB)^3 = E \rangle$$

outer automorphism group generated by

$u_2 : (A, B) \to (ABAB, B^2)$ red

$u_1 : (A, B) \to (ABA^2, B^2AB)$ blue

	E	$BABA$	ABA	A	BAB	AB	A^2	B^2	B	BA^2BAB	AB^2ABA
$\underline{\mathbf{1}}_1$	1	1	1	1	1	1	1	1	1	1	1
$\underline{\mathbf{1}}_2$	1	ω	ω^2	1	ω	ω^2	1	ω	ω^2	1	1
$\underline{\mathbf{1}}_3$	1	ω^2	ω	1	ω^2	ω	1	ω^2	ω	1	1
$\underline{\mathbf{1}}_4$	1	ω	ω	ω^2	ω^2	ω^2	ω	1	1	1	1
$\underline{\mathbf{1}}_5$	1	ω^2	1	ω^2	1	ω	ω	ω	ω^2	1	1
$\underline{\mathbf{1}}_6$	1	1	ω^2	ω^2	ω	1	ω	ω^2	ω	1	1
$\underline{\mathbf{1}}_7$	1	ω^2	ω^2	ω	ω	ω	ω^2	1	1	1	1
$\underline{\mathbf{1}}_8$	1	1	ω	ω	ω^2	1	ω^2	ω	ω^2	1	1
$\underline{\mathbf{1}}_9$	1	ω	1	ω	1	ω^2	ω^2	ω^2	ω	1	1
$\underline{\mathbf{3}}$	3	.	.	.	.	.	.	.	.	3ω	$3\omega^2$
$\underline{\mathbf{3}}^*$	3	.	.	.	.	.	.	.	.	$3\omega^2$	3ω

What are calculable phases?

- consider again a triplet of Higgs doublets $H = (H_1, H_2, H_3) \sim \underline{\mathbf{3}}$ which transforms as

$$\rho(A) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad \rho(B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

- the potential only contains one phase dependent term

$$I \equiv (H_1^\dagger H_2)(H_1^\dagger H_3) + (H_2^\dagger H_3)(H_2^\dagger H_1) + (H_3^\dagger H_1)(H_3^\dagger H_2)$$

  - if coupling λ_4 multiplying I is positive, the global minimum is at $\langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$
    (or a configuration that can be obtained by acting on this vacuum with a group element)
  - if coupling λ_4 is negative, the global minimum is at $\langle H \rangle = \frac{v}{\sqrt{3}}(1, \omega, \omega^2)$
    (or a configuration that can be obtained by acting on this vacuum with a group element)
- These phases do not depend on potential parameters!
  - can this be used to predict (leptonic) CP phases?
  - can they be understood in terms of generalized CP?

Potential Dependence of Phases

- in general you expect two different kinds of vacua of a CP conserving potential
  - either VEV is real, conserves CP and phase does not depend on potential parameters
  - or VEV is complex, breaks CP and phase depends on potential parameters

Example: all parameters real

$$V = m_1^2 \varphi^* \varphi + m_2^2 (\varphi^2 + \varphi^{*2}) + \lambda_1 (\varphi^* \varphi)^2 + \lambda_2 (\varphi^4 + \varphi^{*4})$$
$$= m_1^2 A^2 + m_2^2 A^2 \cos 2\alpha + \lambda_1 A^4 + \lambda_2 A^4 \cos 4\alpha$$

invariant under $\varphi \to \varphi^*$

$$\varphi = A e^{\mathrm{i}\alpha}$$

$$\alpha = 0$$

$$A = -\frac{\sqrt{-\mathrm{m1}^2 - 2\mathrm{m2}^2}}{\sqrt{2}\sqrt{\lambda 1 + 2\lambda 2}}$$

$$\cos^2 \alpha = \frac{2\lambda_2 m_1^2 + \lambda_1 m_2^2 - 2\lambda_2 m_2^2}{4\lambda_2 m_1^2}$$

$$A = \frac{m_1}{\sqrt{2}\sqrt{2\lambda_2 - \lambda 1}}$$

What are calculable phases?

- The vacuum of the form $\langle H \rangle = \frac{v}{\sqrt{3}}(1, \omega, \omega^2)$ leaves invariant the gCP transformation

$$H \to \rho(B^2) H^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} H^*$$

- which is a symmetry of I+I*
    - no surprise there, CP symmetric potential has CP symmetric ground state
- for the other solution $\langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$ there is no group element that leaves H invariant $\langle H \rangle = \rho(g) \langle H \rangle^*$
    - this was interpreted as geometrical CP violation

GEOMETRICAL *T*-VIOLATION

G.C. BRANCO
Instituto Nacional de Investigação Científica, Av. do Prof. Gama Pinto 2, Lisbon, Portugal

and

J.-M. GERARD [1] and W. GRIMUS
CERN, Theory Division, Geneva, Switzerland

[Branco, Gerard and Grimus 1984; de Medeiros Varzielas, Emmanuel-Costa 2011; Battacharyya, de Medeiros Varzielas, Leser 2012]

Calculable Phases as a Result of an accidental generalized CP transformation

- every automorphism corresponds to a generalized CP transformation
- this allows one to search for gCP transformation that leaves $\langle H\rangle = \frac{v}{\sqrt{3}}(\omega^2,1,1)$ invariant and gives a real λ_4
- indeed there is such a gCP transformation:

$$H \rightarrow \tilde{U} H$$

$$\tilde{U} = \begin{pmatrix} 0 & 0 & \omega^2 \\ 0 & 1 & 0 \\ \omega & 0 & 0 \end{pmatrix}$$

$$\Rightarrow$$

$$CP_u[\langle H\rangle] = \langle H\rangle$$

$$CP_u[I] = I$$

$$u\,:\,(A,B) \rightarrow (AB^2AB, AB^2A^2)$$

- potential invariant under a larger symmetry
  - this CP-like trafo does not correspond to physical CP
  - still fixes phases

Calculable Phases as a Result of an accidental generalized CP transformation

- a symmetric potential can have a symmetric ground state
  - phases are dictated by accidental gCP symmetry
  - explains the independence from potential parameters
- this setup is interesting for phenomenlogy:
  - if accidental symmetry only of potential, not of Yukawas, it can be used to predict phases etc.
- need groups with large outer automorphism group
  - notice that shaping symmetries have large outer automorphism groups $|\mathrm{Out}\, Z_4^4| = 1321205760$
- mechanism similar to vacuum alignment mechanisms

T‘ and CP

- T‘ double cover of A_4: $T' = \langle S, T | S^4 = T^3 = (ST)^3 = E \rangle$
- complex Clebsch-Gordon coeffients as a possible new origin of CP violation?[Chen, Mahanthappa 09]
  - vague notions of CP = reality of couplings were used
  - VEVs assumed real $\langle \phi' \rangle = (1,1,1)V', \qquad \langle \phi \rangle = (0,0,1)V \qquad V, V' \in \mathbb{R}$

$$
\begin{aligned}
-\mathcal{L}_{TT} &= y_c TT\phi^2 + y_u TT\phi'^3 + \text{h.c.} \\
&= y_c \frac{3}{2}\frac{2-i}{2}\left\{(1-i)\,T_1T_2\left(\phi_1^2-\phi_2\phi_3\right) + i\,T_1^2\left(\phi_2^2-\phi_1\phi_3\right) + T_2^2\left(\phi_3^2-\phi_1\phi_2\right)\right\} + \\
&+ y_u\frac{1}{3}\left\{\left(2\phi_1'\phi_3' + {\phi_2'}^2\right)\left(iT_1^2\phi_1' + (1-i)T_1T_2\phi_2' + T_2^2\phi_3'\right)\right\} + \text{h.c.}\ ,
\end{aligned}
$$

$$
M_u = y_u \begin{pmatrix} \mathrm{i} & \frac{1-\mathrm{i}}{2} \\ \frac{1-\mathrm{i}}{2} & 1 \end{pmatrix} V'^3 + y_c \begin{pmatrix} 0 & 0 \\ 0 & 1-\frac{\mathrm{i}}{2} \end{pmatrix} V^2 .
$$

CP violation

T' and CP

- Only predictive scenario:
  - impose CP on Lagrangian, break it spontaneously
  - explicit breaking is basis dependent and thus not predictive
- Using the generalized CP formalism we can see that there is exactly one CP transformation which forces VEVs of triplets to be real

$$\underline{\mathbf{1_i}} \to \underline{\mathbf{1_i}}^* \qquad \underline{\mathbf{2_i}} \to \mathrm{diag}(\omega\tilde{\omega}^{-5}, \omega^{-1}\tilde{\omega}^{5})\underline{\mathbf{2_i}}^* \qquad \underline{\mathbf{3}} \to \underline{\mathbf{3}}^*$$

$$u' = \mathrm{conj}(T^2) \circ u$$

$$u : (S,T) \to (S^3, T^2)$$

$\underline{\mathbf{2_1}} : S = A_1, \quad T = \omega A_2$

$\underline{\mathbf{2_2}} : S = A_1 \quad T = \omega^2 A_2;$

$\underline{\mathbf{2_3}} : S = A_1, \quad T = A_2$

$$A_1 = \frac{-1}{\sqrt{3}} \begin{pmatrix} i & \tilde{\omega}\sqrt{2} \\ -\tilde{\omega}^{-1}\sqrt{2} & -i \end{pmatrix} \qquad A_2 = \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}$$

$$\tilde{\omega} = e^{2\pi \mathrm{i}/24} \quad \omega = e^{2\pi \mathrm{i}/3}$$

$$\rho(S) = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix} \qquad \rho(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

T‘ and CP

what does the CP trafo

$$\underline{\mathbf{1}}_{\mathbf{i}} \to \underline{\mathbf{1}}_{\mathbf{i}}^* \qquad \underline{\mathbf{2}}_{\mathbf{i}} \to \mathrm{diag}(\omega\tilde{\omega}^{-5}, \omega^{-1}\tilde{\omega}^{5})\underline{\mathbf{2}}_{\mathbf{i}}^* \qquad \underline{\mathbf{3}} \to \underline{\mathbf{3}}^*$$

imply for

$$\begin{aligned}
-\mathcal{L}_{TT} &= y_c TT\phi^2 + y_u TT\phi'^3 + \text{h.c.} \\
&= y_c \frac{3}{2}\frac{2-i}{2}\left\{(1-i)\,T_1 T_2\left(\phi_1^2 - \phi_2\phi_3\right) + i\,T_1^2\left(\phi_2^2 - \phi_1\phi_3\right) + T_2^2\left(\phi_3^2 - \phi_1\phi_2\right)\right\} + \\
&+ y_u \frac{1}{3}\left\{\left(2\phi_1'\phi_3' + \phi_2'^2\right)\left(iT_1^2\phi_1' + (1-i)T_1T_2\phi_2' + T_2^2\phi_3'\right)\right\} + \text{h.c.}\,,
\end{aligned}$$

$$CP[TT\phi^2] = -\frac{4+3\mathrm{i}}{5}(TT\phi^2)^* \qquad CP[TT\phi'^3] = -\mathrm{i}(TT\phi'^3)^*.$$

$$\begin{aligned}
\arg(y_c) &= -\frac{1}{2}\arg(-4-3\mathrm{i}) \\
&= -\frac{1}{2}\arctan\frac{3}{4}
\end{aligned}$$

$$arg(y_u) = \pi/4$$

- even if you phase rotate T you can only make one of the couplings real, therefore explicit breaking of CP
- agrees with recent finding of [Chen et al. 14]

Conclusions

- Consistency Conditions should be kept in mind when constructing models that contain CP and Flavour Symmetries
- generalized CP transformations may be interpreted as furnishing a representation of the automorphism group
  - physical CP depends on field content
- geometrical CP violation seems to be a consequence of (accidental) generalized CP symmetries of the potential
- maybe automorphisms may be used in model building more generally