## CP in Models with Discrete Flavour Symmetries

based on JHEP 1304 (2013) 122 [arXiv:1211.6953 ] with Manfred Lindner (HD) and Michael A. Schmidt (Melbourne)

FLASY 2014, Brighton
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Heidelberg

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Flasy. Fucking lame ass shit, yo. "Dude, this is flasy. It snowed 2 feet in th crib with the mom and ... 1999-2014 Urban Dictionary ..
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## Gives new insights...

## $\bigcirc$ © flasy $2014-$ Ixquick Web $\times \times$ and Urban Dictionary: Flasy <br> C.


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military
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insults
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flasted
Flasterbate
Flasterbate
Flasterbation
Flasterbat
flastered
flaster-like
Flastibating
flastic

1. Flasy

0
Fucking lame ass shit, yo.
"Dude, this is flasy. It snowed 2 feet in two days, and now I'm stuck in da crib with the mom and pops."
by chia-like December 22, 2009

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Words related to Flasy


## CP Violation and Flavour

○ CP Violation so far only observed in flavour sector

- CP violation in lepton sector within exp. reach

○ flavour symmetries are one possible explanation of the flavour puzzle
O non-abelian discrete symmetries are motivated by close to maximal atmospheric mixing

○ Goal of this talk: Clear up some issues surrounding the compatablility of CP and discrete flavour symmetries


## Motivation

consider the group $\mathrm{A}_{4}$ :

$$
A_{4}=\left\langle S, T \mid S^{2}=T^{3}=(S T)^{3}=E\right\rangle
$$

|  | $E$ | $T$ | $T^{2}$ | $S$ |
| :--- | :---: | :---: | :---: | :---: |
| $\underline{\mathbf{1}}_{\mathbf{1}}$ | 1 | 1 | 1 | 1 |
| $\mathbf{1}_{\mathbf{2}}$ | 1 | $\omega$ | $\omega^{2}$ | 1 |
| $\mathbf{1}_{\mathbf{3}}$ | 1 | $\omega^{2}$ | $\omega$ | 1 |
| $\underline{\mathbf{3}}$ | 3 | 0 | 0 | -1 |

consider a triplet $\chi \sim \underline{\mathbf{3}}$ transforming as

$$
\omega=e^{i \frac{2 \pi}{3}}
$$

$$
\rho_{\mathbf{3}_{\mathbf{1}}}(S)=S_{3} \equiv\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{array}\right){ }_{\rho} \underline{\mathbf{3}}_{\mathbf{1}}(T)=T_{3} \equiv\left(\begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{array}\right)
$$

and a non-trivial singlet $\xi \sim \underline{\mathbf{1}}_{\mathbf{3}}\left(\rho_{\mathbf{1}_{\mathbf{3}}}(S)=1 \quad \rho_{\mathbf{1}_{\mathbf{3}}}(T)=\omega^{2}\right)$
under the CP transformation $\chi \rightarrow \chi^{*} \quad \xi \rightarrow \xi^{*}$

the A4 invariant

$$
I=\xi\left(\chi_{1} \chi_{1}+\omega^{2} \chi_{2} \chi_{2}+\omega \chi_{3} \chi_{3}\right) \sim \underline{1}_{\mathbf{1}}
$$

is mapped to sth. not invariant:

$$
C P[I]=\xi^{*}\left(\chi_{1}^{*} \chi_{1}^{*}+\omega^{2} \chi_{2}^{*} \chi_{2}^{*}+\omega \chi_{3}^{*} \chi_{3}^{*}\right) \sim \underline{1}_{\mathbf{2}}
$$

- CP extends the group A4 and forbids this invariant??
- Is it possible to impose CP without forbidding wanted couplings?


## How to define CP consistently

- Consider the vector made up out of all real(R), pseudo-real ( P ) and complex (C) representations of a given model

$$
\phi=\left(\begin{array}{lllll}
\varphi_{R}, & \varphi_{P}, & \varphi_{P}^{*}, & \varphi_{C}, & \varphi_{C}^{*}
\end{array}\right)^{T}
$$

- under the group G it transforms as $\quad \phi \xrightarrow{G} \rho(g) \phi, \quad g \in G$.
- the (reducible) representation $\rho: G \rightarrow U(N)$ is assumed to be faithful and complex
- if not faithful then real symmetry group of theory is $G / \operatorname{ker} \rho$
- $\quad \rho$ is homomorphism: $\rho\left(\mathrm{a}^{*} \mathrm{~b}\right)=\rho$ (a) $\rho$ (b)
- definition implies the existence of matrix W

$$
\begin{aligned}
& \phi^{*}=W \phi \text { or } \\
& \rho(g)=W \rho(g)^{*} W^{-1}
\end{aligned}
$$

$P: \varphi(t, \vec{x}) \rightarrow \varphi(t,-\vec{x})$
$C: \varphi(t, \vec{x}) \rightarrow \varphi^{*}(t, \vec{x})$

$$
C P: \varphi(t, \vec{x}) \rightarrow \varphi^{*}(t,-\vec{x})
$$

- here only Lorentz-scalars, generalization straightforward


## How to define CP consistently

- A generalized CP (gCP) acts upon the vector

$$
\phi \xrightarrow{C P} U \phi^{*}
$$

[Bernabeu, Branco,
Gronau 86]
where U is unitary, to leave the kinetic term invariant.

## CONSISTENCY CONDITION:

for gauge groups this has been investigated by
[Grimus, Rebelo 95]

- If $G$ is the complete symmetry group, gCP has to close in G : $C P$
$g$



## gCP and the automorphism group

- The consistency condition $U \rho(g)^{*} U^{-1} \in \operatorname{Im} \rho$ defines an automorphism


$$
U \rho(g)^{*} U^{-1}=\rho(u(g))
$$

- the matrcies $\{\mathrm{U}\}$ furnish a representation of the automorphism group

$$
\begin{aligned}
\rho((a \circ b)(g)) & =\rho(a(b(g)))=U(a) \rho(b(g))^{*} U(a)^{-1} \\
& =U(a) W \rho(b(g)) W^{-1} U(a)^{-1} \\
& =U(a) W U(b) \rho(g)^{*} U(b)^{-1} W^{-1} U^{-1}(a)
\end{aligned}
$$

$$
U(a \circ b)=U(a) W U(b)
$$

remember $\rho(g)=W \rho(g)^{*} W^{-1}$
neutral:

$$
U(i d)=W
$$

inverse:
$U\left(u^{-1}\right)=W U^{-1}(u) W^{-1}$

## gCP and the automorphism group

Inverse Direction: : Each automorphism u of G may be represented by such a matrix U .

$$
U \rho(g)^{*} U^{-1}=\rho(u(g))
$$

Proof:

- Construct group extended by automorphism $u$ ( $\mathrm{u}^{\mathrm{n}}=\mathrm{id}$ )

$$
G^{\prime}=G \rtimes_{\theta} Z_{n}^{\theta:\{0, \ldots, n-1\} \rightarrow \operatorname{Aut}(G)} \begin{gathered}
\theta(1)=u \\
\left(g_{1}, z_{1}\right) \star\left(g_{2}, z_{2}\right)=\left(g_{1} \theta_{z_{1}}\left(g_{2}\right), z_{1}+z_{2}\right)
\end{gathered}
$$

- u acts as conjugation within this group

$$
(E, 1) \star(g, 0) \star(E, 1)^{-1}=(u(g), 0)
$$

- Consider representation $\rho^{\prime}: G^{\prime} \rightarrow U(M)$ induced via $\rho^{\prime}(g, 0)=\rho(g)$
- automorphism u is represented by matrix

$$
\left.\begin{array}{l}
\begin{array}{l}
\rho(u(g))
\end{array}=\rho^{\prime}(u(g), 0) \\
\\
\\
\text { automorphism } \mathrm{u} \text { is } \\
\text { represented by matrix }
\end{array} \quad \begin{array}{l}
\rho^{\prime}\left((E, 1) \star(g, 0) \star(E, 1)^{-1}\right) \\
U(u)=\rho^{\prime}((E, 1)) W
\end{array}=\rho^{\prime}((E, 1)) \rho^{\prime}((g, 0)) \rho^{\prime}((E, 1))^{-1}\right)
$$

## Outer automorphism group

- if U is solution of $U \rho(g)^{*} U^{-1}=\rho(u(g))$ then so is $\rho\left(g^{\prime}\right) U$
- corresponds to performing a gCP transformation followed by a group transformation described by $\rho(\mathrm{g})$
- The group transformation corresponds to an inner homomorphism, which does not pose any new restrictions
- therefore interesting gCP transformations correspond to

$$
\operatorname{Out}(G) \equiv \operatorname{Aut}(G) / \operatorname{Inn}(G)
$$

where

$$
\operatorname{Inn}(G)=\left\{u \in \operatorname{Aut}(G) \mid u(g)=A g A^{-1} \text { for some } A \in G\right\}
$$

- aside: continuous groups

$$
\operatorname{Out}(G)=E, Z_{2} \quad \text { except for } \quad \operatorname{Out}(\mathrm{SO}(8))=S_{3}
$$

- outer automorphism groups of small groups can be more involved:

$$
\operatorname{Out}(\Delta(27)) \cong \mathrm{GL}(2,3)
$$

## Physical CP Violation

see talk by A. Trautner

○ not all gCP transformations correspond to physical CP violation
[Nishi 13, Chen et al. 14]
○ additional requirement: the square of a gCP transformation is a symmetry transformation

$$
\rho\left(u^{2}(g)\right)=U U^{*} \rho(g)^{*}\left(U(a) U^{*}\right)^{-1}=\rho\left(g^{\prime}\right)
$$

○ CP has to map irr r to c.c. irr $\mathrm{r}^{*}$
○ U block diagonal [Chen et al. 14]
$\bigcirc \mathrm{u}$ has to be class-inverting
○ "CP-like" trafos useful
$\bigcirc$ to construct $\mathrm{U}(\mathrm{u})$
○ can be used to predict phases

t.f. talk by Trautner

## CP in $\mathrm{A}_{4}$

outer automorphism group is $\mathrm{Z}_{2}$, generated by

$$
\begin{aligned}
& u:(S, T) \rightarrow\left(S, T^{2}\right) \text {. } \\
& \begin{array}{c|cccc} 
& E & T & T^{2} & S \\
\hline \mathbf{1}_{\mathbf{1}} & 1 & 1 & 1 & 1 \\
\imath & \mathbf{1}_{\mathbf{2}} & 1 & \omega & \omega^{2} \\
\mathbf{1}_{\mathbf{3}} & 1 & \omega^{2} & \omega & 1 \\
\underline{\mathbf{3}} & 3 & 0 & 0 & -1
\end{array} \\
& A_{4}=\left\langle S, T \mid S^{2}=T^{3}=(S T)^{3}=E\right\rangle \\
& \text { outer automorphisms } \\
& \text { interchange } \\
& \text { representations and } \\
& \text { conjugacy classes }
\end{aligned}
$$

## CP in $\mathrm{A}_{4}$

on 3-dim representation
$\rho_{\mathbf{3}_{\mathbf{1}}}(S)=S_{3} \equiv\left(\begin{array}{ccc}1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1\end{array}\right) \quad \rho_{\mathbf{3}}(T)=T_{3} \equiv\left(\begin{array}{ccc}0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0\end{array}\right)$
the consistency condition $U_{3} \rho(T)^{*} U_{3}^{-1}=\rho\left(T^{2}\right)$
can be easily seen to require a 2-3 interchange: $U=U_{3} \equiv\left(\begin{array}{ccc}1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0\end{array}\right)$

- this can be easily read off, for more complicated setups it might not be so easy
- sketch of formalism described above:
- construct group extended by outer automorphism, here $S_{4}$
- extra group element gives matrix U


## CP in $\mathrm{A}_{4}$

$\bigcirc$ the , CP transformation' that is trivial with regard to $\mathrm{A}_{4}$ runs into trouble if one considers a non-trivial singlet $\xi \sim \underline{1}_{\mathbf{3}}$ in addition to the triplet $\chi \sim \underline{\mathbf{3}}$
$\cap$ if one would use $\chi \rightarrow \chi^{*}$ and $\xi \rightarrow \xi^{*}$ one finds that the invariant is mapped to sth. non-invariant

$$
\begin{aligned}
& \underline{\mathbf{1}}_{\mathbf{1}} \sim(\chi \chi)_{\mathbf{1}_{\mathbf{2}}} \xi \rightarrow\left(\chi^{*} \chi^{*}\right)_{\mathbf{1}_{\mathbf{2}}} \xi^{*} \sim \underline{\mathbf{1}}_{\mathbf{2}} \\
& \text { with } \underset{(\phi \phi)_{\mathbf{1}}=\frac{1}{\sqrt{3}}\left(\phi_{1} \phi_{1}+\omega^{2} \phi_{2} \phi_{2}+\omega \phi_{3} \phi_{3}\right)}{ }
\end{aligned}
$$

$\bigcirc$ this can be readily understood if one looks at how this ,CP transformation' $\phi \rightarrow \mathrm{U} \phi^{*}$ acts upon $\phi=\left(\xi, \xi^{*}, \chi\right)^{T}$

- naive CP corresponds to $\mathrm{U}=1_{5}$

○ $\mathrm{A}_{4}$ does not close under this CP :

$$
U \rho(T)^{*} U^{-1}=\rho(T)^{*} \notin \rho(G)
$$

$\bigcirc$ the real flavour group is larger, this has to be considered when constructing Lagrangian

$$
\begin{aligned}
& \rho(T)=\operatorname{diag}\left(\omega, \omega^{2}, T_{3}\right. \\
& \rho(S)=\operatorname{diag}\left(1,1, S_{3}\right)
\end{aligned}
$$

often overlooked in literature [Toorop et. al. 2011, Ferreira, Lavoura 2011,....]

## Geometric CP violation in $\Delta$ (27)

$\Delta(27)=\left\langle A, B \mid A^{3}=B^{3}=(A B)^{3}=E\right\rangle$
outer automorphism group generated by $u_{2}:(A, B) \rightarrow\left(A B A B, B^{2}\right) \quad u_{1}:(A, B) \rightarrow\left(A B A^{2}, B^{2} A B\right)$ red


|  | $E$ | $B A B A$ | $A B A$ | A | $B A B$ | $A B$ | $A^{2}$ | $B^{2}$ | B | $B A^{2} B A B$ | $A B^{2} A B A$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underline{1}_{1}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $1_{2}$ | 1 | $\omega$ | $\omega^{2}$ | 1 | $\omega$ | $\omega^{2}$ | 1 | $\omega$ | $\omega^{2}$ | 1 | 1 |
| $1_{3}$ | 1 | $\omega^{2}$ | $\omega$ | 1 | $\omega^{2}$ | $\omega$ | 1 | $\omega^{2}$ | $\omega$ | 1 | 1 |
| $1_{4}{ }^{1}$ | 1 | $\omega$ | $\omega$ | $\omega^{2}$ | $\omega^{2}$ | $\omega^{2}$ | $\omega$ | 1 | 1 | 1 | 1 |
| ${ }^{1}$ | 1 | $\omega^{2}$ | 1 | $\omega^{2}$ | 1 | $\omega$ | $\omega$ | $\omega$ | $\omega^{2}$ | 1 | 1 |
| ${ }^{1} \underline{1}_{6}$ | 1 | 1 | $\omega^{2}$ | $\omega^{2}$ | $\omega$ | 1 | $\omega$ | $\omega^{2}$ | $\omega$ | 1 | 1 |
| ( $\underline{17}_{7}{ }^{1}$ | 1 | $\omega^{2}$ | $\omega^{2}$ | $\omega$ | $\omega$ | $\omega$ | $\omega^{2}$ | 1 | 1 | 1 | 1 |
| $1_{8}$ | 1 | 1 | $\omega$ | $\omega$ | $\omega^{2}$ | 1 | $\omega^{2}$ | $\omega$ | $\omega^{2}$ | 1 | 1 |
| ${ }^{1} 9$ | 1 | $\omega$ | 1 | $\omega$ | 1 | $\omega^{2}$ | $\omega^{2}$ | $\omega^{2}$ | $\omega$ | 1 | 1 |
| $\underline{3}$ | 3 |  |  | . |  |  |  |  |  | $3 \omega$ | $3 \omega^{2}$ |
| $\underline{3}^{*}$ | 3 |  |  | . |  |  |  |  |  | $3 \omega^{2}$ | $3 \omega$ |

## What are calculable phases?

- consider again a triplet of Higgs doublets $H=\left(H_{1}, H_{2}, H_{3}\right) \sim \underline{\mathbf{3}}$ which transforms as

$$
\rho(A)=\left(\begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{array}\right) \quad \rho(B)=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \omega & 0 \\
0 & 0 & \omega^{2}
\end{array}\right)
$$

- the potential only contains one phase dependent term

$$
I \equiv\left(H_{1}^{\dagger} H_{2}\right)\left(H_{1}^{\dagger} H_{3}\right)+\left(H_{2}^{\dagger} H_{3}\right)\left(H_{2}^{\dagger} H_{1}\right)+\left(H_{3}^{\dagger} H_{1}\right)\left(H_{3}^{\dagger} H_{2}\right)
$$

- if coupling $\lambda_{4}$ multiplying I is positive, the global minimum is at (or a configuration that can be obtained by acting on this vacuum with a group element) $\langle H\rangle=\frac{v}{\sqrt{3}}\left(\omega^{2}, 1,1\right)$

- These phases do not depend on potential parameters!
- can this be used to predict (leptonic) CP phases?
- can they be understood in terms of generalized CP?


## Potential Dependence of Phases

$\bigcirc$ in general you expect two different kinds of vacua of a CP conserving potential

- either VEV is real, conserves CP and phase does not depend on potential parameters
$\bigcirc$ or VEV is complex, breaks CP and phase depends on potential parameters
Example:
all parameters real

$$
\begin{aligned}
V & =m_{1}^{2} \varphi^{*} \varphi+m_{2}^{2}\left(\varphi^{2}+\varphi^{* 2}\right)+\lambda_{1}\left(\varphi^{*} \varphi\right)^{2}+\lambda_{2}\left(\varphi^{4}+\varphi^{* 4}\right) \\
& =m_{1}^{2} A^{2}+m_{2}^{2} A^{2} \cos 2 \alpha+\lambda_{1} A^{4}+\lambda_{2} A^{4} \cos 4 \alpha
\end{aligned}
$$

invariant under $\varphi \rightarrow \varphi^{*}$

$$
\varphi=A e^{\mathrm{i} \alpha}
$$

$$
\begin{gathered}
\cos ^{2} \alpha=\frac{2 \lambda_{2} m_{1}^{2}+\lambda_{1} m_{2}^{2}-2 \lambda_{2} m_{2}^{2}}{4 \lambda_{2} m_{1}^{2}} \\
A=\frac{m_{1}}{\sqrt{2} \sqrt{2 \lambda_{2}-\lambda 1}}
\end{gathered}
$$

## What are calculable phases?

- The vacuum of the form $\langle H\rangle=\frac{v}{\sqrt{3}}\left(1, \omega, \omega^{2}\right)$ leaves invariant the gCP transformation

$$
H \rightarrow \rho\left(B^{2}\right) H^{*}=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \omega^{2} & 0 \\
0 & 0 & \omega
\end{array}\right) H^{*}
$$

- which is a symmetry of $\mathrm{I}+\mathrm{I}^{*}$
- no surprise there, CP symmetric potential has CP symmetric ground state
- for the other solution $\langle H\rangle=\frac{v}{\sqrt{3}}\left(\omega^{2}, 1,1\right)$ there is no group element that leaves H invariant $\langle H\rangle=\rho(g)\langle H\rangle^{*}$
- this was interpreted as geometrical CP violation

GEOMETRICAL T-VIOLATION
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and
J.-M. GERARD ${ }^{1}$ and W. GRIMUS

CERN, Theory Division, Geneva, Switzerland
[Branco, Gerard and Grimus 1984; de Medeiros Varzielas, Emmanuel-Costa 2011; Battacharyya, de Medeir Varzielas, Leser 2012

## Calculable Phases as a Result of an accidental generalized CP transformation

○ every automorphism corresponds to a generalized CP transformation

○ this allows one to search for gCP transformation that leaves $\langle H\rangle=\frac{v}{\sqrt{3}}\left(\omega^{2}, 1,1\right)$ invariant and gives a real $\lambda_{4}$
○ indeed there is such a gCP transformation:

$$
\begin{gathered}
H \rightarrow \tilde{U} H \\
\tilde{U}=\left(\begin{array}{ccc}
0 & 0 & \omega^{2} \\
0 & 1 & 0 \\
\omega & 0 & 0
\end{array}\right)
\end{gathered}
$$

$$
\begin{gathered}
C P_{u}[\langle H\rangle]=\langle H\rangle \\
C P_{u}[I]=I \\
\underbrace{:(A, B) \rightarrow\left(A B^{2} A B, A B^{2} A^{2}\right)}
\end{gathered}
$$

○ potential invariant under a larger symmetry
$\bigcirc$ this CP-like trafo does not correspond to physical CP
○ still fixes phases

## Calculable Phases as a Result of an accidental generalized CP transformation

○ a symmetric potential can have a symmetric ground state
○ phases are dictated by accidental gCP symmetry
○ explains the independence from potential parameters
○ this setup is interesting for phenomenlogy:
○ if accidental symmetry only of potential, not of Yukawas, it can be used to predict phases etc.

○ need groups with large outer automorphism group
O notice that shaping symmetries have large outer automorphism groups

$$
\mid \text { Out } Z_{4}^{4} \mid=1321205760
$$

○ mechanism similar to vacuum alignment mechanisms

## T‘ and CP

$\cap \quad \mathrm{T}^{\prime}$ double cover of $\mathrm{A}_{4}: T^{\prime}=\left\langle S, T \mid S^{4}=T^{3}=(S T)^{3}=E\right\rangle$
○ complex Clebsch-Gordon coeffients as a possible new origin of CP violation? [Chen, Mahanthappa 09]

- vague notions of $\mathrm{CP}=$ reality of couplings were used

○ VEVs assumed real $\left\langle\phi^{\prime}\right\rangle=(1,1,1) V^{\prime}, \quad\langle\phi\rangle=(0,0,1) V \quad V, V^{\prime} \in \mathbb{R}$

$$
\begin{aligned}
-\mathcal{L}_{T T} & =y_{c} T T \phi^{2}+y_{u} T T \phi^{\prime 3}+\text { h.c. } \\
& =y_{c} \frac{3}{2} \frac{2-i}{2}\left\{(1-i) T_{1} T_{2}\left(\phi_{1}^{2}-\phi_{2} \phi_{3}\right)+i T_{1}^{2}\left(\phi_{2}^{2}-\phi_{1} \phi_{3}\right)+T_{2}^{2}\left(\phi_{3}^{2}-\phi_{1} \phi_{2}\right)\right\}+ \\
& +y_{u} \frac{1}{3}\left\{\left(2 \phi_{1}^{\prime} \phi_{3}^{\prime}+\phi_{2}^{\prime 2}\right)\left(i T_{1}^{2} \phi_{1}^{\prime}+(1-i) T_{1} T_{2} \phi_{2}^{\prime}+T_{2}^{2} \phi_{3}^{\prime}\right)\right\}+\text { h.c. },
\end{aligned}
$$

$$
M_{u}=y_{u}\left(\begin{array}{cc}
\mathrm{i} & \frac{1-\mathrm{i}}{2} \\
\frac{1-\mathrm{i}}{2} & 1
\end{array}\right) V^{\prime 3}+y_{c}\left(\begin{array}{cc}
0 & 0 \\
0 & 1-\frac{\mathrm{i}}{2}
\end{array}\right) V^{2} .
$$

## T' and CP

○ Only predictive scenario:
○ impose CP on Lagrangian, break it spontaneously
○ explicit breaking is basis dependent and thus not predictive

○ Using the generalized CP formalism we can see that there is exactly one CP transformation which forces VEVs of triplets to be real

$$
\underline{\mathbf{1}}_{\mathbf{i}} \rightarrow \underline{1}_{\mathbf{i}}^{*} \quad \underline{\mathbf{2}}_{\mathbf{i}} \rightarrow \operatorname{diag}\left(\omega \tilde{\omega}^{-5}, \omega^{-1} \tilde{\omega}^{5}\right) \underline{\mathbf{2}}_{\mathbf{i}}^{*} \quad \underline{\mathbf{3}} \rightarrow \underline{\mathbf{3}}^{*} \quad \begin{gathered}
u^{\prime}=\operatorname{conj}\left(T^{2}\right) \circ u \\
u:(S, T) \rightarrow\left(S^{3}, T^{2}\right)
\end{gathered}
$$

$$
\begin{array}{lll}
\underline{\mathbf{2}}_{\mathbf{1}}: S=A_{1}, & T=\omega A_{2} & A_{1}=\frac{-1}{\sqrt{3}}\left(\begin{array}{cc}
i & \tilde{\omega} \sqrt{2} \\
-\tilde{\omega}_{\mathbf{2}}^{-1} \sqrt{2} & -i
\end{array}\right) \\
\underline{\mathbf{2}}_{\mathbf{3}}: S=A_{1} & T=\omega^{2} A_{2} ; & \tilde{\omega}=e^{2 \pi \mathrm{i} / 24}, \\
T=A_{2} & A_{2}=\left(\begin{array}{cc}
\omega & 0 \\
0 & 1
\end{array}\right) & \left.\rho(S)=\frac{1}{3}\left(\begin{array}{ccc}
-1 & 2 \omega & 2 \omega^{2} \\
2 \omega^{2} & -1 & 2 \omega \\
2 \omega & 2 \omega^{2} & -1
\end{array}\right) \quad \rho(T)=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \omega & 0 \\
0 & 0 & \omega^{2}
\end{array}\right) \quad \begin{array}{l}
\text { ( }
\end{array}\right)
\end{array}
$$

## $\mathrm{T}^{‘}$ and CP

## what does the CP trafo

$$
\underline{1}_{\mathbf{i}} \rightarrow \underline{\mathbf{1}}_{\mathbf{i}}^{*} \quad \underline{\mathbf{2}}_{\mathbf{i}} \rightarrow \operatorname{diag}\left(\omega \tilde{\omega}^{-5}, \omega^{-1} \tilde{\omega}^{5}\right) \underline{\mathbf{2}}_{\mathbf{i}}^{*} \quad \underline{\mathbf{3}} \rightarrow \underline{\mathbf{3}}^{*}
$$

imply $-\mathcal{L}_{T T}=y_{c} T T \phi^{2}+y_{u} T T \phi^{\prime 3}+$ h.c.
for

$$
\begin{aligned}
& =y_{c} \frac{3}{2} \frac{2-i}{2}\left\{(1-i) T_{1} T_{2}\left(\phi_{1}^{2}-\phi_{2} \phi_{3}\right)+i T_{1}^{2}\left(\phi_{2}^{2}-\phi_{1} \phi_{3}\right)+T_{2}^{2}\left(\phi_{3}^{2}-\phi_{1} \phi_{2}\right)\right\}+ \\
& +y_{u} \frac{1}{3}\left\{\left(2 \phi_{1}^{\prime} \phi_{3}^{\prime}+{\phi_{2}^{\prime}}^{2}\right)\left(i T_{1}^{2} \phi_{1}^{\prime}+(1-i) T_{1} T_{2} \phi_{2}^{\prime}+T_{2}^{2} \phi_{3}^{\prime}\right)\right\}+\text { h.c. }
\end{aligned}
$$

$$
C P\left[T T \phi^{2}\right]=-\frac{4+3 \mathrm{i}}{5}\left(T T \phi^{2}\right)^{*} \quad C P\left[T T \phi^{\prime 3}\right]=-\mathrm{i}\left(T T \phi^{\prime 3}\right)^{*} .
$$

- even if you phase rotate $T$ you can only make one of the couplings real, therefore explicit breaking of CP

$$
\begin{aligned}
\arg \left(y_{c}\right) & =-\frac{1}{2} \arg (-4-3 \mathrm{i}) \\
& =-\frac{1}{2} \arctan \frac{3}{4} \\
\arg \left(y_{u}\right) & =\pi / 4
\end{aligned}
$$

- agrees with recent finding of [Chen et al. 14]


## Conclusions

○ Consistency Conditions should be kept in mind when constructing models that contain CP and Flavour Symmetries

○ generalized CP transformations may be interpreted as furnishing a representation of the automorphism group
○ physical CP depends on field content
○ geometrical CP violation seems to be a consequence of (accidental) generalized CP symmetries of the potential

○ maybe automorphisms may be used in model building more generally

