FLASY 2014

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Residual discrete symmetries in Froggatt-Nielsen models

Christoph Luhn

Outline

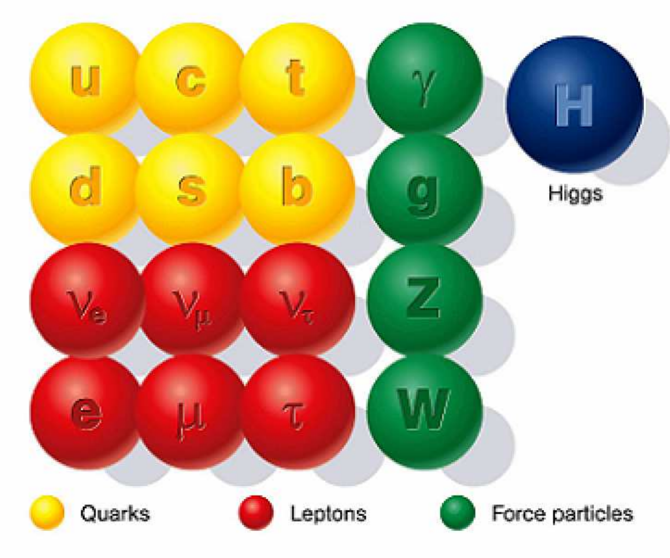
- supersymmetric Standard Model
  - proton decay
  - μ-problem

  $\longrightarrow$ impose discrete symmetry

- Froggatt-Nielsen framework
  - $U(1)_{\mathrm{FN}}$ family symmetry
  - residual discrete Z_N symmetries
  - extend to R-symmetries

- model realisation, i.e. finding viable $U(1)_{\mathrm{FN}}^{R}$ charge assignments
  - anomaly conditions
  - flavon VEV
  - Yukawas and μ-term
  - neutrino masses

Supersymmetric Standard Model



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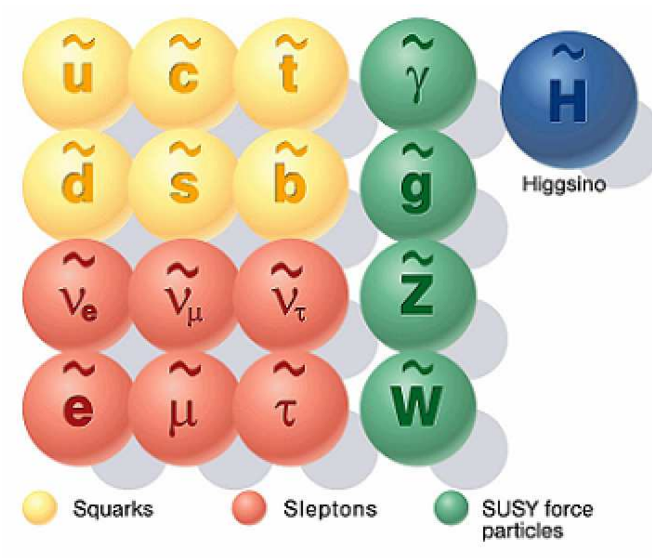

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$$W_{\text{ren}} = \mu\, H_u\, H_d + Y^u_{ij}\, H_u\, Q_i\, U^c_j + Y^d_{ij}\, H_d\, Q_i\, D^c_j + Y^e_{ij}\, H_d\, L_i\, E^c_j$$

$$+ \kappa_i\, H_u\, L_i + \lambda_{ijk}\, L_i\, L_j\, E^c_k + \lambda'_{ijk}\, L_i\, Q_j\, D^c_k + \lambda''_{ijk}\, U^c_i\, D^c_j\, D^c_k$$

- in total > 300 parameters Barbier et al. (hep-ph/0406039)
- impose extra symmetries, e.g. R-parity ∼ matter parity

  → MSSM with 124 parameters Haber (hep-ph/9709450)

R-parity and matter parity

superfield formalism, e.g.

$$\boxed{Q = \widetilde{q} + q\,\theta + F_q\,\theta^2}$$

$q =$ quark

$\widetilde{q} =$ squark

$\theta =$ superspace variable

- R-parity (R_p) defined on component fields, e.g. q and $\widetilde{q}$

R_p	quarks	leptons	Higgs	squarks	sleptons	Higgsino
additive Z_2	0	0	0	1	1	1

- matter parity (M_p) defined on superfields, e.g. Q

M_p	(s)quarks	(s)leptons	Higgs(inos)
additive Z_2	1	1	0

R_p and M_P allow and forbid exactly the same terms in Lagrangian

Matter parity from $U(1)_{B-L}$

matter parity
- forbids (renormalisable) B and L violation
- introduced to stabilise proton
- violated by quantum gravity effects Krauss, Wilczek (1989)
- unless gauge origin, e.g. from breaking $U(1)_{B-L}$

	Q	U^c	D^c	L	E^c	H_u	H_d	ϕ
$U(1)_{B-L}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	1	0	0	$\pm\frac{2}{3}$
$3 \times U(1)_{B-L}$	1	-1	-1	-3	3	0	0	± 2
M_p	1	1	1	1	1	0	0	0

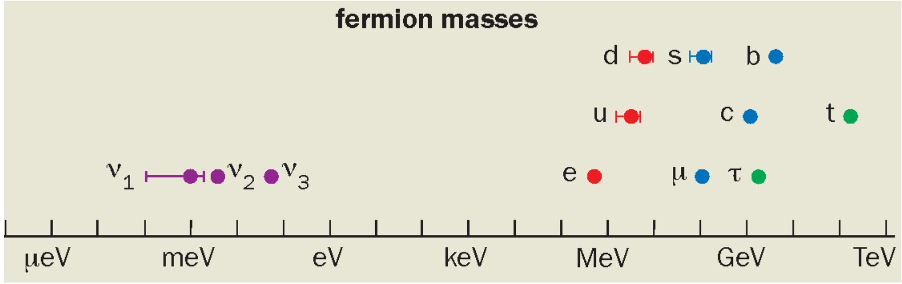
$\rightarrow$ vev of field ϕ breaks $U(1)_{B-L}$ spontaneously down to M_p

"Higgs"-type field
neutral under
SM gauge group

$U(1)_{\mathrm{FN}}$ family symmetry   Froggatt, Nielsen (1979)

Fermion masses and mixings

- fermion mass terms: $M_{ij}\,\overline{\psi}_i\,\psi_j$
- masses = eigenvalues of M
- with $\lambda \approx 0.22$



- quarks

$$m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$$

$$m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1$$

$$U_{\mathrm{CKM}} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

- leptons

$$m_e : m_\mu : m_\tau \sim \lambda^{4\,\mathrm{or}\,5} : \lambda^2 : 1$$

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \begin{cases} \lambda^x : \lambda : 1 \\ 1 : 1 : \lambda^x \\ 1 : 1 : 1 \end{cases}$$

$$U_{\mathrm{PMNS}} \approx \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.37 & 0.57 & 0.70 \\ 0.39 & 0.59 & 0.68 \end{pmatrix}$$

www.nu-fit.org (2013)

Froggatt-Nielsen mechanism

- $U(1)_{\mathrm{FN}}$ family symmetry
- spontaneously broken by flavon field ϕ

$$\boxed{Y_{ij}\,\overline{\psi}_i\,\psi_j H} \quad \text{forbidden} \quad \longrightarrow \quad \boxed{c_{ij}\,\overline{\psi}_i\,\psi_j H\left(\frac{\phi}{\Lambda}\right)^{x_{ij}}} \quad \text{allowed}$$

- flavon VEV generates Yukawas $Y_{ij} = c_{ij}\left(\frac{\langle\phi\rangle}{\Lambda}\right)^{x_{ij}}$
- $\mathcal{O}(1)$ coefficients c_{ij} not fixed

fields	Q_1	Q_2	Q_3	D_1^c	D_2^c	D_3^c	H_d	ϕ
$U(1)_{\mathrm{FN}}$	6	4	0	5	3	3	-3	-2

$$\frac{\langle\phi\rangle}{\Lambda} \sim \lambda$$

$$c_{ij}\,Q_i\,D_j^c\,H_d\left(\frac{\phi}{\Lambda}\right)^{x_{ij}} \quad \to \quad c_{ij}\,Q_i\,D_j^c\,H_d\,\lambda^{x_{ij}} \quad \to \quad Y_d \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}$$

- hierarchies arise from spontaneous breakdown of $U(1)_{\mathrm{FN}}$
- often residual discrete Z_N symmetry, e.g. matter parity

Discrete symmetries from $U(1)_{\text{FN}}$

- simple conditions to obtain a particular Z_N symmetry from $U(1)_{\text{FN}}$
- in $U(1)_{\text{FN}}$ charge normalisation with $X_\phi = -1$

	$X_{H_d} - X_{L_1}$	$3X_{Q_1} + X_{L_1}$
M_p	integer $- \frac{1}{2}$	integer
P_6	integer $- \frac{1}{2}$	integer $\pm \frac{1}{3}$
B_3	integer	integer $\pm \frac{1}{3}$

Dreiner, Luhn, Murayama, Thormeier (2006)

M_p lightest SUSY particle (LSP) is stable $\rightarrow$ dark matter candidate
allows for non-renormalisable operator $QQQL$ $\rightarrow$ proton decay

P_6 LSP is stable $\rightarrow$ dark matter candidate
forbids $QQQL$ $\rightarrow$ proton stabilised

B_3 LSP not stable $\rightarrow$ less missing E_T
forbids $QQQL$ $\rightarrow$ proton stabilised
renormalisable L violation $\rightarrow$ neutrino masses without seesaw

Extension to R-symmetries

A unique Z_4^R symmetry for the MSSM

- no μ-term in superpotential
- (discrete) anomaly cancellation adopting Green-Schwarz mechanism
- compatible with $SU(5)$ $[SO(10)]$

$\longrightarrow$ requires R-symmetry

- minimal [unique] solution

	Q	U^c	D^c	L	E^c	H_u	H_d	θ
Z_4^R	1	1	1	1	1	0	0	1

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg Vaudrevange (2010)

- superspace variable θ charged under R-symmetry
- forbids $H_u H_d$ as well as $QQQL$
- both generated when Z_4^R gets broken to M_p

Z_4^R from $U(1)_{\rm FN}^R$

- define $U(1)_{\rm FN}^R$ charges on superfields, e.g. $Q = \widetilde{q} + \theta q + \theta^2 F_q$
- fix normalisation such that

$$\boxed{R_\phi = -1 \quad \rightarrow \quad R_\theta}$$

- terms allowed by $U(1)_{\rm FN}^R$ have

$$R_{\rm term} = \begin{cases} 2R_\theta + \text{integer} & \text{(superpotential)} \\ 0 + \text{integer} & \text{(Kähler potential)} \end{cases}$$

- require: superpotential $\longrightarrow$ Yukawas $Q_i U_j^c H_u,\ Q_i D_j^c H_d,\ L_i E_j^c H_d$
  Kähler potential $\longrightarrow$ μ-term $H_u H_d$

- each term $\left\{ \begin{matrix} \text{allowed} \\ \text{forbidden} \end{matrix} \right\}$ by Z_4^R also $\left\{ \begin{matrix} \text{allowed} \\ \text{forbidden} \end{matrix} \right\}$ by $U(1)_{\rm FN}^R$ if

$R_{H_d} - R_{L_1}$	$3R_{Q_1} + R_{L_1}$	R_θ	R_ϕ
integer $\pm \frac{1}{4}$	integer	integer $\pm \frac{1}{4}$	-1

Dreiner, Luhn, Opferkuch (2013)

Model realisation

Anomalies and $U(1)^R_{\mathrm{FN}}$ breaking

- anomaly cancellation via Green-Schwarz
- gauge coupling unification $g_C = g_W = \sqrt{\frac{5}{3}} g_Y$

$$\boxed{\mathcal{A}_{CCR} \;=\; \mathcal{A}_{WWR} \;=\; \tfrac{3}{5}\,\mathcal{A}_{YYR} \;\neq\; 0 \qquad \mathcal{A}_{YRR} \;=\; 0}$$

- $U(1)^R_{\mathrm{FN}}$ charges enter with, e.g., $R_{Q_1} - R_\theta$

- "anomalous" nature of $U(1)^R_{\mathrm{FN}}$
- Fayet-Iliopoulos term $\xi\, D$ with $\xi \propto \mathcal{A}_{CCR}\, M^2_{\mathrm{grav}}$
- unbroken supersymmetry requires D-flatness
- induces VEV of flavon field

$$\boxed{\frac{\langle\phi\rangle}{M_{\mathrm{grav}}} \;=\; \frac{g_C}{4\pi}\sqrt{\mathcal{A}_{CCR}} \;\stackrel{!}{\approx}\; \lambda \;\approx\; 0.22}$$

Yukawas and μ-term

- encode fermion mass hierarchies in $U(1)^R_{\mathrm{FN}}$ charges, e.g.

$$Y^u_{11} \approx \lambda^8 \quad \longrightarrow \quad R_{Q_1} + R_{U^c_1} + R_{H_u} = 8 + 2R_\theta$$

$$Y^u_{22} \approx \lambda^4 \quad \longrightarrow \quad R_{Q_2} + R_{U^c_2} + R_{H_u} = 4 + 2R_\theta$$

- μ-term from Kähler potential via Giudice-Masiero mechanism
- introduce $U(1)^R_{\mathrm{FN}}$ neutral hidden sector field Z

$$\mathcal{L} \ \sim \ \int d^2\theta \int d^2\bar{\theta} \left(\frac{Z^*}{M_{\mathrm{grav}}}\right) \cdot \left(\frac{\phi}{M_{\mathrm{grav}}}\right)^{R_{H_u}+R_{H_d}} H_u H_d$$

- SUSY broken by $\langle F_Z \rangle = m_{\mathrm{soft}}\, M_{\mathrm{grav}} \quad [Z^R_4 \rightarrow M_p]$

$$\int d^2\theta \underbrace{\int d^2\bar{\theta} \left(\frac{\langle F_Z\rangle^* \ \bar{\theta}^2}{M_{\mathrm{grav}}}\right)}_{m_{\mathrm{soft}}} \cdot \lambda^{|R_{H_u}+R_{H_d}|} \ H_u H_d$$

Neutrino sector

- without right-handed neutrinos
  - $H_u L_i$ & $L_i Q_j D_k^c$ & $L_i L_j E_k^c$ - forbidden by Z_4^R ☹
  - $\frac{1}{M_{\text{grav}}} L_i\, H_u\, L_j\, H_u$ - too small ☹
- with right-handed neutrinos
  - Dirac term $L_i N_j^c H_u$ from superpotential

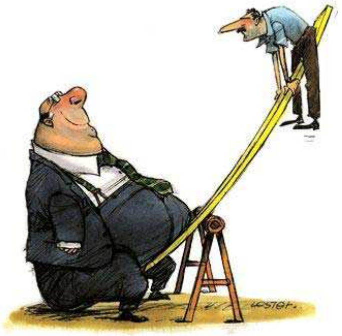
    $$(R_{L_i} + R_{N_j^c} + R_{H_u} - 2R_\theta) \in \mathbb{N} \quad \to \quad {m_{\text{D}}}^{ij} \sim \langle H_u \rangle \cdot \lambda^{R_{L_i} + R_{N_j^c} + R_{H_u} - 2R_\theta}$$

  - Majorana term $N_i^c N_j^c$ from superpotential

    $$(R_{N_i^c} + R_{N_j^c} - 2R_\theta) \in \mathbb{N} \quad \to \quad {M_{\text{M}}}^{ij} \sim M_{\text{grav}} \cdot \lambda^{R_{N_i^c} + R_{N_j^c} - 2R_\theta}$$

  - seesaw mechanism

$$\boxed{{m_\nu}^{ij} \sim \frac{\langle H_u \rangle^2}{M_{\text{grav}}}\, \lambda^{R_{L_i} + R_{L_j} + 2R_{H_u} - 2R_\theta}}$$



Example & summary

i	R_{Q_i}	$R_{U_i^c}$	$R_{D_i^c}$	R_{L_i}	$R_{E_i^c}$
1	$\frac{69}{20}$	$\frac{109}{20}$	$\frac{13}{20}$	$-\frac{7}{20}$	$\frac{89}{20}$
2	$\frac{69}{20}$	$\frac{29}{20}$	$-\frac{27}{20}$	$-\frac{27}{20}$	$\frac{69}{20}$
3	$\frac{29}{20}$	$-\frac{11}{20}$	$-\frac{27}{20}$	$-\frac{27}{20}$	$\frac{29}{20}$

R_ϕ	$=$	-1
R_θ	$=$	$-\frac{1}{4}$
R_{H_u}	$=$	$-\frac{7}{5}$
R_{H_d}	$=$	$-\frac{3}{5}$

MSSM + $U(1)_{\text{FN}}^R$
- ▶ discrete symmetry Z_4^R
- ▶ family structure

- proton stabilised
- μ-term via Giudice-Masiero mechanism
- neutrino masses from seesaw

Thank you