Fluctuations and the QCD phase diagram

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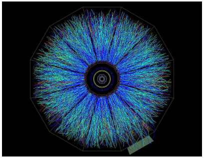
Hot Quarks 2014, Las Negras, Spain

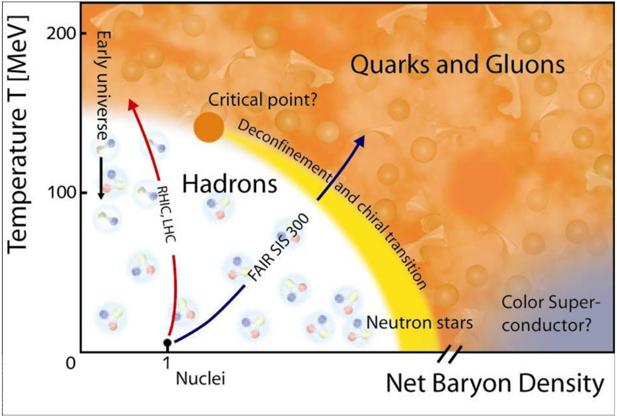
QCD phase diagram - Heavy-ion collisions

- One of the main goals of heavy-ion collisions is to understand the phase structure of hot and dense strongly interacting matter.

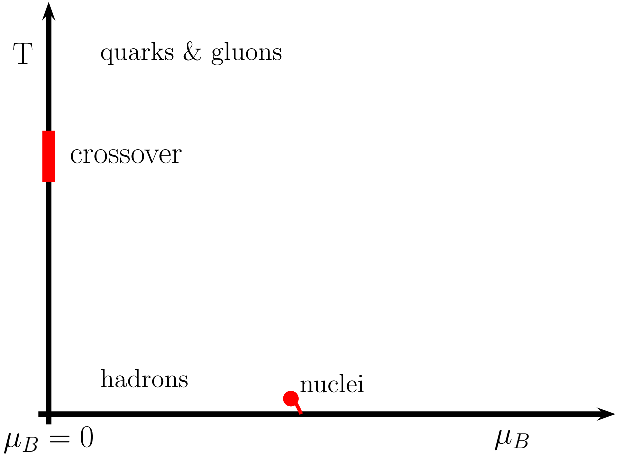


- Can we experimentally produce a deconfined phase with colored degrees of freedom?
- What are the properties of this phase?
- What is the nature of the phase transition between deconfined and hadronic phase?

QCD phase diagram - visionary plot

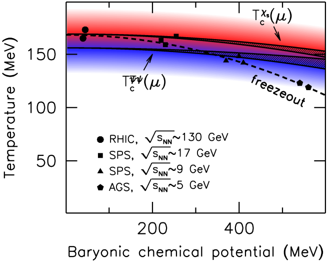


QCD phase diagram - current knowledge

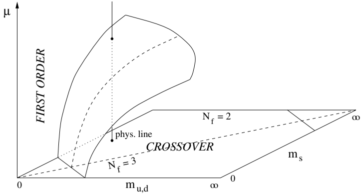


QCD phase diagram - Lattice QCD

- Liberation of color degrees of freedom at high temperatures.
- Crossover at $\mu_B = 0$ and $T = [145, 165]$ MeV

  Wuppertal-Budapest JHEP 1009 (2010),HotQCD PoS LATTICE2010 (2010)
- Fermionic sign problem at $\mu_B \neq 0 \rightarrow$ usual importance sampling fails
- 

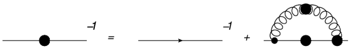
  Wuppertal-Budapest (2002)&(2011)

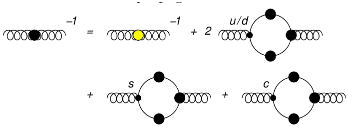
  F. Karsch et al. (2004)
- Methods to extend to finite μ_B: **Taylor expansion**, imaginary μ_B, reweighting, (complex Langevin equation) $\rightarrow$ no critical point for small μ_B / T.

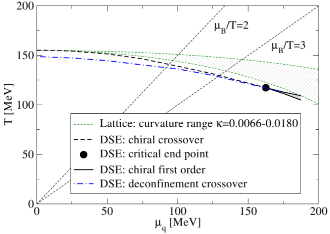
QCD phase diagram - Dyson-Schwinger equations

DSE for quarks



DSE for gluons

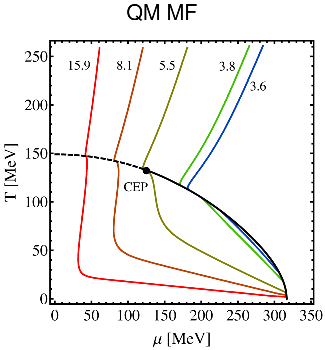


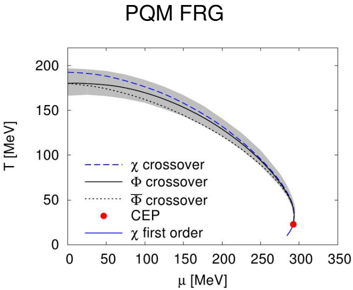


C. Fischer, J. Luecker, C. Welzbacher, 1405.4762

- T-dependent lattice data for the quenched gluon propagator + implement back reaction of the quark sector by adding loops to the gluon DSE.
- Sophisticated truncation schemes and approaches to the vertex strength are necessary!

QCD phase diagram - Effective models



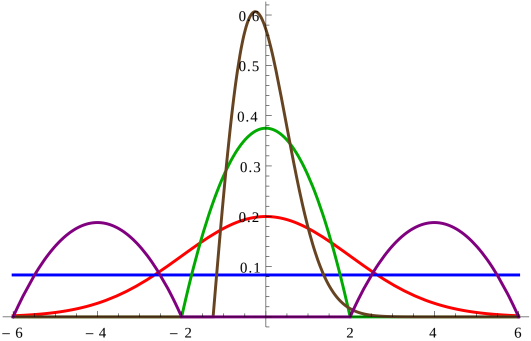


E. Nakano, B. -J. Schaefer, B. Stokic, B. Friman, K. Redlich, PLB **682** (2010); T. Herbst, J. Pawlowski, BJ. Schaefer, PLB **696** (2011)

- Crossover (CO), critical point (CP), first order phase transition (FO) in (P)QM models.
- Location (T_c, μ_c) of the CP is not universal: depends on model parameters and the approach to solve the model.

Fluctuations

- Mean values are only first-order information about observables.
- Distributions with the same mean can take very different forms.



- In some cases, the mean value of a distribution corresponds to a very rare event.
- Higher-order moments/cumulants: variance : $\sigma^2 = \langle \Delta N^2 \rangle$,

$$\text{skewness} : S = \langle \Delta N^3 \rangle / \langle \Delta N^2 \rangle^{3/2}, \quad \text{kurtosis} : \kappa = \langle \Delta N^4 \rangle / \langle \Delta N^2 \rangle^2 - 3$$

Fluctuations - thermodynamically

Microcanonically System is isolated $\rightarrow$ Energy is exactly conserved

Canonically System can exchange energy with heat bath $\rightarrow$ Energy is only conserved on average
Energy fluctuations: $\langle E^2 \rangle - \langle E \rangle^2 = kT^2 \partial \langle E \rangle / \partial T$.

Grandcanonically System can exchange energy and particles with heat bath $\rightarrow$ Energy and particle number are only conserved on average
Energy fluctuations: $\langle E^2 \rangle - \langle E \rangle^2 = kT^2 \frac{\partial \langle E \rangle}{\partial T} + kT \sum_i \mu_i \frac{\partial \langle E \rangle}{\partial \mu_i}$

Particle number fluctuations: $\langle N_i^2 \rangle - \langle N_i \rangle^2 = kT \frac{\partial \langle N_i \rangle}{\partial \mu_i}$.

NOTE: Averages in all ensembles are the same, fluctuations are not!

Fluctuations - what are they good for?

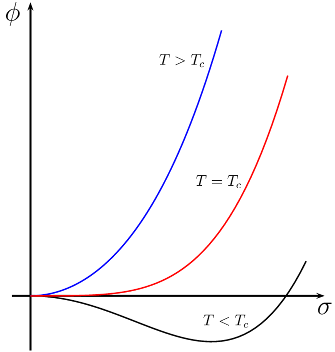
- Is the matter created a (locally) thermal QGP or hadronic system?
  Talk by Miki Sakaida, this session!
- Can we learn something about the chemical freeze-out properties of the system?
  Talk by Marcus Bluhm, this session!
- Phase transitions... This talk!

Fluctuations - at a critical point

In mean-field, Ginzburg-Landau theory of phase transitions, effective potential (free energy):

$$\Phi(T;\sigma) = \Phi_0(T) + a(T - T_c)\sigma^2 + b\sigma^4$$

- order parameter σ
- $m_\sigma^2 = \frac{\partial^2 \Phi}{\partial \sigma^2} \to 0$
- correlation length diverges $\xi = \frac{1}{m_\sigma} \to \infty$
- universality classes
  for QCD: $\mathcal{O}(4)$ Ising model in 3d
  $\Rightarrow \langle \sigma^2 \rangle \propto \xi^2$
- beyond mean-field: renormalization group
- critical opalescence

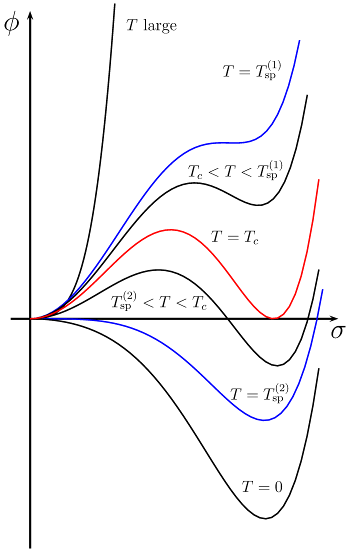


$\Rightarrow$ Large event-by-event fluctuations in thermal systems!

Fluctuations - at a first order phase transition

- two degenerate minima separated by a barrier
- latent heat
- phase coexistence
- supercooling effects in nonequilibrium situations
- nucleation
- spinodal decomposition

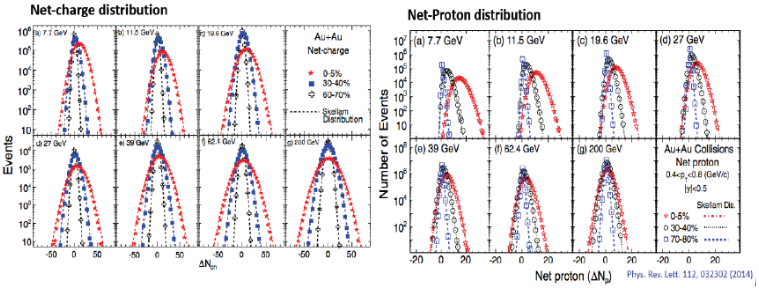
  I.N.Mishustin, PRL **82** (1999); Ph.Chomaz, M.Colonna, J.Randrup, Physics Reports **389** (2004))



$\Rightarrow$ (Large) fluctuations in single events in nonequilibrium situations!

Fluctuations - event-by-event

In experiment the measured number of particles varies from event to event $\rightarrow$ event-by-event fluctuations



STAR collaboration, 2013/2014

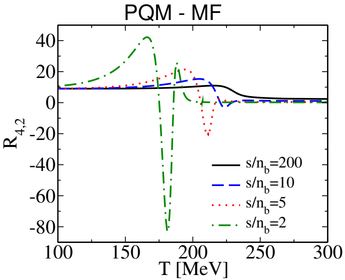
Why net-charge? $\Rightarrow$ Net-electric charge is a conserved quantity of QCD!

Why net-proton?

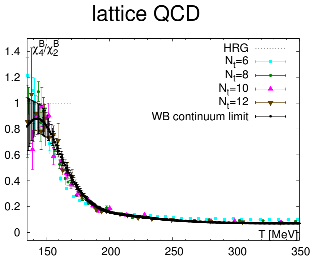
Net-baryon vs. net-proton (1)

- In lattice QCD and effective models one looks at net-baryon number fluctuations via susceptibilities, which relate to measured fluctuations:

$$\chi_n^{(i)} = \left.\frac{\partial^n (P/T^4)}{\partial(\mu_i/T)^n}\right|_T = \langle(\Delta N_i)^n\rangle_c \qquad n \geq 2$$



Skokov et al.



Wuppertal-Budapest, PRL (2013)

- In experiment neutrons are not detected → isospin fluctuations $(p(n) + \pi \rightarrow \Delta \rightarrow n(p) + \pi)$ M. Kitazawa, M. Asakawa, PRC 85 and 86 (2012)

Net-baryon vs. net-proton (2)

Does it matter for detecting critical fluctuations in experiment? $\rightarrow$ depends on what the critical mode is...

- If the critical mode is the sigma field: via the coupling to two pions $g_\pi \sigma\pi\pi$, kaons $g_K \sigma\pi\pi$ and protons $g_p \sigma\bar{p}p \Rightarrow$ fluctuations in these multiplicity fluctuations directly:

$$\langle(\delta N)^2\rangle \propto \langle(\Delta\sigma)^2\rangle \propto \xi^2$$

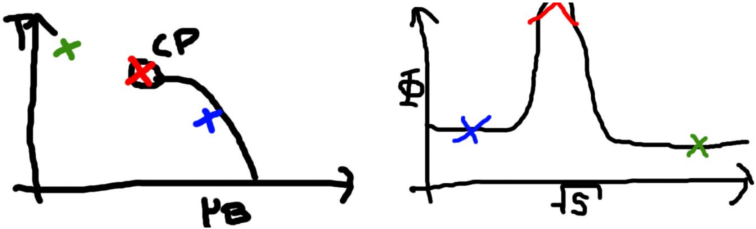
  ξ: correlation length, diverges at the CP

  M. Stephanov, K. Rajagopal, E. Shuryak, PRL **81** (1998), PRD **60** (1999)

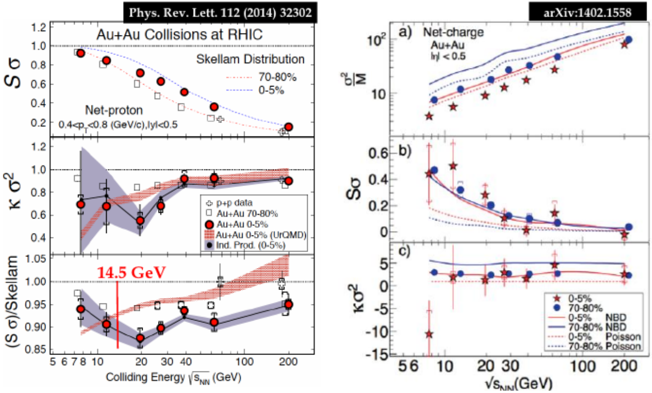
- Higher cumulants are more sensitive to the CP
  skewness ($S\sigma$: $\langle(\delta N)^3\rangle / \langle(\delta N)^2\rangle \propto \xi^{4.5}$
  kurtosis ($\kappa\sigma^2$): $\langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \propto \xi^7$
- But sigma mode might only be a subdominant contribution to the critical mode: In real-time dynamics, criticality is manifested in a vanishing net-baryon diffusion rate. M. Stephanov, D. Son PRD **70** (2004)

Naive expectation

Non-monotonic behavior of an appropriate fluctuation observable Φ as a function of the beam energy $\sqrt{s}$:



Experimental results

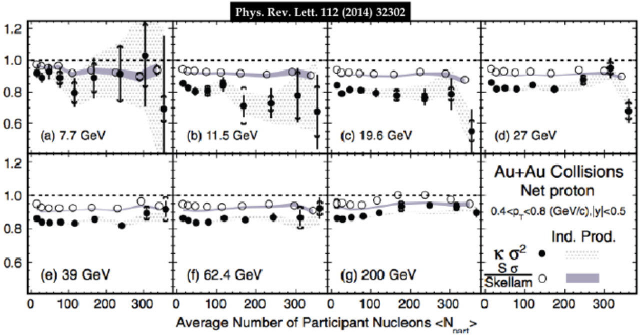


STAR collaboration, 2013/2014

- Deviations from Poisson-distribution, but consistent with independent production.
- Dip in the kurtosis around $\sqrt{s} = 19$ GeV.
- With the 14.5GeV point, new data with the TOF and BES phase II: fill in the gap, reduce errors and hopefully resolve some details!
- (Errors on net-charge fluctuations not limited by statistics - will likely not reduce...)

Experimental results - centrality dependence

?



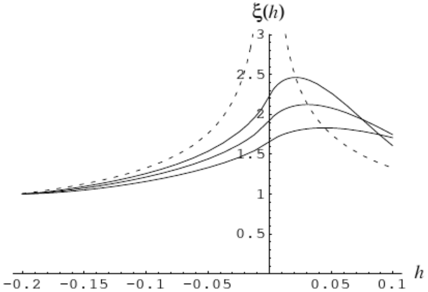
Fluctuations at the critical point - dynamically

- Long relaxation times near a critical point
  $\Rightarrow$ the system is driven out of equilibrium (critical slowing down)!
- Phenomenological equation:

$$\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)})$$

with $\Gamma(m_\sigma) = \frac{A}{\xi_0}(m_\sigma \xi_0)^z$
$z = 3$
(dynamical) critical exponent from model H in Hohenberg-Halperin

$\Rightarrow \xi \sim 1.5 - 2.5$ fm



B. Berdnikov and K. Rajagopal, PRD **61** (2000)); D.T.Son, M.Stephanov, PRD **70** (2004); M.Asakawa, C.Nonaka, Nucl. Phys. A774 (2006))

Nonequilibrium chiral fluid dynamics (NχFD)

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks (QM model)

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- For PQM: phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\rm eff}}{\partial \ell} = \xi_\ell$$

- Fluid dynamical expansion of the quark fluid = heat bath, including energy-momentum exchange

$$\partial_\mu T_{\rm q}^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu} \,, \quad \partial_\mu N_{\rm q}^\mu = 0$$

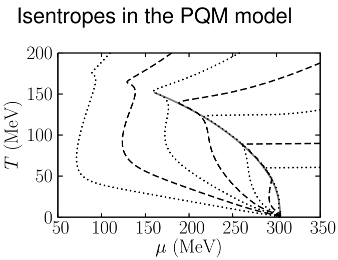
  $\Rightarrow$ includes a stochastic source term!

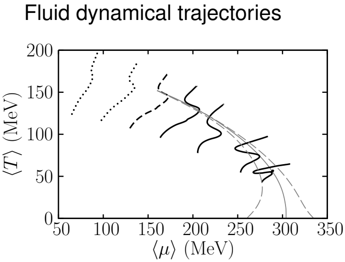
- Nonequilibrium equation of state $p = p(e, \sigma)$

Selfconsistent approach within the 2PI effective action!

MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC **84** (2011); PLB **711** (2012); JPG **40** (2013)
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013); NPA**925** (2014)

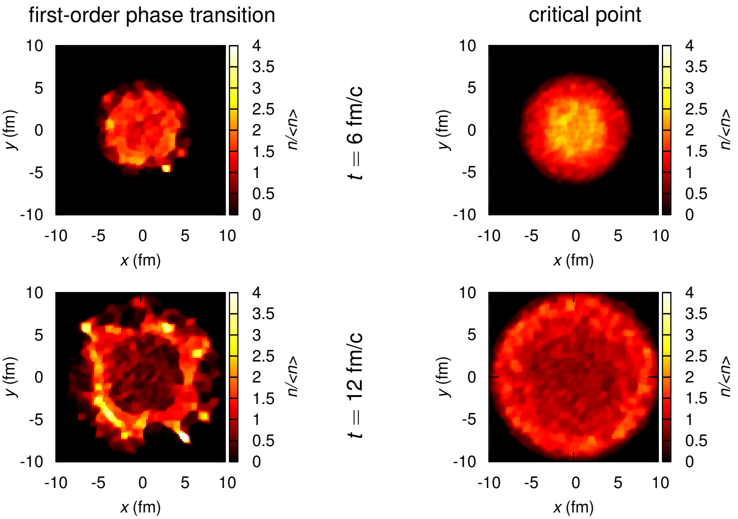
NχFD at finite μ_B





- Fluid dynamical trajectories differ from the isentropes due to interaction with the fields.
- No significant features in the trajectories left of the critical point.
- Right of the critical point: system spends significant time in the spinodal region! $\Rightarrow$ possibility of spinodal decomposition!

NχFD - bubble formation

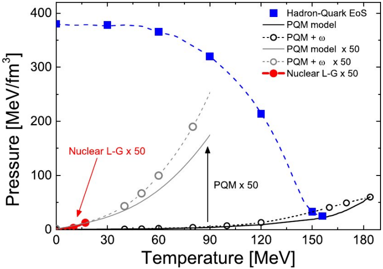


C. Herold, MN, I. Mishustin, M. Bleicher, NPA**925** (2014)

NχFD - bubble formation

Can we expect experimental evidences for the first-order phase transition from bubble formation?

- Do the irregularities survive when a realistic hadronic phase is assumed?
- While several eos lead to similar pressures at $\mu_B \approx 0$, they differ at large μ_B.
- A strong pressure could transform the coordinate-space irregularities into momentum-space Fourier-coefficients of baryon-correlations $\Rightarrow$ enhanced higher flow harmonics at a first-order phase transition? Very eos dependent!



J. Steinheimer, J. Randrup, V. Koch PRC**89** (2014)

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

Stochastic viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} + \Delta T^{\mu\nu}_{\mathrm{visc}} + \Xi^{\mu\nu}$$
$$N^{\mu} = N^{\mu}_{\mathrm{eq}} + \Delta N^{\mu}_{\mathrm{visc}} + \mathrm{I}^{\mu}$$

P. Kovtun, J.Phys. A**45** (2012); C. Chafin and T. Schäfer, PRA**87** (2013); P. Romatschke and R. E. Young, PRA**87** (2013); K. Murase, T. Hirano, arXiv:1304.3243; C. Young et al. arxiv:1407.1077

Fluid dynamical fluctuations

The noise terms are such that averaged quantities exactly equal the conventional quantities:

$$\langle T^{\mu\nu} \rangle = T^{\mu\nu}_{\text{eq}} + \Delta T^{\mu\nu}_{\text{visc}} \qquad \text{with} \quad \langle \Xi^{\mu\nu} \rangle = 0$$
$$\langle N^{\mu} \rangle = N^{\mu}_{\text{eq}} + \Delta N^{\mu}_{\text{visc}} \qquad \text{with} \quad \langle \mathrm{I}^{\mu} \rangle = 0$$

The two formulations will, however, differ when one calculates correlation functions!

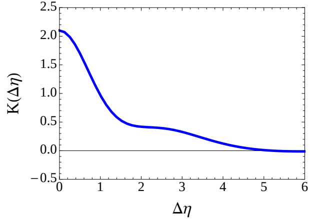
In linear response theory the retarded correlator

- $\langle T^{\mu\nu}(x) T^{\mu\nu}(x') \rangle$ gives the viscosities and
- $\langle N^{\mu}(x) N^{\mu}(x') \rangle$ the charge conductivities

via the dissipation-fluctuation theorem (Kubo-formula)!

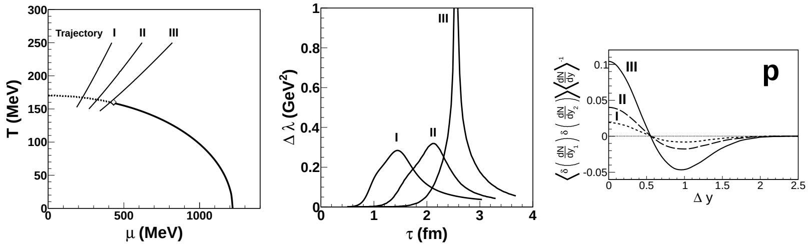
It means that when dissipation is included also fluctuations need to be included!

Example: Bjorken expansion



Particle number correlations extend over more than one unit in rapidity.

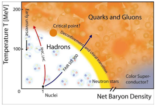
J. Kapusta, B. Mueller, M. Stephanov PRC**85** (2012)



On the flyby near the CP the thermal conductivity is enhanced $\Rightarrow$ enhancement of the rapidity correlator of protons.

J. Kapusta, J. Torres-Rincon PRC**86** (2012)

Summary



- Many theoretical approaches to the QCD phase diagram conjecture a critical point.
- Fluctuation observables are promising tools to understand the phase structure of QCD.
- Need to combine purely theoretical approaches with the dynamics of heavy-ion collisions.
- Nonequilibrium effects can become strong enough to develop signals of the first order phase transition (NχFD).
- Fluid dynamical fluctuations play an important role at the critical point!

? Many unresolved questions at finite μ_B: equation of state, transport coefficients, initial conditions..., hadronic interactions..., efficiency corrections...