# Non-Extensive Thermodynamical Approach of Hadronization in High Energy Collisions

G.G. Barnaföldi & T.S. Biró & K. Ürmössy & P. Ván

Wigner RCP of the Hungarian Academy of Sciences

Eur. Phys. J. A49 (2013) 110,  Physica A 392 (2013) 3132

Hot Quarks, Las Negras, Spain, 27th  September 2014

This talk is not an overview talk,
but
aims to give an overview...

# ...from a slightly different viewpoint


See M. Greif's talk for the other view...

# ...from a slightly different viewpoint


**...for those who want to be the part of the 'Big Picture'**

# O U T L I N E

- Motivation...
  - Is there physics behind the parameters of FFs?
  - How about the $p_T$ power of the tail?
  - Can we understand an experimental parameter, T, which we use to fit to low the $p_T$ spectra?

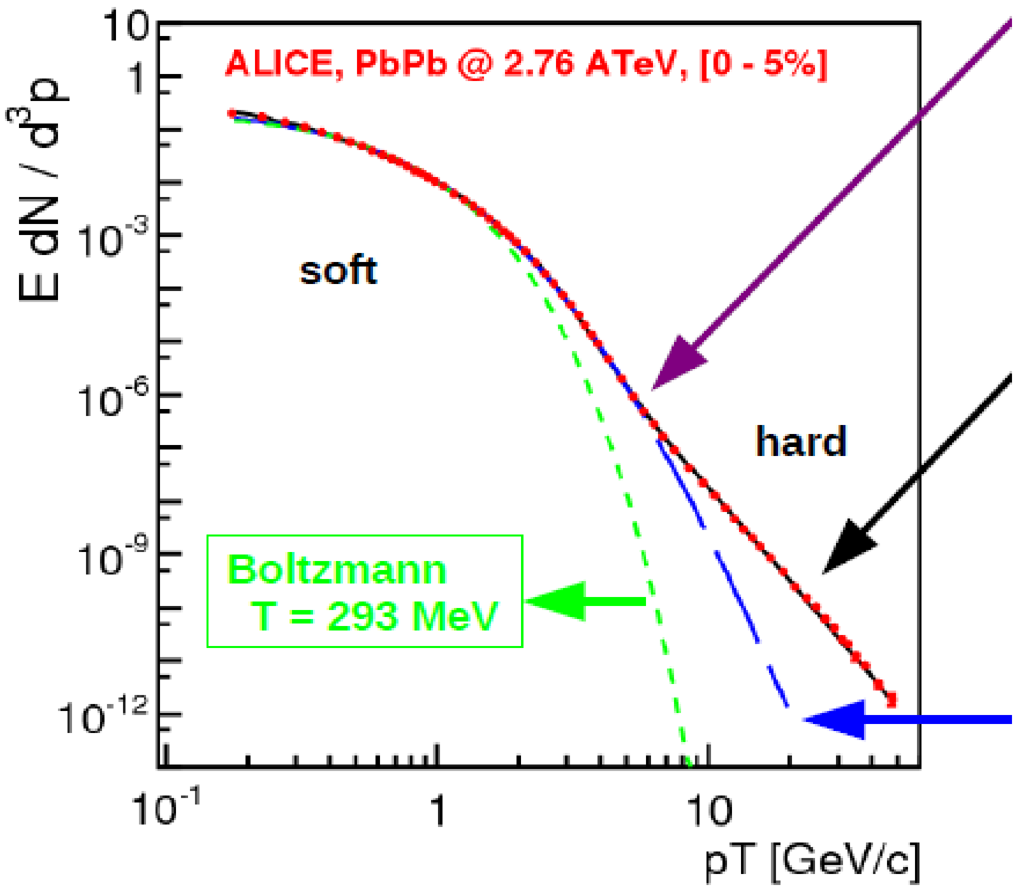
- For 'hard' guys: Derivation of the parameter q


The phyiscal meaning of the 'mysterious q' by deriving Tsallis/Rényi-like entropies from the first principles

- For 'soft' guys: What can be the parameter T?

An application: a simple Bag model to get QGP temperature

# MOTIVATION

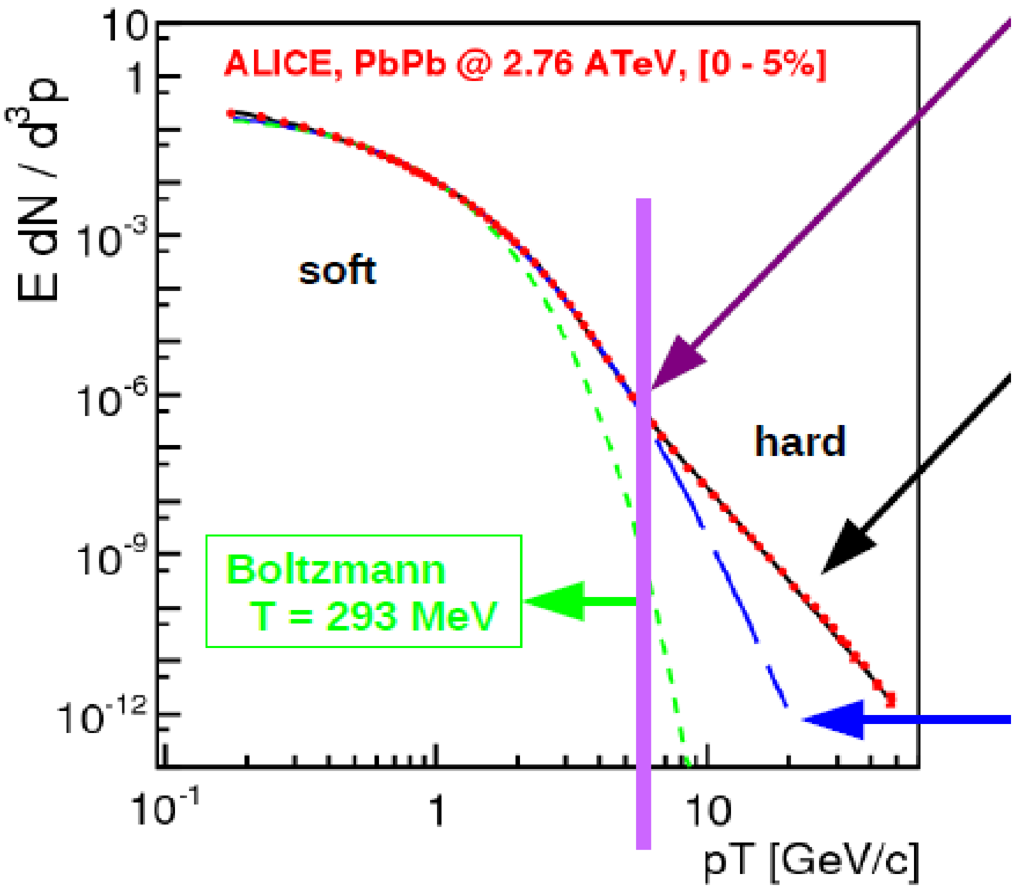



The power of the spectra changes at around 6 GeV/c

Power-law with $p^{-6.08}$ dependence

Tsallis spectra with $p^{-13.7}$

# MOTIVATION

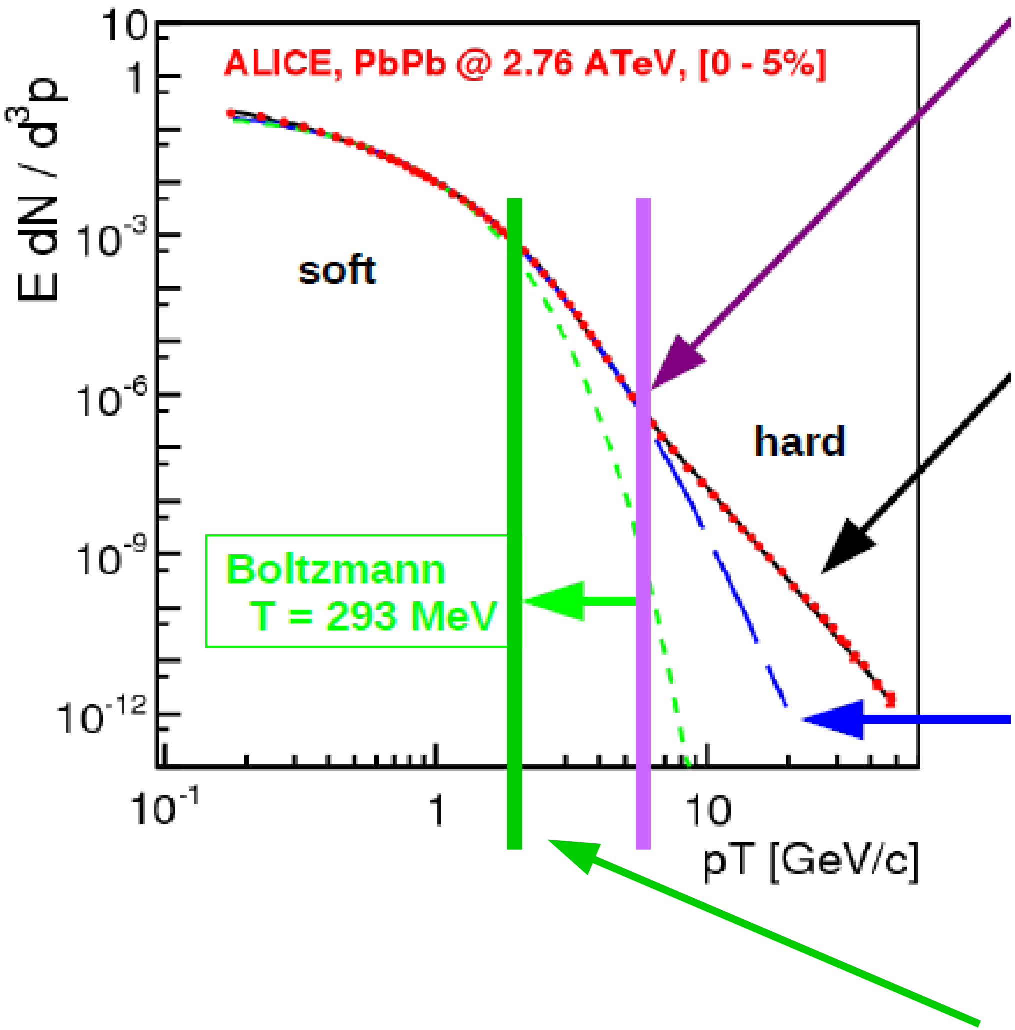



The power of the spectra changes at around 6 GeV/c

Power-law with $p^{-6.08}$ dependence

Tsallis spectra with $p^{-13.7}$

# MOTIVATION




The power of the spectra changes at around 6 GeV/c

Power-law with $p^{-6.08}$ dependence

Tsallis spectra with $p^{-13.7}$

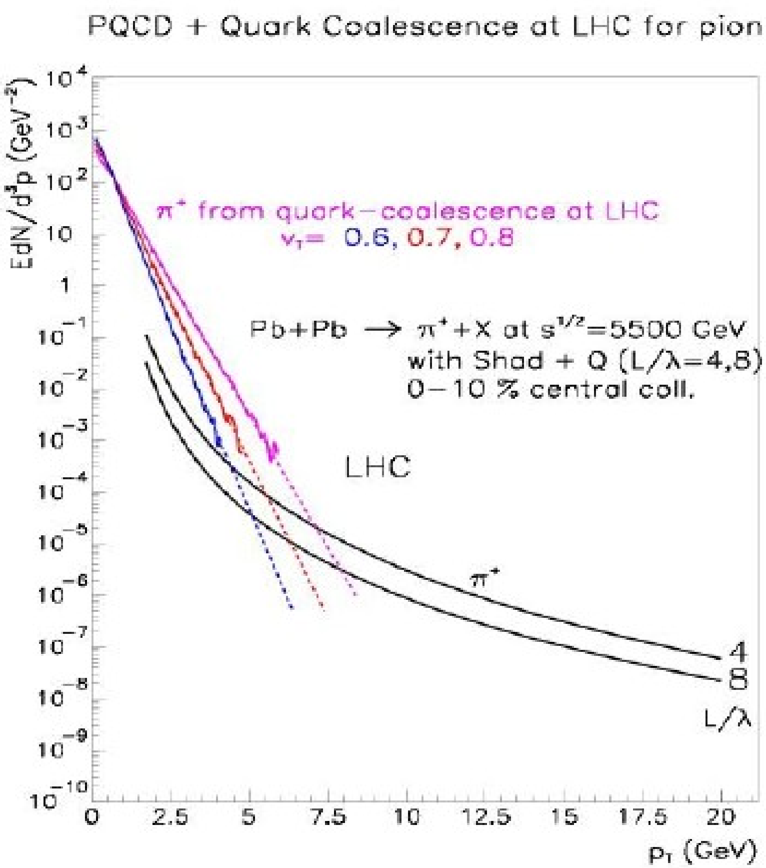
Handling soft/hard regime with a new approach, using not only the temperature, T

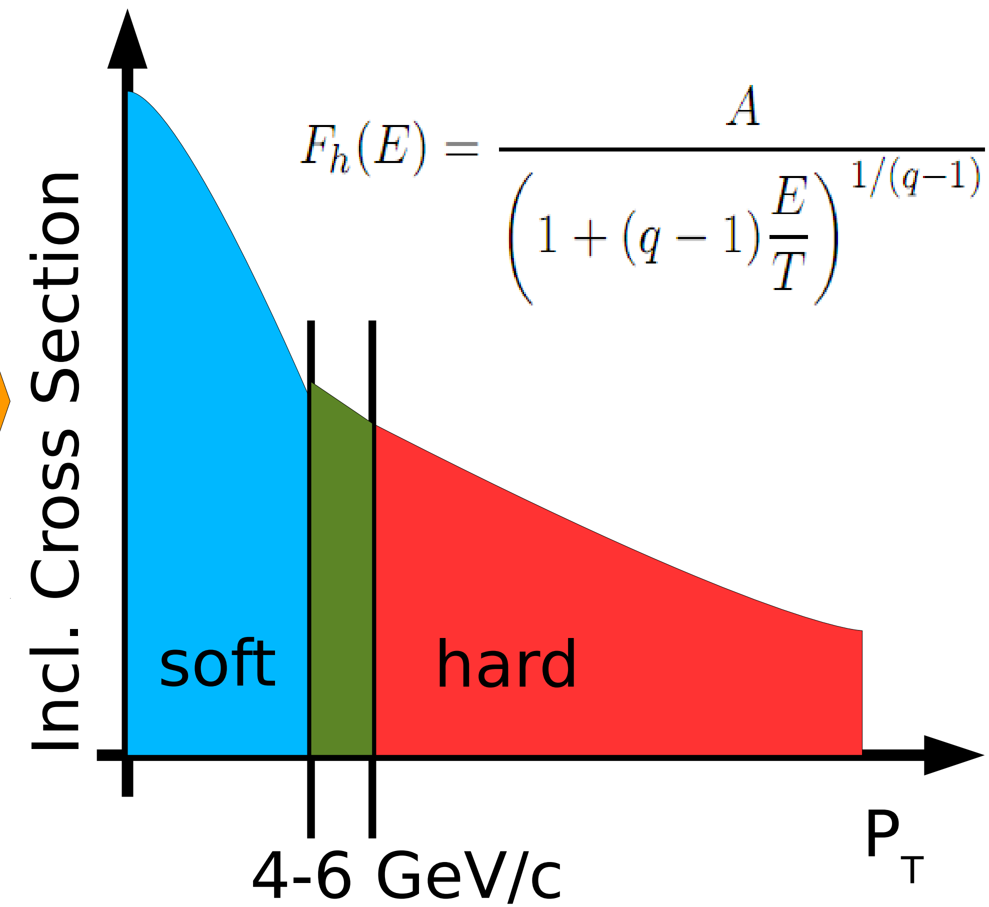
# What? Why? Where?

- What is '*non-extensive*'?
  - We do not like to give up energy *additivity*: $E_1+E_2 = E_{tot}$
  - But e.g. in case of (strongly inteacting forces) *entropy is not additive*: $S(E_1)+S(E_2)+S(E_1E_2)= S(E_{12})$
  - We would like to transform this to an additive one, with a *generalized entropy*: $L(S(E_1))+L(S(E_2))= L(S(E_{12}))$
- Why to use '*Tsallis-like*' entropies?
  - Tsallis entropy is first order correction to Boltzmann.
  - We use 'like' just because there are not only one kind.
- Where to use?
  - Certainly in high-energy collisions, but there are many other fields as well. See all in C. Tsallis' bibliography http://tsallis.cat.cbpf.br/biblio.htm

# Tsallis Entropy has Tsallis distribution

- Simplest and best fit to hadron spectra at low-$p_T$ & high-$p_T$




P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

# The Derivation of Tsallis/Rényi Entropy and the Physical Meaning of the 'q'

Eur. Phys. J. A49 (2013) 110,  Physica A 392 (2013) 3132

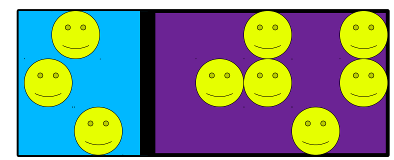
# General derivation as inproved canonical

The story is about...

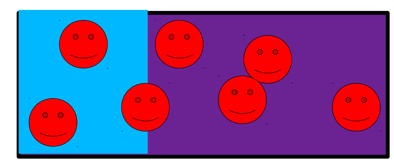
- Two body thermodynamics:

  1 subsystem *($E_1$)* +one  reservoir *($E$-$E_1$)*

- Finite system, finite energy → microcanonical description
  - microcanonical $\sum_j \epsilon_j = E$



  - canonical $\sum_j < \epsilon_j > = E$



- Maximize a monotonic function of the Boltzmann-Gibbs entropy, *L(S)* (0th law of thermodynamics)

- Taylor expansion of the *L(S)* = max, principle beyond $-\beta E$

# Description of a system & reservoir

- For generalized entropy function $L(S_{12}) = L(S_1) + L(S_2)$
- In order to exist $\beta$ of the system $L\left(S(E_1)\right) + L\left(S(E - E_1)\right) = \max$

TS Biró P. Ván: Phys Rev. E84 19902 (2011)

- Thermal contact between system *($E_1$)* & reservoir *($E$-$E_1$)*, requires to eliminate $E_1$ :

$$\begin{aligned}\beta_1 &= L'\left(S(E_1)\right) \cdot S'(E_1) \\ &= L'\left(S(E - E_1)\right) \cdot S'(E - E_1)\end{aligned}$$

- This is usually handled in canonical limit, but now, we keep higher orders in the Taylor-expansion in $E_1/E$

$$\beta_1 = L'\left(S(E)\right) \cdot S'(E) - \left[S'(E)^2 L''(S(E)) + S''(E) L'(S(E))\right] E_1 + \ldots$$

# Description of a system & reservoir

- Assuming $\beta_1 = \beta$, the Lagrange multiplicator become familiar for us:

$$\beta = L'(S(E)) \cdot S'(E) = L'(S) \cdot \frac{1}{T}$$

- To satisfy this, need simply to solve

$$\frac{L''(S)}{L'(S)} = -\frac{S''(E)}{S'(E)^2}$$

- *Universal Thermostat Independence (UTI)* Principle: l.h.s. must be as an $S$-independent constant for solving $L(S)$,

$$\frac{L''(S)}{L'(S)} = a$$

- Based on $L(S) \rightarrow S$ for small $S$, coming from 3rd law of the thermodynamics $L'(0)=1$ and $L(0)=0$

$$L(S) = \frac{e^{aS} - 1}{a}$$

- EoS derivatives do have physical meaning:

$$S'(E) = 1/T$$
$$S''(E) = -1/CT^2$$

# Description of a system & reservoir

- Assuming $\beta_1=\beta$, the Lagrange multiplicator become familiar for us:

$$\beta = L'\left(S(E)\right)\cdot S'(E) = L'(S)\cdot\frac{1}{T}$$

- To satisfy this, need simply to solve

$$\frac{L''(S)}{L'(S)} = -\frac{S''(E)}{S'(E)^2}$$

- *Universal Thermostat Independence (UTI)* Principle: l.h.s. must be as an *S*-independent constant for solving *L(S)*,

$$\frac{L''(S)}{L'(S)} = a$$

- Based on *L(S)* →*S* for small *S*, coming from 3rd law of the thermodynamics *L'(0)=1* and *L(0)=0*

$$L(S) = \frac{e^{aS}-1}{a}$$

- Simly the heat capacity of the reservoir:

$$a = 1/C$$

# From two system to many...

- Analogue to Gibbs ensamble generalize

$$S = -\sum_i P_i \ln P_i \quad \rightarrow \quad L(S) = \sum_i P_i L(-\ln P_i)$$

- 

- The *L*-additive form of a generally non-additive entropy, given by:

$$L(S(E_1)) - \beta E_1 = \frac{1}{a}\left(e^{aS(E_1)} - 1\right) - \beta E_1 = \max.$$

- Introducing $a = 1/C(E) \quad \rightarrow \quad L(S(E_1)) = L(-\ln P_1) = \frac{1}{a}\left(P_1^{-a} - 1\right)$

- we need to maximize: $\frac{1}{a}\sum_i \left(P_i^{1-a} - P_i\right) - \beta \sum_i P_i E_i - \alpha \sum_i P_i = \max.$

which, results Tsallis:

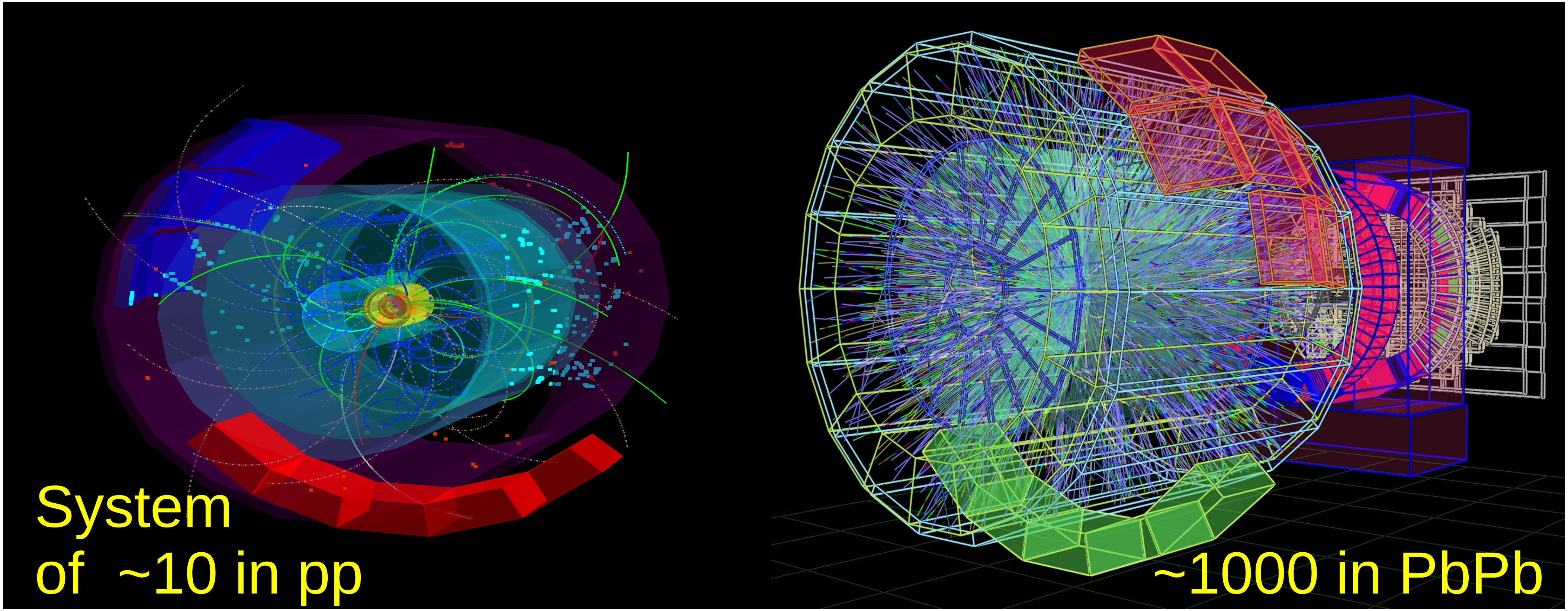
and its inverse Rényi:

$$S_{\text{Tsallis}} := L(S) = \frac{1}{q-1}\sum_i \left(P_i - P_i^q\right)$$

$$S_{\text{Rényi}} := S = \frac{1}{1-q}\ln \sum_i P_i^q$$

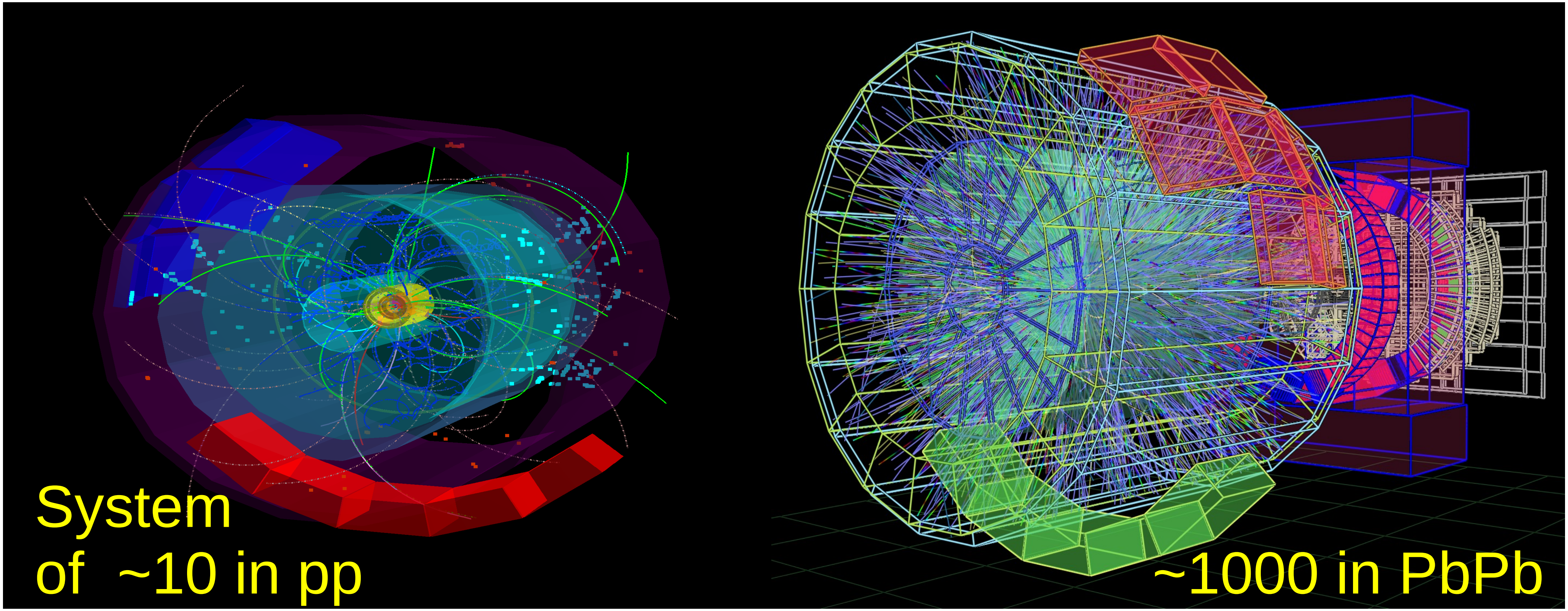
# What is the meaning of T?
# a.k.a.
# Application:
# Quark Gluon Plasma temperature

# De-MOTIVATION




How can we measure the temperature ?

# De-MOTIVATION




How can we measure the temperature of what?

This is NOT a system of $10^{23}$ particles, but 1000.

# Experimental data fits by $T_{slope}(E)$

- Taking the $T_{slope}(E)$ fit using $T_{\rm slope}(E_i) = \left(-\frac{d}{dE_i}\ln P_i\right)^{-1} = T_0 + E_i/C,$
- Fitted data
  - RHIC@200GeV AuAu: $T_0$= 48 MeV, C=4.5

    T.S. Biró, K. Ürmössy, Zs. Schram: JPG36 064044 (2009)
    T.S. Biró, K. Ürmössy: JPG37, 0940027 (2010),
    K. Ürmössy, T.S. Bíró: PL B689 14 (2010)

  - ALICE@900GeV pp: $T_0$= 55 MeV, C=8

    J. Cleymans, D. Worku: JPG39, 025006 (2012)

The obtained values are surprizingly low!!! Why????

- Findings: *K*=2 (mesons) and *K*=3 (baryons)

$$P_{\rm hadron}(E) = P_i^K(E/K) \quad \text{and} \quad T_{\rm slope}^{\rm hadron}(E) = T_{\rm slope}^{\rm quark}(E/K)$$

# Thermal model to heavy-ion collisions

- Test of $T_0$ in physical models, in a finite termostats,

  small subsystem: $\lim_{C \to \infty} T_0 = T_1$ and $T_1 = 1/\beta_1 = T e^{-S/C}$

- Taking Stefan-Boltzmann in a bag, with a fix volume, *V* and bag constant, *B*

$$E/V = \sigma T^4 + B \qquad p = \tfrac{1}{3}\sigma T^4 - B \qquad S = \tfrac{4}{3}\sigma V T^3$$

- The heat capacity is:

$$C = \frac{dE}{dT} = 4\sigma V T^3 + \left(\sigma T^4 + B\right)\frac{dV}{dT}$$

# Thermal model to heavy-ion collisions

- Let's discuss some specific cases:

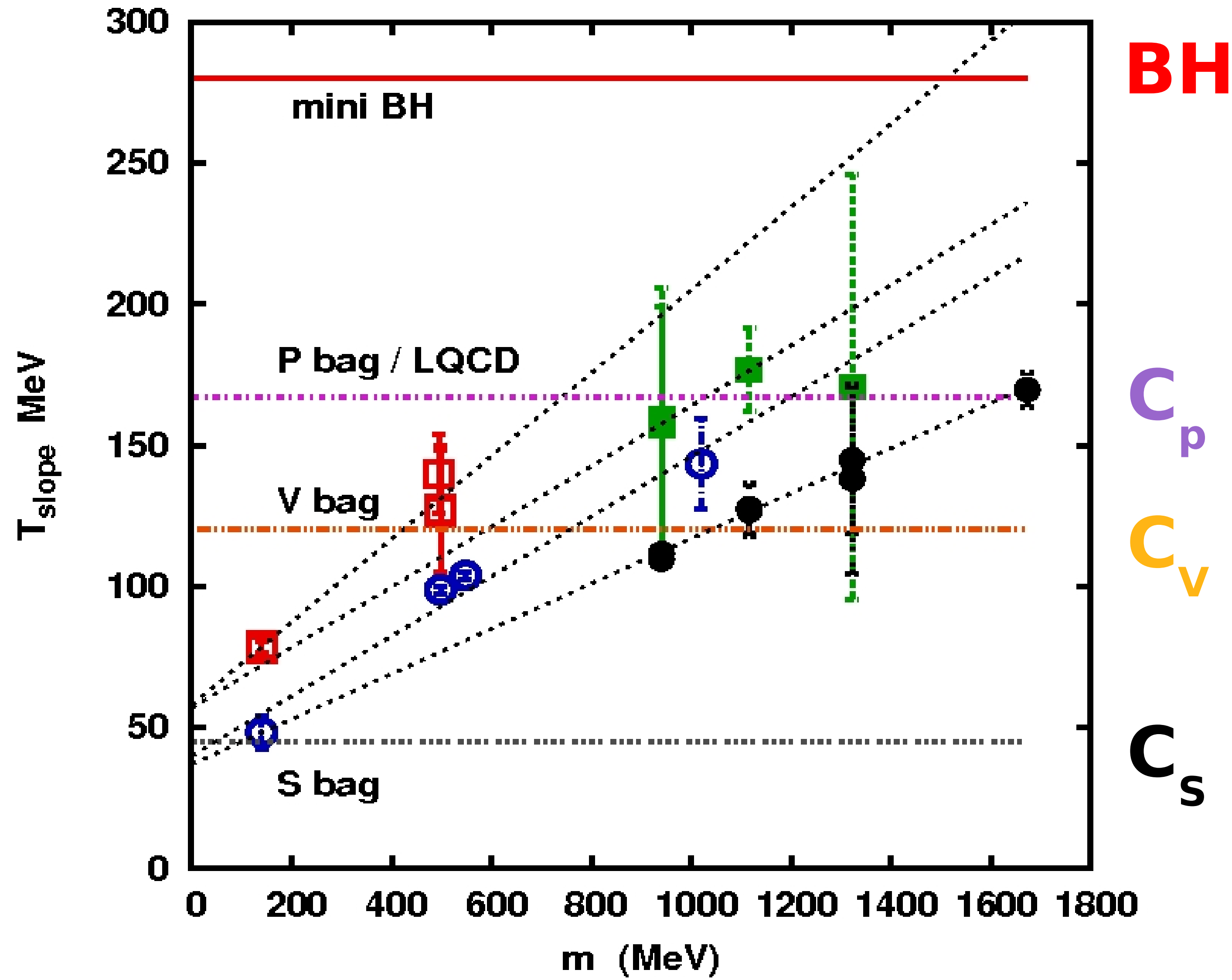
| | Heat capacity | Subsystem's T | Note |
|---|---|---|---|
| $C_V$ | $C_V = 4\sigma V T^3 = 3S$ | $T_{1V} = T e^{-1/3}$ | |
| $C_p$ | $C_p = \infty$ | $T_{1P} = T$ | |
| $C_S$ | $C_S = 3S(1 - T_*^4/T^4)/4$ | $T_{1S} \leq T e^{-4/3}$ | $C_S \leq 3S/4$ |
| BH | $C = -2S$ | $T_1 = T e^{1/2}$ | |

- Taking the lattice QCD value T=167 MeV, $T_{slopes}$ are:

$$T_{1P} = T = 167 \text{ MeV}, T_{1V} = T e^{-1/3} \approx 120 \text{ MeV and } T_{1S} \leq T e^{-4/3} \approx 45 \text{ MeV}$$

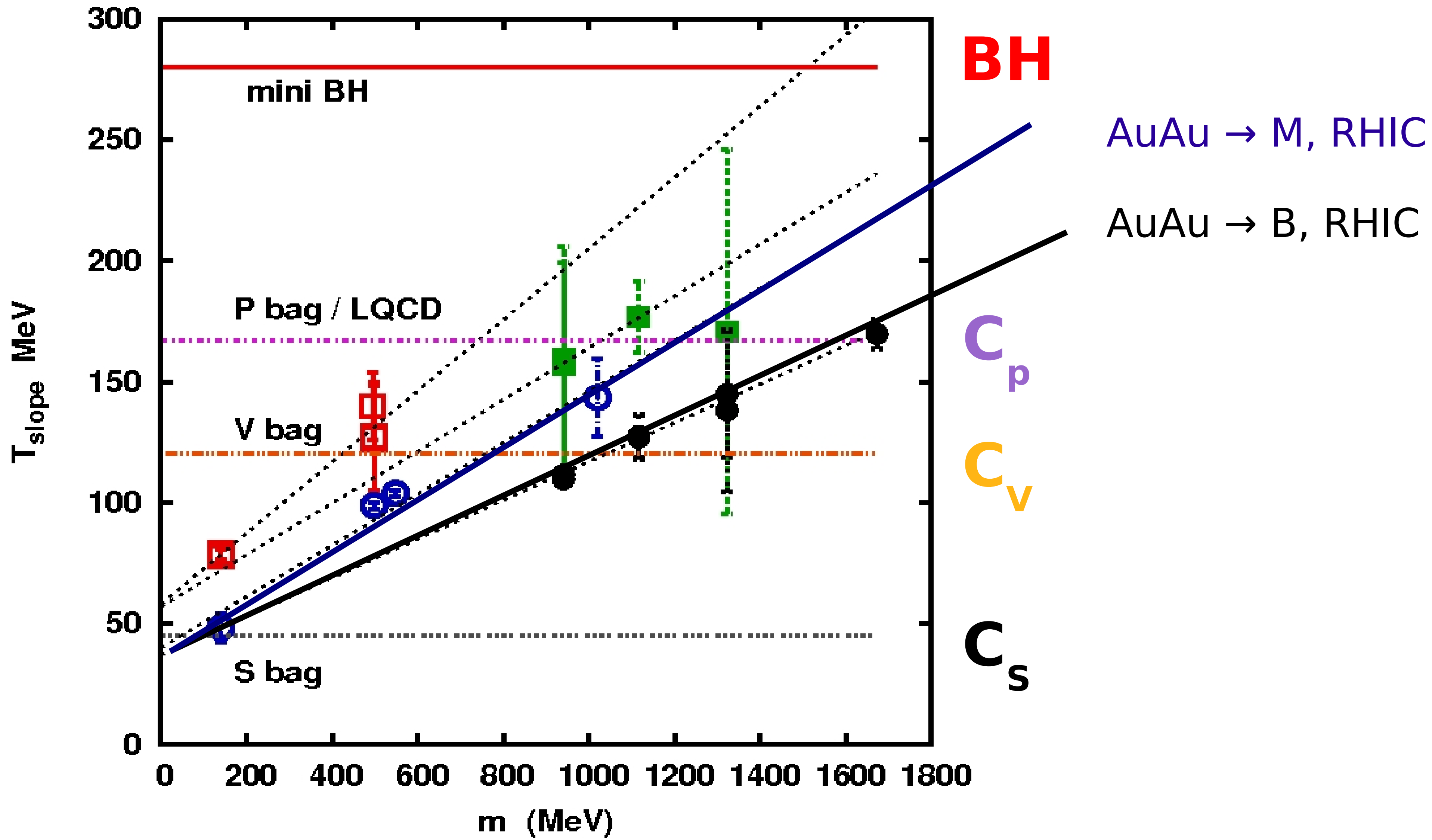
for Tsallis distribution of valence quarks

# The temperature slope for different models



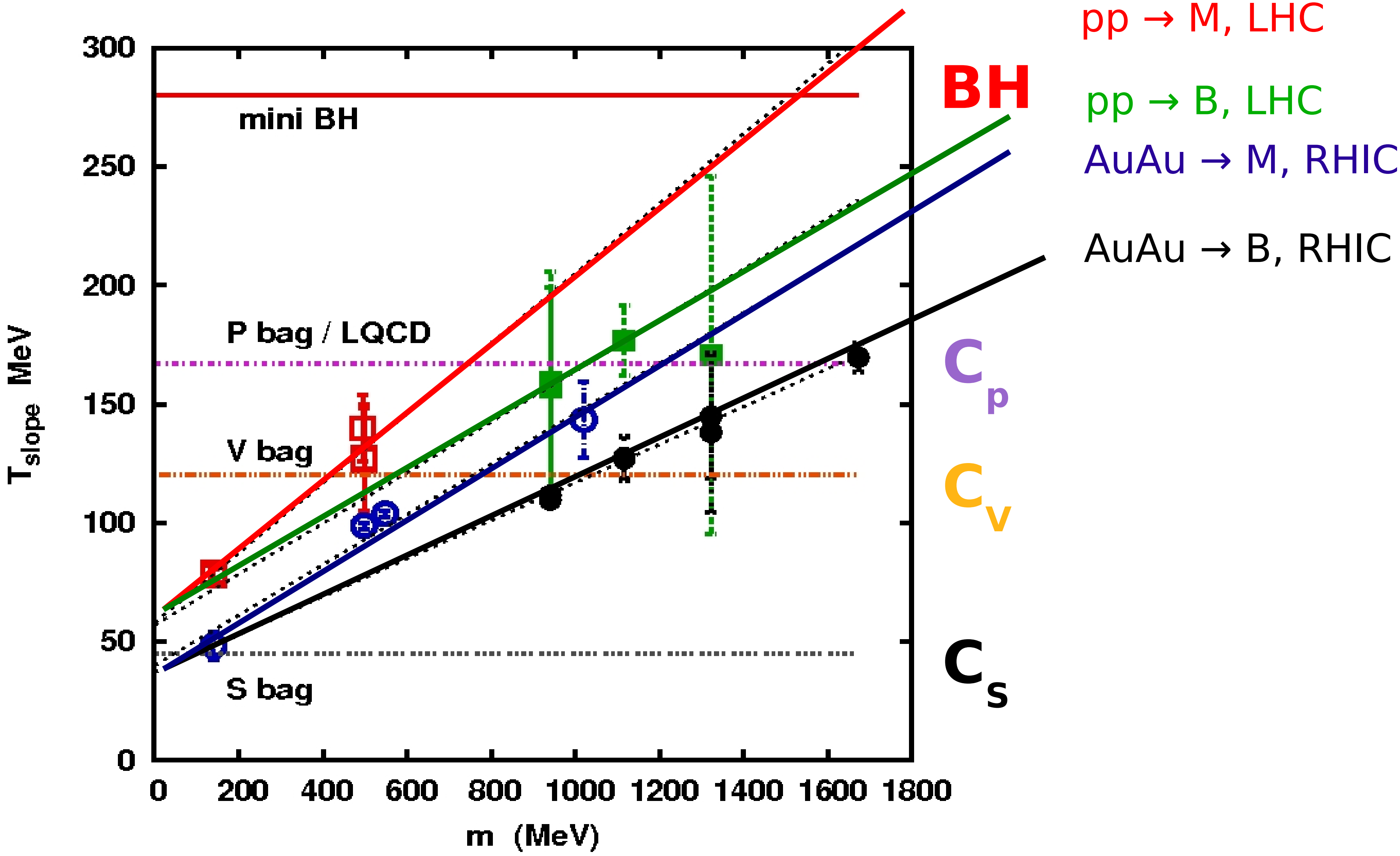

TS Biró, GGB, P. Ván, EPJ A49 (2013) 110

# The temperature slope for different models



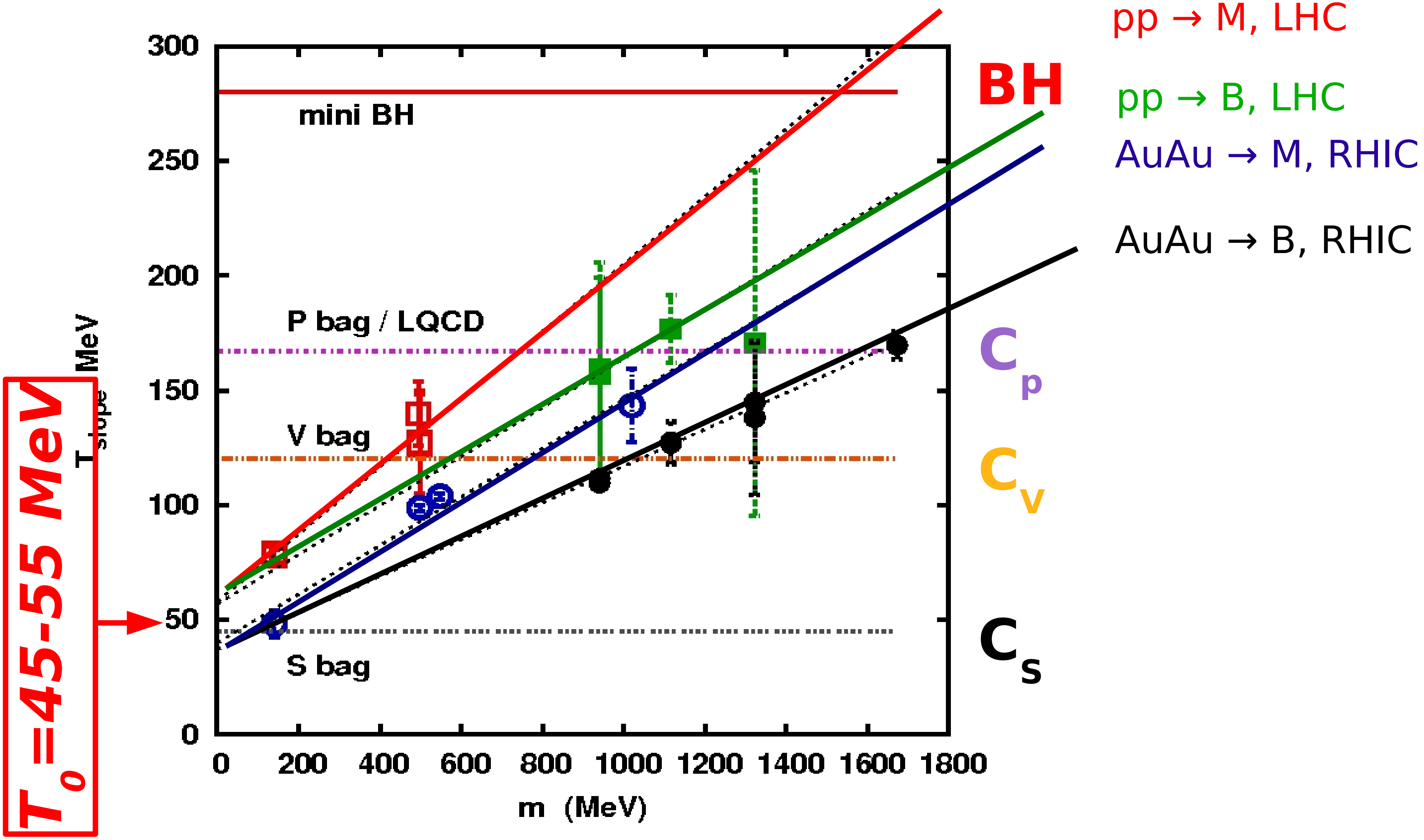

TS Biró, GGB, P. Ván, EPJ A49 (2013) 110

# The temperature slope for different models




TS Biró, GGB, P. Ván, EPJ A49 (2013) 110

# The temperature slope for different models




TS Biró, GGB, P. Ván, EPJ A49 (2013) 110

## If these parameters have physical meaning, then can Tsallis-Pareto-like distribution work as a Fragmentation Function?

# Fragmentation in Parton Model

In a pQCD based parton model, fragmentation functions (FF) gives how parton (*a*) fragment into a hadron (*h*), $D_{h/a}(z,Q^2)$.

DGLAP scale evolution:

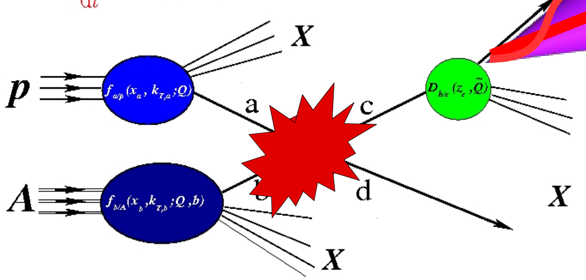
$$\frac{\partial}{\partial \ln Q^2} D_i^h(x,Q^2) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{4\pi} P_{ji}\left(\frac{x}{z}, Q^2\right) D_i^h(z,Q^2)$$

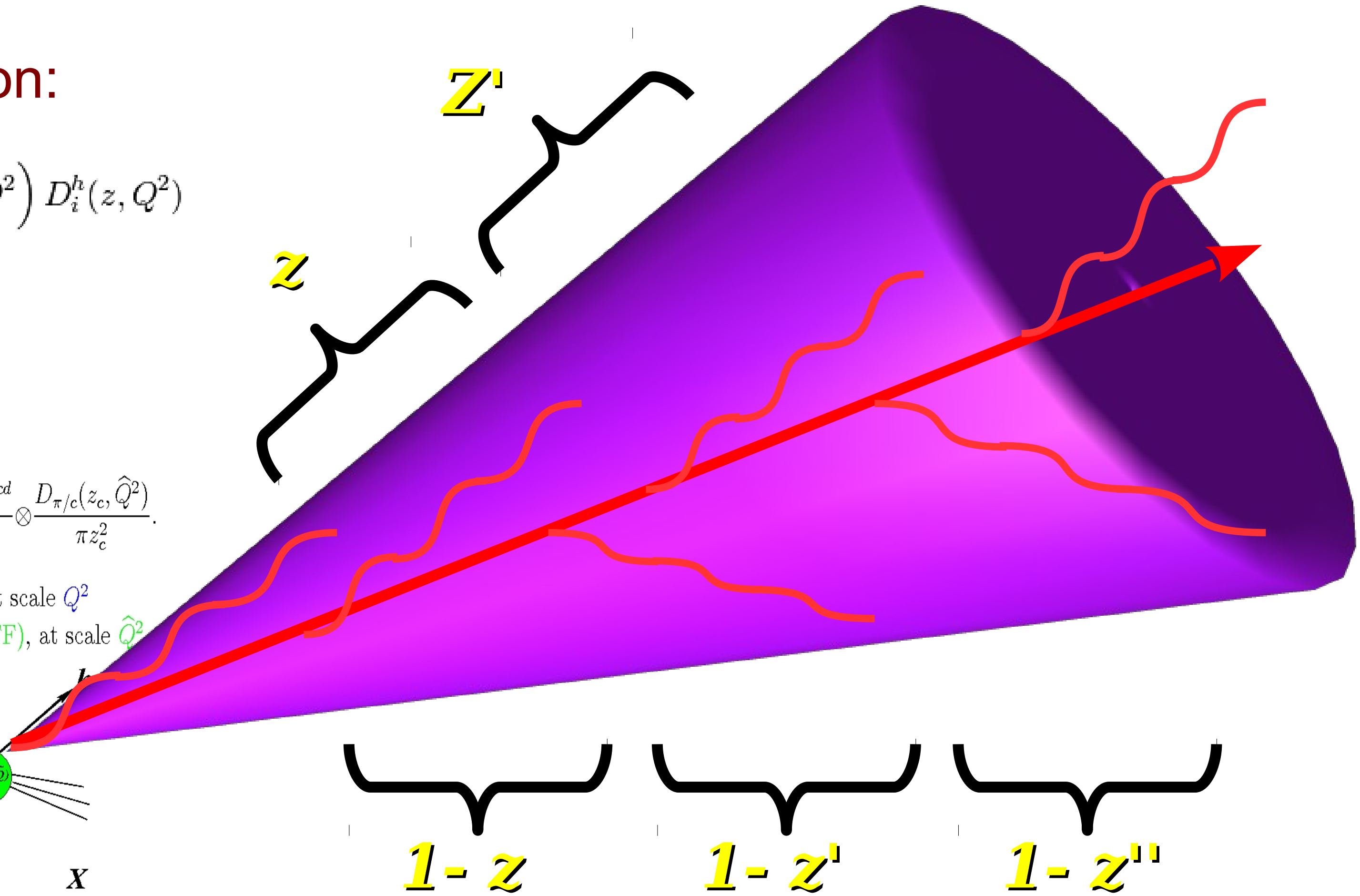
$$E_\pi \frac{d\sigma_\pi^{pA}}{d^3p_\pi} \sim f_{a/p}(x_a,Q^2;k_T) \otimes f_{b/A}(x_b,Q^2;k_T,b) \otimes \frac{d\sigma^{ab\to cd}}{d\hat{t}} \otimes \frac{D_{\pi/c}(z_c,\hat{Q}^2)}{\pi z_c^2}.$$

$f_{b/A}(x_a,Q^2;k_T,b)$ : Parton Dist. Function (PDF), at scale $Q^2$

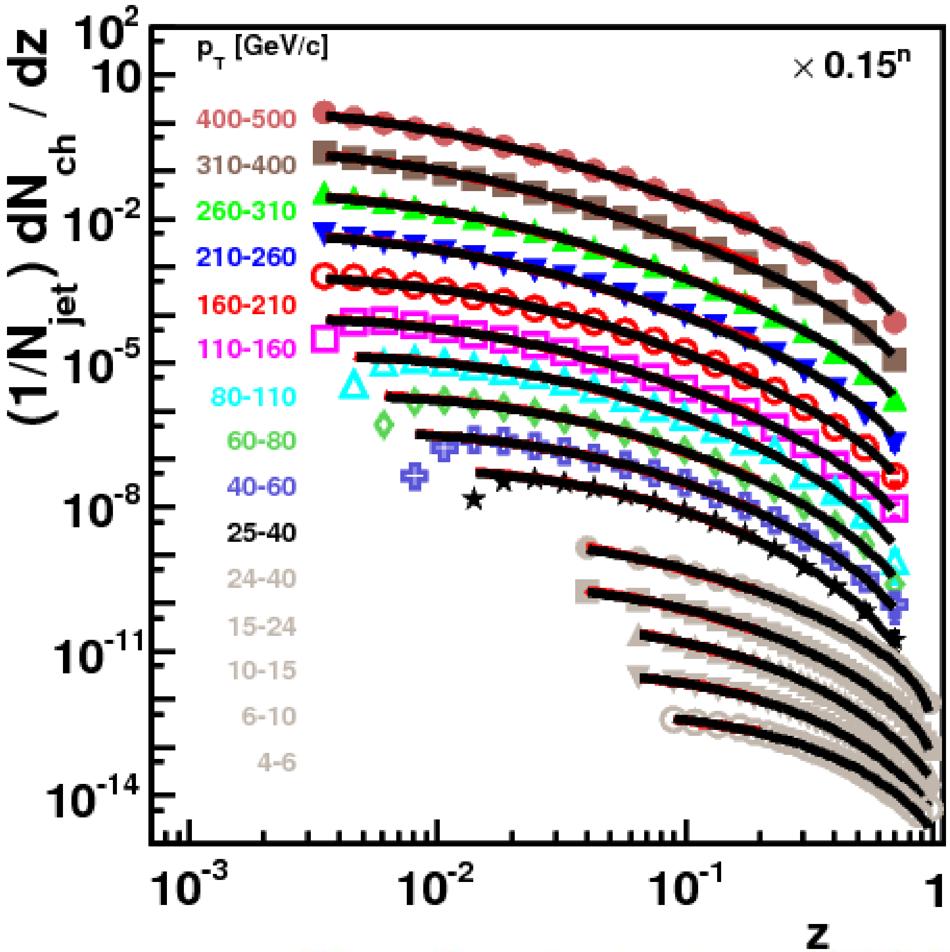
$D_{\pi/c}(z_c,\hat{Q}^2)$ : Fragmentation Function for $\pi$ (FF), at scale $\hat{Q}^2$

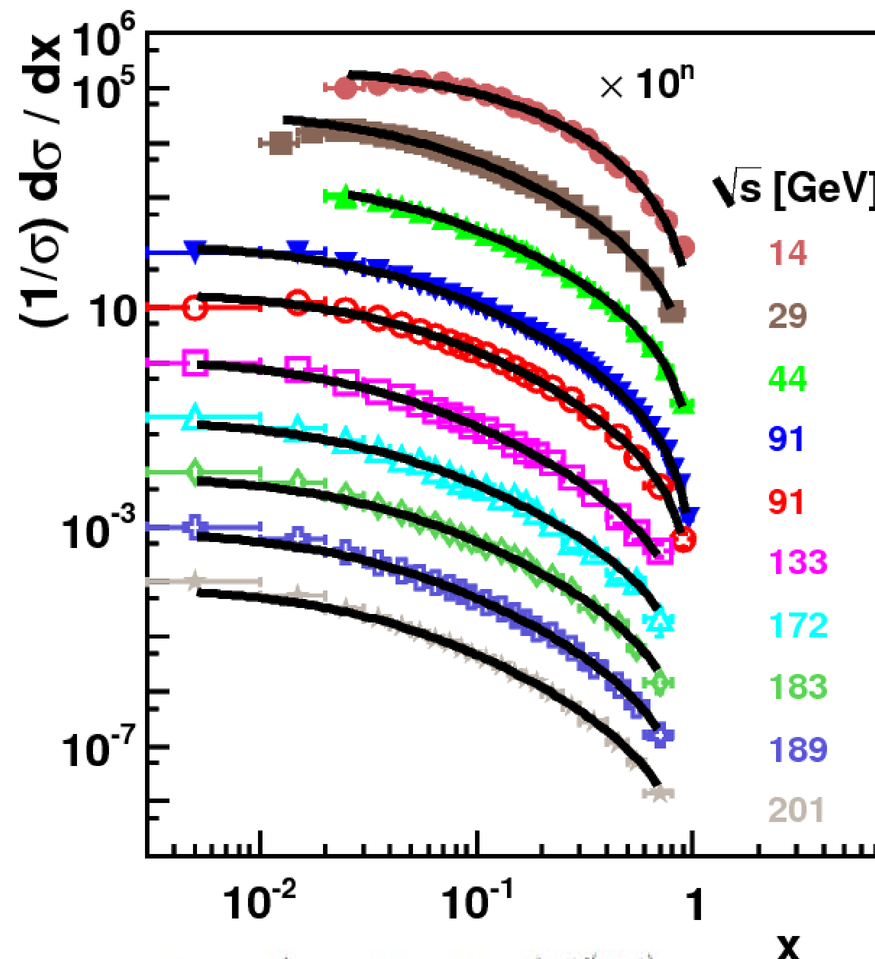
$\frac{d\sigma^{ab\to cd}}{d\hat{t}}$ : Partonic cross section



# Fits for jet spectra in pp (left) and $e^+e^-$ (right)



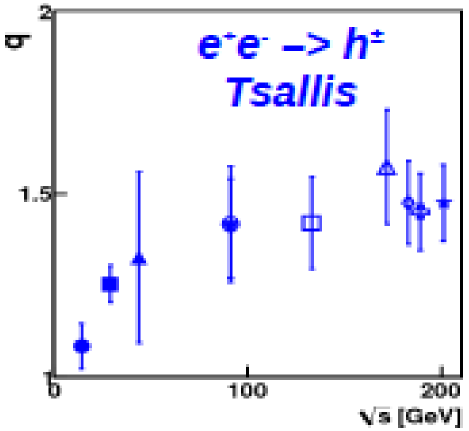

$$\frac{d\sigma}{dx} \propto \left(1 - \frac{q-1}{T/(\sqrt{s}/2)} \ln(1-x)\right)^{-1/(q-1)} \rightarrow \left(1 + \frac{q-1}{T/(\sqrt{s}/2)} x\right)^{-1/(q-1)}$$

Ref: K Ürmössy, GGB, TS Biró, PLB 710 (2011) 111, PLB 718 (2012) 125.

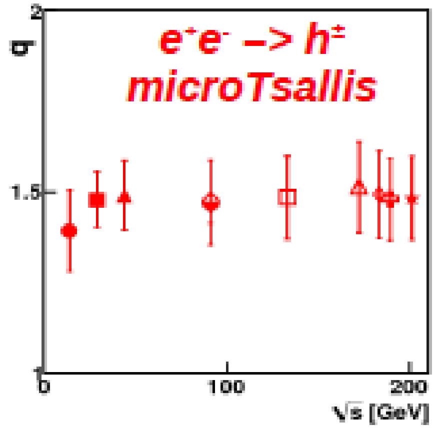
# Scale Evolution of the parameter q

**Fitts:**

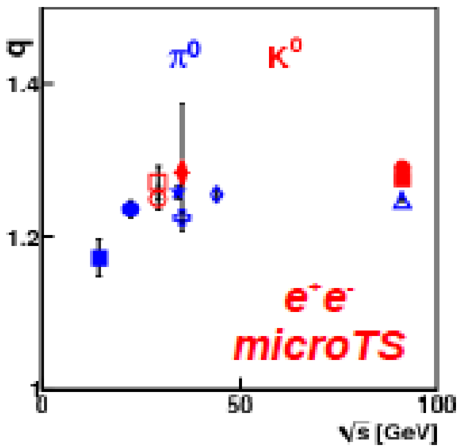
*1)*



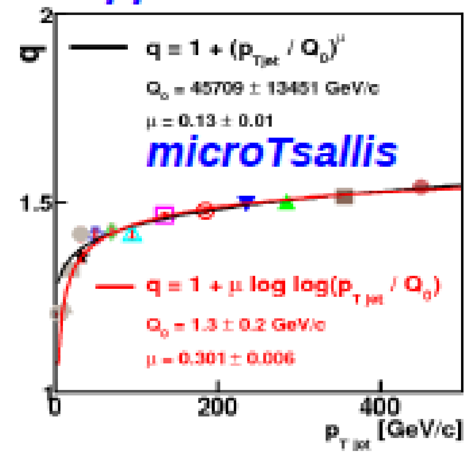

*2)*




*3)*

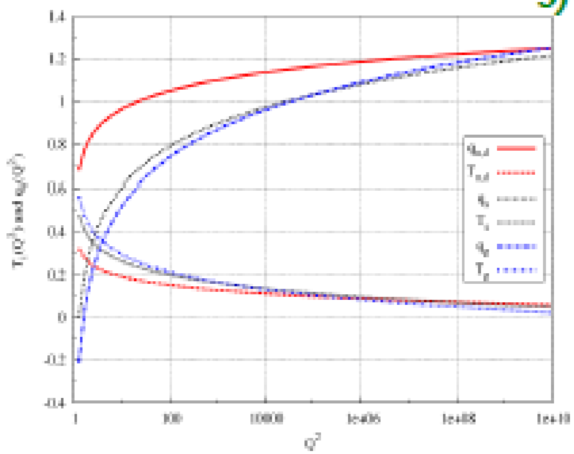



*pp –> Jet –> h±* *4)*




**Theory:** ***Scale evolution of q, T from fits to AKK Frag. Funcs:***

*5)*



$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

$$q_i = q_{0i}+q_{1i}\ln\left(\ln\left(Q^2\right)\right)$$

*1-2)* U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

*3)* T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

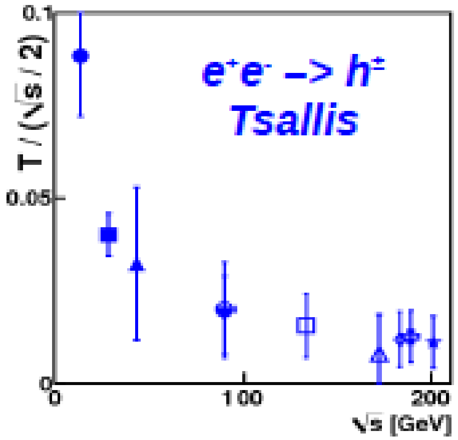
*4)* U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

*5)* Barnaföldi etal., *Gribov-80 Conf.* C10-05-26.1, p.357-363

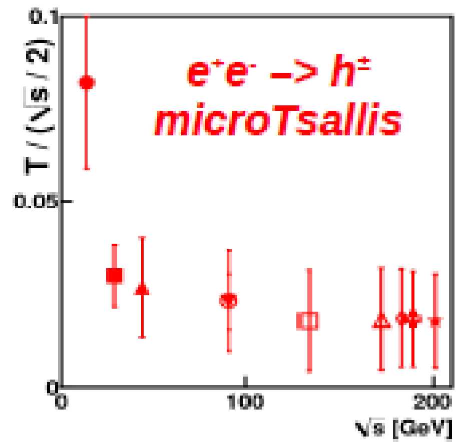
# Scale Evolution of the parameter T

**Fitts:**

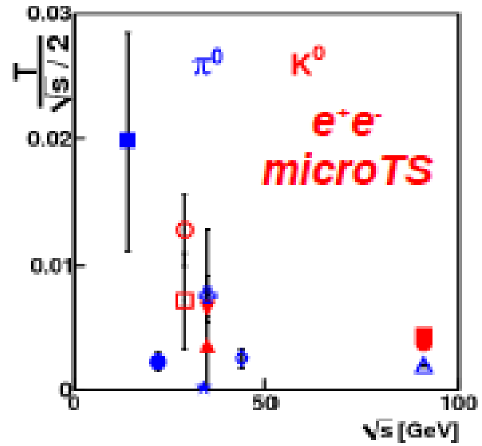
1)



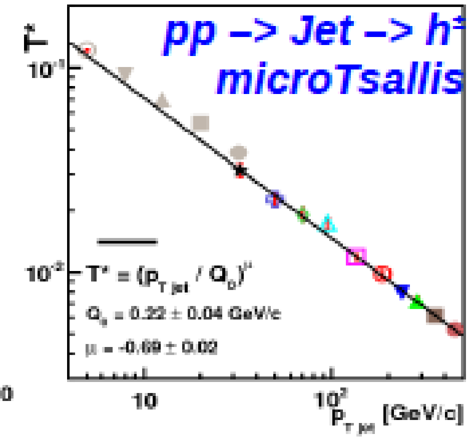

2)




3)

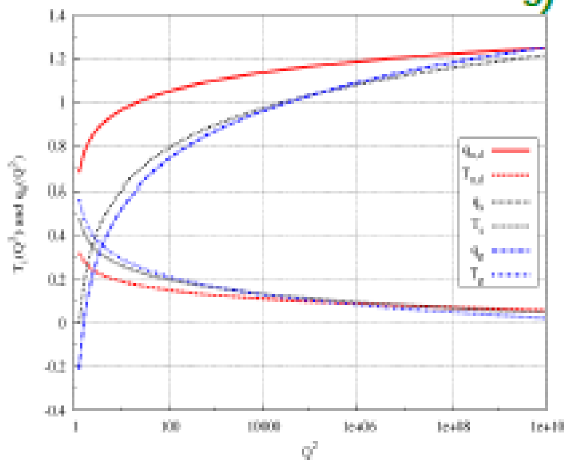



4)




**Theory:** Scale evolution of *q*, *T from fits to AKK Frag. Funcs:*

5)




$$D_{p_i}^{\pi^{+}}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

$$T_i = T_{0i}+T_{1i}\ln(\ln(Q^2))$$

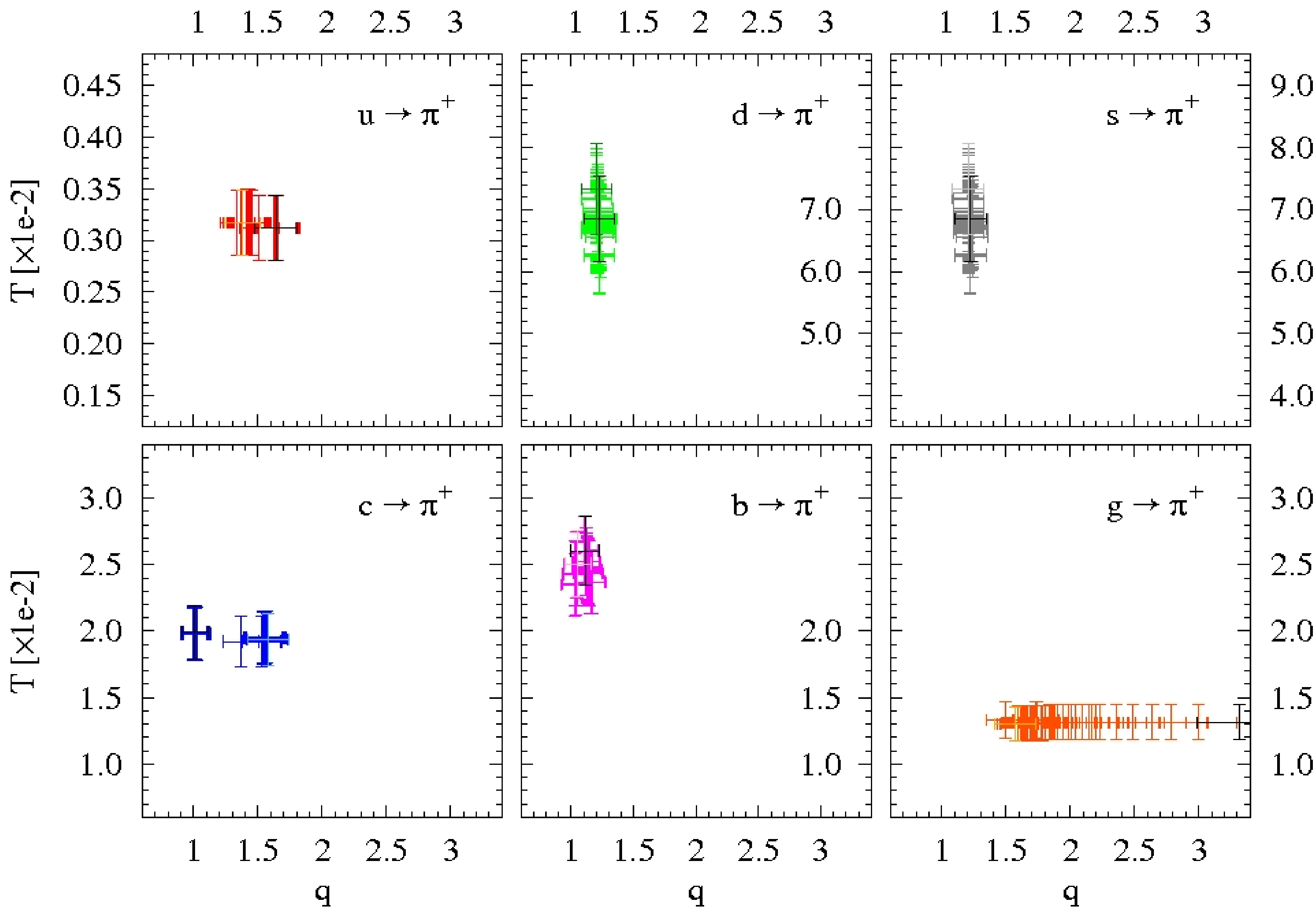
***1-2)*** U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

***3)*** T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

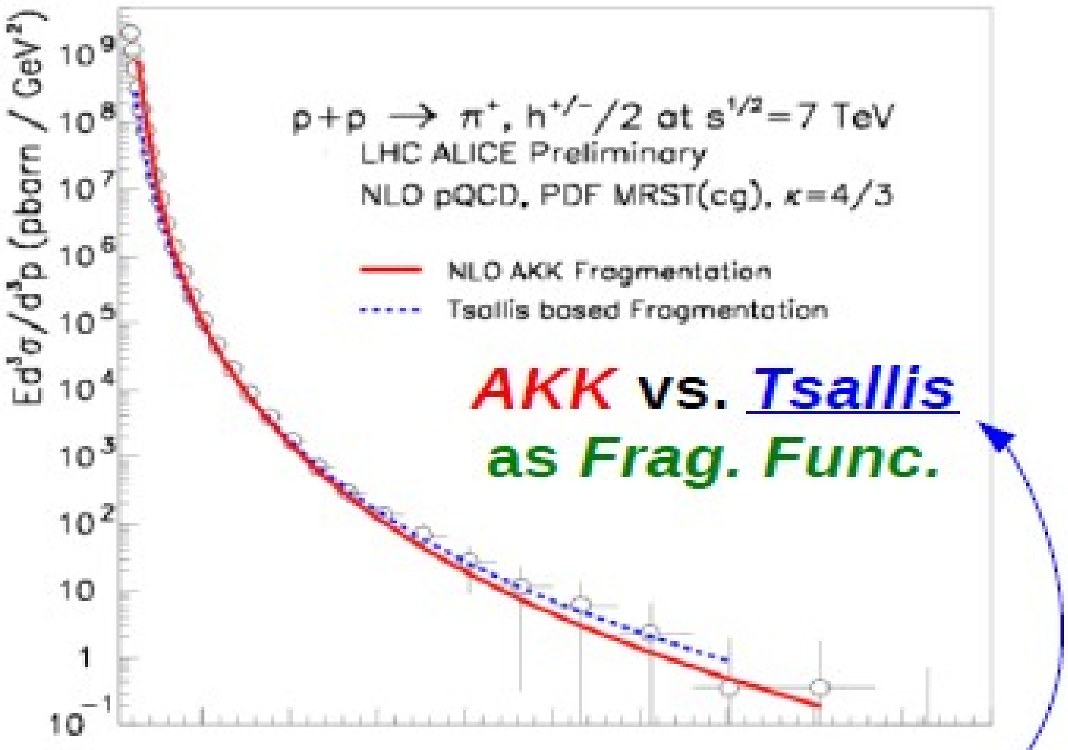
***4)*** U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

***5)*** Barnaföldi etal., *Gribov-80 Conf.* C10-05-26.1, p.357-363

# Full calculation of fitted FFs with DGLAP



# Test of the FF via NLO pQCD code (kTpQCDv20)




$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

# S U M M A R Y

- Derivation
  - Obtained Tsallis/Rényi entropies from the first principles.
  - Not only assumption, but rather a recipe.
  - Providing phyiscal meaning of the 'mysterious q',
  - $q=1-1/C=1-a$
  - *Boltzmann Gibbs limit* $C \to \infty$, $a \to 0$ $(q \to 1)$, $L(S) \to S$
  - *and more see arXiv: 1405.3813,*
- Application
  - Ideal gas TSB Physica A392 (2013) 3132
  - for Bag model the QGP temperature
    TSB, GGB, PV: EPJ A49 (2013) 110
  - FFs based on Tsallis fits to ee, pp
- It seems we can the theory works, at least, we feel...

# … to have experimental 'things' in good hands!

# B A C K U P

# Related publications..


1. arXiv:1409.5975: Statistical Power Law due to Reservoir Fluctuations and the Universal Thermostat Independence Principle

2. arXiv:1405.3963 Disentangling Soft and Hard Hadron Yields in PbPb Collisions at $\sqrt{s_{NN}}$ = 2.76 ATeV

3. arXiv:1405.3813 New Entropy Formula with Fluctuating Reservoir, Physica A (in Print) 2014

4. arXiv:Statistical Power-Law Spectra due to Reservoir Fluctuations

5. arXiv:1209.5963 Nonadditive thermostatistics and thermodynamics, Journal of Physics, Conf. Ser. V394, 012002 (2012)

6. arXiv:1208.2533 Thermodynamic Derivation of the Tsallis and Rényi Entropy Formulas and the Temperature of Quark-Gluon Plasma, EPJ A 49: 110 (2013)

7. arXiv:1204.1508 Microcanonical Jet-fragmentation in proton-proton collisions at LHC Energy, Phys. Lett. B, 28942 (2012)

8. arXiv:1101.3522 Pion Production Via Resonance Decay in a Non-extensive Quark-Gluon Medium with Non-additive Energy Composition Rule

9. arXiv:1101.3023 Generalised Tsallis Statistics in Electron-Positron Collisions, Phys.Lett.B701:111-116,2011

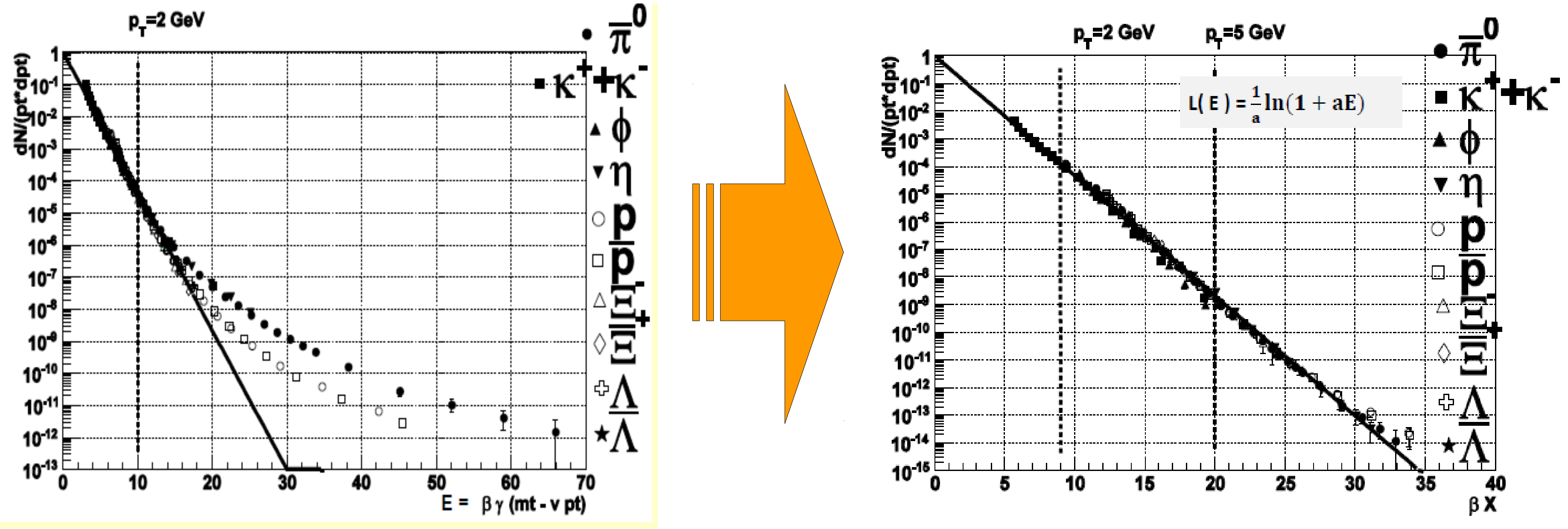
10. arXiv:0802.0381 Pion and Kaon Spectra from Distributed Mass Quark Matter, J.Phys.G35:044012,2008

# Experimental data fits by $T_{slope}(E)$

- Findings: *K=2* (mesons) and *K=3* (baryons)

$$P_{\rm hadron}(E) = P_i^K(E/K) \quad \text{and} \quad T_{\rm slope}^{\rm hadron}(E) = T_{\rm slope}^{\rm quark}(E/K)$$

This finding is coming from the scaling of the PID-spectra...




T.S.Biró, K.Ürmössy, JPhysG 36, 064044, 2009

# The temperature slope

- Taking $P_i$ weights of system, $E_i$ , results cut power law:

$$P_i = \left(Z^{1-q} + (1-q)\frac{\beta}{q}E_i\right)^{\frac{1}{q-1}} = \frac{1}{Z}\left(1 + \frac{Z^{-1/C}\,\mathrm{e}^{S/C}}{C-1}\frac{E_i}{T}\right)^{-C}$$

- Partition sum is related to Tsallis entropy, $L(S_1)$ and $E_1$

$$\ln_q Z := C\left(Z^{1/C} - 1\right) = L\left(S_1\right) - \frac{1}{1-1/C}\beta E_1$$

- In $C \to \infty$ limit, the inverse log slope of the energy distribution:

$$T_{\mathrm{slope}}(E_i) = \left(-\frac{d}{dE_i}\ln P_i\right)^{-1} = T_0 + E_i/C, \quad \text{with} \quad T_0 = T\mathrm{e}^{-S/C}Z^{1/C}(1-1/C)$$

# Fragmentation via associative composition

Program:

1) Search and fit Tsallis-Pareto distribution to data.

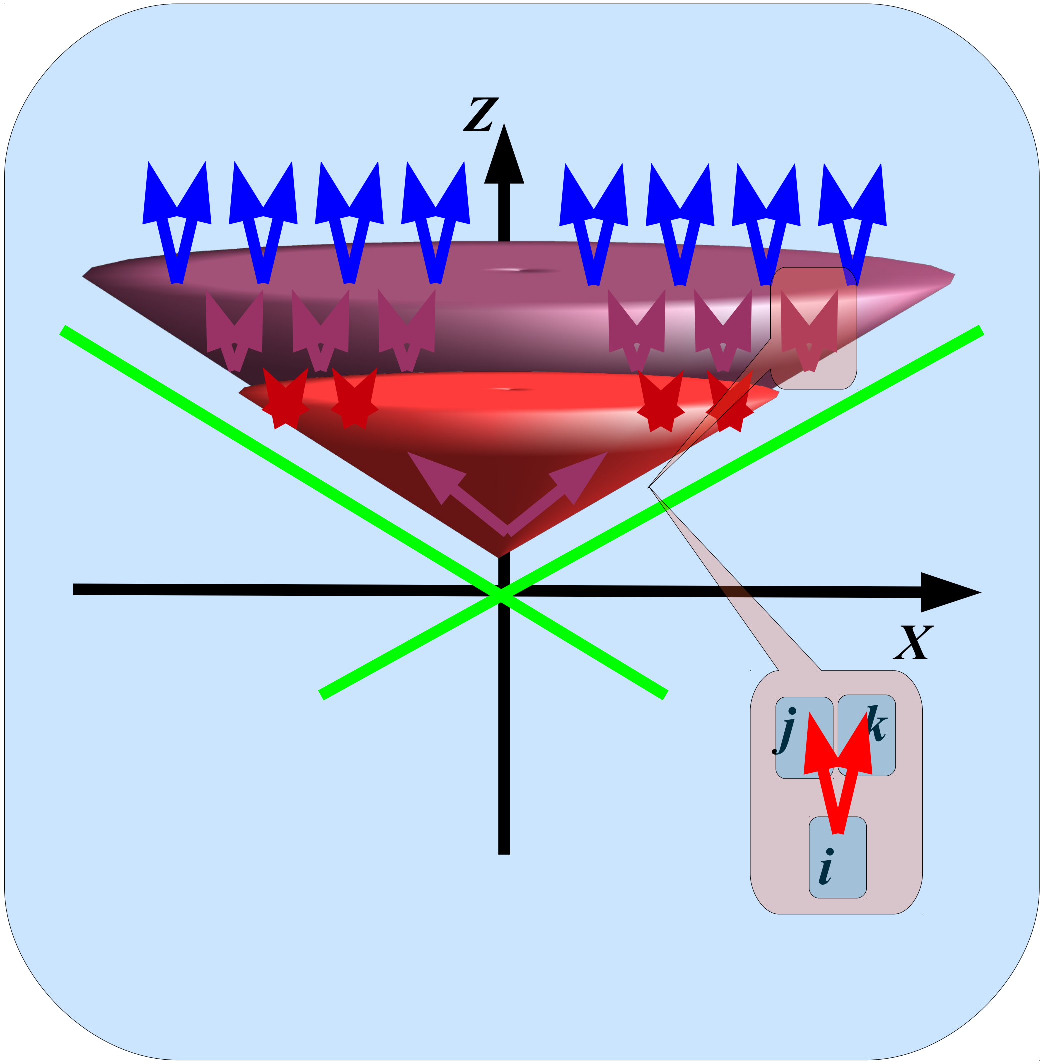
2) Serach for physical meaning of T and q parameters.

3) Components of the sub-systems are e.g. 'splitting functions' $P_{qg}$, $P_{gg}$

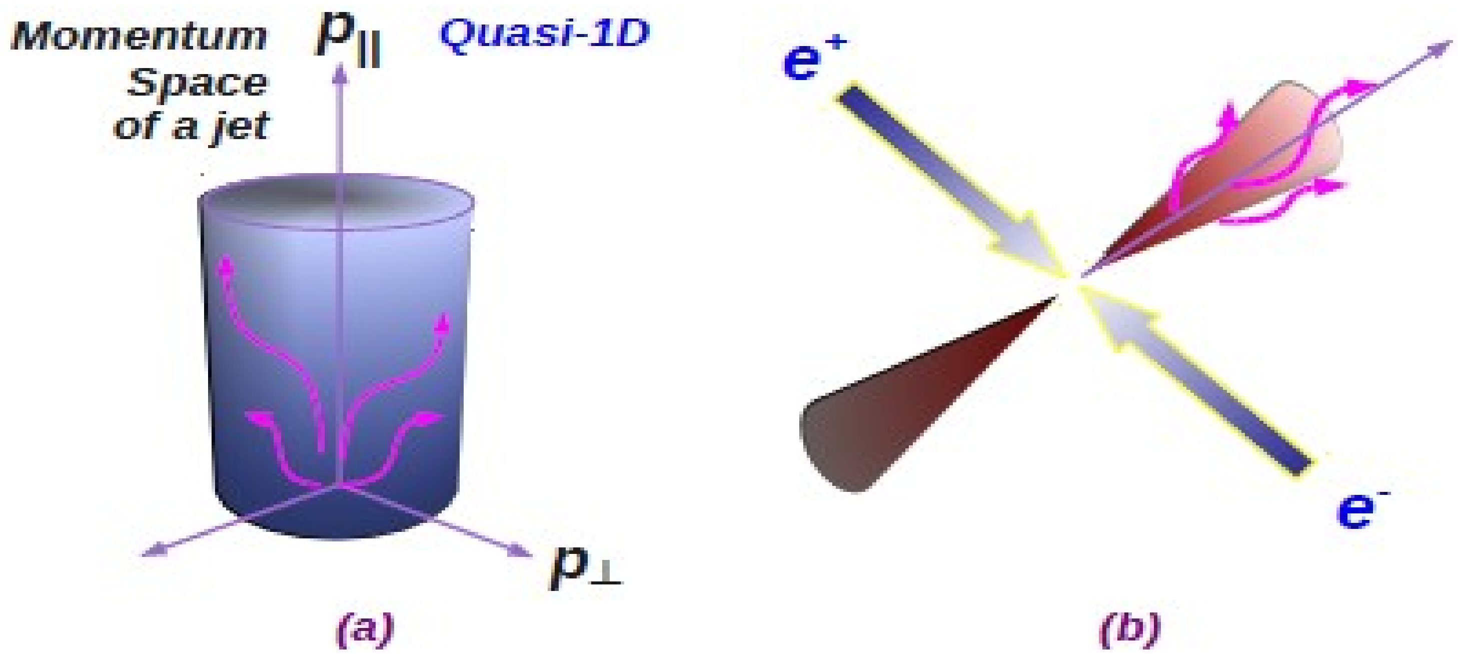
4) Test: can a DGLAP-like evolution equation be obtained?

$$D(x,Q^2) \sim f(E,T,q) * f(\ln(Q^2))$$

$$D(x,Q^2) \sim f(E,T(\ln(Q^2)),q(\ln(Q^2)))$$



# The 'Thermodynamics of Jets'




K. Ürmössy, G.G. Barnaföldi, T.S. Bíró:

- Microcanonical Jet-Fragmentation in pp at LHC energies:

  Phys. Lett. B701 (2011) 111

- Generalized Tsallis distribution in $e^+e^-$ collisons

  Phys. Lett. B718 (2012) 125

# New Directions to Investigate..

Formulated questions from the theory...

- What is responsible for the power law tail measured at high-$p_T$?
- Can we assume thermodynamical equilibrium for high-$p_T$ particles?
- What is the origin of the ’collectivity’? Is it coming from ’quark level’ or ’hadron level’?
- Is there difference between baryon and meson formation? What is the statistical origin of this (e.g coalescence, fragmentation, etc.)?

The VHMPID LoI (2013) arXiv:1309.5880

# Why to use Tsallis/Rényi entropy formula?

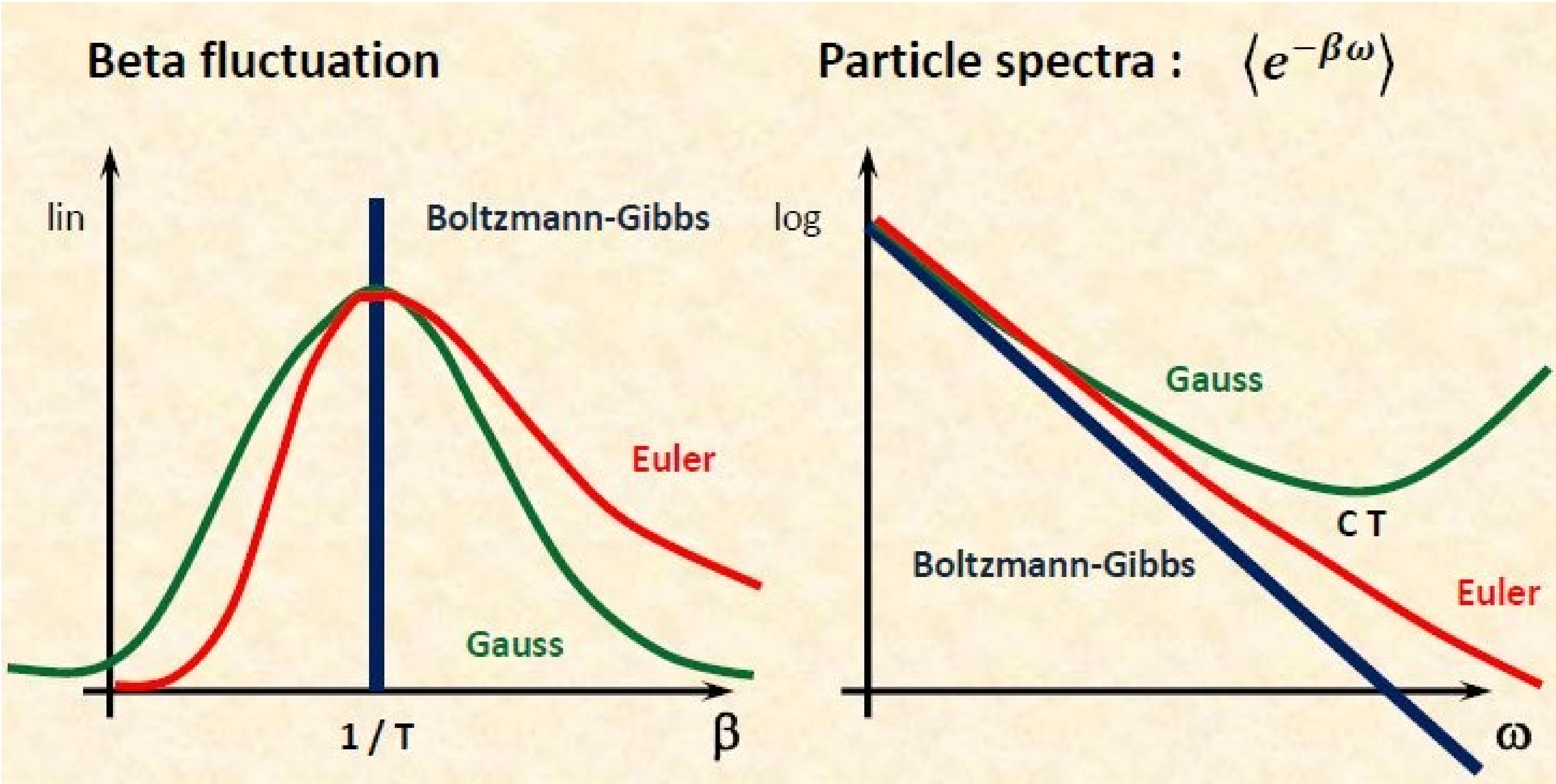
- It generalizes the Boltzmann-Gibbs-Shanon formula.
- It treats statistical entanglement between subsystem and reservoir (due to conservation).
- It claims to be universal: applicable for whatever material quantity of the reservoir.
- It leads to a cut power law energy distribution in the canonical treatment.

# Why NOT to use Tsallis/Rényi formulas?

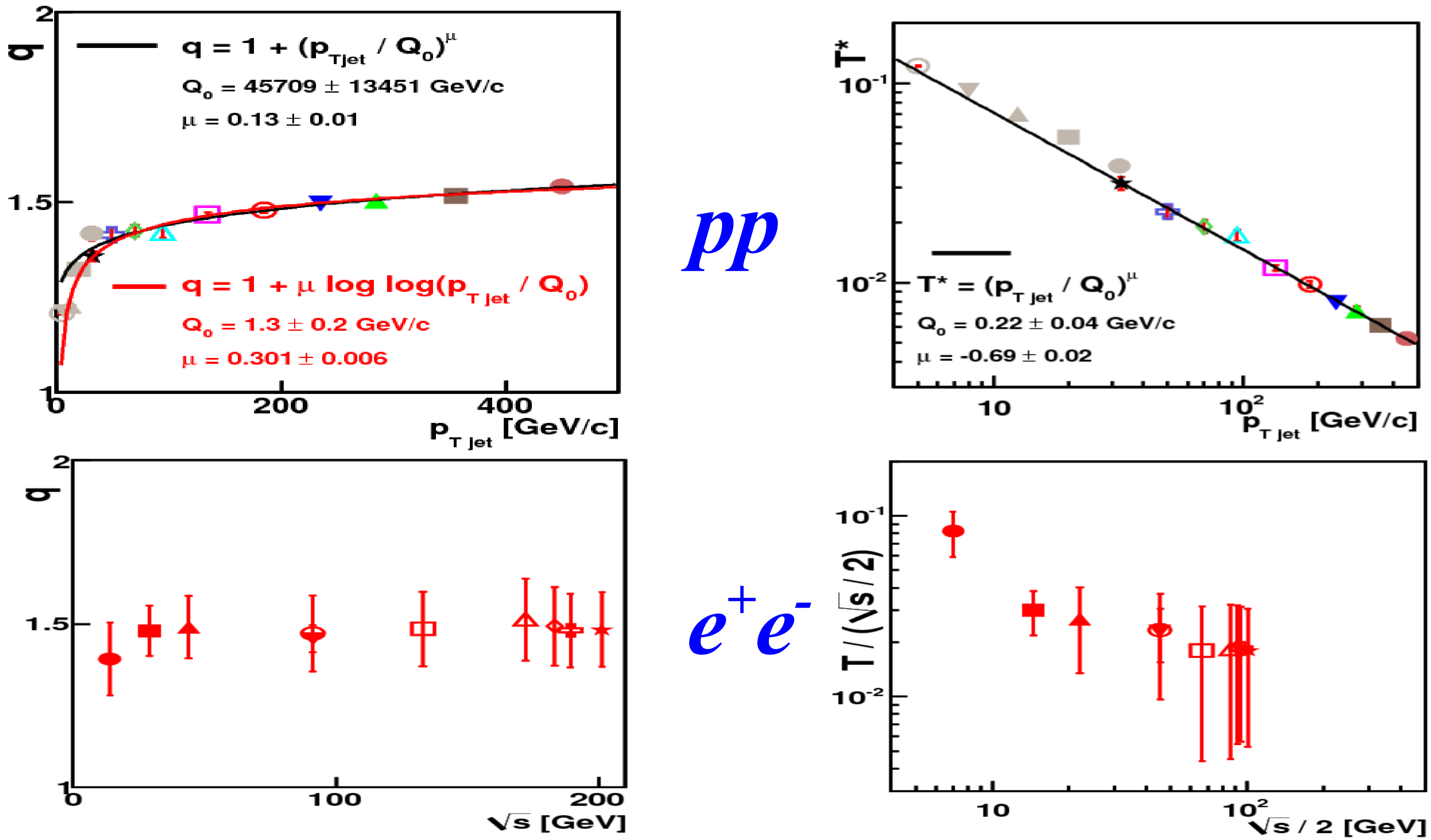
- They lack 300 years of classical thermo-dynamic foundation
- Tsallis is NOT additive, Rényi is NOT linear
- There is an extra parameter: the mysterious q
- How do different q systems equilibrated?
- Why this and not other?
- It looks pretty formal....

So here is some input to get rid of bad feelings...

# What do we measure as temperature?



# The evolution of parameters q and T




Ref: K Ürmössy, GGB, TS Biró, PLB 710 (2011) 111, PLB 718 (2012) 125.

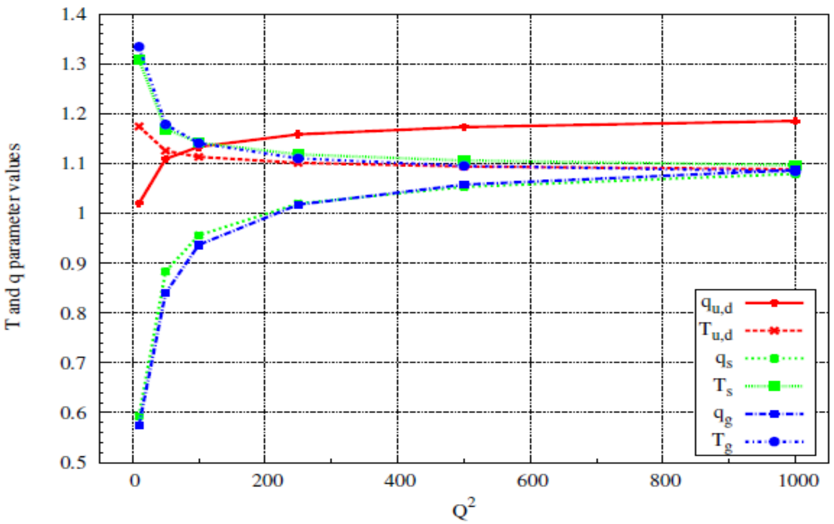
# Hadronization with parameter Evolution

Ref: GGB, G Kalmár, K Ürmössy, TS Biró, Proc. of Gribov 80. (2011):

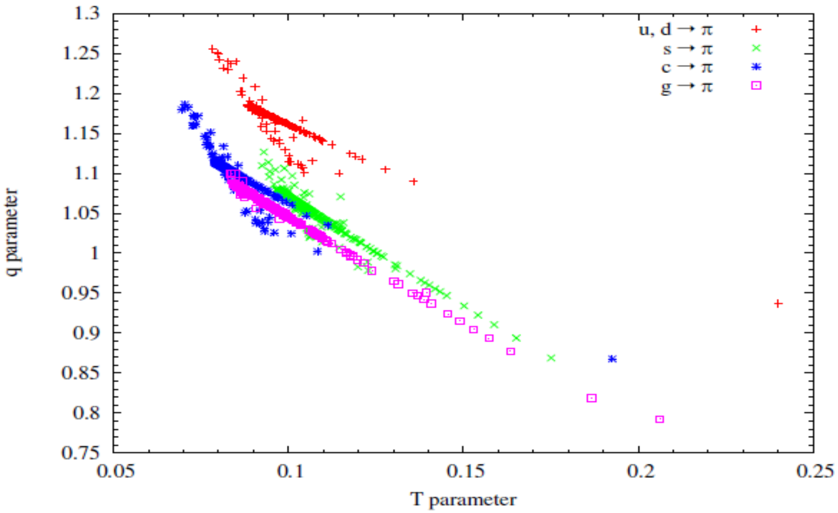
Tsallis based hadronization for p:

$$\sim \left(1 + (q_i - 1) \cdot \frac{z}{T_i}\right)^{-1/(q_i-1)}$$

Tsallis–Pareto parameters can be extracted for hadronization:




| parton, $i$ | $T_{i1}$ | $T_{i0}$ | $q_{i1}$ | $q_{i0}$ |
|---|---|---|---|---|
| $u, \bar{u}, d, \bar{d}$ | -0.057753 | 0.239825 | 0.124000 | 0.860351 |
| $s, \bar{s}$ | -0.093988 | 0.343175 | 0.265042 | 0.384453 |
| $c, \bar{c}$ | -0.048170 | 0.205408 | -0.40198 | 2.142750 |
| $b, \bar{b}$ | -0.033599 | 0.156249 | 0.103565 | 0.803255 |
| $g$ | -0.118556 | 0.394749 | 0.318477 | 0.253205 |




$$T_i \longrightarrow T_i(Q^2) = T_{i1} \cdot \ln\left(\ln(Q^2)\right) + T_{i0} \,,$$
$$q_i \longrightarrow q_i(Q^2) = q_{i1} \cdot \ln\left(\ln(Q^2)\right) + q_{i0} \,.$$

# $^{60}$Co decay scheme