Renormalization of the jet-quenching parameter

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OUTLINE

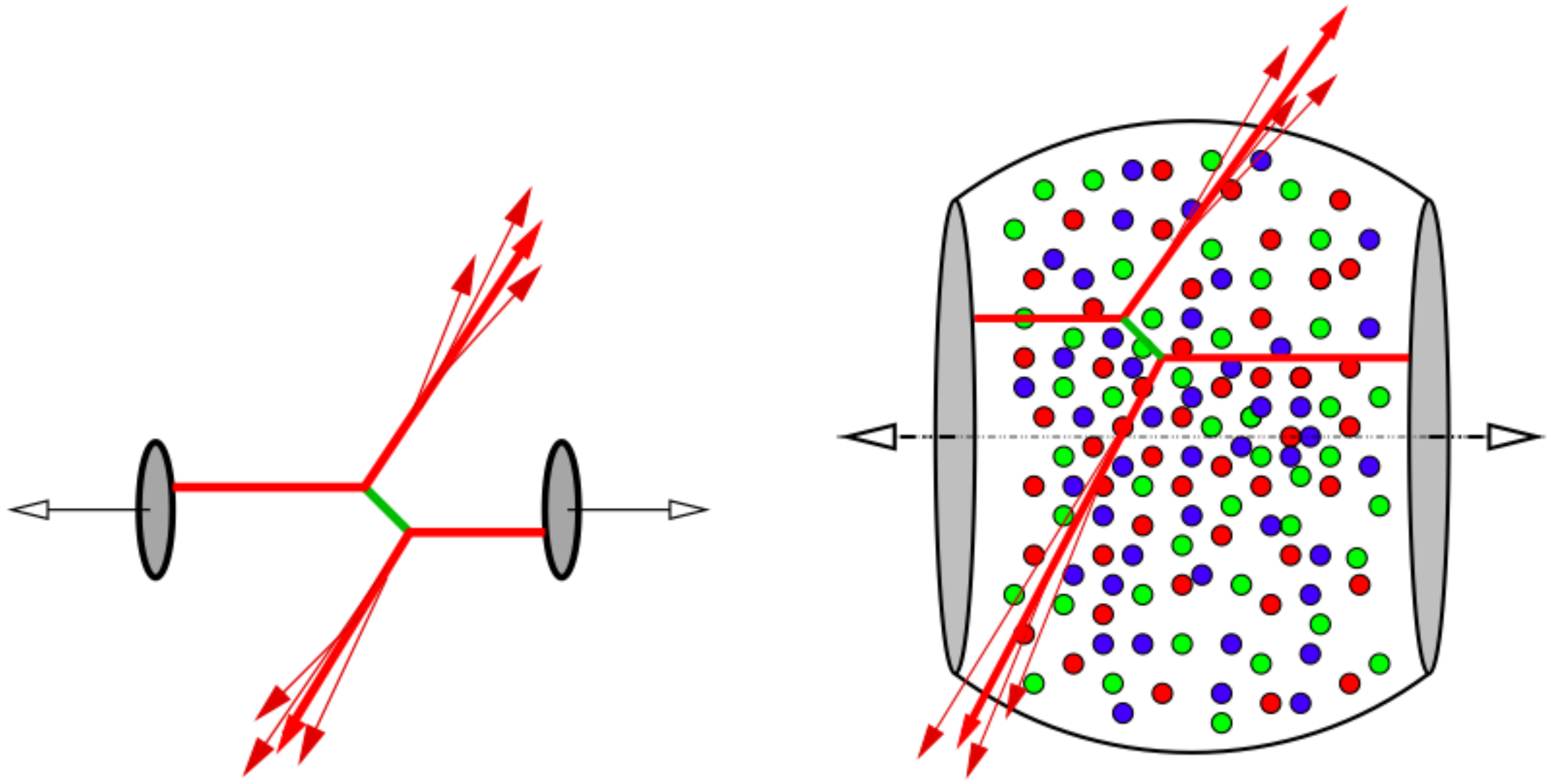
- The jet-quenching parameter in pt-broadening and radiative energy loss in a QGP
- Radiative corrections to pt-broadening and radiative energy loss (universality)
- Renormalization of the jet-quenching parameter

Based on: J. -P. Blaizot and YMT, Nucl. Phys. A 929 (2014) arXiv:1403.2323 [hep-ph]

- Jet-quenching is characterized by the transport coefficient

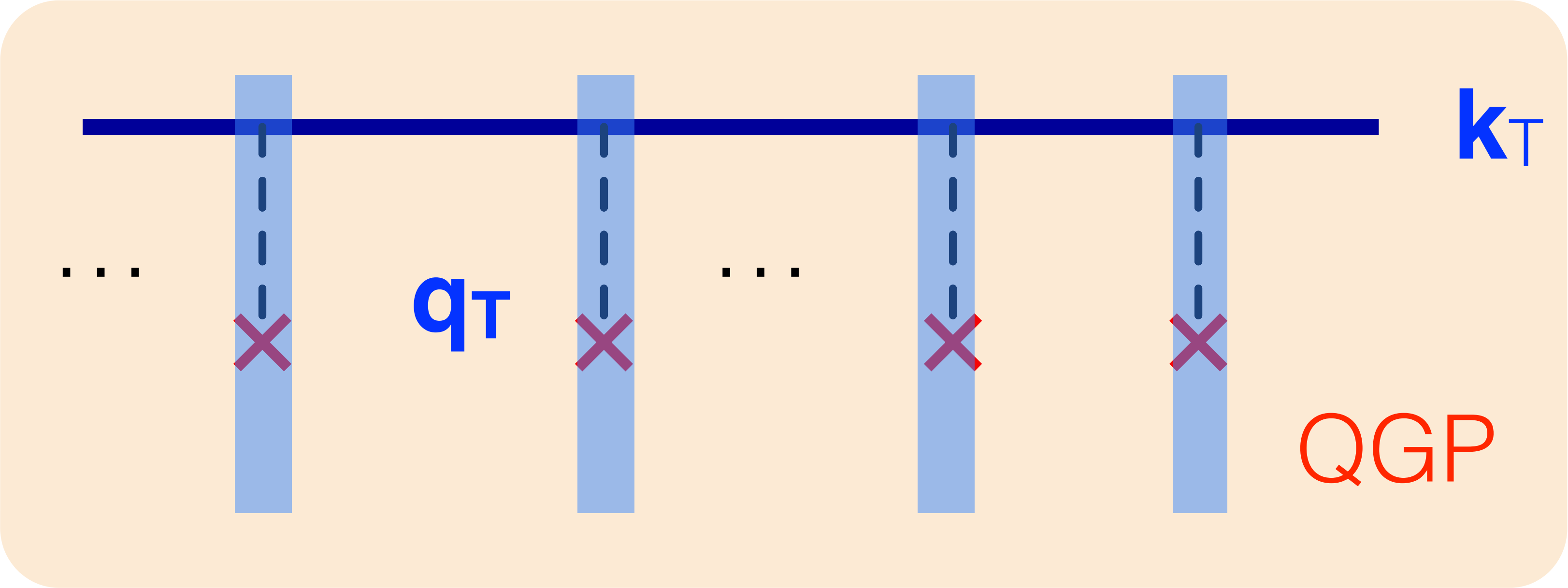
$$\hat{q} \sim \langle \mathbf{k}_T^2 \rangle / L$$

~ 1 - 6 GeV^2/fm [pQCD, phenomenology, Lattice]



pt-broadening in a plasma

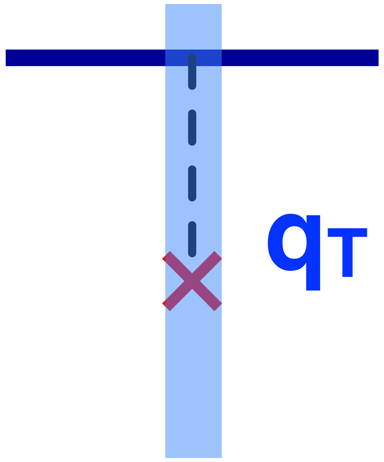
Diffusion approximation: Independent multiple scatterings



correlation length ≪ mean-free-path ≪ L

Local interaction: elastic cross-section

$$\frac{d\sigma_{\rm el}}{d^2\boldsymbol{q}} \simeq \frac{g^4}{\boldsymbol{q}^4} \quad (\text{for } \ q_\perp \gg m_D)$$



pt-broadening in a plasma

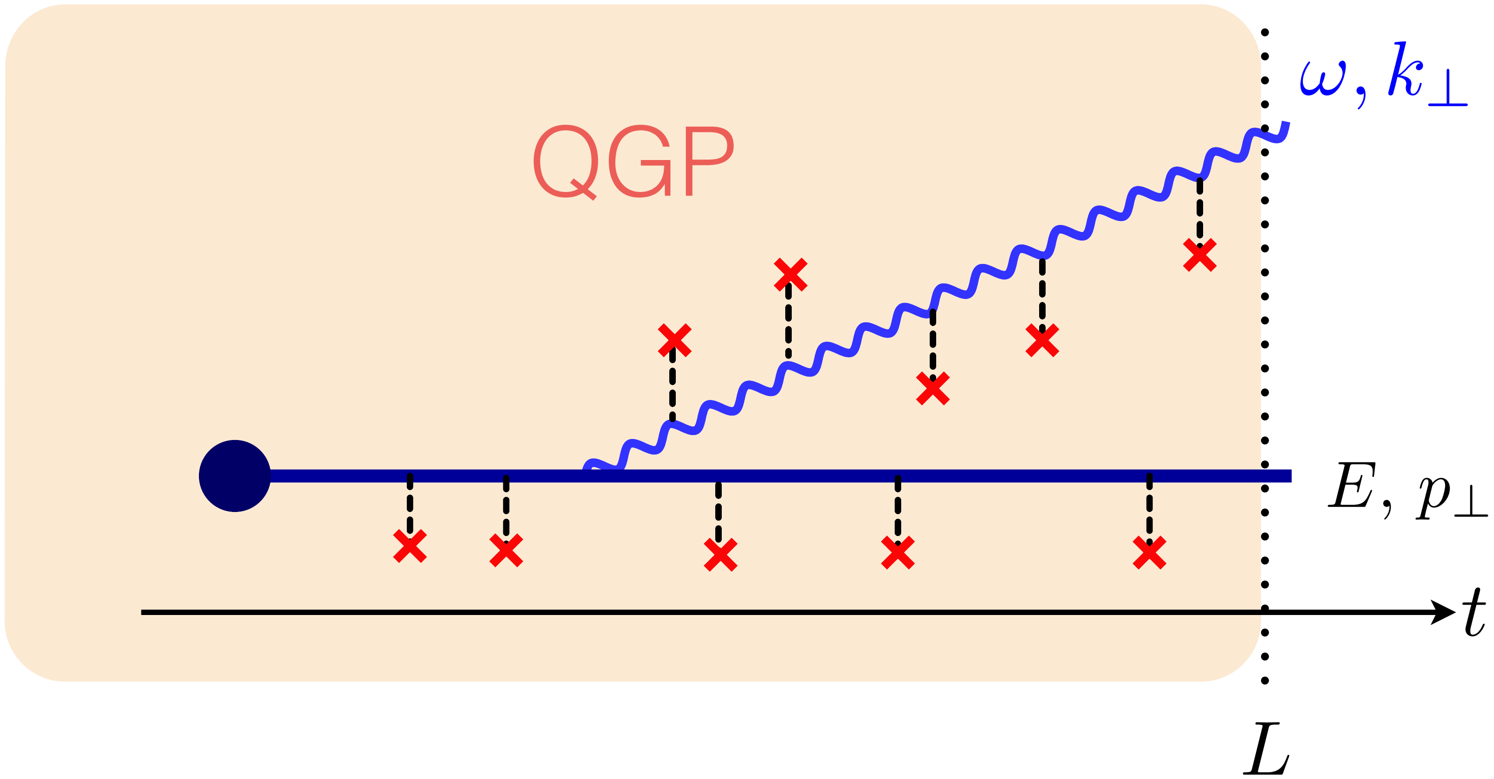
The quenching parameter is defined as a (local) diffusion coefficient

$$\hat{q} \equiv \frac{d\langle q_\perp^2 \rangle}{dt} = n \int_{\boldsymbol{q}} \boldsymbol{q}^2 \, \frac{d\sigma_{\rm el}}{d^2\boldsymbol{q}} \; \sim \; \alpha_s^2 \, C_R \; n \ln \frac{\boldsymbol{k}^2}{m_D^2}$$

The probability to acquire a transverse mom. $k_\perp$ after a time t is given by a Fokker-Planck equation

$$\mathcal{P}(\boldsymbol{k}, L) = \frac{4\pi}{\hat{q}\, L} \, \exp\left(- \frac{\boldsymbol{k}^2}{\hat{q}\, L} \right) \qquad\qquad \langle k_\perp^2 \rangle_{\rm typ} \equiv \hat{q}\, L$$

Radiative Energy Loss



- Scatterings with the medium can induce gluon radiation
- The radiation mechanism is linked to transverse momentum broadening, i.e., to $\hat{q}$

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

Radiative Energy Loss

How does it happen? After a certain number of scatterings coherence between the parent quark and gluon fluctuation is broken and the gluon is formed (decoherence is faster for softer gluons)

$$t_f \equiv \frac{\omega}{\langle q_\perp^2 \rangle} \simeq \frac{\omega}{\hat{q}\, t_f} \quad \Rightarrow \quad t_f = t_{\rm br} \equiv \sqrt{\frac{\omega}{\hat{q}}}$$

maximum frequency for this mechanism $\omega_c = \frac{1}{2}\hat{q}\, L^2$

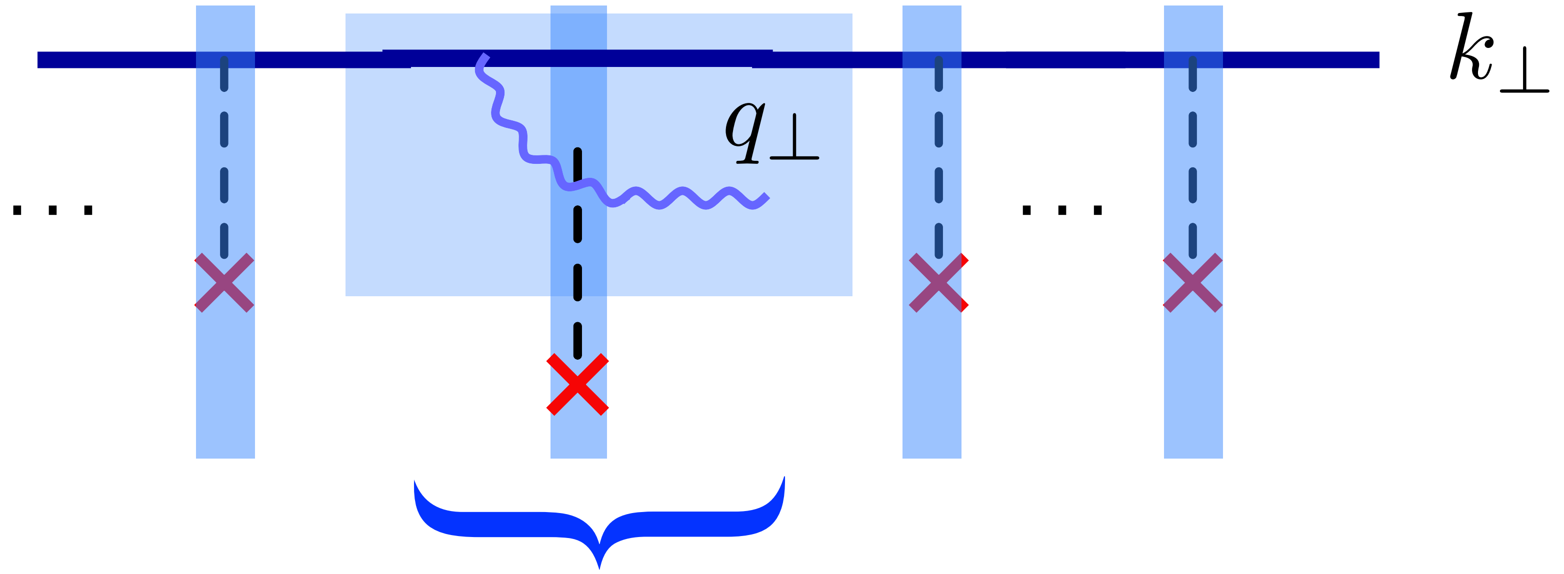
corresponding to $t_{\rm br} \sim L$

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

Radiative corrections to pt-broadening

The radiative correction exhibits a logarithmic singularity when the duration of the gluon fluctuation $\tau \to 0$

The leading logarithmic contribution requires that the time scale of the fluctuation $\boldsymbol{\tau}$ to be smaller than the radiative coherence length $t_{br} \sim (\omega/\hat{q})^{1/2}$



$1/m_D \ll \boldsymbol{\tau} \ll t_{br} \ll L$

Radiative correction to pt-broadening

The radiative correction exhibits a logarithmic singularity when the duration of the gluon fluctuation $\tau \to 0$

- Possible interpretation of the fluctuation as local
- Double Log corrections to the jet-quenching parameter

$$\Delta\hat{q}(L,\boldsymbol{k}^2) \equiv \frac{\alpha_s N_c}{\pi} \int_{\tau_0}^{L} \frac{d\tau}{\tau} \int_{\hat{q}\tau}^{\boldsymbol{k}^2} \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \hat{q}(\tau_0)$$

DL: Single scattering : $\mathbf{q} \gg \hat{q}\,\boldsymbol{\tau}$ (LPM suppression)

Radiative correction to pt-broadening

To logarithmic accuracy $k_\perp^2 \sim \hat{q} L$

$$\Delta\hat{q}(L) \simeq \frac{\alpha_s N_c}{\pi}\,\hat{q}\,\ln^2\frac{L}{\tau_0}$$

[T. Liou, A. H. Mueller, B. Wu, (2013) J.P. Blaizot, F. Dominguez, E. Iancu, YMT (2013)]

The size of the fluctuation extends up the length of the medium (non local correction)

Can we still talk about independent multiple scattering (2-D diffusion) ? Yes, to logarithmic accuracy

Independent multiple radiative corrections

Hence, radiative corrections to the broadening probability can be reabsorbed in a redefinition of the jet-quenching parameter

$$\hat{q}_0 \to \quad \hat{q}(L) \equiv \hat{q}_0 \left(1 + \frac{\bar{\alpha}}{2} \ln^2 \frac{L}{\tau_0} \right)$$

leaving the diffusion picture intact

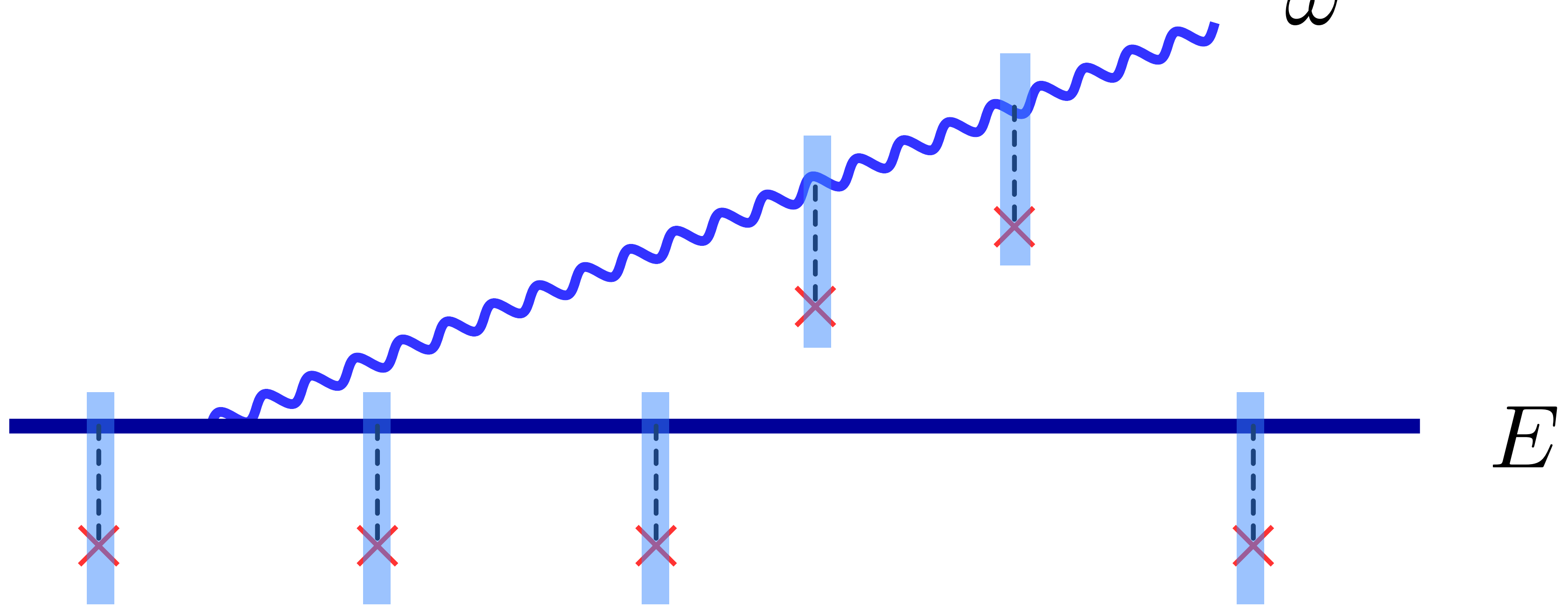
$$\mathcal{P}(\boldsymbol{k}, L) = \frac{4\pi}{\hat{q}(L)\, L} \exp\left(- \frac{\boldsymbol{k}^2}{\hat{q}(L)\, L} \right)$$

Radiative corrections to Energy loss

What about the radiative corrections to the medium-induced gluons spectrum which is also is also function of the quenching parameter?

The BDMPS spectrum (soft gluon radiation $\omega << E$)

$$\omega \frac{dN}{d\omega} \sim \bar{\alpha} \sqrt{\frac{\hat{q}}{\omega}} L$$

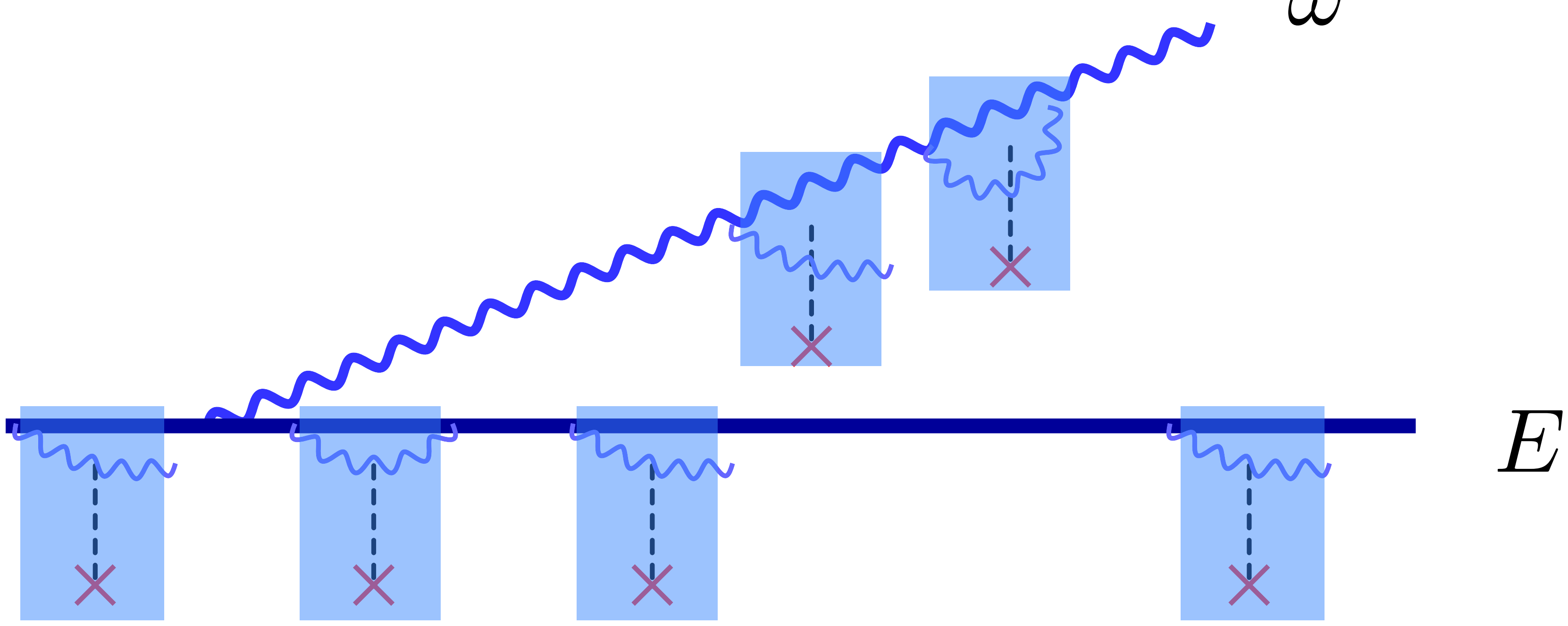


Radiative corrections to Energy loss

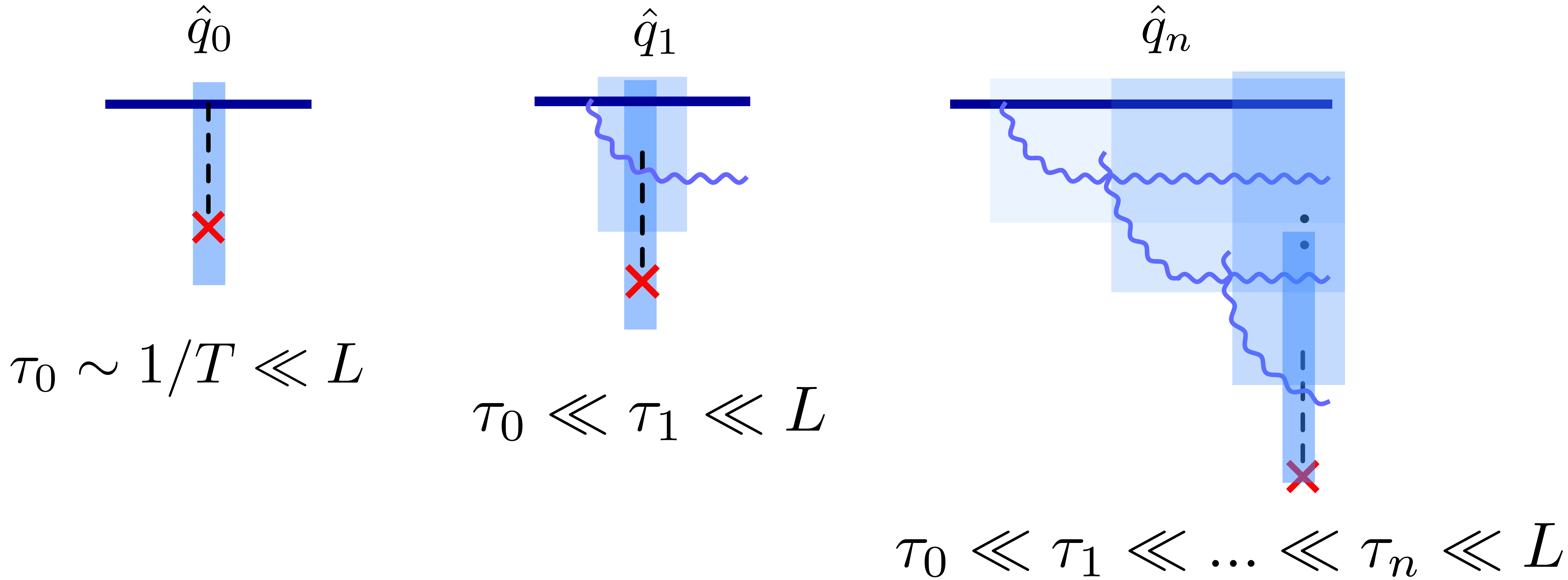
The Double logarithmic divergence can be absorbed in a redefinition of q̂

The BDMPS spectrum (soft gluon radiation $\omega << E$)

$$\omega \frac{dN}{d\omega} \sim \bar{\alpha} \sqrt{\frac{\hat{q} + \Delta \hat{q}}{\omega}} L$$



The first $\boldsymbol{\alpha}_s$ correction is enhanced by a double log (DL) which is resummed with strong ordering in formation time (or energy) and transverse mom. of overlapping successive gluon emissions



Renormalization Group Equation for the quenching parameter

$$\frac{\partial \hat{q}(\tau, Q^2)}{\partial \ln(\tau/\tau_0)} = \int_{\hat{q}\tau}^{Q^2} \bar{\alpha}(\boldsymbol{q}) \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2}\, \hat{q}(\tau, \boldsymbol{q}^2)$$

with initial condition $\hat{q}(\tau_0) \equiv \hat{q}_0$

Universality of the double logs

As a consequence to the renormalization of the quenching parameter, the DL's not only enhance the pt-broadening but also the radiative energy loss expectation:

For large media (asymptotic behavior)

$$\langle k_\perp^2 \rangle \propto L^{1+\gamma}$$

anomalous dimension

$$\Delta E \propto L^{2+\gamma} \qquad \gamma = \sqrt{\frac{4\alpha_s N_c}{\pi}}$$

To be compared to N=4 SYM (strong-coupling) estimate $\Delta E \sim L^3$

[Gubser et al, Hatta et al, Chesler et Yaffe (2008)]

SUMMARY

- We have shown that to Double-Log accuracy radiative corrections can be reabsorbed in a renormalization of the jet-quenching parameter without altering the classical picture of independent scatterings
- For large media the renormalized quenching parameter increases compared to the standard perturbative estimate and exhibits an anomalous scaling $\Rightarrow$ consequences for phenomenological

  studies that aim to probe the nature of the QGP in HIC

Backup

Radiative Energy Loss

Spectrum = bremsstrahlung x effective number of scatterers

The mean-energy loss dominated by « hard » but rare emissions $\omega \sim \omega_c$ (maximum coherence length ~ L)

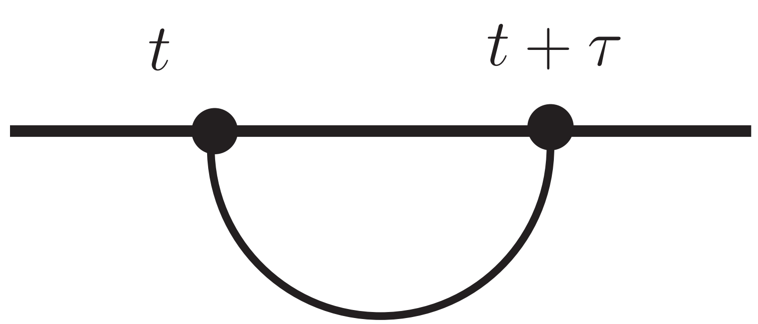
$$\Delta E = \int^{\omega_c} d\omega\, \omega \frac{dN}{d\omega} \simeq \alpha_s C_R\, \hat{q}\, L^2 \propto \omega_c$$

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

Independent multiple radiative corrections

1-dim model:

Introducing a correction (radiative) with a logarithmic phase space (non-local corrections)



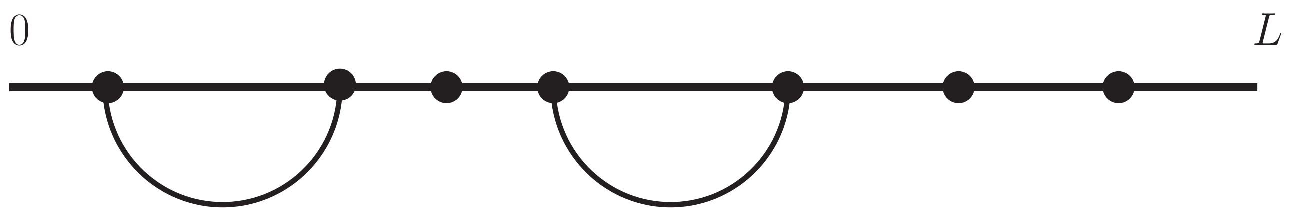
$$\equiv \sigma_0 \, \alpha \int_{\tau_0}^{L} \frac{d\tau}{\tau}$$

and consider the large medium limit such that

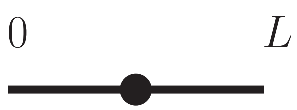
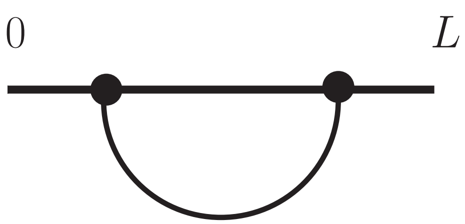
$$\alpha \ln \frac{L}{\tau_0} \sim 1 \quad \text{and} \quad \alpha \ll 1$$

Independent multiple radiative corrections

To logarithmic accuracy, only disconnected graphs contribute



The survival probability reads

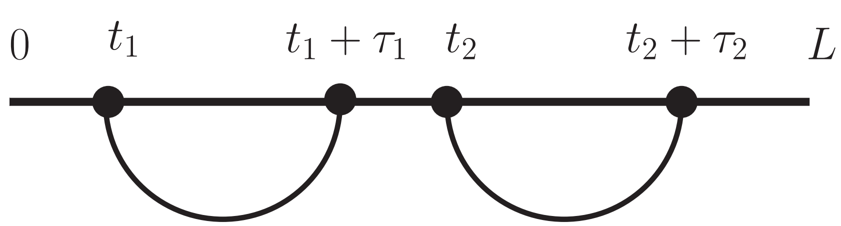
$$S(L) \to \sum_n \frac{1}{n!} \Bigg|$$  $+$  $$\Bigg|^{n}$$

$$\sigma_0 \to \sigma(L) = \sigma_0 \left(1 + \alpha \ln \frac{L}{\tau_0}\right)$$

Therefore, independent scattering approximation is still valid even if radiative corrections are non-local

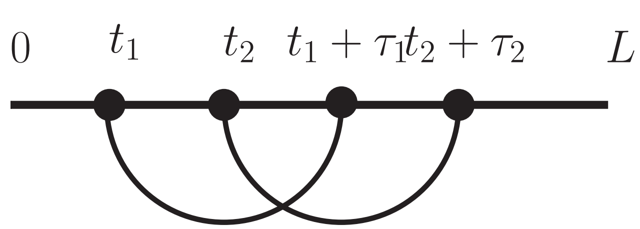
Independent multiple radiative corrections

the factorized contribution (disconnected graph)



$$\simeq \frac{1}{2!}\left(\sigma_0\,\alpha\,L\ln\frac{L}{\tau_0}\right)^2$$

connected graph



$$\simeq \frac{\pi^2}{12}\left(\sigma_0\,\alpha\,L\right)^2$$ (suppressed)