Transport coefficients for bulk viscous evolution in the relaxation-time approximation

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in collaboration with
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Hot Quarks 2014
Las Negras, Andalucia, Spain
September 24, 2014

Motivation

Applicability of relativistic viscous hydrodynamics

Kovtun, Son, and Starinets, Phys.Rev.Lett. 94, 111601 (2005)

- relativistic hydrodynamics plays an important role in the SM of HIC
- strongly-coupled $\mathcal{N} = 4$ SYM theory impose lower bound of the $\eta/\mathcal{S}$
  $\Rightarrow$ dissipative corrections important
- relativistic viscous hydrodynamics describes experimental data very well
- recently the properties of the QGP are studied more precisely (i.e. shear viscosity, ...)

- thermodynamic gradients ⇒ transport phenomena ⇒ transport coefficients
- form of kinetic coefficients entering relaxation-type equations of motion for the shear-stress tensor $\pi^{\mu\nu}$ and bulk viscous pressure Π must be derived within a certain framework
- various methods available: Israel-Stewart, Grad's 14-moment approximation, Chapman-Enskog method
  ⇒ large uncertainties concerning second-order transport coefficients

- research devoted mainly to the extraction of the $\eta/\mathcal{S}$, a systematic and self-consistent study of the effect of bulk viscosity has not been performed so far
- at large T the coupling is weak, theory is nearly conformal, the bulk viscosity is expected to be small
  $\Rightarrow$ however QCD is non-conformal theory

- canonical treatment based on an expansion of the general distribution function around local equilibrium state (corrections give rise to dissipative currents)

$$f(x,p) = \underbrace{f_{\rm iso}\left(\frac{p^\mu u_\mu}{T(x)}\right)}_{\rm LO} + \underbrace{\delta f(x,p)}_{\rm NLO}$$

  $\Rightarrow$ early thermalization required

- large anisotropy at early times predicted by microscopic models (CGC, AdS/CFT, ...)
- studied systems are subject to rapid longitudinal expansion

  $\Rightarrow$ large viscous corrections to the ideal energy-momentum tensor
    $\Rightarrow$ canonical expansion breaks down
      $\Rightarrow$ may cause unphysical results

- goal:
  assess efficacy of various dissipative hydrodynamic approaches by comparing their predictions with exact solutions of the underlying kinetic theory equations
- it is possible using relaxation time approximation for collisional kernel and simple boost-invariant transversely homogeneous symmetry of the system

- Boltzmann equation in the relaxation time approximation

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)] \qquad C[f] = p^\mu u_\mu \, \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

background distribution (Boltzmann statistics)

$$f^{\rm eq} = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{p^\mu u_\mu}{T}\right)$$

- for transversely homogeneous boost-invariant system

$$w = t p_\parallel - zE \qquad v = tE - z p_\parallel \qquad (Bialas, Czyz)$$

$$\frac{\partial f}{\partial \tau} = \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

$$f^{\rm eq}(\tau, w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{\sqrt{w^2 + (m^2 + p_\perp^2)\tau^2}}{T\tau}\right)$$

- formal solution

$$f(\tau, w, p_\perp) = D(\tau,\tau_0) f_0(w,p_\perp) + \int\limits_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} D(\tau,\tau')\, f^{\rm eq}(\tau', w, p_\perp)$$

$$D(\tau_2,\tau_1) = \exp\left[-\int\limits_{\tau_1}^{\tau_2} \frac{d\tau''}{\tau_{\rm eq}(\tau'')}\right]$$

- initial condition (Romatschke-Strickland form)

$$f_0(w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left[-\frac{\sqrt{(1+\xi_0)w^2 + (m^2+p_\perp^2)\tau_0^2}}{\Lambda_0\, \tau_0}\right]$$

$\xi_0 = \xi(\tau_0)$ - initial value of the anisotropy parameter
$\Lambda_0 = \Lambda(\tau_0)$ - initial transverse-momentum scale

- particle density, energy density, transverse and longitudinal pressure

$$
\begin{aligned}
n(\tau) &= g_0 \int dP \, \frac{v}{\tau} \, f(\tau, w, p_\perp) \\
\mathcal{E}(\tau) &= g_0 \int dP \, \frac{v^2}{\tau^2} \, f(\tau, w, p_\perp) \\
\mathcal{P}_T(\tau) &= g_0 \int dP \, \frac{p_T^2}{2} \, f(\tau, w, p_\perp) \\
\mathcal{P}_L(\tau) &= g_0 \int dP \, \frac{w^2}{\tau^2} \, f(\tau, w, p_\perp)
\end{aligned}
$$

- determination of effective temperature (Landau matching)

$$
\begin{aligned}
u_\mu T^{\mu\nu} &= u_\mu T^{\mu\nu}_{\rm eq} \\
\mathcal{E}(\tau) &= \mathcal{E}^{\rm eq}(\tau) \\
&= g_0 \int dP \, \frac{v^2}{\tau^2} \, f^{\rm eq}(\tau, w, p_\perp) \\
&= \frac{g_0 T m^2}{\pi^2} \left[3 T K_2\left(\frac{m}{T}\right) + m K_1\left(\frac{m}{T}\right) \right]
\end{aligned}
$$

$$
\begin{aligned}
T^{\mu\nu} &= (\mathcal{E} + \mathcal{P}_T) u^\mu u^\nu - \mathcal{P}_T g^{\mu\nu} + (\mathcal{P}_L - \mathcal{P}_T) z^\mu z^\nu \\
T^{\mu\nu}_{\rm eq} &= (\mathcal{E}_{\rm eq} + \mathcal{P}_{\rm eq}) u^\mu u^\nu - \mathcal{P}_{\rm eq} g^{\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
T^{\mu\nu}_{\rm LRF} &= \mathrm{diag}(\mathcal{E}, \mathcal{P}_T, \mathcal{P}_T, \mathcal{P}_L) \\
T^{\mu\nu}_{\rm eq;LRF} &= \mathrm{diag}(\mathcal{E}_{\rm eq}, \mathcal{P}_{\rm eq}, \mathcal{P}_{\rm eq}, \mathcal{P}_{\rm eq})
\end{aligned}
$$

W. Florkowski, R. Ryblewski, M. Strickland, Phys. Rev. C88, 024903 (2013)
W. Florkowski, E. Maksymiuk, R. Ryblewski, M. Strickland, Phys. Rev. C89, 054908 (2014)

$$
\begin{aligned}
Tm^2\left[3TK_2\left(\frac{m}{T}\right)+mK_1\left(\frac{m}{T}\right)\right] = \frac{g_s}{4}\Bigg[& D(\tau,\tau_0)\Lambda_0^4\widetilde{\mathcal{H}}_2\left(\frac{\tau_0}{\tau\sqrt{1+\xi_0}},\frac{m}{\Lambda_0}\right) \\
& + \int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{\rm eq}(\tau')}D(\tau,\tau')T'^4\widetilde{\mathcal{H}}_2\left(\frac{\tau'}{\tau},\frac{m}{T'}\right)\Bigg]
\end{aligned}
$$

- numerical (iterative) method
  1) use a trial function $T' = T(\tau')$ on the RHS of the dynamic equation
  2) the LHS of the dynamic equation determines the new $T = T(\tau)$
  3) use the new $T(\tau)$ as the trial one
  4) repeat steps 1-3 until the stable $T(\tau)$ is found

G. Denicol, S. Jeon, and C. Gale, arXiv:1403.0962
A. Jaiswal, R. Ryblewski, M. Strickland, arXiv:1407:4767

energy and momentum continuity equation (zero net charge, no charge diffusion)

$$\partial_\mu T^{\mu\nu}_{\rm visc} = 0 \qquad T^{\mu\nu}_{\rm visc} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P}_{\rm eq} + \Pi) + \pi^{\mu\nu}$$

relaxation-type evolution equations for bulk viscous pressure and shear stress tensor in relaxation-time approximation

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}$$

relaxation time approximation imposes $\tau_\Pi = \tau_\pi = \tau_{\rm eq}$

- form of transport coefficients may depend on the method employed (Grad's 14-moment approximation, Chapman-Enskog-like method, Israel-Stewart ...)
- some kinetic coefficients ($\tau_{\pi\pi}$, $\lambda_{\Pi\pi}$ and $\lambda_{\pi\Pi}$) are absent in the traditional Israel-Stewart viscous hydrodynamics

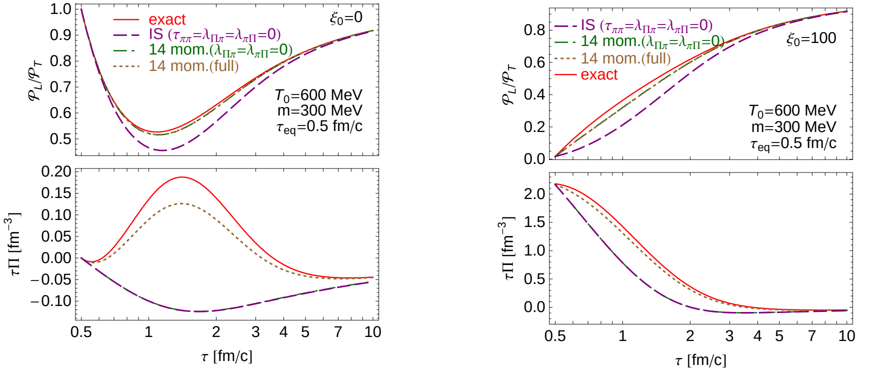
W. Israel, J. M. Stewart, Ann. Phys. (N.Y.) 118, 341 (1979)
Muronga, Phys.Rev.C 69, 034903 (2004)
Heinz, Song, Chaudhuri, Phys.Rev.C 73, 034904 (2006)
Jaiswal, Bhalerao, Pal, Phys.Rev.C 87, 021901 (2013)

Results

Importance of kinetic coefficients in the second-order viscous hydrodynamics

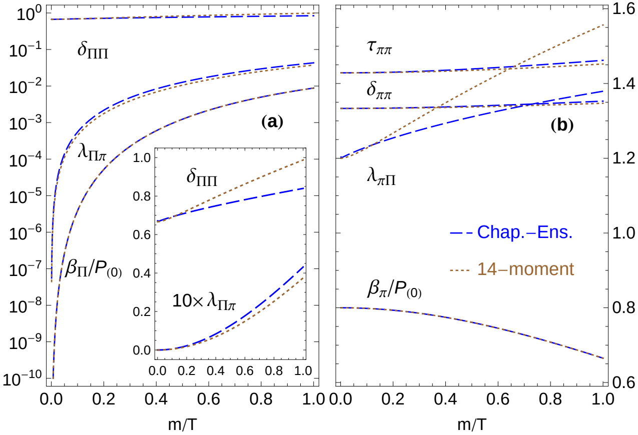
$$T^{\mu}{}_{\mu\,\mathrm{LRF}} = T^{\mu}{}_{\mu\,\mathrm{visc;LRF}}$$
$$\downarrow$$
$$\Pi = \frac{1}{3}\left[\mathcal{P}_{\parallel}(\tau) + 2\mathcal{P}_{\perp}(\tau) - 3\mathcal{P}_{\mathrm{eq}}(\tau)\right]$$

G. Denicol, H. Niemi, E. Molnar and D. H. Rischke, Phys. Rev. D 85, 114047 (2012)
G. Denicol, S. Jeon, C. Gale, arXiv:1403:0962
G. Denicol, W. Florkowski, R. Ryblewski, M. Strickland, arXiv:1407:4767



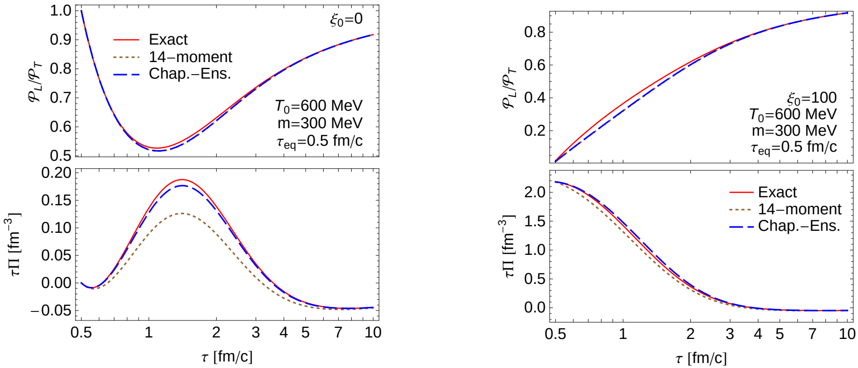
- $\tau_{\pi\pi}$ is extremely important for correct description of shear stress corrections (20% discrepancy, warning for IS users!)
- shear–bulk couplings ($\lambda_{\Pi\pi}$ and $\lambda_{\pi\Pi}$) are crucial for correct description of the bulk viscous correction

A. Jaiswal, Phys. Rev. C 87, 051901 (2013)
A. Jaiswal, R. Ryblewski, M. Strickland, arXiv:1407:4767



- Chapman-Enskog method and 14-moment approximation provide slightly different form of second-order kinetic coefficients

A. Jaiswal, R. Ryblewski, M. Strickland, arXiv:1407:4767



- kinetic coefficients obtained within Chapman-Enskog method provides even better description of bulk pressure evolution than 14-moment approximation

L. Tinti, W. Florkowski, Phys. Rev. C89 034907 (2014)
M. Nopoush, R. Ryblewski, M.Strickland, Phys. Rev. C90 (2014) 014908

- anisotropic hydrodynamics → one expands around an anisotropic background, momentum-space anisotropies are built into the LO

$$f(x,p) = \underbrace{f_{\rm iso}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\Lambda(x)}\right)}_{\rm LO} + \underbrace{\delta\tilde{f}(x,p)}_{\rm NLO}$$

spheroidal ansatz for $\Xi_{\mu\nu}$ give (LRF) $p^\mu \Xi_{\mu\nu} p^\nu = p_x^2 + p_y^2 + (1+\xi)p_z^2$ (R-S form)

anisotropy tensor decomposition

$$\Xi^{\mu\nu} u^\mu u^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu}\Phi$$

$$u_\mu \xi^{\mu\nu} = 0 \qquad u_\mu \Delta^{\mu\nu} = 0 \qquad \xi^\mu{}_\mu = 0 \qquad \Delta^\mu{}_\mu = 3$$

$$\xi^{\mu\nu} = \mathrm{diag}(0, \boldsymbol{\xi}) \qquad \boldsymbol{\xi} \equiv (\xi_x, \xi_y, \xi_z)$$

L. Tinti, W. Florkowski, Phys. Rev. C89 034907 (2014)
M. Nopoush, R. Ryblewski, M.Strickland, Phys. Rev. C90 (2014) 014908

equations of motion for ξ_z, Φ, λ, T for (0+1)d case are obtained by taking moments of the Boltzmann equation in the relaxation time approximation

$$p^\mu \partial_\mu f = p^\mu \frac{u_\mu}{\tau_{\rm eq}}(f^{\rm eq} - f) \qquad \rightarrow \qquad \partial_{\mu_1} \int dP\, p^{\mu_1} ... p^{\mu_{n+1}} f = u_{\mu_1} \int dP\, p^{\mu_1} ... p^{\mu_n} \frac{1}{\tau_{\rm eq}}(f^{\rm eq} - f)$$

0th moment (1 eq.)

$$\partial_\mu N^\mu = \frac{u_\mu}{\tau_{\rm eq}}(N^\mu_{\rm eq} - N^\mu)$$

1st moment (2 eq.)

$$u_\nu \partial_\mu T^{\mu\nu} = u_\nu \frac{u_\mu}{\tau_{\rm eq}}(T^{\mu\nu}_{\rm eq} - T^{\mu\nu})$$

$$u_\mu T^{\mu\nu}_{\rm eq} = u_\mu T^{\mu\nu}$$

2nd moment (1 eq.)

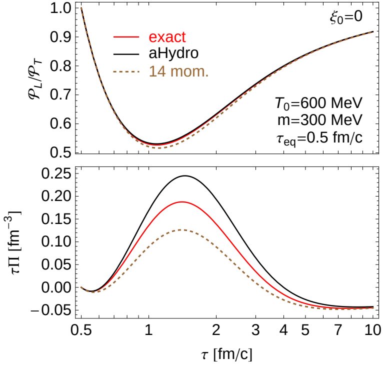
$$X^i_\mu X^i_\nu \partial_\lambda \Theta^{\lambda\mu\nu} = X^i_\mu X^i_\nu \frac{u_\lambda}{\tau_{\rm eq}}\left(\Theta^{\lambda\mu\nu}_{\rm eq} - \Theta^{\lambda\mu\nu}\right)$$

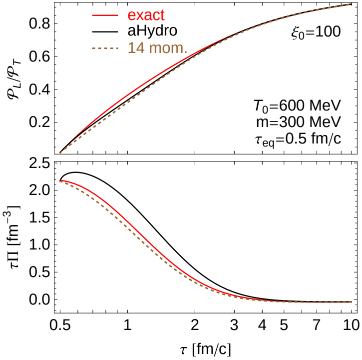
$$i = 0, 1, 2, 3$$

...

- anisotropic hydrodynamics has various appealing features (no negative pressures, reproduced free-streaming limit, kinetic coefficients included implicitly ...)

M. Nopoush, R. Ryblewski, M. Strickland, Phys. Rev. C90 (2014) 014908
G. Denicol, W. Florkowski, R. Ryblewski, M. Strickland, arXiv:1407:4767





- anisotropic hydrodynamics better captures the $\mathcal{P}_L/\mathcal{P}_T$ behavior but does not describe bulk correction as good as viscous hydrodynamics
- kinetic coefficients implicitly included

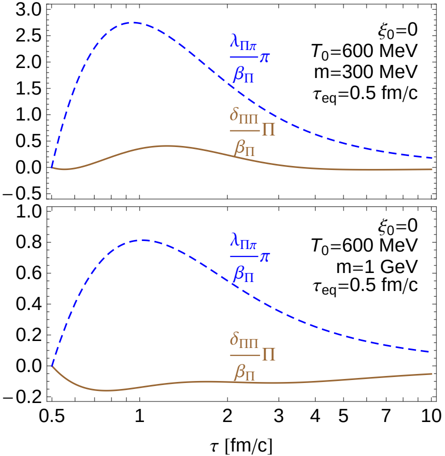
- we applied the iterative Chapman-Enskog method to derive second-order viscous hydrodynamic equations and the associated transport coefficients for a massive gas in relaxation time approximation
- we used exact solution of RTA Boltzmann kinetic equation for testing various approximation schemes
- we found that:
  - commonly used Israel-Stewart 2nd order viscous hydrodynamics equations do not describe early-time evolution of bulk viscous pressure and shear stress correctly (shear–bulk couplings and $\tau_{\pi\pi}$ are crucial!)
  - Chapman-Enskog method provides equations which give the best overall agreement with exact solutions
  - anisotropic hydrodynamics provides the best description of $\mathcal{P}_L/\mathcal{P}_T$ evolution
- NOTE:
  there are new exact solutions of the RTA Boltzmann equation now available for conformal systems employing so called Gubser symmetry

G. Denicol, U. Heinz, M. Martinez, J. Noronha, M. Strickland, arXiv:1408.5646
G. Denicol, U. Heinz, M. Martinez, J. Noronha, M. Strickland, arXiv:1408.7048

Thank you for your attention!

Backup slides

A. Jaiswal, R. Ryblewski, M. Strickland, arXiv:1407:4767



- the evolution of bulk viscous pressure is dominated by its coupling to the shear