Electric Conductivity and Heat Conductivity of the Quark-Gluon Plasma

Hot Quarks 2014

Moritz Greif, in collaboration with: Ioannis Bouras, Armando Puglisi, Christian Wesp, Zhe Xu, Florian Senzel, Gabriel Denicol and Carsten Greiner

mainly based on: arXiv:1408.7049

26.09.2014

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Overview

1. Electric Conductivity
2. Numerical methods
3. Results
4. Heat Conductivity of the QGP
5. Appendix

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Overview


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TABERNA EL 22
Ensalada del 22
Ensalada de Pasta
LASAÑA de CARNE
LASAÑA VEGETAL
MUSSAKA GRIEGA
CANELONES
ESPAGUETI BOLOÑESA
TALLARINES CARBONARA
BACALAO "a la RIOJANA"
LOMO con PIMIENTOS
PIMIENTOS del PIQUILLO rellenos de BACALAO
TORTILLA ESPAÑOLA
ESPINACAS al QUESO
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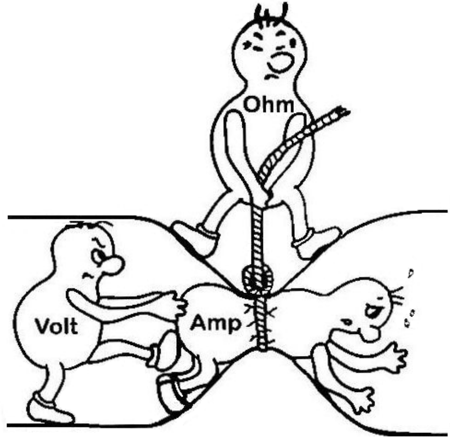
What is the Electric Conductivity σ_{el}?



$$U = R \cdot I$$

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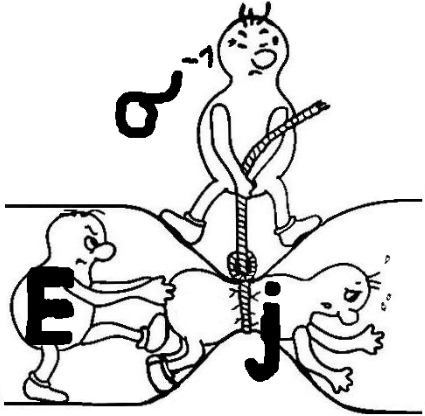

Basic definition

$$\vec{j} = \sigma_{\text{el}} \vec{E}$$

- $\vec{E}$ electric field (unit $\mathrm{GeV/fm}$)
- q electric charge (e.g. $1/3\mathrm{e}$)
- $\vec{j}$ electric current density (unit $[\mathrm{GeV/fm^2}]$, $\mathrm{e} = \sqrt{4\pi/137}$)

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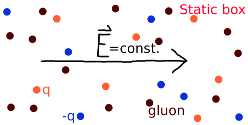
Well known "Drude"-formula for σ_{el}



$$\tau = \frac{1}{n\sigma}$$

"F. Reif/Wikipedia derivation":

- Uniform, constant, **small** electric field $\vec{E}$
- Charged particles, charge q, density n
- Time between collisions : τ
- After collision: $\langle p \rangle = 0$ **!RESISTANCE!**
- Momentum kicks between collisions (every τ seconds): $\mathrm{d}p = qE\tau$
- Average momentum: $\langle p \rangle = qE\tau$
- Electric current density : $j = nq \langle p \rangle / m = \frac{nq^2\tau}{m} E$

"Drude" Electric conductivity: $\boxed{\sigma_{\text{el}} = \frac{nq^2\tau}{m}}$

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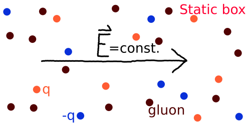

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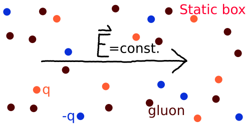

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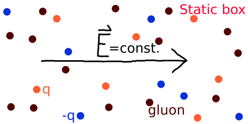

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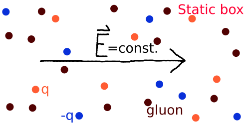

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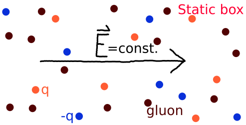

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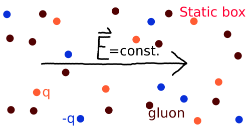

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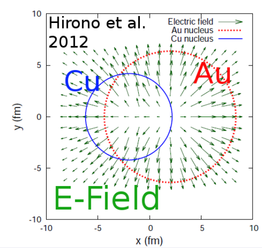
Why is electric conductivity interesting?

Hirono et al., arXiv:1211.1114

"..the charge-dependent directed flow of hadrons is sensitive to the charge dipole in the medium and is useful in estimating the electric conductivity of the QGP."



A. Rybicki and A. Szczurek, arXiv:1405.6860v1

"...the spectator-induced electromagnetic interaction on the directed flow of charged pions.[...]"
"...a baseline for studies of other phenomena, like those related to the electric conductivity of the quark-gluon plasma."

Other effects: diffusion of magnetic fields, soft dilepton production rate

Analytic expressions for the Electric Conductivity σ_{el}...

Relaxation time parametrisation: $\tau \sim (\text{Density} \cdot \text{Cross Section})^{-1}$

non-relativistic Drude

$\sigma_{\text{el}} = \frac{nq^2\tau}{m}$ (with charged particle density n, charge q, mass m)

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Ultrarelativistic cases:

Linearized collision term, $0+1$ energy expansion

$\sigma_{\text{el}} \cdot T\sigma_{\text{tot. cross section}} = \frac{2}{13}q_u^2 + \frac{4}{13}q_d^2$ (u,d,s, physical deg.)

M.G., G.Denicol, Details later in this talk...

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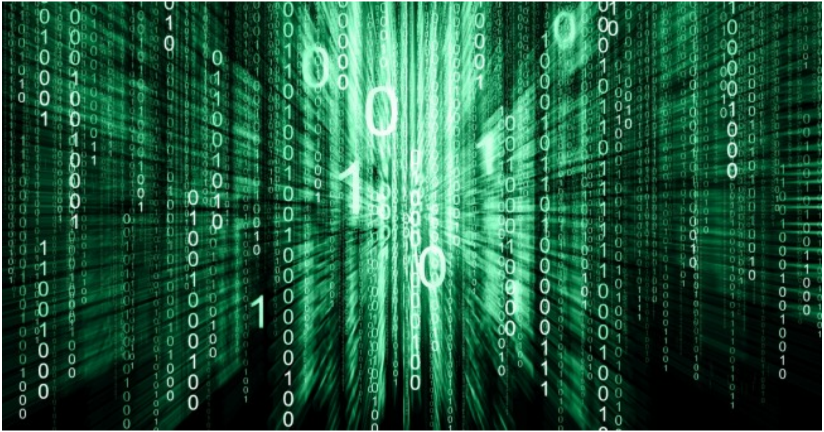
AMY, for pQCD cross sections

$$\sigma_{\text{el}} = \frac{1}{g^4}\left(\frac{(\text{sum of charges})^2}{\text{some number}}\right)T$$

P.Arnold, G.D. Moore, L.G. Yaffe. JHEP2000 + own changes

How to get the Electric Conductivity of a QGP numerically?



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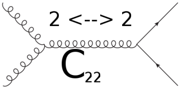
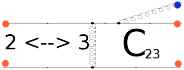
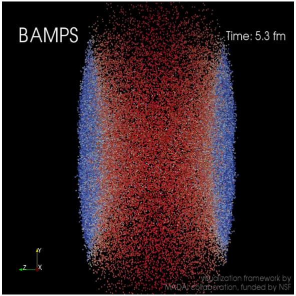
Partonic cascade BAMPS

$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{22}[f] + \mathcal{C}_{23}[f]$$







Stochastic Collision Probability,
Cross Sections σ_{22}, σ_{23}

$$P_{22} = v_{rel} \frac{\sigma_{22}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

$$P_{23} = v_{rel} \frac{\sigma_{23}}{N_{test}} \frac{\Delta t}{\Delta^3 x}, \quad P_{32} = \ldots$$

- ~ 1000 cells $\Delta^3 x$ with ~ 20 particles/cell
- ~ 30000 timesteps Δt
- massless particles, several species

Z. Xu & C. Greiner, Phys. Rev. C 71 (2005) 064901

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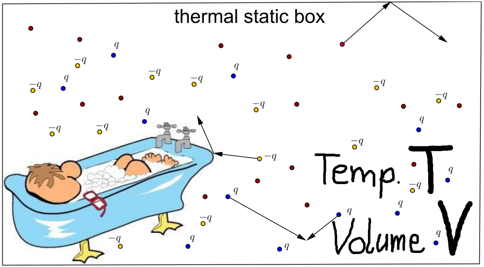
1.) Electric Conductivity via the **Green-Kubo formula**



- Thermal/chemical equilibrium
- Extract classical current-current-correlator $\langle J_x(0) J_x(t) \rangle$
- Use in Kubo-Formula

Kubo-Formula

$$\sigma_{el} = \frac{1}{TV} \int_0^\infty \mathrm{d}t \quad \underbrace{\langle J_x(0) J_x(t) \rangle}_{\text{Current-Current-Correllator}}$$

with electric current in x-direction $J_x(t)$, time t

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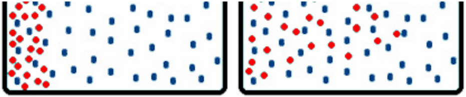
Green-Kubo formula, **What is electric current?**

$$N_k^\nu = \int \frac{d^3p}{p^0} p^\nu f_k(x,p)$$

Green-Kubo formula, **What is electric current?**

Non-Relativistically: $j = nqv$

In general: **Net-Charge Diffusion Current**:



$$j_\mu = (g_{\mu\nu} - u_\mu u_\nu) \sum_{k=1}^{\text{Species}} q_k N_k^\nu$$

with particle current density (for species k):

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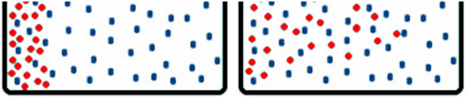

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$$N_k^\nu = \int \frac{d^3p}{p^0} p^\nu f_k(x,p)$$

- 4-velocity u^μ, p^μ 4-Momentum, $f_k(x,p)$ distribution
- Discrete version: sum up particle momenta...
- Alternative def: $j_k^\mu = q_k N_k^\mu - (n_k/n_{\text{tot}}) q_k N_{\text{tot}}^\mu$

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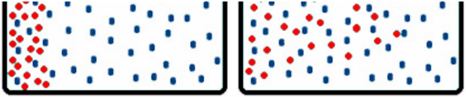

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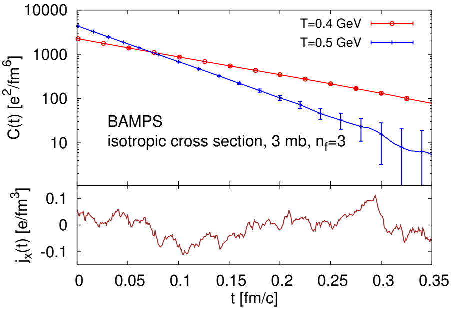
Take home message

Diffusion current of species: current with respect to flow of all species

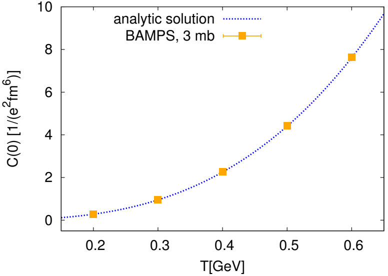
Green-Kubo formula... **How to get correlations?**

Green-Kubo relation: $\sigma_{\text{el}} \sim \int dt\, C(t)$

$J^x(0)J^x(t)$-Correlator: $C(t) = \frac{1}{s_{\text{max}}} \sum_{s=0}^{s_{\text{max}}} J^x(s)J^x(s+t)$



(a) Typical $C(t)$ @ $T = 0.4/0.5$ GeV



(b) Variance $C(0)$

Figure : Examples of Green-Kubo Analysis

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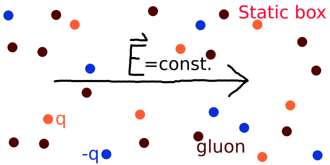
2.) Electric Conductivity via the **external force method**: *"Simple picture"*



1. Additional[1] momentum for each particle i, (charge q_i,timestep Δt), using small electric field E^x,

$$p_i^x \longrightarrow p_i^x + (\Delta t E^x q_i)$$

2. Wait until static, non-zero current has established
3. **Read off electric conductivity σ_{el}**

$$\sigma_{\text{el}} = \frac{J^x}{E^x}$$

[1] First done by Cassing et al.,arXiv:1302.0906

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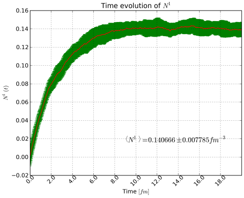

sketch

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Method 1) and 2) also good for shear viscosity and heat conductivity!

Shear viscosity η/s:

- Green-Kubo formalism: Christian Wesp et al., PRC 84, 2011
- Velocity gradient method: Felix Reining et al., PRE 85, 2012

Heat conductivity $\kappa_{\mathbf{heat}}$:

- Temperature gradient method: MG et al., PRE 87, 2013
- Green-Kubo: this work, see later

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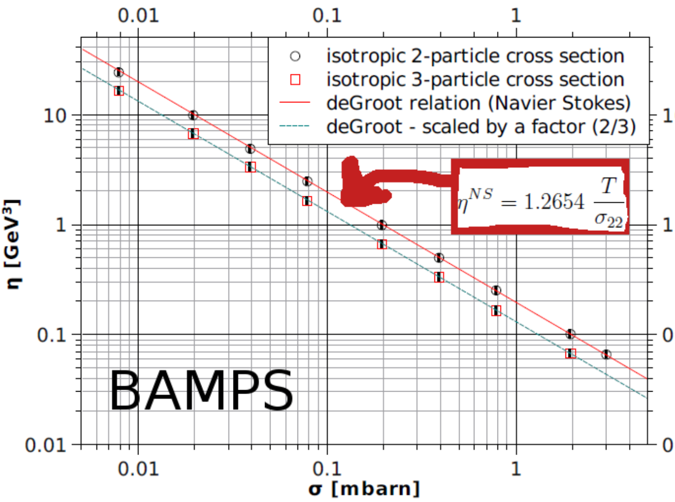
Some results to cross-check the method

First some 2011/2012 results for shear viscosity
Wesp et al, Reining et al., Plumari et al.,...

Previous results for η, **Constant isotropic cross section**



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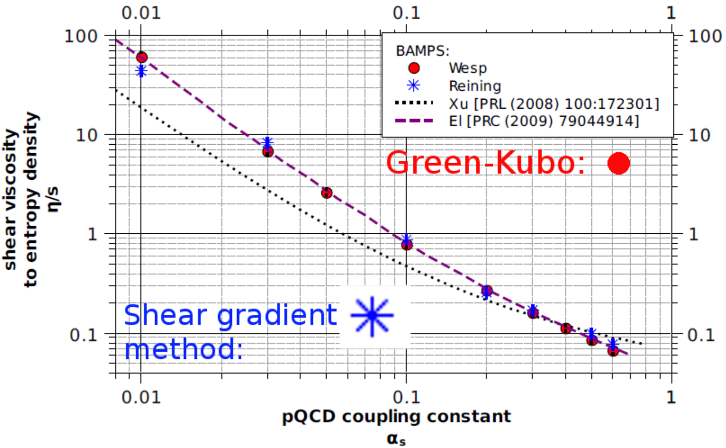
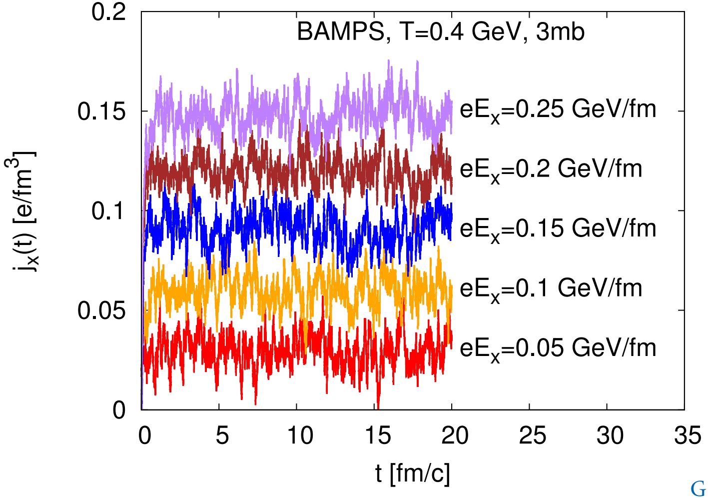


Some results to cross-check the method

Now for the electric conductivity...

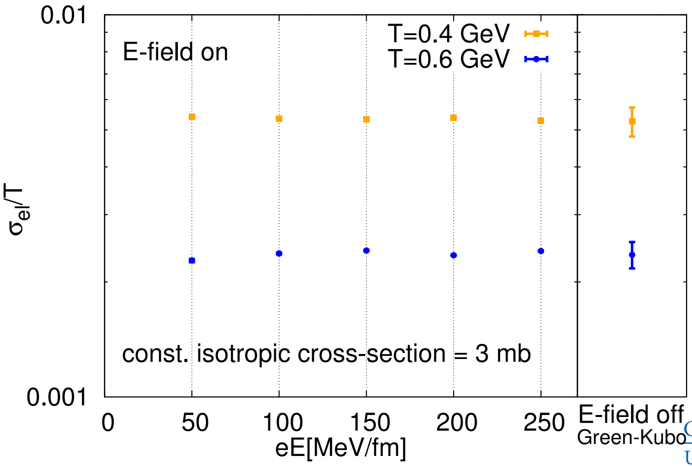
Vary the electric field...okaj...



Same result as A. Puglisi et al., Catania

Compare Green-Kubo with Efield method...okaj...

Compare BAMPS-results obtained by methods **1)** and **2)**:



About the notion of *relaxation time* τ

1) *Mean time between collisions* τ

Drude-model, non-relativistic derivation, $\sigma_{\text{el}} = \frac{nq^2\tau}{m}$

2) *Exponential relaxation towards equilibrium with slope* τ

Simplified collision term in Boltzmann equation $\frac{p^\mu u_\mu}{\tau}(f_{\text{eq}} - f)$

3) *Current-Current time-correlation, exponential slope* τ

Used for Green-Kubo analysis; current in thermal equilibrated system:
$\langle j_x j_x \rangle = A e^{-t/\tau}$

All three are the same quantity!**What is the value of τ?**

Phenomenologically

$\tau = \frac{1}{n \cdot \text{cross section}}$

Numerically

Fit $\langle j_x j_x \rangle$

Analytic

$\sigma_{\text{el}} \longrightarrow \tau_{\text{el}}$

Constant Isotropic Cross Sections

Compare BAMPS to analytics?

Relativistic, analytic calculations for σ_{el} start all with:

$$p^\mu \partial_\mu f(\vec{k}, x) + qF^{\mu\nu} k_\nu \partial/\partial k^\mu f(\vec{k}, x) = \text{Collision term} \tag{1}$$

Collision term:

- Linearized, $L[f]$, so far NOTHING ON THE MARKET
  - Motivation: $f = f_{\text{equilibrium}} + \delta f$, soon results!
- *Anderson-Witting*, $\tau^{-1}(f - f_{\text{eq}})$
  - Drude-formula:

$$\sigma_{\text{el}} = \frac{1}{3T} \left(\frac{\sum_{\text{species k}} q_k^2 n_k}{\sum_{\text{species k}} n_k} \right) \tau \tag{2}$$

  - Chapman-Enskog calculation from Cercignani and Kremer, neglects gluon-gluon scattering...

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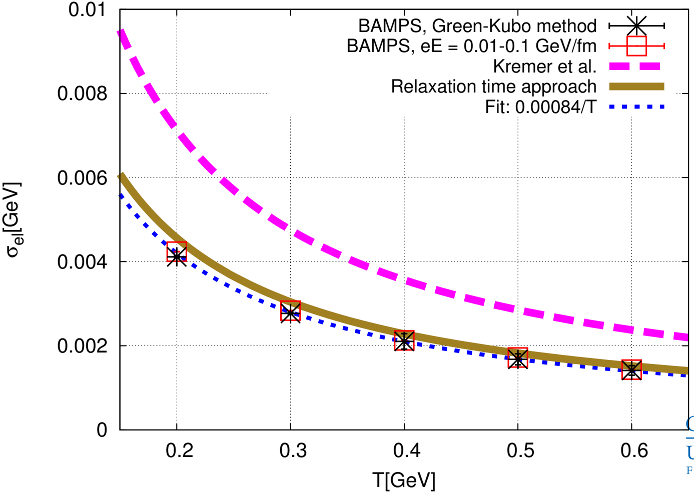
Constant Isotropic Cross Sections!

Compare BAMPS-results with analytic calculations:
Remember Armando Puglisi's talk on Wednesday!



Constant Isotropic Cross Sections,linearized 22-Collision Operator!

Idea: Solve the Boltzmann equation for $f_k(x,p)$ after small perturbation, get σ_{el} from f_k. Expect high precision!

- Boltzmann equation, **Linearized collision operator** $\rightarrow \sigma_{\text{tot}}$
- External force-term: $qF^{\mu\nu}k_\nu \partial/\partial k^\mu f(\vec{k},x)$ (E-field inside)
- Expand... $f = f_0 + \delta f$
- Current $j \sim \int \mathrm{d}P \delta f ... \times E$
- Expand δf in energy: $\sum_n a_n E^n$
- Solve for $a_n' s$

Results so far

0+1's order in Energy is 25% off BAMPS, 2nd almost done :-)

Constant Isotropic Cross Sections,linearized 22-Collision Operator!

Example: Result for order 0 in energy:

$$\sigma_{\text{el}} = \frac{1}{\sigma_{\text{tot}}} \frac{4\, n_u\, {q_u}^2 + 4\, n_d\, {q_d}^2}{(10\, n_u + 5\, n_g + 10\, n_d)\; T}$$

- n_u, n_d, n_g : densities of ups, downs, gluons
- q_u, q_d : electric charge of ups, downs
- T Temperature

Warning! next order 75% different - look for convergence!

Results for more realistic physics

- Use elastic ($2 \leftrightarrow 2$) pQCD-Cross Sections
- Use Full-pQCD-Cross Sections (also $2 \leftrightarrow 3$ processes)
- Compare with lattice and other models

$\Rightarrow \alpha_s$: check **running** coupling vs. **fixed** coupling

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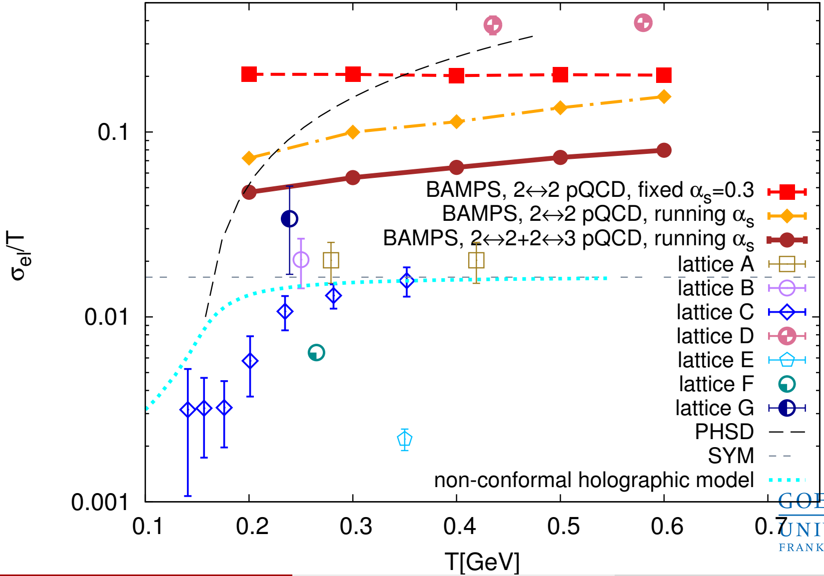
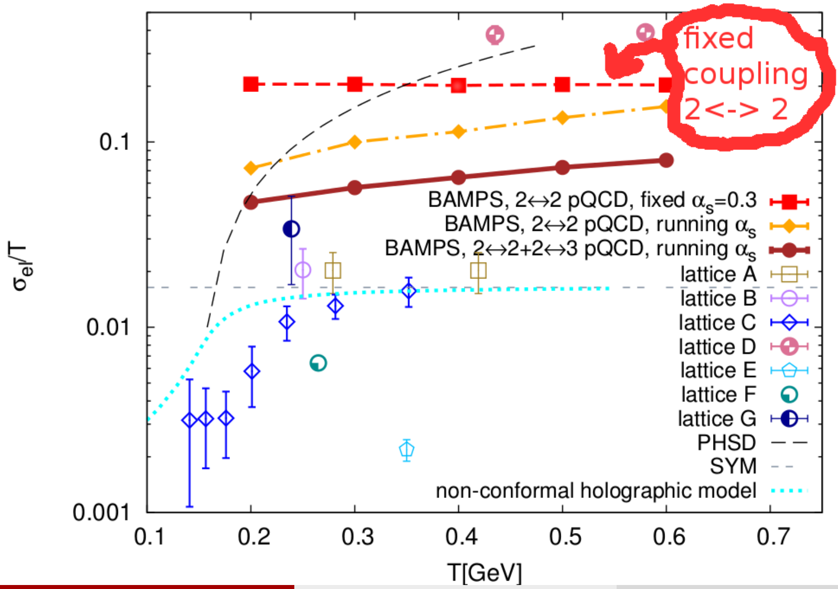
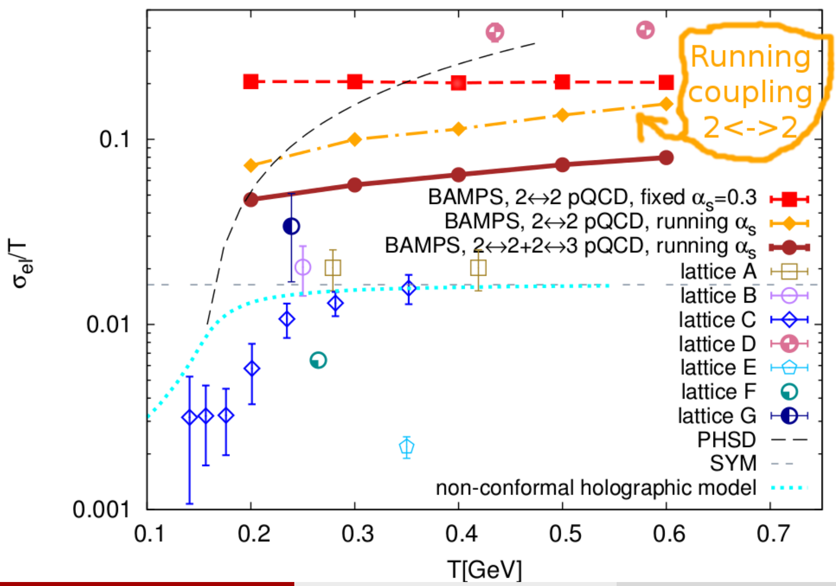
BAMPS results compared to lattice



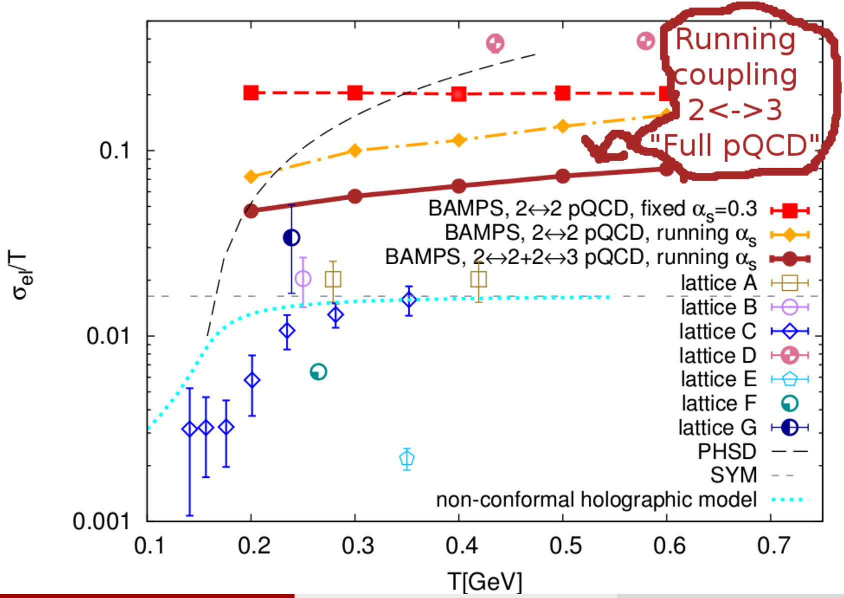
BAMPS results compared to lattice



BAMPS results compared to lattice

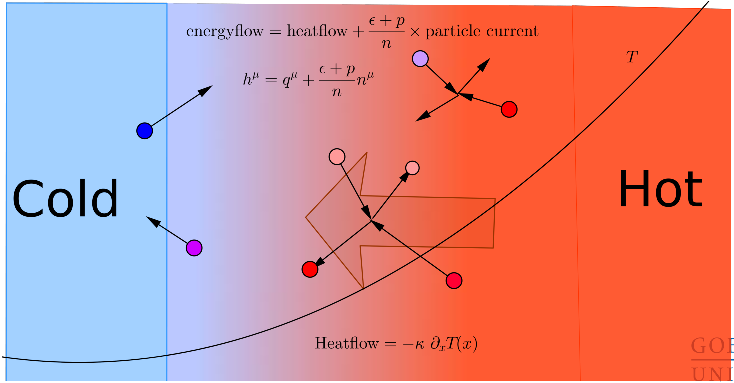


BAMPS results compared to lattice



Heat flow in relativistic Navier-Stokes theory

- Heatflow: Energy transfer through *collisions* of particles
- Non-relativistic definition: $Q = -\kappa \bigtriangledown T$
- Navier-Stokes **heat conductivity** κ



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Numerical results for elastic cross-sections

Use textbook-picture-method:

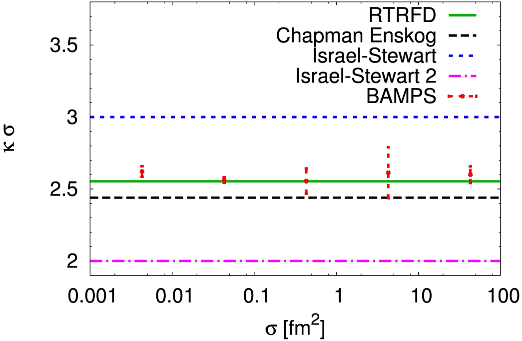

Figure : MG et al., Phys. Rev. E 87, 033019 (2013)

Check: Green-Kubo gives the same result!

Numerical results for full inelastic cross-sections

Realistic heat conductivity estimation for the QGP

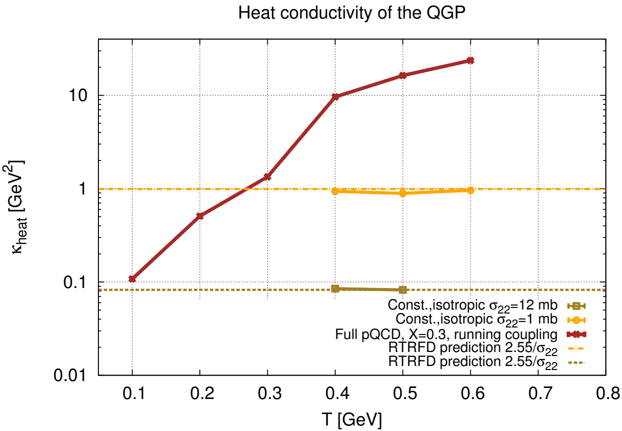

Figure : Green-Kubo method. Still PRELIMINARY. No correct errors yet.

QGP Heat Conductivity, Other Calculations!

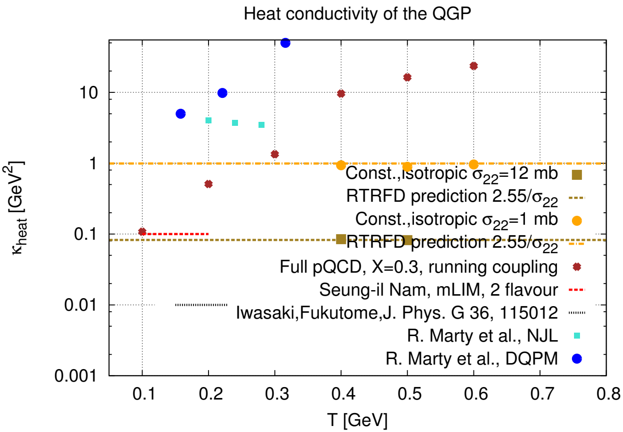

Figure : Comparisons of the heat conductivity coefficient κ

Thank you for listening!

Conclusion

- Electric conductivity using BAMPS
- Two methods confirm each other
- Analytical calculation: soon
- Relaxation time estimation ...okaj
- Full inelastic pQCD scattering rates from BAMPS
- Comparison of transport coefficients amongst different groups difficult

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Thank you for listening!

Special thanks for excellent teamwork and support goes to:

- Carsten Greiner
- Ioannis Bouras
- Armando Puglisi
- Jan Uphoff
- Christian Wesp
- Florian Senzel
- Hendrik van Hees
- and many more ...

Appendix 1) The AMY Electric Conductivity σ_{el}

Paper:

$$\sigma_{\text{el}} = \left(\frac{\text{number} \times N_{\text{leptons}}}{3\pi^2 + 32 N_{\text{species}}} \right) \frac{T}{e^2 \ln e^{-1}} \tag{3}$$

P.Arnold, G.D. Moore, L.G. Yaffe. Transport coefficients in high temperature gauge theories (I): leading-log results. Journal of High Energy Physics,Nov 2000

The AMY Electric Conductivity σ_{el}

Paper:

$$\sigma_{\text{el}} = \left(\frac{\text{number} \times N_{\text{leptons}}}{3\pi^2 + 32 N_{\text{species}}} \right) \underbrace{\frac{T}{e^2 \ln e^{-1}}}_{n\tau q^2 T^{-1}} \tag{4}$$

with $n \sim T^3$, $m \sim T$, lepton charge q and
transport relaxation time $\tau = (e^4 T \ln e^{-1})^{-1}$

Note!

- Neglects quark contribution to electric curent!
- ... Departures from $f_{\text{equilibrium,quarks}}$ small
- ... Rate of strong qq interactions higher than EM

Direct comparison to BAMPS difficult

example

For $u, d, s + e, \mu$: $\sigma_{\text{el}} = 12.3 \frac{T}{e^2 \ln e^{-1}} \approx 1.3T$ with $e^2 = 4\pi/137$

Appendix 2) Anderson-Witting Model

The Anderson-Witting model is a model for the collision term,

$$p^\mu \partial_\mu f_q + qF^{\alpha\beta} p_\beta \frac{\partial f_q}{\partial p^\alpha} = -\frac{p^\mu u_\mu}{\tau} \left(f_q - f_{\mathrm{eq},q}\right). \tag{5}$$

It allows for a relatively easy calculation of the quark distribution f_q after applying an external electric field. The gluon distribution remains thermal due to the above arguments $f_g = f_{\mathrm{eq},g}$. The result is

$$\sigma_{\mathrm{el}} = \frac{\tau_{qg} q^2 n x_g x_q}{4T} = \frac{\tau_{qg} q^2 n_g x_q}{4T} = \frac{q^2 x_q}{4\sigma_{22} T}. \tag{6}$$

Kremer et al. start as well from (5) and obtain a similar result,

$$\sigma_{\mathrm{el}} = \frac{q^2 \tau_{qg} n_q}{12 n T} \left(3n_e + 4n_g\right) = \frac{q^2 x_q}{4\sigma_{22} T} \left(\frac{n_q}{n_g} + \frac{4}{3}\right). \tag{7}$$

This expression was calculated taking partial heat fluxes and the cross-effects between heat and electric conductivity into account, which were neglected in the first derivation.

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Appendix 3) Principle of Lattice-QCD calculations of σ_{el}

Lattice observable:

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \left\langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0}) \right\rangle$$ **Euklidean correllator**

Vector spectral function:

$$G_{\mu\nu}(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\vec{p},T) \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Kubo-formula:

$$\frac{\sigma_{\text{el}}}{T} = \left(\sum_{a=1}^{N_f} q_a^2 \right) \frac{1}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

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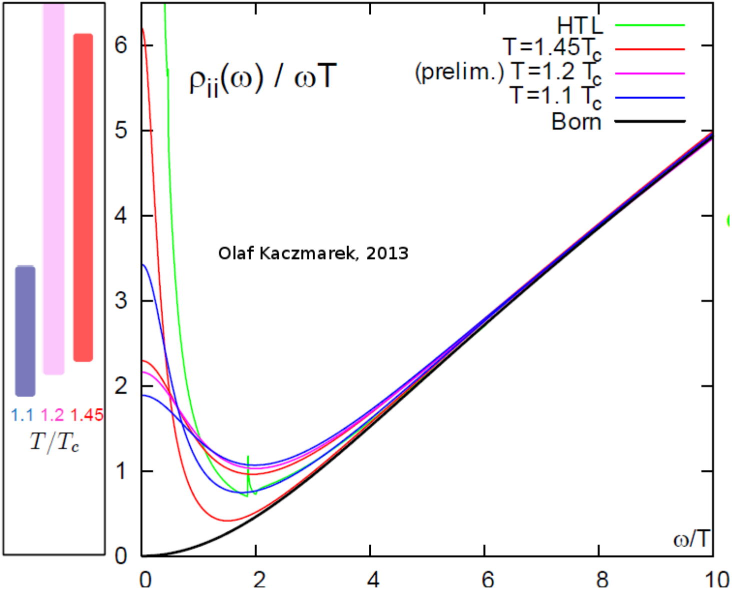
Spectral function from Lattice



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Different approaches in the lattice framework

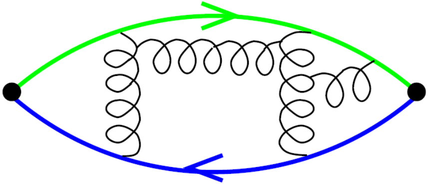
Quenched approximation

Reason: Only computer power

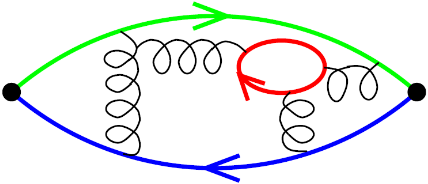
Physics: Turn off vaccuum polarization effects of quark loops

Speak: $N_f = 0$, no sea quarks, no dynamical quarks

Formula: $S = S_{\text{gauge}} + S_{\text{quarks}} = \int d^4x \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \underbrace{\sum_{\text{flavors}} \log(\det M_i)}_{\equiv 0}$



(A) Quenched QCD: quark loops neglected



(B) Full QCD

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Appendix 4) The Analytic Variance: $C(0)$ is known!

- Invented and applied by Christian Wesp for $\langle T^{xy}T^{xy}\rangle$.
- Check of the numerical value of $C(0)$

$$\mathcal{V}(N^1) = \mathcal{V}\left(\sum_i^N \frac{p_i^1}{p_i^0}\frac{1}{V}\right) = \sum_i^N \mathcal{V}\left(\frac{1}{V}\frac{p_i^1}{p_i^0}\right) = \frac{n}{V}\frac{1}{3},$$

in equilibrium: $n = \frac{d_{\text{species}}}{\pi^2}T^3$

- Local in time: Interactions irrelevant!

High-precision integration

$$C(t) = C(0)e^{-t/\tau} = q^2\frac{n}{3V}e^{-t/\tau}$$

Errors only in τ!

Appendix 5) Electric conductivity of the QGP: Applications

1. Diffusion of magnetic fields...

governed by $\Delta\vec{B} = \sigma_{\text{el}}\partial_t\vec{B}$
B-fields with $L \sim \sqrt{\frac{t}{4\pi\sigma_{\text{el}}}}$ are damped in the universe

Baym,Heiselberg, Phys.Rev. D56 (1997) 5254-5259
Tuchin, arXiv:1301.0099

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Electric conductivity of the QGP: Applications

2. Thermal emission rate of $\gamma' s$ (and dileptons)

$$E\frac{dR}{d^3p} = \frac{-2}{(2\pi)^3}\mathrm{Im}\Pi^{ret,\mu}_{\mu}\frac{1}{e^{E/T}-1}$$

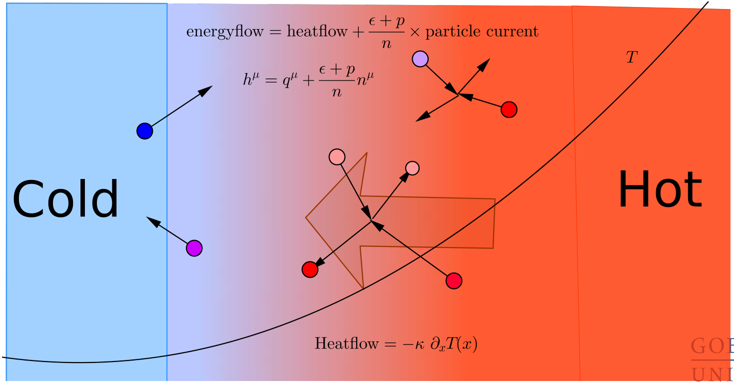
$$\frac{-2}{e^{E/T}-1}\mathrm{Im}\tilde{\Pi}^{\mathrm{ret}}(k) = \tilde{\Pi}^{<}(k)$$
$$= i\frac{1}{Z}\sum_{f,i} e^{-\beta H_i}(2\pi)^4\delta(p_i - p_f - k)\left\langle i|j^{\dagger}_{\mu}(0)|f\right\rangle \left\langle f|j_{\nu}(0)|i\right\rangle$$

Green-Kubo Formula

$$\sigma = \frac{1}{6}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x e^{i\omega t}\left\langle [j_i(t,\vec{x}), j_i(0)]\right\rangle$$

Appendix 5) Heat flow in relativistic Navier-Stokes theory

- Heatflow: Energy transfer through *collisions* of particles
- Non-relativistic definition: $Q = -\kappa \bigtriangledown T$
- Navier-Stokes **heat conductivity** κ



Wanted: value for κ

Useful form for q^μ:

$$q^\mu = -\kappa \frac{nT^2}{\epsilon + p} \nabla^\mu \left(\frac{\mu}{T}\right) = \kappa \left(\nabla^\mu T - \frac{T}{\epsilon + p} \nabla^\mu p \right)$$

(Valid for a small Knudsen number $\lambda_{\textbf{mfp}}/L_{mac}$ and first order in deviation from equilibrium)

AND

$$q^\mu \equiv \Delta^{\mu\alpha} u^\beta T_{\alpha\beta}$$

with the dissipative energy-momentum tensor $T_{\alpha\beta}$.

$\nabla^\mu = \partial^\mu - u^\mu D$: space-like Gradient, $D = u^\mu \partial_\mu$: comoving time derivative, $\Delta^{\mu\alpha} = u^\mu u^\alpha - g^{\mu\alpha}$

Const. pressure, static $0+1$-dim. system:

$$\kappa = \frac{q^x}{\gamma^2 \partial_x T(x)}$$

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Numerical results for elastic cross-sections

This work: $\kappa\sigma_{22} = 2.59 \pm 0.07$, Denicol et al, RTRFD: $\kappa\sigma_{22} = 2.5536$
($\sigma_{22} = 0.043\ mb - 430\ mb$, elastic, ultrarelativistic Boltzmann particles)

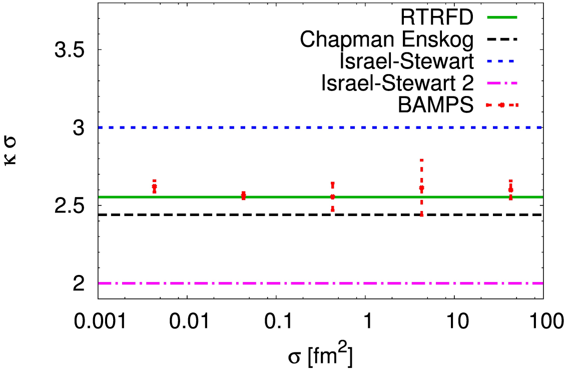
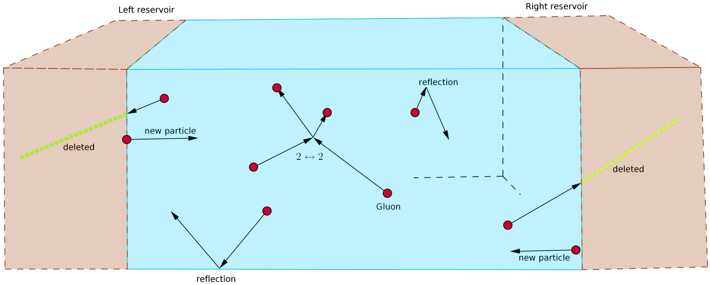


Figure : MG et al., Phys. Rev. E 87, 033019 (2013)

BAMPS - partonic cascade



- Different temperatures in reservoirs ($T_l = 0.5\ GeV,\ T_r = 0.3\ GeV$)
- Fugacity in left reservoir was (arbitrarily) set to 1
- We required $p =$ const. everywhere
- ...$\epsilon_l, \epsilon_r, n_l, n_r$ follow via $\epsilon = 3p = 3nT$

(See arXiv:hep-ph/0406278v2, arXiv:1003.4380v1, ...)

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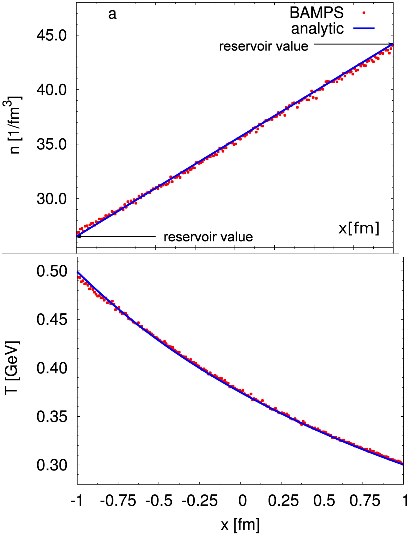
Numerical details: How to set up a Temp.-Gradient

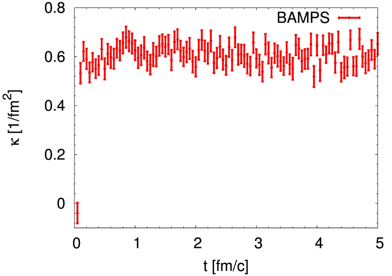


Reservoirs have different density n:

$$n(x) = ax + b$$

$\Rightarrow$ Temperature (const $= p = nT$)

$$T(x) = p/(ax + b)$$



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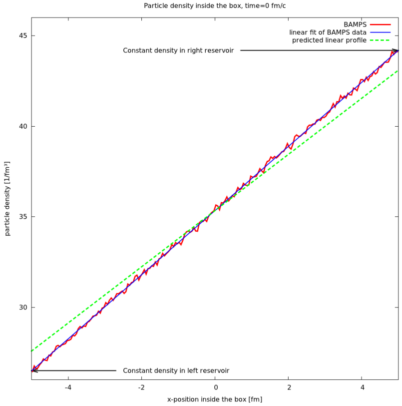
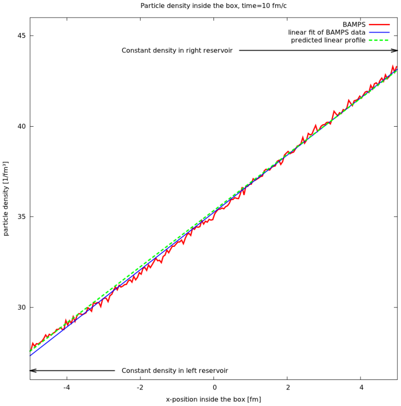
Linear density profile in ultrarelativistic cascade

$$n(x) = \frac{n_r - n_l}{L + 2\lambda_{mfp}} x + \frac{n_r - n_l}{2} + n_l, \quad \lambda_{\text{mfp}} = \frac{1}{n\sigma} \sim 0.65 \;\Rightarrow\; Kn = \frac{\lambda_{\text{mfp}}}{L} \sim 0.065 \tag{8}$$



(a) (initialised) density at $t = 0\ fm/c$



(b) density at $t = 10\ fm/c$

Figure : $\sigma = 0.43mb$, density evolution: BAMPS vs. eq.(8)

Appendix 6 Simple Relaxation-Time model

$$p^{\mu}\partial_{\mu}f_q + qF^{\alpha\beta}p_{\beta}\frac{\partial f_q}{\partial p^{\alpha}} = -\frac{p^{\mu}u_{\mu}}{\tau}\left(f_q - f_{\mathrm{eq},q}\right). \tag{9}$$

Assume

- assumes an exponential relaxation towards f_{eq}
- local rest frame of the fluid $u = (1, \vec{0})$
- Boltzmann-distribution of species a: $f_{\mathrm{eq,a}} = d_a e^{-\beta p^0}$
- No spatial gradients at all
- Relaxation time: $\tau \;\to\; \tau_{qg} = \frac{1}{n_g \sigma_{22}}$
- Expansion $f(x, \vec{p}, t) = f_{\mathrm{eq}} + f_{\mathrm{eq}}\phi$

Relaxation-Time model A

Field-Strength tensor

$$F^{\mu\nu} = u^{\nu} E^{\mu} - u^{\mu} E^{\nu} - B^{\mu\nu} \tag{10}$$

Assume

- $B^{\mu\nu} = 0$

Then directly

$$\Rightarrow \quad \phi = \tau \beta q \vec{E} \cdot \frac{\vec{p}}{p^0} \tag{11}$$

And the current:

$$j^x = q \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{p^x}{p^0} f_{\mathsf{eq}} \phi = d_q \tau \frac{8}{3} \frac{\pi q^2}{(2\pi)^3 \beta^2} E^x$$

Electric conductivity, model A

$$\sigma_{el} = d_q \frac{q^2}{3\pi^2} \frac{T^2}{n_g \sigma_{qg}} = \frac{1}{3} \frac{d_q}{d_g} \frac{q^2}{\sigma_{qg} T} \tag{12}$$

Relaxation-Time model B

$$p^\mu \partial_\mu f_q + qF^{\alpha\beta} p_\beta \frac{\partial f_q}{\partial p^\alpha} = -\frac{p^\mu u_\mu}{\tau} \left(f_q - f_{\mathrm{eq},q}\right). \tag{13}$$

Steps:

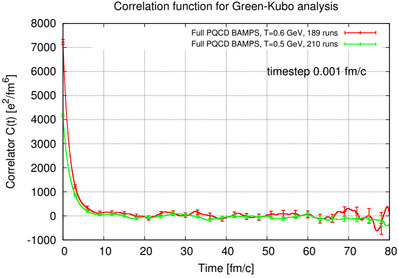
1. Calculate 2^{nd} moment to obtain $\partial_\mu T^{\mu\nu}$
2. Neglect partial heat fluxes
3. Use equilibrium-$T^{\mu\nu}$ for gradient
4. Project on spatial direction
5. Merge with $\partial_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha$

Yields

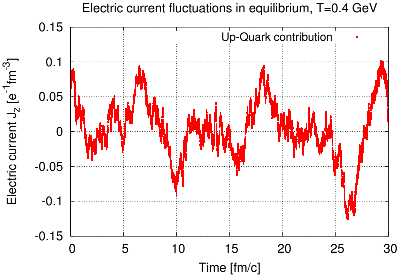
Electric conductivity, model B

$$\sigma_{el} = \tau q^2 \frac{n_g n_q}{n} \frac{1}{4T} = \frac{1}{4} \frac{d_q}{d_g + d_q} \frac{q^2}{\sigma_{qg} T} \tag{14}$$

Green-Kubo formula... **Get a feeling**



(a) Typical corr., $T = 0.5/0.6$ GeV



(b) Current fluctuation

$$\sigma_{el} = \frac{1}{TV}\int_0^\infty \mathrm{d}t \quad \underbrace{\langle J_x(0) J_x(t)\rangle}_{\text{Current-Current-Correllator}}$$

with electric current in x-direction $J_x(t)$, time t

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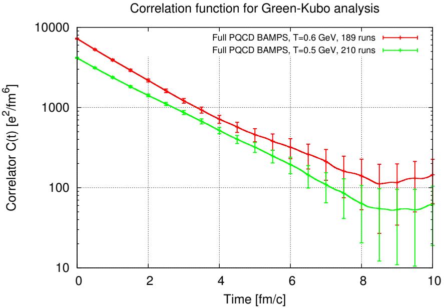
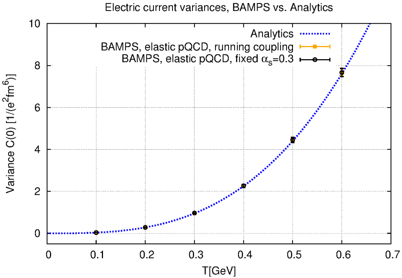
Green-Kubo formula… **How to get correlations?**

Green-Kubo relation: $\sigma_{\text{el}} \sim \int dt\, C(t)$

$J^x(0)J^x(t)$-Correlator: $C(t) = \frac{1}{s_{\text{max}}} \sum_{s=0}^{s_{\text{max}}} J^x(s)J^x(s+t)$



(c) Typical $C(t)$ @ $T = 0.5/0.6$ GeV



(d) Variance $C(0)$

Figure : Examples of Green-Kubo Analysis