

Beam energy scan using a viscous hydro+cascade model

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for Advanced Studies



“Stages of Heavy Ion Collision”



- 1 Initial(pre-thermal) stage
 - ▶ Thermalization
- 2 Hydrodynamic expansion
 - ▶ Quark-gluon plasma phase
 - ▶ Phase transition
 - ▶ Hadron Gas phase
 - ▶ Chemical freeze-out
 - ▶ End of hydrodynamic regime



- 3 Hadronic rescatterings, resonance decays
Kinetic freeze-out
⇓
Free streaming, then hadrons are detected

Ingredients of hydro+cascade model:

- 1 Initial stage model
Enforced thermalization
- 2 Hydrodynamic solution
 - ▶ Equation of state for hydrodynamics
 - ▶ transport coefficients
- 3 Particlization and switching to a cascade



scripts to analyze generated events

Some lessons from Au-Au RHIC/ Pb-Pb LHC

- Hydrodynamic/hybrid approach works very well
- $\eta/s \approx 1/(4\pi)$ at top RHIC with Monte Carlo Glauber IC,
- and somewhat larger value for 2.76 TeV PbPb LHC
- However, there are large uncertainties in η/s extraction from the initial state model, quantities to fit, model parameters.

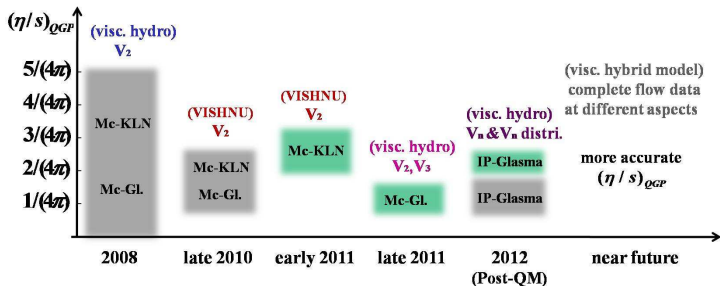
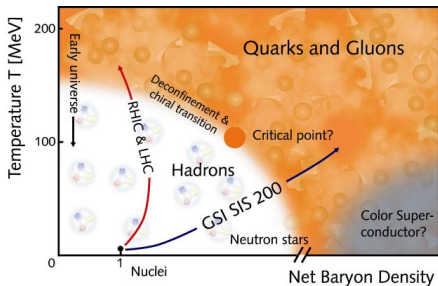


Figure: extracted η/s for top RHIC (grey bands), 2.76 TeV LHC (green bands).

Taken from Huichao Song, QM2012 proceedings arXiv:1210.5578

Motivation: apply a hybrid for RHIC BES, FAIR/NICA

to understand whether fluid is created at lower energies, and find its properties: transport coefficients ($\eta/s, \dots$) and EoS.

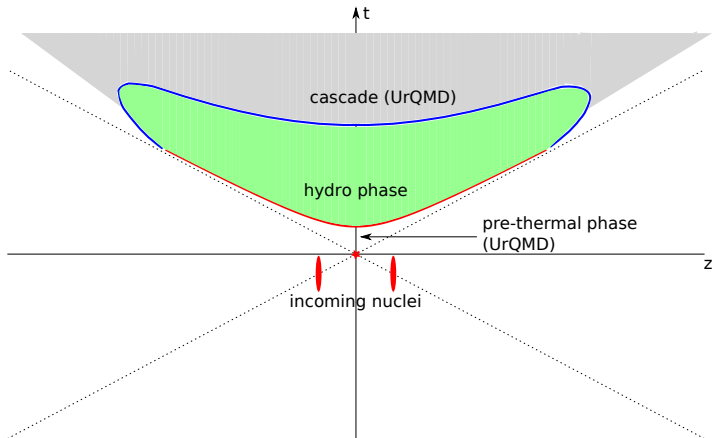


For Beam energy scan, we need a more elaborate model:

- 1 3D (non-boost-invariant) fluctuating initial state
- 2 Baryon and electric charge densities
 - ▶ obtained from an initial state model
 - ▶ propagated in hydro phase and included in EoS
 - ▶ taken into account in particlization procedure

The model

Initial conditions for hydro phase



Pre-thermal phase: UrQMD

Hydro starts at $\tau = \sqrt{t^2 - z^2} = \tau_0$ (red curve):

$\{T^{0\mu}, N_b^0, N_q^0\}$ of fluid = averaged $\{T^{0\mu}, N_b^0, N_q^0\}$ of particles

$$\tau_0 = \frac{2R}{\gamma v_z}$$

Hydrodynamic phase

The hydrodynamic equations: local energy-momentum and charge conservation

$$\partial_{;v} T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma_{\nu\lambda}^\mu T^{\nu\lambda} + \Gamma_{\nu\lambda}^\nu T^{\mu\lambda} = 0, \quad \partial_{;v} N^v = 0 \quad (1)$$

where (we choose Landau definition of velocity)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (p + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu} \quad (2)$$

Evolutionary equations for shear/bulk, coming from **Israel-Stewart** formalism:

$$\langle u^\gamma \partial_{; \gamma} \pi^{\mu\nu} \rangle = -\frac{\pi^{\mu\nu} - \pi_{\text{NS}}^{\mu\nu}}{\tau_\pi} - \frac{4}{3} \pi^{\mu\nu} \partial_{; \gamma} u^\gamma \quad (3a)$$

where $\langle A^{\mu\nu} \rangle = (\frac{1}{2} \Delta_\alpha^\mu \Delta_\beta^\nu + \frac{1}{2} \Delta_\alpha^\nu \Delta_\beta^\mu - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta}) A^{\alpha\beta}$

* Bulk viscosity $\zeta = 0$, charge diffusion=0

vHLL code, see [arXiv:1312.4160](https://arxiv.org/abs/1312.4160) for the details of the code and its testing.

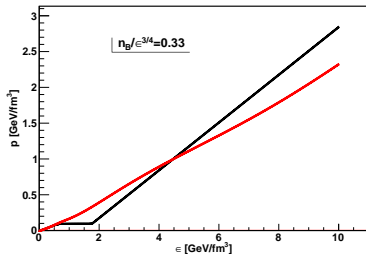
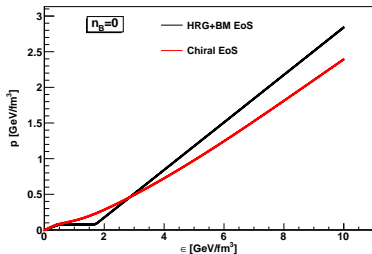
Equations of state for hydrodynamic phase

- Chiral model

- ▶ coupled to Polyakov loop to include the deconfinement phase transition
- ▶ good agreement with lattice QCD data at $\mu_B = 0$, also applicable at finite baryon densities
- ▶ (current version) has **crossover type PT** between hadron and quark-gluon phase at all μ_B

- Hadron resonance gas + Bag Model (a.k.a. EoS Q)

- ▶ hadron resonance gas made of u, d quarks including repulsive meanfield
- ▶ the phases matched via Maxwell construction, resulting in **1st order PT**

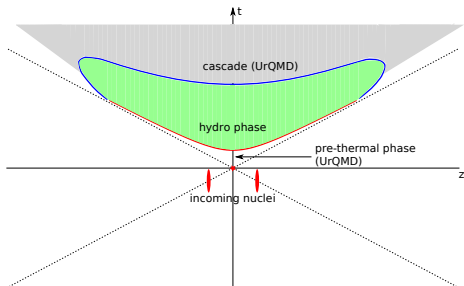


refs: J. Steinheimer, S. Schramm and H. Stoecker, J. Phys. G 38, 035001 (2011);
P.F. Kolb, J. Sollfrank, and U. Heinz, Phys.Rev. C 62, 054909 (2000).

Fluid → particle transition

$\varepsilon = \varepsilon_{SW} = 0.5 \text{ GeV/fm}^3$ (blue curve):

$\{T^{0\mu}, N_b^0, N_q^0\}$ of hadron-resonance gas = $\{T^{0\mu}, N_b^0, N_q^0\}$ of fluid



▷ Momentum distribution from Landau/Cooper-Frye prescription:

$$p^0 \frac{d^3 n_i}{d^3 p} = \int (f_{i,\text{eq.}}(x, p) + \delta f(x, p)) p^\mu d\sigma_\mu$$

▷ Cornelius subroutine* is used to compute $\Delta\sigma_i$ on transition hypersurface.

▷ UrQMD cascade is employed after particlization surface.

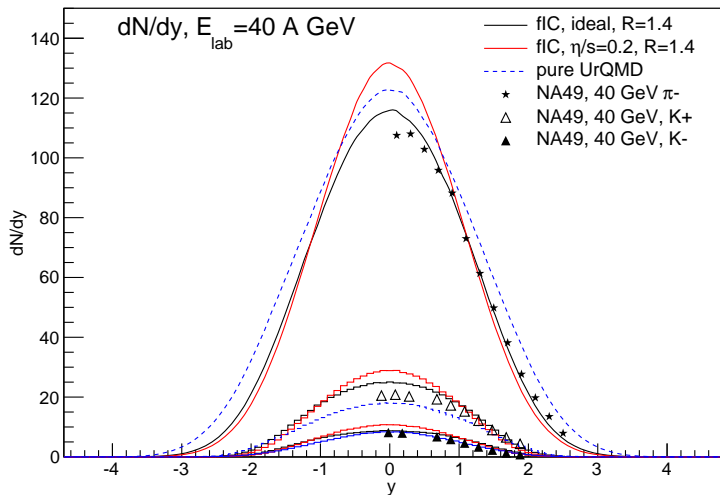
*Huovinen P and Petersen H 2012, *Eur.Phys.J. A* **48** 171

Results:

shear viscosity and EoS dependence

Results: $E_{\text{lab}} = 40$ A GeV Pb-Pb (SPS)

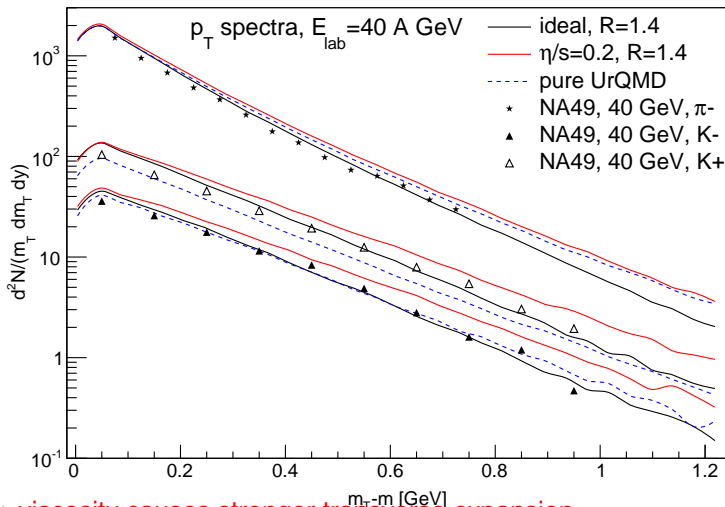
$\sqrt{s_{NN}} = 8.8$ GeV, 0-5% central collisions ($b = 0 \dots 3.4$ fm) (Chiral EoS only)



→ viscous entropy production

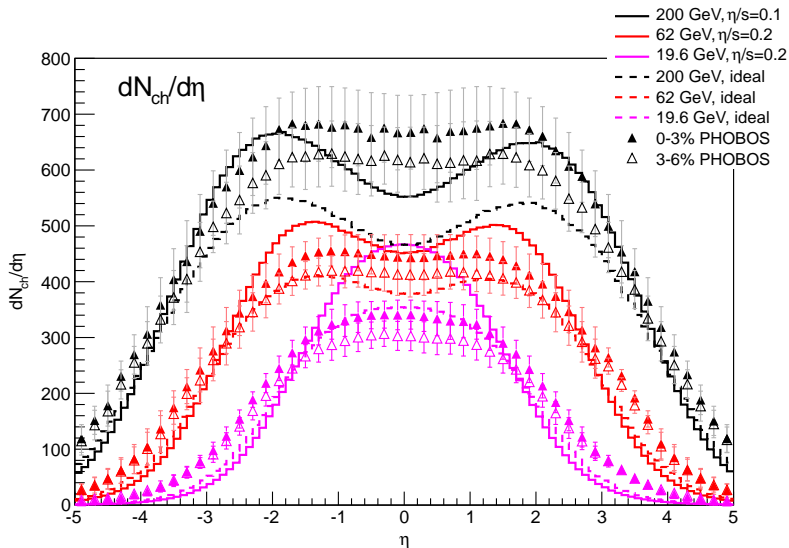
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→ viscosity causes stronger transverse expansion

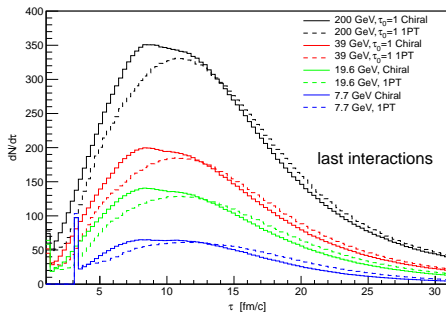
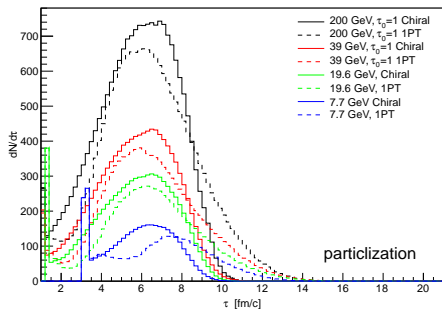
$dN/d\eta$ for RHIC ($\sqrt{s_{NN}} = 19.6, 62.4, 200$ GeV Au-Au)



Fine tuning is required for every energy individually.

Effects of the EoS Q compared to Chiral EoS?

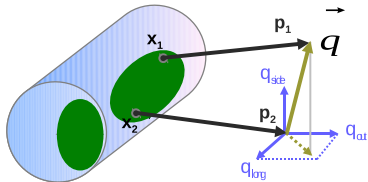
Yes: hydro phase in average lasts longer with EoS Q



Can we see it in femtoscopy (HBT), or any other observables?

HBT(interferometry) measurements

The only tool for space-time measurements at the scales of 10^{-15}m , 10^{-23}s



$$\vec{q} = \vec{p}_2 - \vec{p}_1$$

$$\vec{k} = \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$$

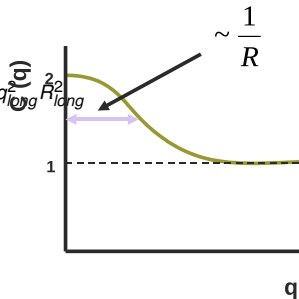
$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = \frac{\text{real event pairs}}{\text{mixed event pairs}}$$

Gaussian approximation of CFs ($q \rightarrow 0$):

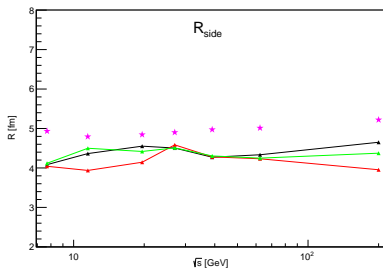
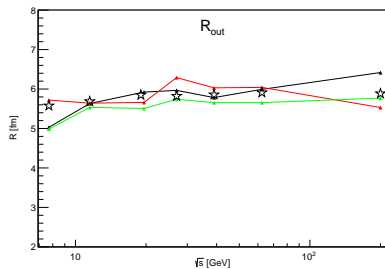
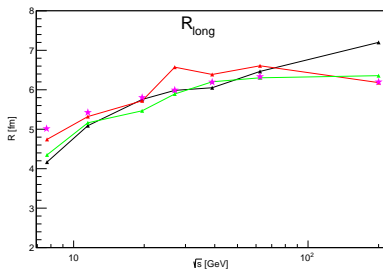
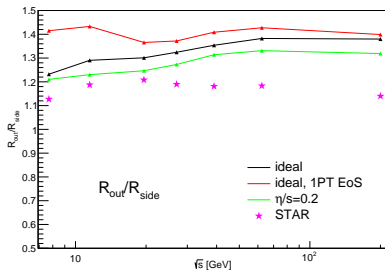
$$C(\vec{k}, \vec{q}) = 1 + \lambda(k) e^{-q_{out}^2 R_{out}^2 - q_{side}^2 R_{side}^2 - q_{long}^2 R_{long}^2}$$

$R_{out}, R_{side}, R_{long}$ (HBT radii) correspond to *homogeneity lengths*, which reflect the space-time scales of emission process

In an event generator, BE/FD two-particle amplitude (anti)symmetrization must be introduced

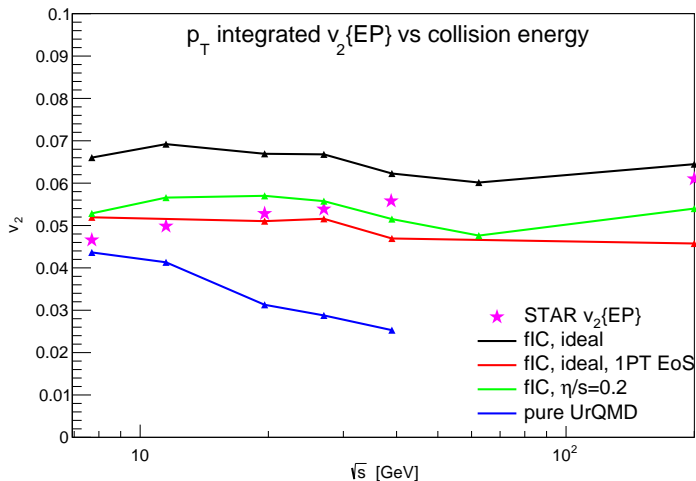


Femtoscopic radii: ideal hydro/Chiral EoS, ideal hydro/EoS Q, visc.hydro/Chiral EoS



Previous results for EoS dependence of HBT in hybrid UrQMD, see Q. Li et al., Phys.Lett.B674:111,2009

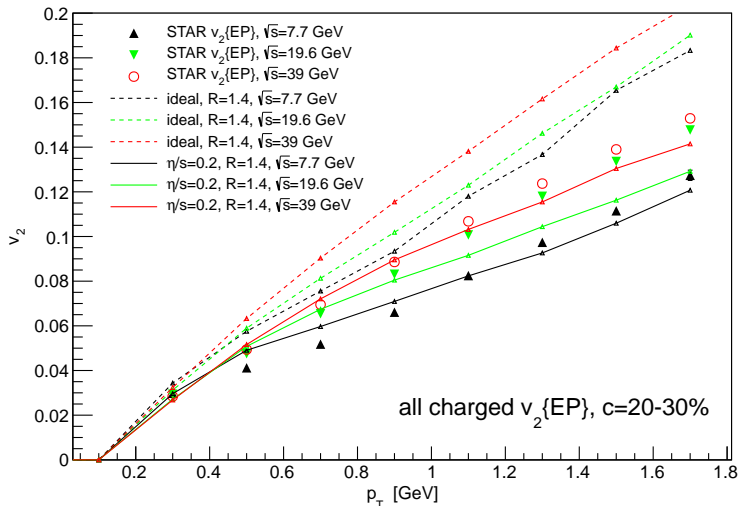
Elliptic flow, $\sqrt{s_{NN}} = 7.7 \dots 200$ GeV Au-Au



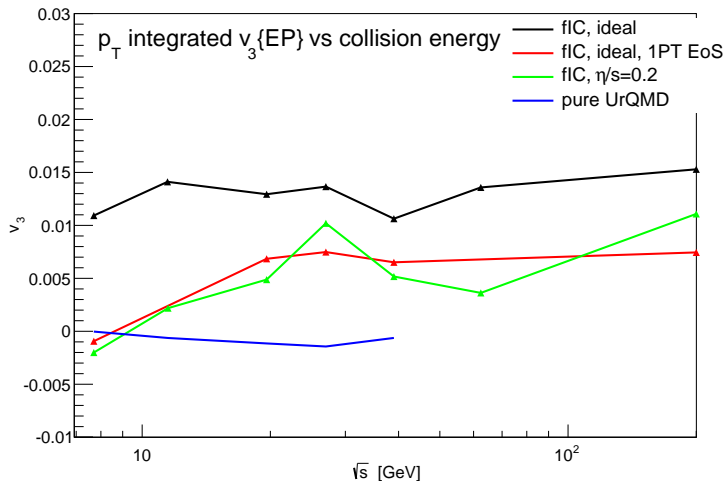
Shear viscosity suppresses the elliptic flow (as expected)
 However, EoS Q also results in lower v_2 as compared to Chiral EoS.

ρ_T differential v_2

gets systematically better as well with viscous hydro phase



Triangular flow, $\sqrt{s_{NN}} = 7.7 \dots 200$ GeV Au-Au

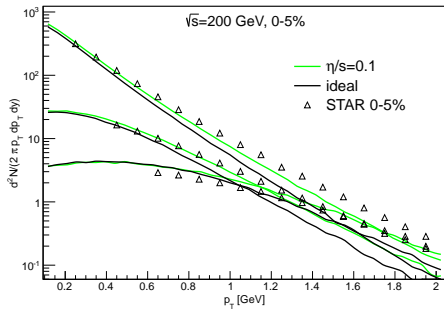
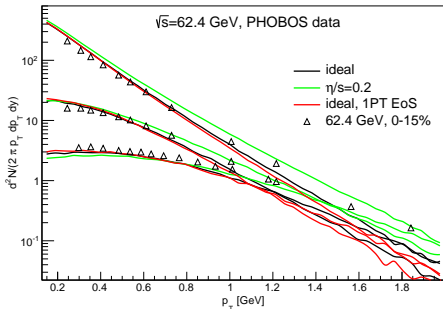
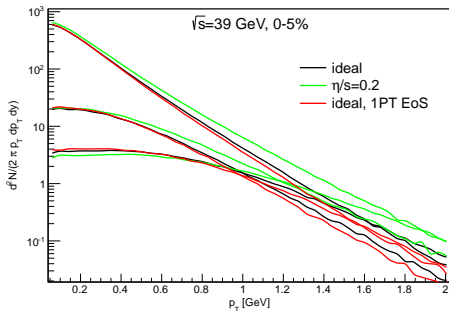


Looks like that v_3 is similarly suppressed, however the statistics (number of events) might not be enough.

p_T spectra:

Radial flow is:

- increased by shear viscosity
- less affected by EoS change

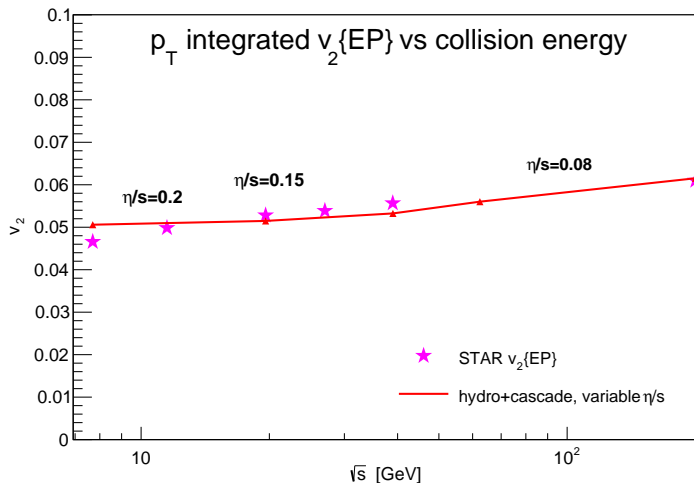


Results:

fit to BES data

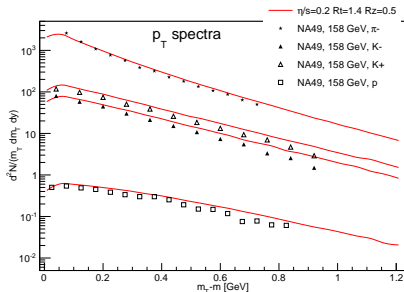
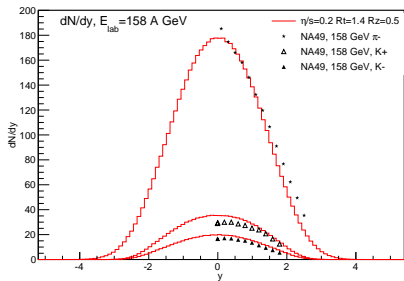
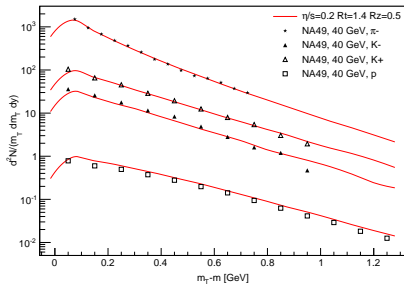
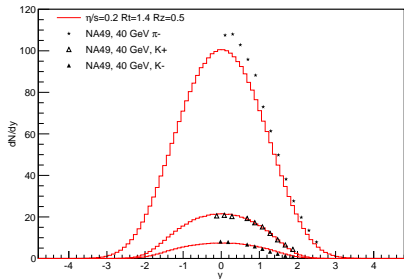
A fit to BES elliptic flow data

consistently with dn/dy and p_T at available data points

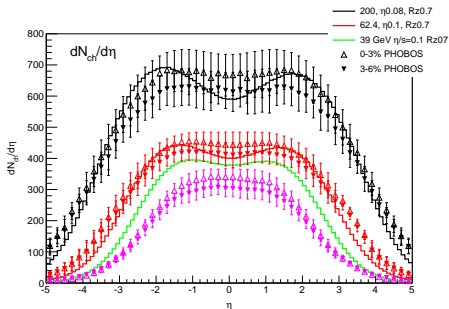
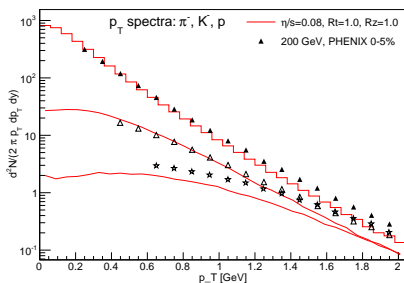
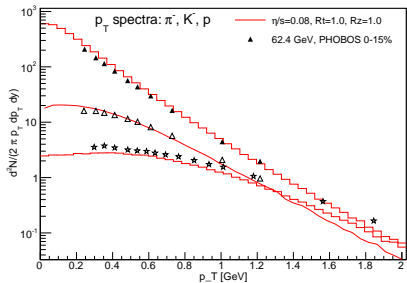


Variable $\eta/s = 0.2 \rightarrow 0.08$ when $\sqrt{s} = 7.7 \rightarrow 200$ GeV is used

The fit to 40 + 158 A GeV SPS



The fit to $\sqrt{s} = 62.4, 200 \text{ GeV}$



Summary

Viscous hydro + UrQMD model:

- pre-thermal stage: UrQMD
- 3+1D viscous hydrodynamics
- EoS at finite μ_B : Chiral model, EoS Q

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Conclusions:

- Model applied for $\sqrt{s_{NN}} = 7.7 \dots 200$ GeV A+A collisions.
- A fit to experimental data suggests $\eta/s = 0.2 \rightarrow 0.08$ when $\sqrt{s} = 7.7 \rightarrow 200$ GeV, modulo initial state (UrQMD) and EoS (Chiral model) used.
- This hints for μ_B dependent η/s or $\eta/(\varepsilon + p)$ being appropriate quantity.
- EoS Q seems to be disfavored by the data (too small v_2 + too small $dN/d\eta$, slightly worse for HBT. Harder to fix by readjusting other model parameters)
- As usual, more experimental data is needed to extract η/s and other model parameters less ambiguously.

Summary

Viscous hydro + UrQMD model:

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Outlook: Different initial state model Work in progress.

Thank you for your attention!

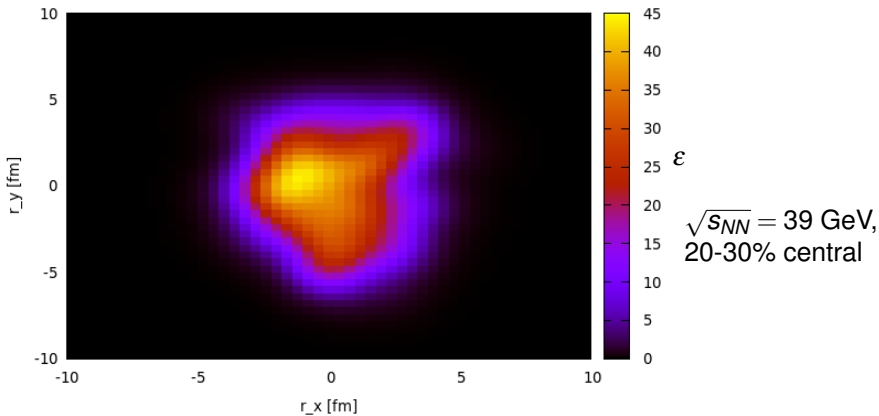
Extra slides

Initial conditions for hydro phase (2)

Fluctuating, but smoothed initial state:

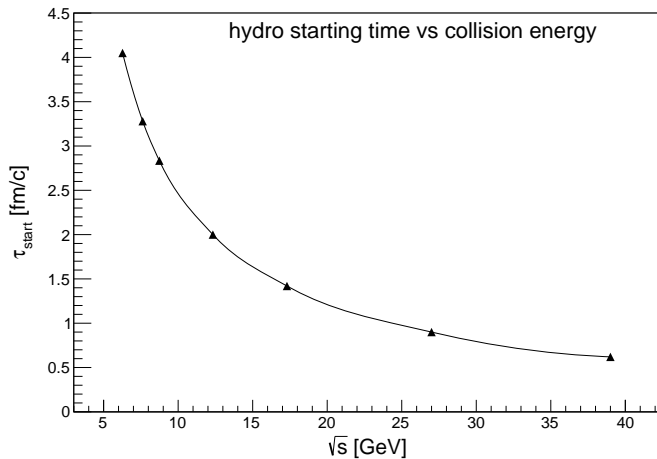
$$E \propto \exp\left(-\frac{(x-x_{part})^2+(y-y_{part})^2+\gamma_z^2(z-z_{part})^2}{R^2}\right), \text{ where } R = 1 \text{ fm}$$

see e.g. H. Petersen et al., Phys.Rev. C78 (2008) 044901



Hydro starting time

Graphical representation (plot) of the $\tau_0 = \frac{2R}{\gamma v_z}$ formula



Coordinate transformations (hydro phase)

Milne coordinates

The coordinate system is defined as follows:

$$0) \tau = \sqrt{t^2 - z^2}$$

$$1) x = x$$

$$2) y = y$$

$$3) \eta = \frac{1}{2} \ln \frac{t+z}{t-z}$$

$$g^{\mu\nu} = \text{diag}(1, -1, -1, -1/\tau^2)$$

Nonzero Christoffel symbols are:

$$\Gamma_{\tau\eta}^{\eta} = \Gamma_{\eta\tau}^{\eta} = 1/\tau, \quad \Gamma_{\eta\eta}^{\tau} = \tau$$

$$T^{\mu\nu} = (\varepsilon + p)u^{\mu}u^{\nu} - p \cdot g^{\mu\nu}, \text{ where}$$

$$u^{\mu} = \left\{ \cosh(\eta_f - \eta) \cosh \eta_T, \sinh \eta_T \cos \phi, \sinh \eta_T \sin \phi, \frac{1}{\tau} \sinh(\eta_f - \eta) \cosh \eta_T \right\}$$

$$\text{(cf. } u_{\text{Cart}}^i = \left\{ \cosh(\eta_f) \cosh \eta_T, \sinh \eta_T \cos \phi, \sinh \eta_T \sin \phi, \sinh(\eta_f) \cosh \eta_T \right\})$$

EM conservation equations are

$$\partial_{;\nu} T^{\mu\nu} = 0$$

or

$$\mu = 0: \quad \partial_{\nu} T^{\tau\nu} + \tau T^{\eta\eta} + \frac{1}{\tau} T^{\tau\tau} = 0$$

$$\mu = 1: \quad \partial_{\nu} T^{x\nu} + \frac{1}{\tau} T^{x\tau} = 0$$

$$\mu = 2: \quad \partial_{\nu} T^{y\nu} + \frac{1}{\tau} T^{y\tau} = 0$$

$$\mu = 3: \quad \partial_{\nu} T^{\eta\nu} + \frac{3}{\tau} T^{\eta\tau} = 0$$

Additional transformations:

$$\begin{aligned} T^{\mu\eta} &\rightarrow T^{\mu\eta}/\tau, \quad \mu \neq \eta, \\ T^{\eta\eta} &\rightarrow T^{\eta\eta}/\tau^2 \end{aligned}$$

$\Downarrow\Downarrow$

$$\partial_{\nu}(\tau T^{\tau\nu}) + \frac{1}{\tau}(\tau T^{\eta\eta}) = 0$$

$$\partial_{\nu}(\tau T^{x\nu}) = 0$$

$$\partial_{\nu}(\tau T^{y\nu}) = 0$$

$$\partial_{\nu}(\tau T^{\eta\nu}) + \frac{1}{\tau}\tau T^{\eta\tau} = 0$$

Conservative variables are

$$Q^{\mu} = \tau \cdot T^{\tau\mu}$$

Closer to numerics:

$$\partial_\mu (T_{\text{id}}^{\mu\nu} + \delta T^{\mu\nu}) = S^\nu, \quad S = \text{geometrical source terms}$$

$$\underbrace{\partial_\tau (T_{\text{id}}^{\tau i} + \delta T^{\tau i})}_{Q_i} + \underbrace{\partial_j (T^{ji})}_{\text{id.flux}} + \underbrace{\partial_j (\delta T^{ji})}_{\text{visc.flux}} = \underbrace{S_{\text{id}}^\nu + \delta S^\nu}_{\text{source terms}}$$

Finite-volume realization:

$$\frac{1}{\Delta\tau} (Q_{\text{id}}^{n+1} + \delta Q^{n+1} - Q_{\text{id}}^n - \delta Q^n) + \frac{1}{\Delta X} (\Delta F_{\text{id}}^{n+1/2} + \Delta \delta F^{n+1/2}) = S_{\text{id}}^{n+1/2} + \delta S^{n+1/2}$$


now, a small trick:

$$\frac{1}{\Delta\tau} (Q_{\text{id}}^{n+1} + \delta Q^{n+1} - \underbrace{Q_{\text{id}}^{*n+1} + Q_{\text{id}}^{*n+1}}_{=0} - Q_{\text{id}}^n - \delta Q^n) + \frac{1}{\Delta X} (\Delta F_{\text{id}} + \Delta \delta F) = S_{\text{id}} + \delta S$$

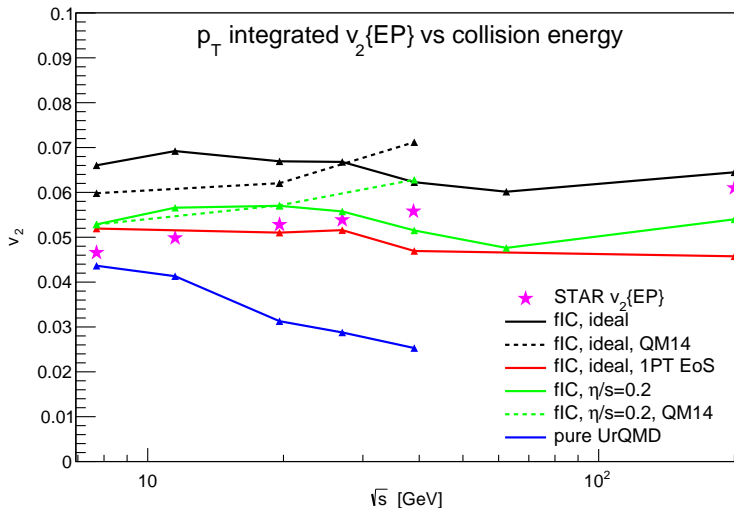
Then, split the equation into two parts¹:

$$\frac{1}{\Delta t} (Q_{\text{id}}^{*n+1} - Q_{\text{id}}^n) + \frac{1}{\Delta X} \Delta F_{\text{id}} = S_{\text{id}} \quad (\text{using finite volume, HLLE approx}) \quad (4)$$

$$\frac{1}{\Delta t} (Q_{\text{id}}^{n+1} + \delta Q^{n+1} - Q_{\text{id}}^{*n+1} - \delta Q^n) + \frac{1}{\Delta X} \Delta \delta F = \delta S \quad (\text{Lax-Wendroff}) \quad (5)$$

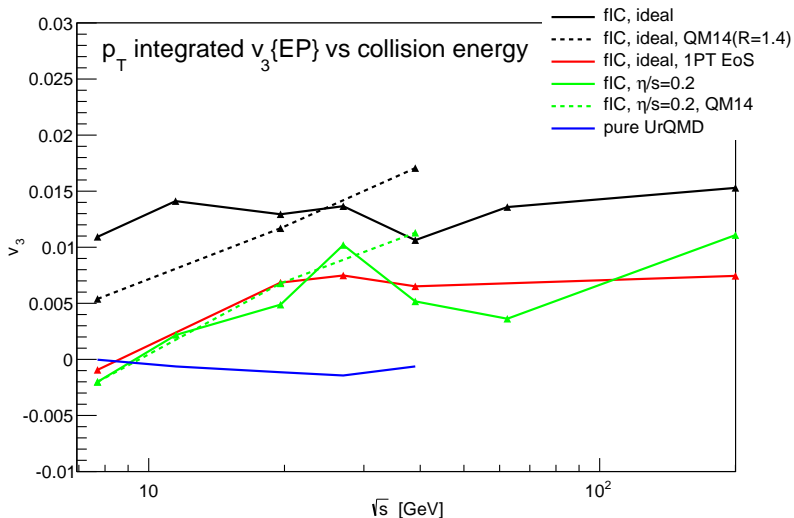
¹Makoto Takamoto, Shu-ichiro Inutsuka, J.Comput.Phys. 230 (2011), 7002 

Comparison to QM14 results: v_2



QM14 results (dashed): hydro starting time is not limited from below

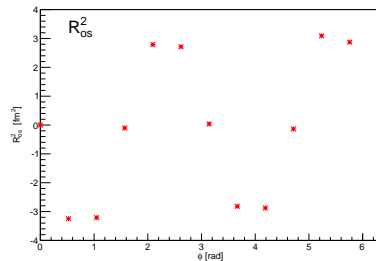
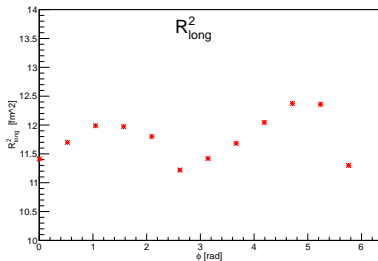
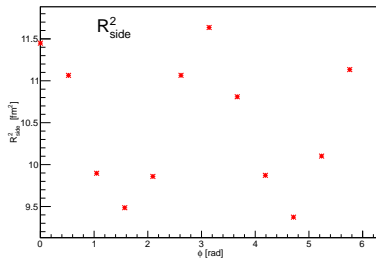
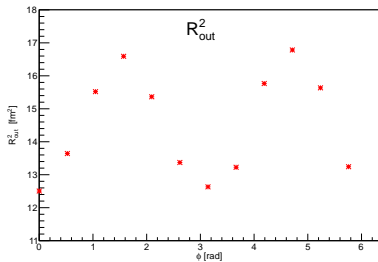
Comparison to QM14 results: v_3



QM14 results (dashed): hydro starting time is not limited from below

Azimuthally-sensitive femtoscopy

$\sqrt{s_{NN}} = 7.7$ GeV, 10-30% central AuAu; $p_T = 0.15 \dots 0.6$ GeV; $\phi = \psi_{\text{pair}} - \Psi_{\text{RP}}$



Azimuthally-sensitive femtoscopy

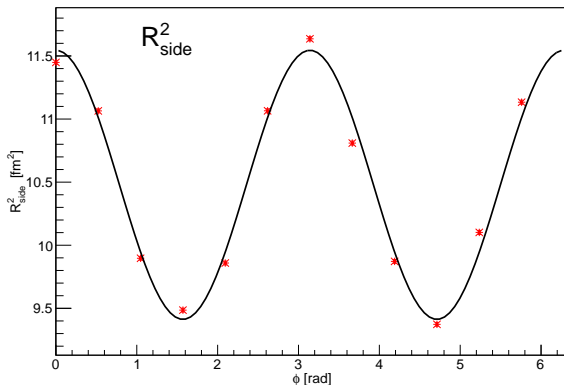
$$R_i^2(\phi) = R_{i,0}^2 + 2 \sum_{n=2,4,6\dots} R_{i,n}^2 \cos(n\phi), \quad i = \text{out, side, long}$$

$$R_i^2(\phi) = 2 \sum_{n=2,4,6\dots} R_{i,n}^2 \sin(n\phi), \quad i = \text{os}$$

solid curve:

$$R_{s,0}^2 + 2R_{s,2}^2 \cos(2\phi) \Rightarrow$$

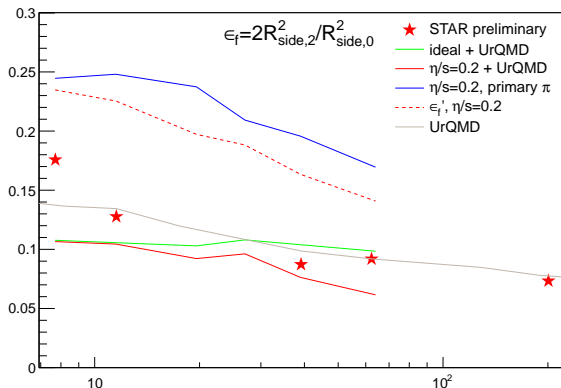
$$\varepsilon_f = 2 \frac{R_{\text{side},2}^2}{R_{\text{side},0}^2}$$



F. Retiere and M. Lisa, Phys.Rev. **C70:044907**, 2004

Azimuthally-sensitive femtoscopy

using averaged initial state, single shot hydro



STAR: C. Anson,
J.Phys. G38:124148,2011

10-30% central AuAu,
 $p_T = 0.15 \dots 0.6$ GeV

$$\epsilon' = \frac{\int (y^2 - x^2) u^\mu d\sigma_\mu}{\int (y^2 + x^2) u^\mu d\sigma_\mu} \quad 1$$

Rescatterings and
resonance decays
decrease the
eccentricity

¹ C. Shen, U. Heinz, Phys.Rev. C 85, 054902 (2012)

² UrQMD: M.A. Lisa, et al., New J.Phys.13:065006,2011