Shear Viscosity and Electric Conductivity of QGP in a Kinetic Theory approach

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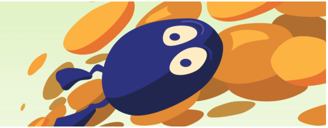
arxiv:1407.2559, **arxiv:1408.7043**
Phys. Rev. C 86, 054902(2012)

Hot Quarks 2014, Las Negras
21-28 Sept 2014



Outline

1. Motivations
   - QGP and Transport Coefficients
2. Transport Theory
   - Relativistic Boltzmann Transport Equation
   - Green-Kubo relations
3. Shear Viscosity
4. Electric Conductivity

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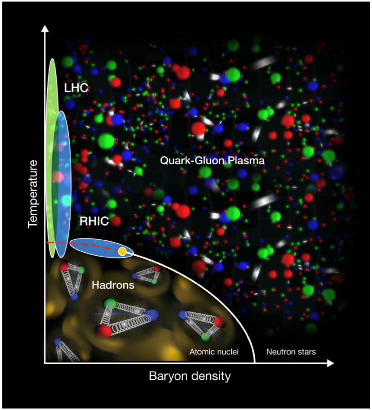
QCD phase diagram

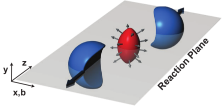


$$\frac{1}{p_T}\frac{d^3N}{dp_T\,dy\,d\phi} = \frac{1}{2\pi p_T}\frac{d^2N}{dp_T\,dy}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T, y)\cos[n(\phi - \psi_R)]\right)$$

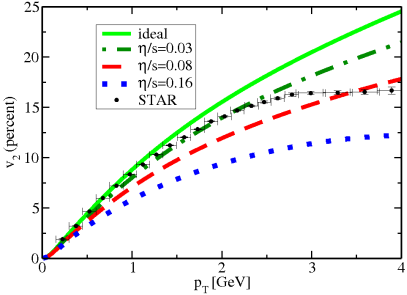
H. G. Ritter and R. Stock, arXiv:1408.4296

Elliptic Flow: $n = 2$

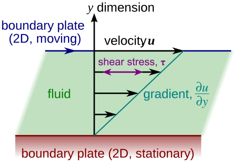
$$v_2(p_T, b) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



P. Romatschke and U. Romatschke, Phys. Rev. Lett 99, 172301 (2007)



Shear Viscosity



Shear Stress

$$\tau = \frac{F_x}{A_{yz}} = -\eta \frac{\partial u_x}{\partial y}$$

fluid	$P\,[Pa]$	$T\,[K]$	$\eta\,[Pa \cdot s]$	$\eta/s\,[\hbar/k_B]$
H_2O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	8.2
^4He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	1.9
H_2O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	2.0
^4He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	0.7
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\leq 5 \cdot 10^{11}$	≤ 0.4

T.Schäfer, D. Teaney, Rep. Prog. Phys **72** (2009) 126001

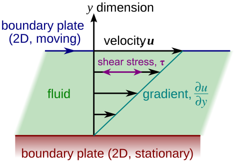
Viscosity ⟷ microscopic details ?

$$\eta \rightarrow \lambda, \sigma, \langle p \rangle \ldots$$

Pitch drop experiment

- started in 1927
- 8th drop on 28th November 2000
- $\eta_{pitch} = 2.3 \cdot 10^{11}\ \eta_{H_2O} \sim 6.67 \cdot 10^7\ Pa\,s$
- $\eta_{pitch} \ll \eta_{QGP}$
- η/s

Shear Viscosity



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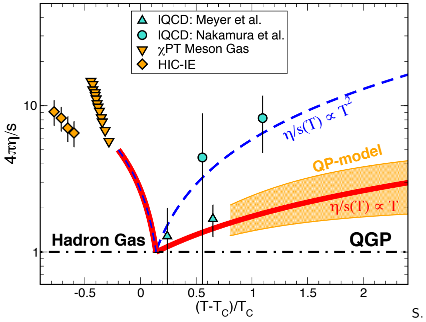


Plumari et al: arXiv:1304.6566v1

- QM: $\eta/s \simeq \frac{4}{15}\langle p \rangle \tau \Rightarrow \eta/s > \frac{1}{15}$
- AdS/CFT: $\eta/s = \frac{1}{4\pi}$

Electric Conductivity

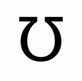
Ohm's Law

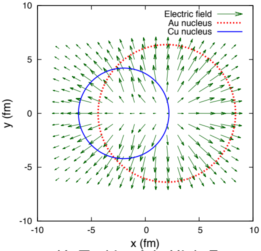
$$I = \frac{V}{R} \Rightarrow j = \sigma_{el} E$$

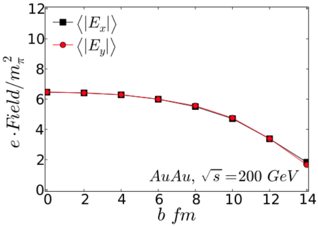
$$eE \simeq (m_\pi)^2 \rightarrow E \sim 10^{21}\, V/cm$$

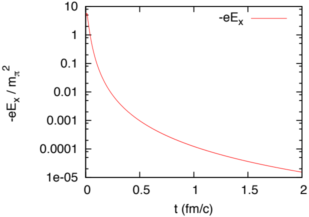
Old unit:

Mho









K. Tuchin, Adv.High Energy Phys. 2013 (2013) 490495   Y. Hirono, M. Hongo, T. Hirano, arXiv:1211.1114 (2012)

Direct Flow

$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$

Photon rate

$$\omega \frac{d\Gamma_\gamma}{d^3p} = \frac{\alpha_{EM}}{\pi^2 e^2} \frac{\sigma_{el}\omega}{e^{\omega/T} - 1}$$

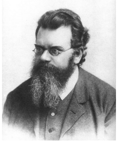
Lattice QCD

$$G_{\mu\nu}(\tau, T) = \int d^3x \, \langle J_\mu(\tau, \mathbf{x}) J_\nu(0, \mathbf{0})^\dagger \rangle$$

Relativistic Transport Equation

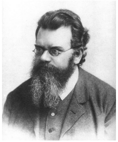
General Transport Equation

$$\left(p^\mu \partial_\mu + (m^*(x)\partial_\mu m^*(x))\partial_p^\mu\right) f(x,p) = \mathcal{C}[f](x,p)$$



- free-streaming
- Mean Field: long range interactions $\rightarrow \epsilon - 3P \neq 0$
- Collisions: short range interactions $\rightarrow \eta, \sigma_{el}, \cdots$

Relativistic Transport Equation



General Transport Equation

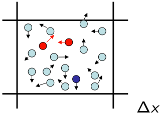
$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{22}[f]$$

$$\mathcal{C}_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 \times |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1+p'_2-p_1-p_2)$$

$$-\frac{1}{2E_1}\times\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 \times |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p'_1-p'_2)$$

gain

loss



Test-particle method

$$f(x,p) = \sum_{i=1}^{N} \delta^4(x_i(t)-x)\delta^4(p_i(t)-p)$$

$$N = N_{real} \times N_{test} \quad , \quad \sigma \to \sigma/N_{test}$$

Stochastic method

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\frac{\sigma_{22}}{N_{test}}\frac{\Delta t}{\Delta^3 x}$$

if $P_{22} > rand()$ collision takes place

$v_{rel} = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}/E_1 E_2$

Z. Xu and C. Greiner, Phys.Rev. C71 (2005) 064901

Transport Coefficients: Green-Kubo

Transport coefficients: $\eta, \zeta, k, D, \sigma_{el}$ characterize the non-equilibrium behaviour of a system.

Fluctuation-Dissipation Theorem

It establishes a relation between equilibrium fluctuations of a physical observable and a dissipative process that takes place when the system is perturbed from equilibrium.

Green-Kubo Relation

$$A = \frac{V}{T} \int_0^\infty dt \, \langle J(t) J(0) \rangle$$

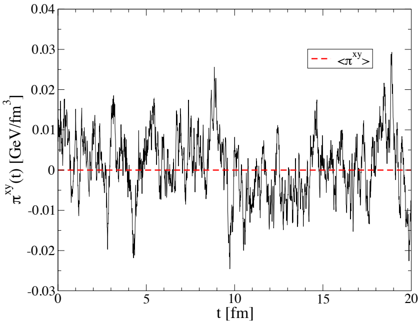
$$\eta \Rightarrow J = \pi^{xy} = -\eta \frac{\partial u_x}{\partial y}$$

$$\sigma_{el} \Rightarrow J = \sigma_{el} E$$

$$D \Rightarrow J = -D \nabla \rho$$

$$\zeta \Rightarrow J = P$$

$$k \Rightarrow J = Q = -k \nabla T$$

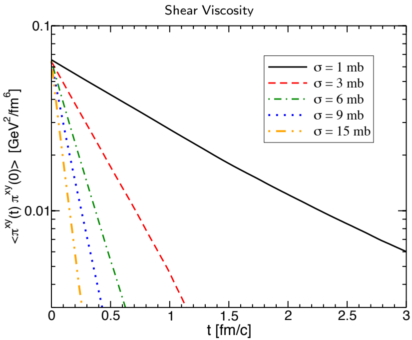


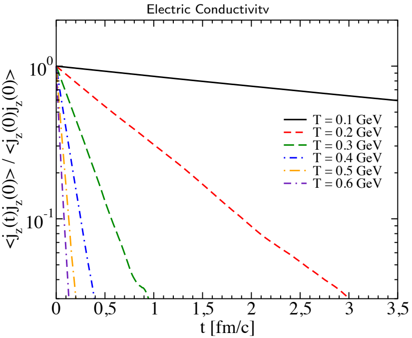
Transport Coefficients: Green-Kubo

Correlation Functions

$$\langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \langle \pi^{xy}(0)^2\rangle e^{-t/\tau} \Longrightarrow \eta = \frac{V}{T}\int_0^\infty dt\, \langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \frac{V}{T}\langle \pi^{xy}(0)^2\rangle \tau$$

$$\langle J_z(t)J_z(0)\rangle = \langle J_z(0)^2\rangle e^{-t/\tau} \Longrightarrow \sigma_{el} = \frac{V}{T}\int_0^\infty dt\, \langle J_z(t)J_z(0)\rangle = \frac{V}{T}\langle J_z(0)^2\rangle \tau$$





C. Wesp et al, Phys.Rev. C84 (2011) 054911, S. Plumari, A. Puglisi et al., Phys.Rev. C86 (2012) 054902, A. Puglisi et al, arXiv: 1408.7043, M. Greif et al., arXiv:1408:7049

Shear Viscosity: analytic formulas

Relaxation Time Approxiamtion (RTA)

$$p^\mu \partial_\mu f = \mathcal{C}[f] \approx \frac{f - f^{eq}}{\tau} \Rightarrow \eta = \frac{4}{15}\rho\langle p\rangle\tau_{tr} = \frac{4}{15}\rho\langle p\rangle\frac{1}{\langle\rho\sigma_{tr}v_{rel}\rangle} = \frac{4}{15}\frac{\langle p\rangle}{\langle h(a)\rangle\sigma_{tot}}$$

$\sigma_{tr} = \int d\sigma^{gg\to gg}\sin^2\theta_{cm} = \sigma_{tot}h(a) < \frac{2}{3}\sigma_{tot}$

with $h(a) = 4a(a+1)[(2a+1)ln(1+a^{-1}) - 2]$ $\quad a = mD^2/s,$ $\quad h(m_D \to \infty) = 2/3$

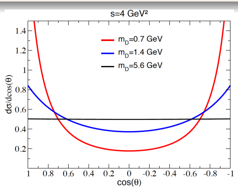
Chapman-Enskog (CE)

$$\eta^{1st} = \frac{4}{15}\rho\langle p\rangle\tau_\eta = \frac{4}{15}\frac{\langle p\rangle}{g(a)\sigma_{tot}} \quad with \quad g(a) = \frac{1}{50}\int dy\, y^6\left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right]h\left(\frac{a^2}{y^2}\right)$$

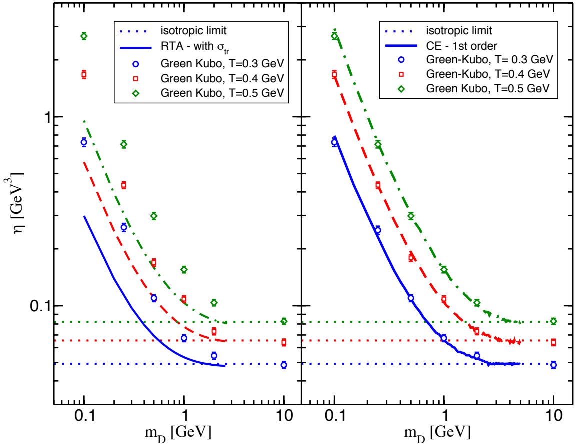
$a = m_D/2T,$ $\quad g(m_D \to \infty) = 2/3$

pQCD angular dependence

$$\frac{d\sigma^{gg\to gg}}{d\Omega} \propto \frac{\alpha_s^2}{\left(q^2(\theta)+m_D^2\right)^2} \to \sigma_{tot} = \frac{9\pi\alpha_s^2}{2m_D^2}$$



Shear Viscosity: anisotropic σ



S. Plumari et al, Phys.Rev. C86 (2012) 054902

RTA

$$\eta = \frac{4}{5}\frac{T}{\langle h(a)\rangle \sigma_{tot}}$$

CE

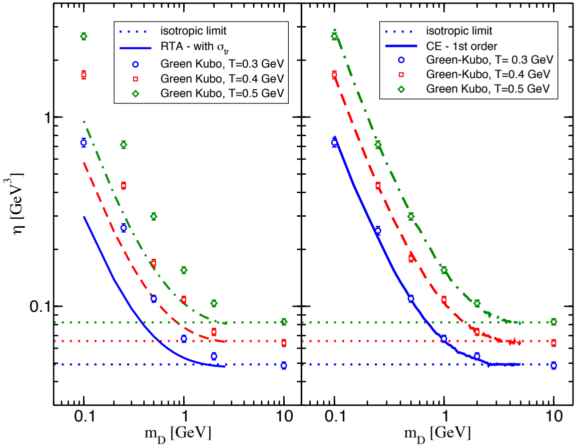
$$\eta^{1st}_{CE} = \frac{4}{5}\frac{T}{g(a)\sigma_{tot}}$$

RTA vs CE: anisotropic case: $g = m_D/T = 0.1\ GeV/0.5\ GeV = 0.2 \rightarrow$ factor 2

$m_D \sim 1\ GeV$ to 20% discrepancy

However: RTA vs CE: isotropic limit $\rightarrow$ ok

Shear Viscosity: anisotropic σ



S. Plumari et al, Phys.Rev. C86 (2012) 054902

RTA

$$\eta = \frac{4}{5} \frac{T}{\langle h(a) \rangle \sigma_{tot}}$$

CE

$$\eta_{CE}^{1st} = \frac{4}{5} \frac{T}{g(a) \sigma_{tot}}$$

η_{CE}^{1st} *OK!* in all m_D range.

RTA vs CE: anisotropic case: $g = m_D/T = 0.1\ GeV/0.5\ GeV = 0.2 \rightarrow$ factor 2

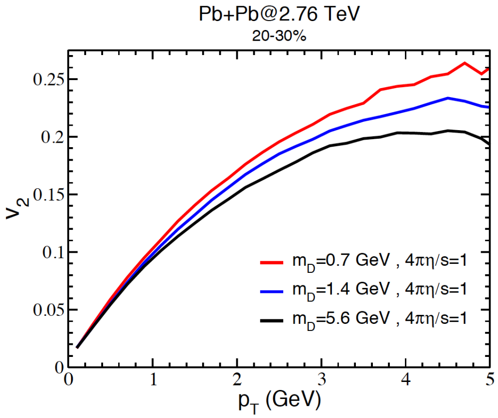
$m_D \sim 1\ GeV$ *to* 20% discrepancy

However: RTA vs CE: isotropic limit $\rightarrow$ ok

Elliptic flow

S. Plumari, A. Puglisi, et al. J.Phys.Conf.Ser. 446 (2013) 012025

$$v_2(p_T, b) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



How to fix η?

$$\sigma_{tot}(\rho(\mathbf{x}), T) = \frac{1}{5}\frac{T}{g(a)}\frac{1}{\eta/s}$$

- σ locally computed
- $\eta = cost$ changing σ

Initial Condition

- coordinate space: Glauber model
- momentum space: thermalized up to $p_T = 2\ GeV$
- $T^0_{max} = 2T_c\ (3T_c)$

- $v_2(p_T < 1.5\ GeV)$: η/s
- $v_2(p_T > 1.5\ GeV)$: sensitive to microscopic details of interaction

Shear Viscosity: Quasi-particle model

Relaxation Time Approximation

$$\eta = \sum_q \frac{1}{15T}\tau_q \int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E^2}\gamma_q f(p) + \frac{1}{15T}\tau_g \int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E^2}\gamma_g f(p)$$

$$\tau_q^{-1} = \sum_j \langle \rho_j \sigma^{qj} v_{rel}^{qj} \rangle \qquad \tau_g^{-1} = \sum_j \langle \rho_j \sigma^{gj} v_{rel}^{gj} \rangle$$

C. Sasaki and K. Redlich, Phys.Rev. C79 (2009) 055207

Quasi-Particle Model

- $E = \sqrt{p^2 + m^2(T)}$
- non-perturbative effects
- ϵ, P from Lattice-QCD

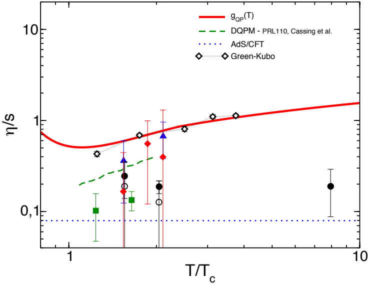
M. Bluhm et al., Phys. Rev. C 84, 025201 (2011),

S. Plumari et al. Phys. Rev. D **84**, 094004 (2011)

$$g^2_{QP}(T) = \frac{48\pi^2}{(11N_c - 2N_f)\log\left[\lambda\left(\frac{T}{T_c} - \frac{T}{T_s}\right)\right]}$$

$$\sigma^{ij}_{tot}(s) \sim \beta^{ij}\frac{\alpha_s^2}{m_D^2(T)}\frac{s}{(s+m_D^2(T))}$$

	β
$qq \to qq$	$2\frac{8\pi}{9}$
$q\bar{q} \to q\bar{q}$	$\frac{8\pi}{9}$
$qg \to qg$	2π
$gg \to gg$	9π



Marty R et al. 2013 *Phys. Rev.* C **88** 045204

$$m_g^2(T) = \frac{1}{6}g^2\left(N_c + \frac{1}{2}N_f\right)T^2$$

$$m_q^2(T) = \frac{N_c^2-1}{8N_c}g^2T^2$$

- GK vs RTA
- QP: $\eta/s \sim 6 \times (1/4\pi)$
- QP vs. DQPM: anisotropic cross-sections $\sim$ 2 factor

Electric Conductivity in the box

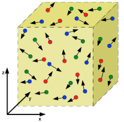
Boltzmann Equation

$$p^\mu \partial_\mu f(x,p) + qF^{\alpha\beta} p_\beta \frac{\partial}{\partial p^\alpha} f(x,p) = \mathcal{C}[f]$$

E-field method

$$j_z = \sigma_{el} E_z, \qquad \frac{d}{dt} p_z^i = q_i e E_z$$

$$j_z(t) = \frac{1}{V} \sum_i e q_i \frac{p_z^i(t)}{E_i} \Rightarrow \sigma_{el} = \frac{j_z^{eq}}{E_z}$$



Green-Kubo method

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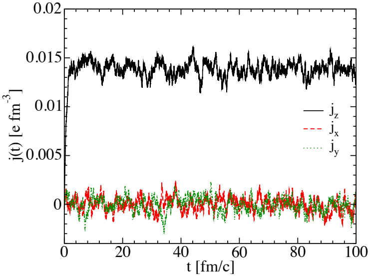

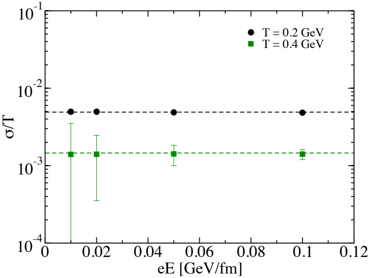

Cassing W et al., 2013 *Phys. Rev. Lett.* **110** 182301, A. Puglisi et al. arXiv:1408.7043, M. Greif et al., arXiv:1408.7049

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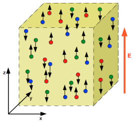
$$j_z(t) = \frac{1}{V}\sum_i eq_i \frac{p_z^i(t)}{E_i} \Rightarrow \sigma_{el} = \frac{j_z^{eq}}{E_z}$$



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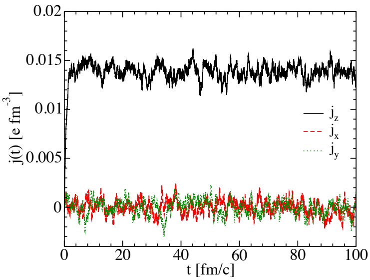
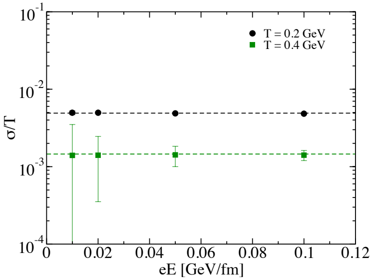




Cassing W et al., 2013 *Phys. Rev. Lett.* **110** 182301, A. Puglisi et al. arXiv:1408.7043, M. Greif et al., arXiv:1408.7049

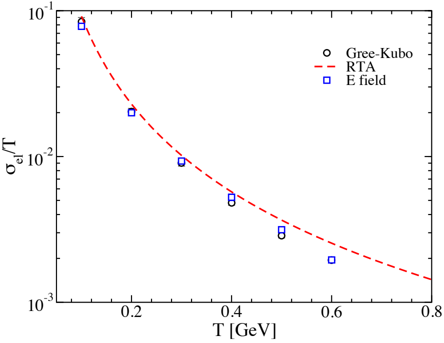
Electric Conductivity: $m = 0$ and isotropic scatterings

Electric Conductivity: RTA

$$\tau^{-1}_{tr,i} = \sum_{j=q,\bar{q},g} \langle \rho_j v^{ij}_{rel} \sigma^{ij}_{tr} \rangle = \frac{2}{3}\sigma_{tot}(\rho_q + \rho_{\bar{q}} + \rho_g), \qquad \rho_{q,g} = \gamma_{q,g} T^3/\pi^2$$

$$\frac{\sigma_{el}}{T} = \frac{e^2}{3T^2} \sum_{j=q,\bar{q}} q_j^2 \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{E^2} \tau_j f_{eq} = \frac{e^2_\star}{3T^2} \left\langle \frac{p^2}{E^2} \right\rangle \tau_q \rho_q = \frac{e^2_\star}{3T^2} \frac{\gamma_q}{6\gamma_q + \gamma_g} \frac{1}{\frac{2}{3}\sigma_{tot}}$$

$e^2_\star = e^2 \sum_{j=q,\bar{q}} q_j^2 = 4e^2/3,\ q_j = \pm 1/3, \pm 2/3$



Box Setup: $V = 5^3\ fm^3$, $\sigma_{tot} = 3\ mb$

Green-Kubo

$$\frac{\sigma_{el}}{T} = \frac{V}{T^2} \langle J_z(0)^2 \rangle \tau$$

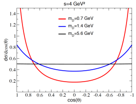
- GK vs. E-field method: agreement
- RTA overestimates σ_{el} for $T > 0.4\ GeV$: $> 30\%$

A. Puglisi et al., arXiv:1408.7043, M. Greif et al., arXiv:1408.7049

Electric Condictivity: Anisotropic cross-section

pQCD angular dependence

$$\frac{d\sigma^{gg\to gg}}{d\Omega} \propto \frac{\alpha_s^2}{\left(q^2(\theta)+m_D^2\right)^2} \to \sigma_{tot} = \frac{9\pi\alpha_s^2}{2m_D^2}$$

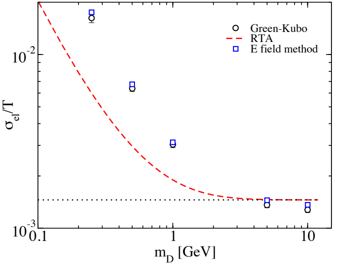


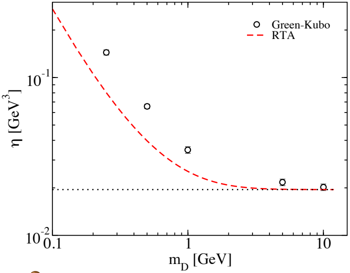
Green-Kubo

$$\frac{\sigma_{el}}{T} = \frac{V}{T^2}\langle J_z(0)^2\rangle\tau$$

Electric Conductivity: RTA

$$\tau_{i,tr}^{-1} = \sum_j \langle \rho_j \sigma_{tr}^{ij} v_{rel}^{ij}\rangle = \sigma_{tot}\langle v_{rel} h(a)\rangle(\rho_q+\rho_{\bar q}+\rho_g), \qquad \frac{\sigma_{el}}{T} = \frac{e_\star^2}{3T^2}\left\langle \frac{p^2}{E^2}\right\rangle \tau_{q,tr}\rho_q$$





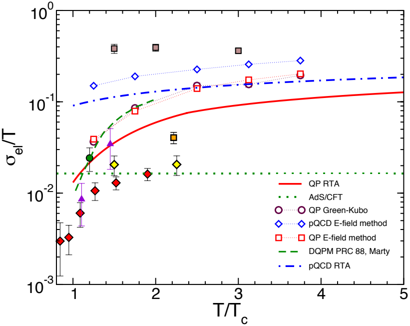
- Green-Kubo vs. E-field method: agreement
- RTA underestimation: $\sim$ 40% for $m_D = 1\ GeV$
- Green-Kubo vs RTA: $\sim$ 20% for $m_D = 1\ GeV$
- RTA better for η

Electric Conductivity: Quasi-Particle Model

$$\frac{\sigma_{el}}{T} = \frac{e_\star^2}{3T^2}\left\langle \frac{p^2}{E^2}\right\rangle \tau_{q,tr}\rho_q, \quad \tau_{q,tr}^{-1} = \sum_j \langle \rho_j \sigma_{tr}^{qj} v_{rel}^{qj}\rangle, \quad \rho_q = \frac{\gamma_q}{2\pi^2} T^3 \left(\frac{m(T)}{T}\right)^2 K_2\left(\frac{m(T)}{T}\right)$$

$$m_g^2(T) = \frac{1}{6} g^2 \left(N_c + \frac{1}{2}N_f\right) T^2, \quad m_q^2(T) = \frac{N_c^2-1}{8N_c} g^2 T^2$$

A. Puglisi et al., arXiv:1408.7043, M. Greif et al., 1408.7049



Marty R et al. 2013 *Phys. Rev.* C **88** 045204,

AdS/CFT: S. C. Huot et al., JHEP12(2006)015

$$g_{QP}^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\log\left[\lambda\left(\frac{T}{T_c} - \frac{T}{T_s}\right)\right]^2}$$

$$g_{pQCD}^2 = \frac{16\pi^2}{\left(11 - \frac{2}{3}N_f\right)\log\left[\frac{2\pi T}{\Lambda_{QCD}}\right]^2}, \quad \Lambda_{QCD} = 0.2\ GeV$$

$$\sigma_{tot}^{ij}(s) \sim \beta^{ij}\frac{\alpha_s^2}{m_D^2(T)}\frac{s}{(s+m_D^2(T))}, \quad m_D^2(T) = 4\pi\alpha_s T^2$$

$$\beta^{qq} = 16\pi/9,\ \beta^{q\bar{q}} = 8\pi/9,\ \beta^{qg} = 2\pi,\ \beta^{gg} = 9\pi$$

$$\sigma_{el}^{QGP}(T = 0.2\ GeV) \simeq 0.004\ GeV \simeq 7 \times 10^{11} S/m \sim 10^4 \times \sigma_{el}^{Ag}$$

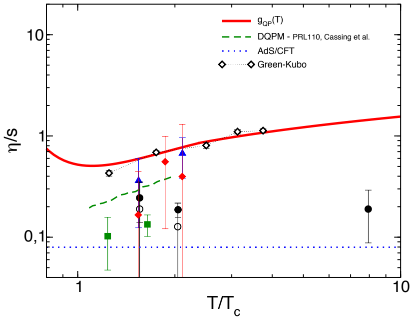
- GK vs. E-field method: agreement
- RTA understimates numerical results: $\sim$ 1.7 factor
- $\sigma_{el}^{QP} \sim 5 \times \sigma_{el}^{Lattice}$

Renormalizing cross-section to reproduce the minimum of η/s: K factor

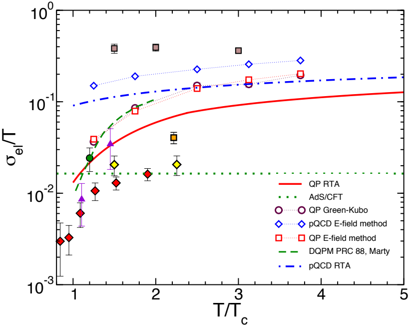
$$\sigma^{ij}_{tot}(s) \sim K\beta^{ij} \frac{\alpha_s^2}{m_D^2(T)} \frac{s}{(s+m_D^2(T))}$$

$$\eta/s = \frac{1}{15Ts}\left\langle \frac{p^4}{E^2} \right\rangle (\tau_q \rho_q^{tot} + \tau_g \rho_g)$$

$$\frac{\sigma_{el}}{T} = \frac{e_\star^2}{3T^2}\left\langle \frac{p^2}{E^2} \right\rangle \tau_q \rho_q$$



Marty R et al. 2013 *Phys. Rev.* C **88** 045204



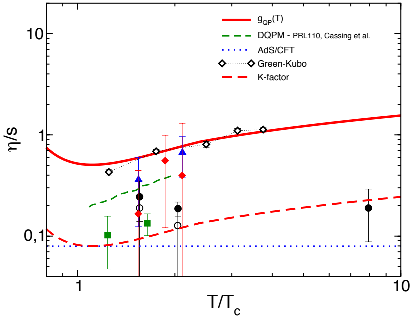
Lattice: A. Amato et al. 2013 *Phys. Rev. Lett.* **111** 172001

Renormalizing cross-section to reproduce the minimum of η/s: K factor ~ 6.36

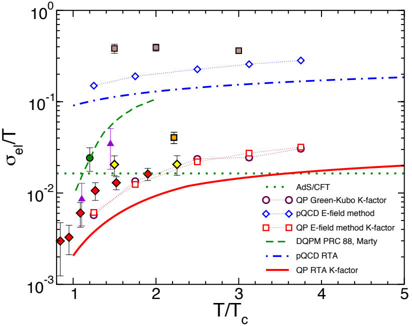
$$\sigma^{ij}_{tot}(s) \sim K\beta^{ij}\frac{\alpha_s^2}{m_D^2(T)}\frac{s}{(s+m_D^2(T))}$$

$$\eta/s = \frac{1}{15Ts}\left\langle\frac{p^4}{E^2}\right\rangle(\tau_q\rho_q^{tot}+\tau_g\rho_g)$$

$$\frac{\sigma_{el}}{T} = \frac{e_\star^2}{3T^2}\left\langle\frac{p^2}{E^2}\right\rangle\tau_q\rho_q$$



Marty R et al. 2013 *Phys. Rev.* C **88** 045204



Lattice: A. Amato et al. 2013 *Phys. Rev. Lett.* **111** 172001

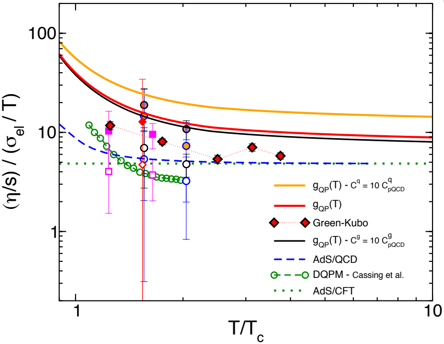
Shear Viscosity η to Electric Conductivity σ_{el} ratio

Ratio

$$\frac{\eta/s}{\sigma_{el}/T} = \frac{6}{5}\,\frac{T\langle p^2/E^2\rangle^{-1}}{s e_\star^2}\left\langle \frac{p^4}{E^2}\right\rangle\left(1+\frac{\tau_g}{\tau_q}\,\frac{\rho_g}{\rho_q^{tot}}\right), \qquad \frac{\tau_g}{\tau_q} = \frac{C^q + \frac{\rho_g}{\rho_q}}{6 + C^g\,\frac{\rho_g}{\rho_q}}$$

- Fixed by Thermodynamics: Lattice QCD
- τ_g/τ_q Dynamical: unknown
- $C^q = (\beta^{qq} + \beta^{q\bar{q}} + 2\beta^{q\bar{q}'} + 2\beta^{qq'})/\beta^{qg}$
- $C^g = \beta^{gg}/\beta^{qg}$

- $(\eta/s)/(\sigma_{el}/T) \sim g(T)$
- $C^q|_{pQCD} = \frac{28}{9} \simeq 3.1$
- $C^g|_{pQCD} = \frac{9}{2}$



A. Puglisi et al., arXiv:1407.2559,

Marty R et al. 2013 *Phys. Rev.* C **88** 045204

AdS/QCD: J. Noronha et al., Phys. Rev. D 89, 106008 (May 2014)

AdS/CFT: S. C. Huot et al., JHEP12(2006)015

Ratio

- independent on K and α_s
- sensitive only on C^q
- increases near T_c
- constant value for $T \gg T_c$

Conclusions

Shear Viscosity results

- Gluon system

  Relaxation Time Approximation good only for isotropic scatterings.

  Chapman-Enskog in agreement with numerical results for all m_D and T of interest

- Mixture

  Relaxation Time Approximation vs. Green-Kubo results: agreement

  Quasi-particle model: high η/s

Electric Conductivity results

- E-field method vs. Green-Kubo: agreement
- RTA isotropic scatterings: good approximation
- RTA anisotropic scatterings: underestimates numerical results
- Quasi-particle model vs. Lattice QCD: lower η/s lower σ_{el}/T
- $(\eta/s)/(\sigma_{el}/T)$ dependent on quark-quark(-antiquark) over quark-gluon scatterings

Thank you!

Shear Viscosity of a mixture

Quasi-Particle Model

- Description of Lattice results in terms of quasiparticles quarks and gluons.
- interaction generates quasiparticle mass $m(T)$

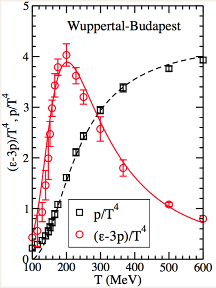
$$E = \sqrt{p^2 + m^2(T)}$$

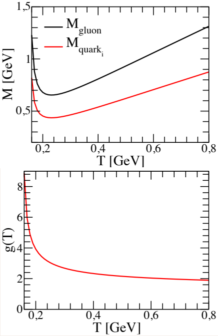
- $\Rightarrow$ quasiparticle are weakly interacting.

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ \ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

$$m_g^2(T) = \frac{1}{6} g^2 \left(N_c + \frac{1}{2} N_f\right) T^2$$

$$m_q^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 T^2$$





Relaxation Time Approximation

$$\eta = \sum_q \frac{1}{15T} \tau_q \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E^2} \gamma_q f(p) + \frac{1}{15T} \tau_g \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E^2} \gamma_g f(p)$$

$$\tau_q^{-1} = \sum_j \langle \rho_j \sigma^{qj} v_{rel}^{qj} \rangle \qquad \tau_g^{-1} = \sum_j \langle \rho_j \sigma^{gj} v_{rel}^{gj} \rangle$$

C. Sasaki and K. Redlich, Phys.Rev. C79 (2009) 055207

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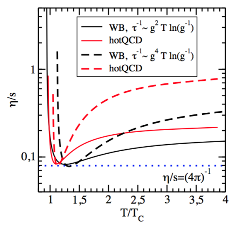
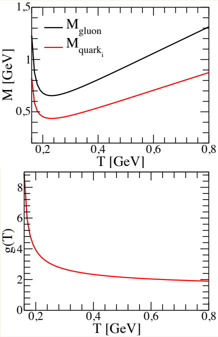


S. Plumari et all. Phys. Rev. D **84**, 094004 (2011)



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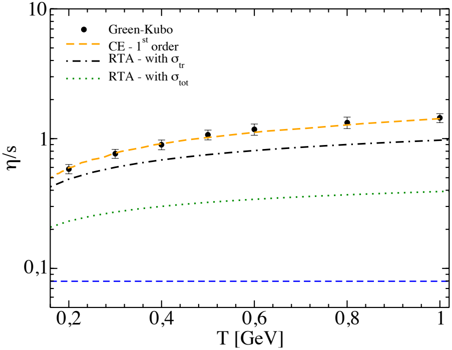
C. Sasaki and K. Redlich, Phys.Rev. C79 (2009) 055207

η/s of a gluon plasma

pQCD: anisotropic and energy-dependent cross-section

$$\frac{d\sigma^{gg\to gg}}{d\Omega} \propto \frac{\alpha_s^2}{\left(q^2(\theta)+m_D^2\right)^2} \quad \to \quad \sigma_{tot} = \frac{9\pi\alpha_s^2}{2m_D}\frac{s}{s+m_D^2}$$

$$m_D(T) = T\sqrt{4\pi\alpha_s} \qquad \alpha_s(T) = \frac{4\pi}{11\ln\left(\frac{2\pi T}{\Lambda}\right)^2}, \qquad \Lambda = 200\ MeV$$



$$(\eta/s)_{RTA} = \frac{1}{5}\frac{T}{\langle h(a)\rangle \rho\sigma_{tot}}, \qquad (\eta/s)^{1st}_{CE} = \frac{1}{5}\frac{T}{g(a)\rho\sigma_{tot}}$$

RTA often used in literature: factor ~ 1.5

RTA*: 30% discrepancy

Green-Kubo vs CE: 3%

S. Plumari, A. Puglisi et al., Phys.Rev. C86 (2012) 054902

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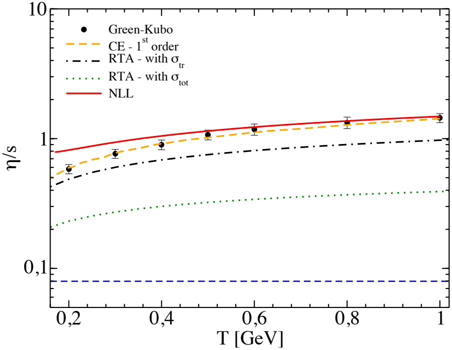

S. Plumari, A. Puglisi et al., Phys.Rev. C86 (2012) 054902

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RTA often used in literature: factor ~ 1.5

RTA*: 30% discrepancy

Green-Kubo vs CE: 3%

NLL P. Arnold et al: JHEP05(2003)051

$$\eta \sim \frac{T^3}{g^4 \ln g^{-1}}$$

- u,t,s channels and interference terms
- longitudinal and transverse components of gluon propagator

Electric Conductivity: analytic formula

Boltzmann Equation

$$p^\mu \partial_\mu f(x,p) + qF^{\alpha\beta} p_\beta \frac{\partial}{\partial p^\alpha} f(x,p) = \mathcal{C}[f]$$

RTA

$$\mathcal{C}[f] \simeq -\frac{p^\mu u_\mu}{\tau}(f - f_{eq})$$

$$f(x,p,t) = f_{eq}(x,p)(1+\phi), \quad \phi \ll f_{eq}$$

τ relaxation time

Electric Conductivity

$$j^\mu = q\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p^0} f = q\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p^0} f_{eq}(1+\phi) = \sigma_{el} E$$

$$\sigma_{el} = \frac{e^2}{3T}\sum_{j=q,\bar{q}} q_j^2 \int \frac{d^3p}{(2\pi)^3}\frac{p^2}{E^2}\tau_j f_{eq} = \frac{e_\star^2}{3T}\left\langle \frac{p^2}{E^2}\right\rangle \tau_q \rho_q$$

with: $q = (\pm 1/3, \pm 2/3)$, ρ_q quark density, $e_\star^2 = e^2 \sum_{j=q,\bar{q}} q_j^2 = 4e^2/3$

Green-Kubo Relations

Green-Kubo relations give the exact expression for transport coefficients.

Green-Kubo Relations

$$L = \int_0^\infty dt \, \langle J(t) J(0) \rangle = \int_0^\infty dt \, C(t)$$

- J current;
- $\langle \dots \rangle$ ensemble average
- $C(t)$ time correlation function

Physical Observable $A \rightarrow$ current $J \rightarrow$ correlation function $J(t)J(0) \rightarrow$ transport coefficients L

Time correlation function

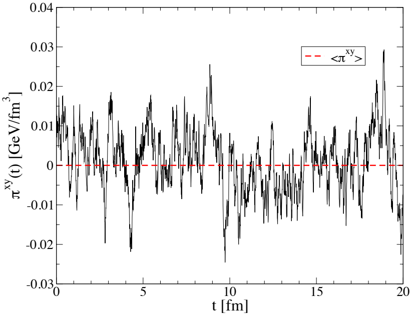
$$C(t) = \left\langle \lim_{T\to\infty} \frac{1}{T} \int_0^T dt' J(t+t') J(t') \right\rangle$$

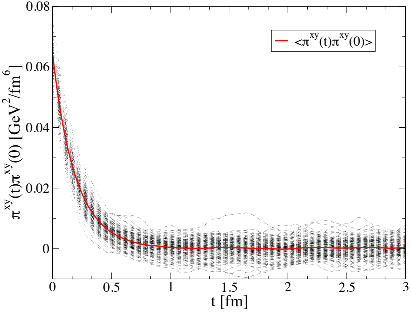
Shear Viscosity

$$J \equiv \pi^{xy} = -\eta \frac{\partial u_x}{\partial y}$$

Green-Kubo Correlator

$$\eta = \frac{V}{T}\int_0^\infty dt\,\langle \pi^{xy}(t)\pi^{xy}(0)\rangle \quad , \quad \pi^{xy}(t) = \frac{1}{V}\sum_{i=1}^{N}\frac{p_i^x p_i^y}{E_i}$$





Green-Kubo Correlator

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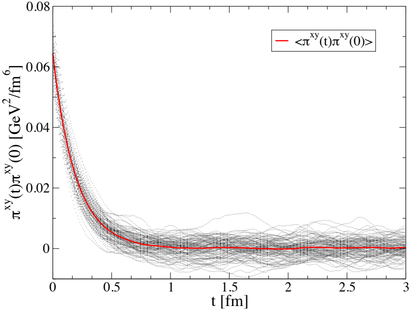
$$\langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \langle \pi^{xy}(0)^2\rangle e^{-t/\tau}$$

$$\eta = \frac{V}{T}\langle \pi^{xy}(0)^2\rangle \tau$$



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$$\eta = \frac{V}{T}\int_0^\infty dt\,\langle \pi^{xy}(t)\pi^{xy}(0)\rangle \quad , \quad \pi^{xy}(t) = \frac{1}{V}\sum_{i=1}^{N}\frac{p_i^x p_i^y}{E_i}$$

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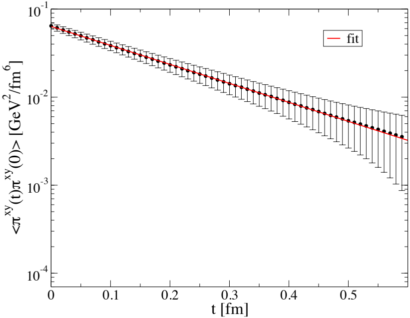
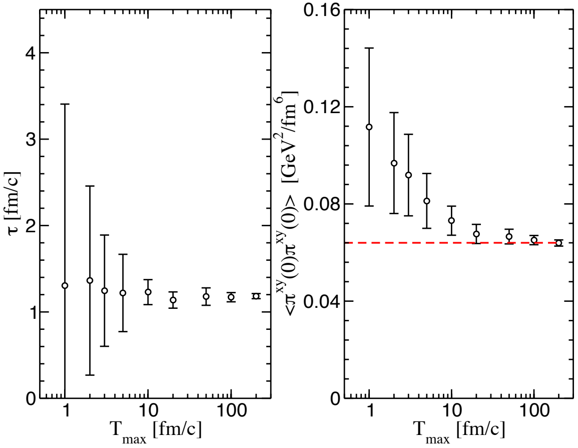


Correlator Convergence: T_{max}

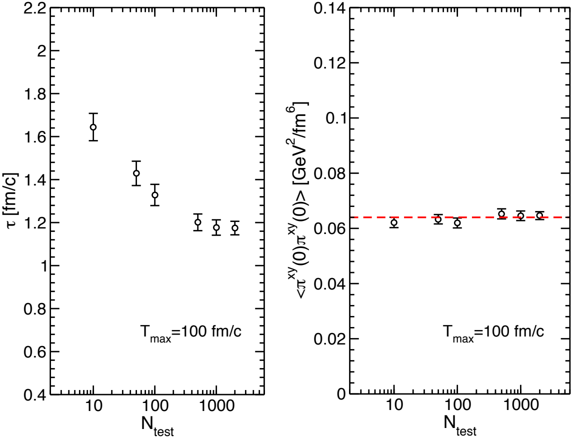


$$\langle \pi^{xy}(0)^2 \rangle = \frac{4}{15} \frac{\epsilon T}{V}$$

$$\eta = \frac{V}{T} \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \tau$$

$$T_{max} = 100\ fm/c$$

Correlator Convergence: N_{test}



$$\langle \pi^{xy}(0)^2 \rangle = \frac{4}{15}\frac{\epsilon T}{V}$$

$$\eta = \frac{V}{T}\langle \pi^{xy}(0)\pi^{xy}(0)\rangle\,\tau$$

$N_{test} = 1000$

Collision Rate

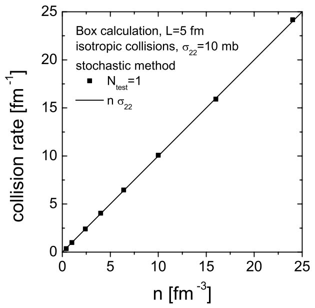
Collision Rate

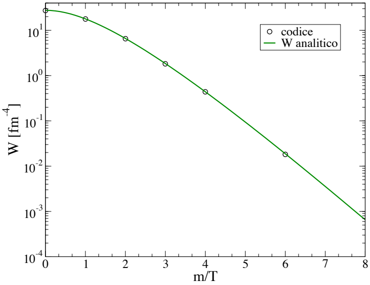
$$R = \rho \langle \sigma v_{rel} \rangle$$

$m = 0 \rightarrow R = \rho\sigma$

Collision rate per unit volume: $W = \frac{\Delta N_{coll}}{V \Delta t}$

$$W = \frac{8T^6}{\pi^4}\sigma \int_{2m/T}^{\infty} dx\,(x^2 - (2m/T)^2) K_1(x)$$





Relaxation to equilibrium

- non-equilibrium initial distribution function:

$$\frac{dN}{N dp_T dp_z} = \delta(p_{xy} - p_{xy}^{init})\delta(p_z)$$

$$p_{xy} = \sqrt{p_x^2 + p_y^2}$$

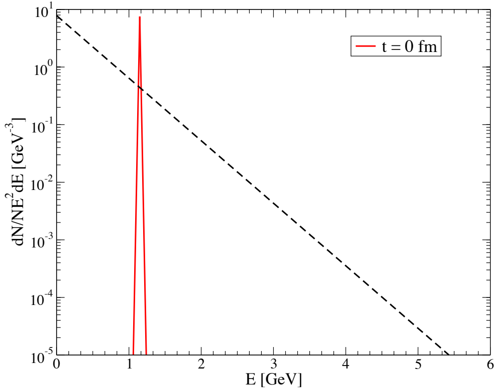
- equilibrium distribution function (Boltzmann):

$$\frac{dN}{NE^2 dE} = \frac{1}{2T^3} e^{-E/T}$$

$$E = 3T$$

$$p_{xy}^{init} = 1.2\ GeV \rightarrow T = 0.4\ GeV$$

$$\sigma = 10\ mbarn,\ V = 3^3\ fm^3$$



Relaxation to equilibrium

- non-equilibrium initial distribution function:

$$\frac{dN}{N dp_T dp_z} = \delta(p_{xy} - p_{xy}^{init})\delta(p_z)$$

$$p_{xy} = \sqrt{p_x^2 + p_y^2}$$

- equilibrium distribution function (Boltzmann):

$$\frac{dN}{NE^2 dE} = \frac{1}{2T^3} e^{-E/T}$$

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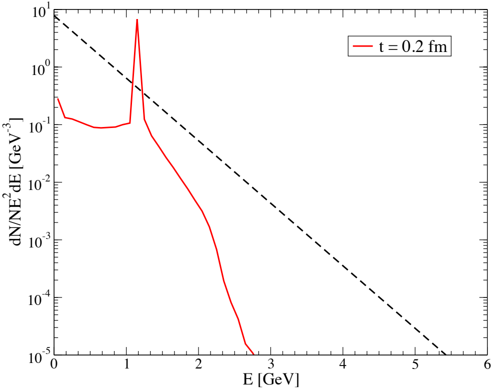

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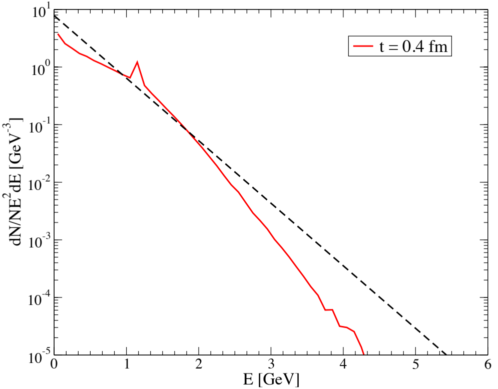

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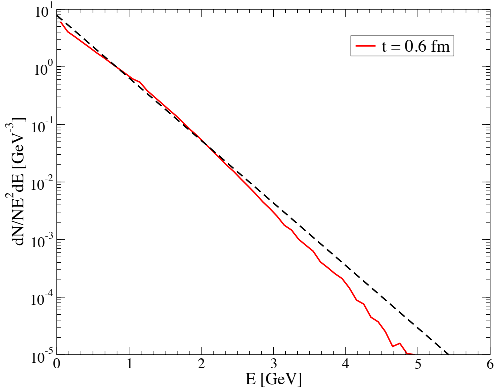

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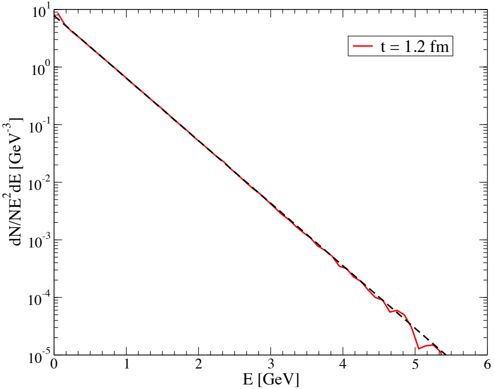


Thermodynamics test

Relativistic Boltzmann gas

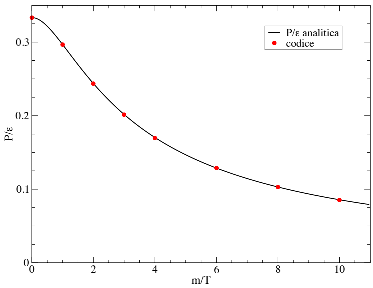
$$f(\mathbf{p}) = e^{-E/T}, \qquad E = \sqrt{\mathbf{p}^2 + m^2}$$

EOS

$$\frac{P}{\epsilon} = \frac{1}{3 + \frac{m}{T} K_1(m/T)/K_2(m/T)}$$

energy density

$$\epsilon = \rho m \left(\frac{K_1(m/T)}{K_2(m/T)} + 3\frac{T}{m} \right)$$



Shear Viscosity: massive case

Why consider $m \neq 0$?

Quasi-particle model: $M(T)$, reproduce E.O.S from Lattice QCD.

Chapman-Enskog: first order

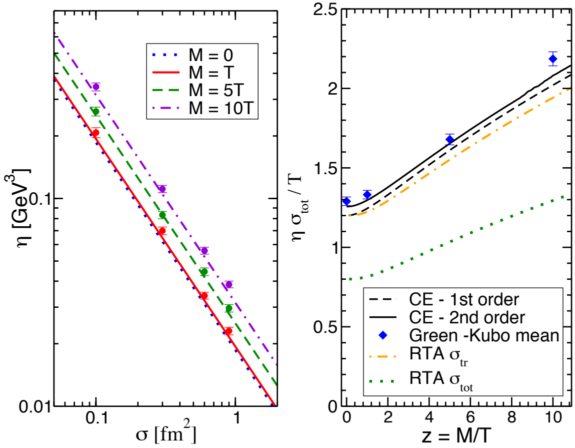
$$\eta = \frac{T}{\sigma_{tot}} h(z) \qquad z = \frac{M}{T}$$

$$h(z) = \frac{15}{16} \frac{z^4 \left[K_3(z)\right]^2}{(15z^2 + 2)K_2(2z) + (3z^3 + 49z)K_3(2z)}$$

N. Moroz, arXiv:1112.0277 1 dicembre (2011)

A. Wiranata and M. Prakash *Phys.Rev. C***85** (2012) 054908

Shear Viscosity: massive case



CE first order

$$\eta = \frac{T}{\sigma_{tot}} h(z)$$

Green-Kubo

$$\eta = \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle \tau$$

Chapman-Enskog

Chapman-Enskog: first order

$$\eta_{CE}^{1st} = 10T \left[\frac{K_3(z)}{K_2(z)}\right]^2 \frac{1}{c_{00}} = g(m_D, T)\frac{T}{\sigma_{tot}}$$

$$c_{00} = 16\left[\omega_2^{(2)} - z^{-1}\omega_1^{(2)} + (3z^2)^{-1}\omega_0^{(2)}\right]$$

$$\omega_i^{(2)} = \frac{z^3}{[K_2(z)]^2}\int_1^\infty dy(y^2-1)^7 y^i K_j(2zy)\,\sigma_{tr}$$

$j = \frac{5}{2} + \frac{1}{2}(-1)^i, \quad y = \frac{\sqrt{s}}{2M}$

$\sigma_{tr} = \int d\Omega\, \sigma(s,\Theta)\sin^2\Theta$