A new shock capturing numerical scheme for ideal hydrodynamics

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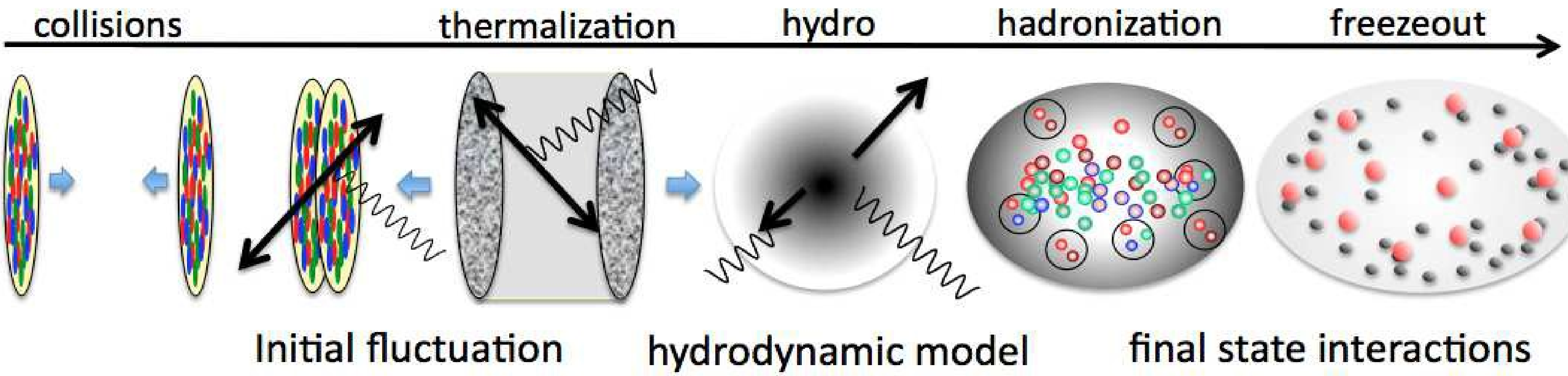
Hot Quarks,
21-29. 9. 2014

A heavy-ion collision



C. Nonaka, M. Asakawa, arXiv:1204.4795v2 [nucl-th]

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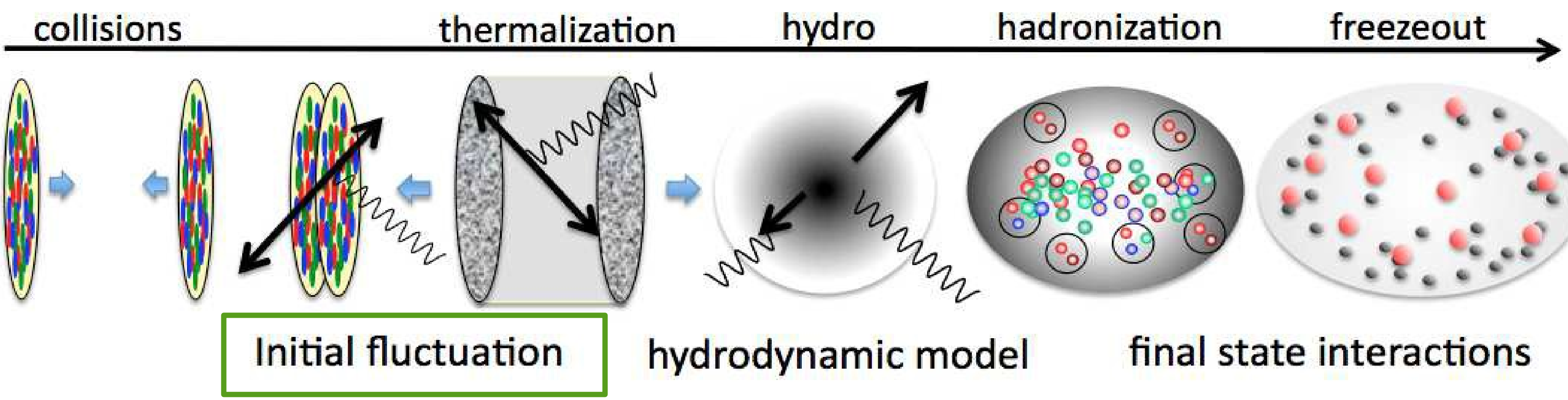

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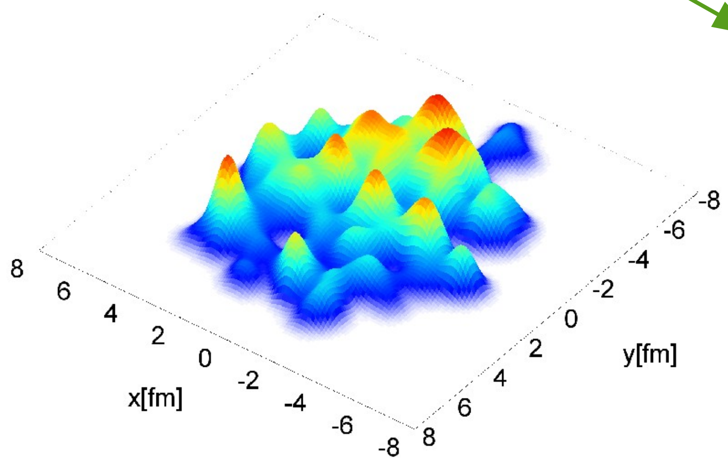
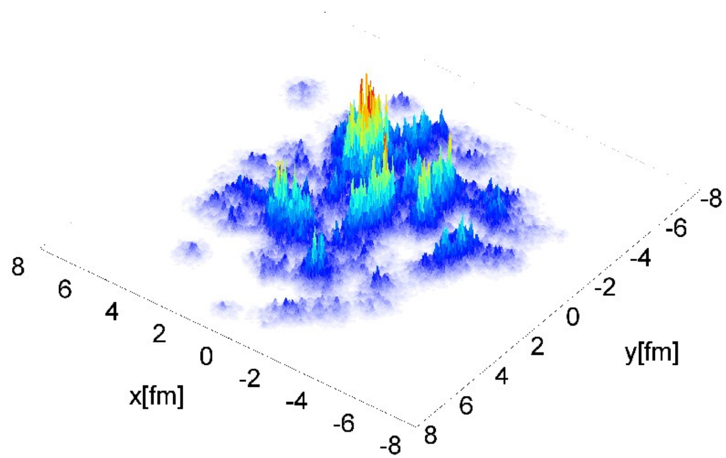


MC Glauber



IP Glasma

B. Schenke et al., Phys. Rev. Lett. 108 (2012) 252301

Relativistic hydrodynamics

- Ideal hydrodynamics:

$$\partial_\mu n^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$p = p(\epsilon, n)$$

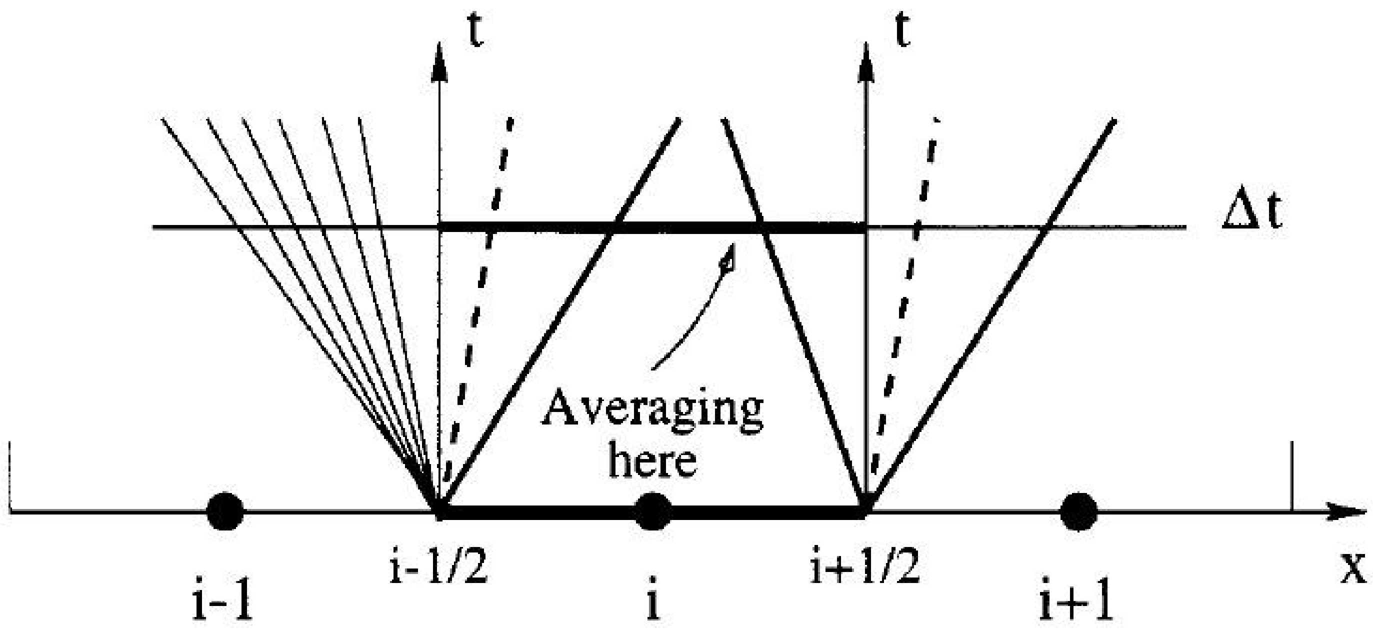
$$T^{\mu\nu}_{(0)} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

- Viscous hydrodynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}$$

Our numerical scheme

- Godunov method: computing the flow of conserved variables on cell boundaries solving the Riemann problem
- Exact solution of relativistic Riemann problem with an arbitrary EoS



Testing the scheme

- Sound wave propagation: precision and numerical viscosity
  -> initial conditions:

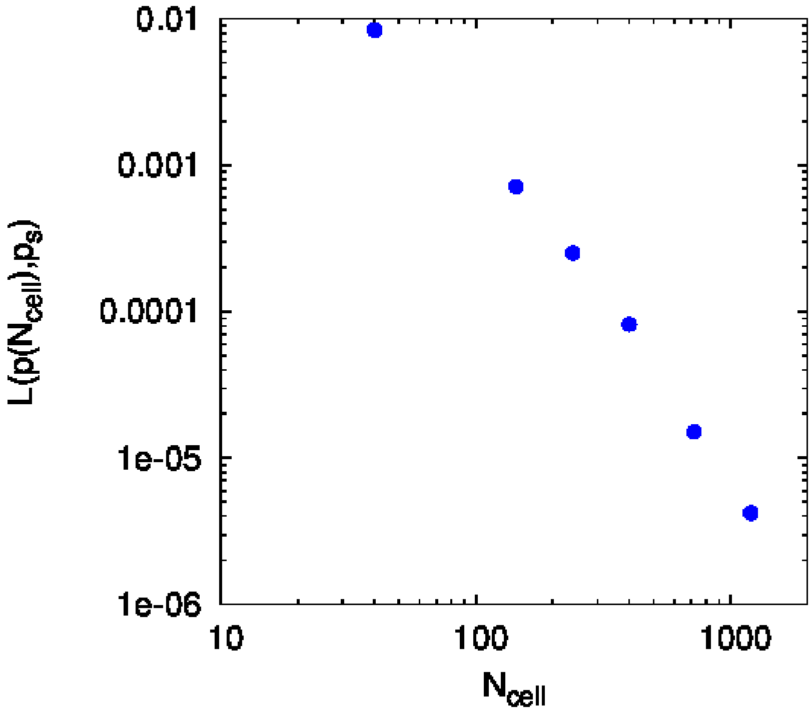
$$p_{init}(x) = p_0 + \delta p \sin(2\pi x/\lambda), v_{init}(x) = \frac{\delta p}{c_{s0}(e_0 + p_0)} \sin(2\pi x/\lambda)$$

$$p_0 = 10^3 fm^{-4}, \delta p = 10^{-1} fm^{-4}, \lambda = 2 fm$$

Sound wave propagation

L1 norm:

$$L(p(N_{cell}), p_s) = \sum_{i=1}^{N_{cell}} |p(x_i, \lambda/c_s; N_{cell}) - p_s(x_i, \lambda/c_s)| \frac{\lambda}{N_{cell}}$$



Our numerical scheme

Sound wave propagation

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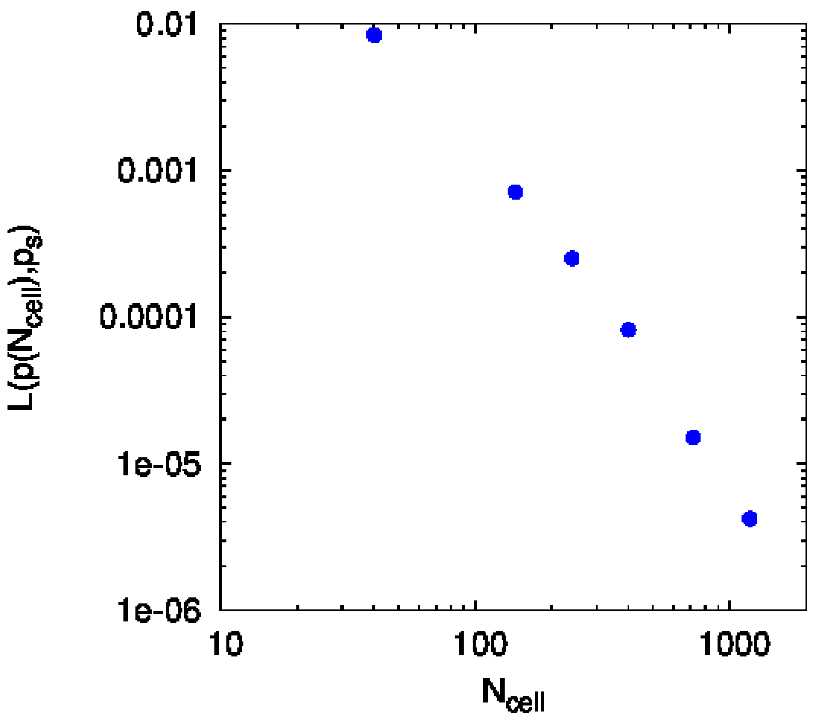

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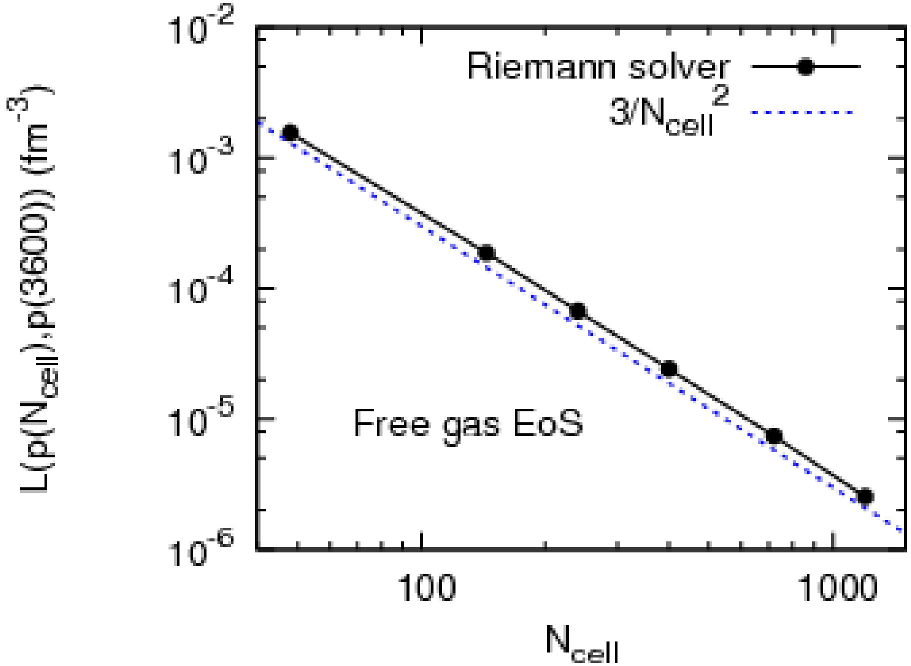
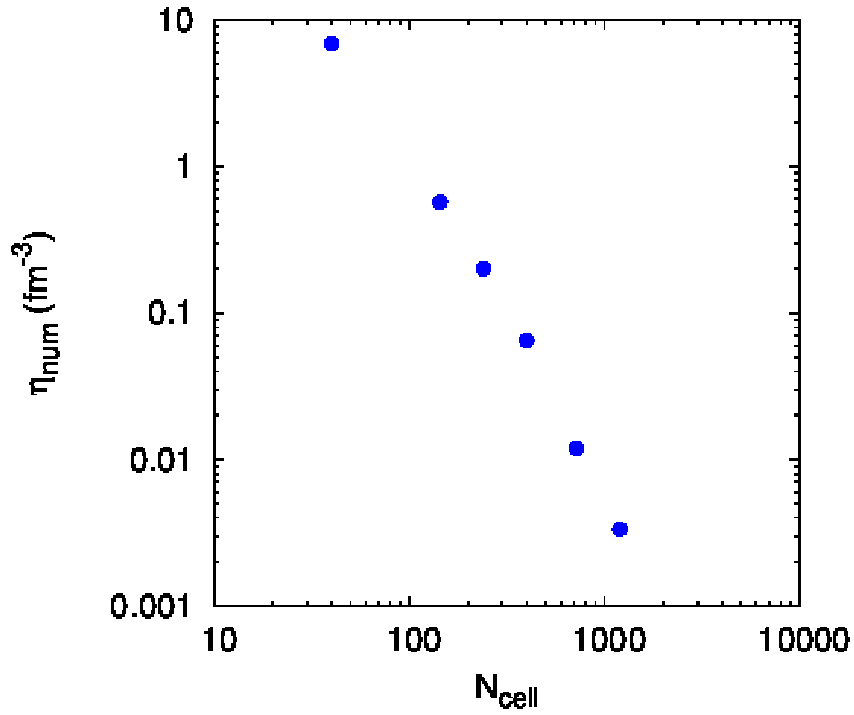


Y. Akamatsu et al., J. Comput. Phys. 256 (2014) 34

Sound wave propagation

Numerical viscosity

$$\eta_{num}=\frac{-3\lambda}{8\pi^2}c_{s0}(e_0+p_0)\ln[1-\frac{\pi}{2\lambda\delta p}L(p(N_{cell},p_s))]$$



Our numerical scheme

Sound wave propagation

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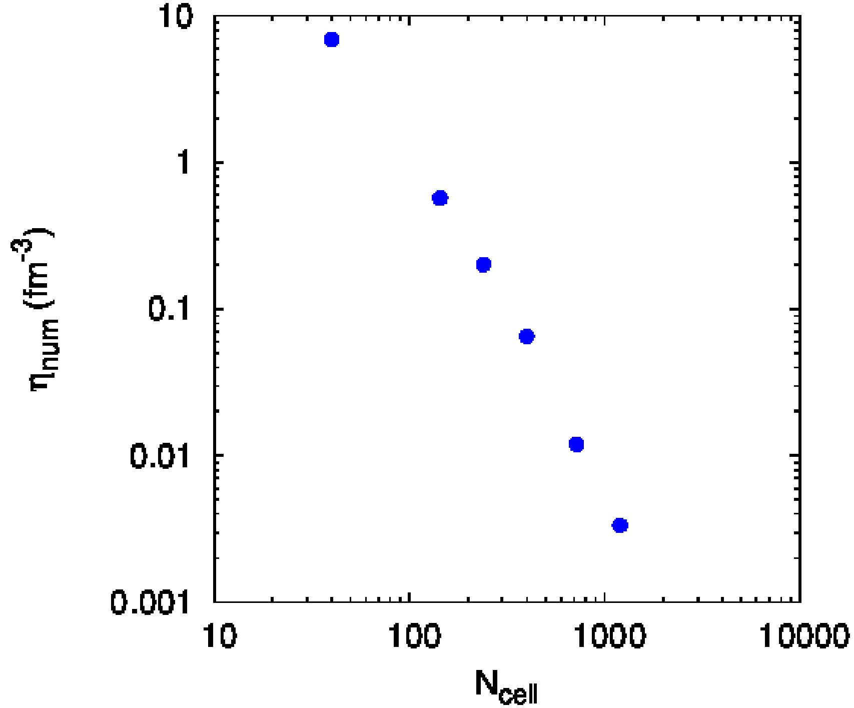

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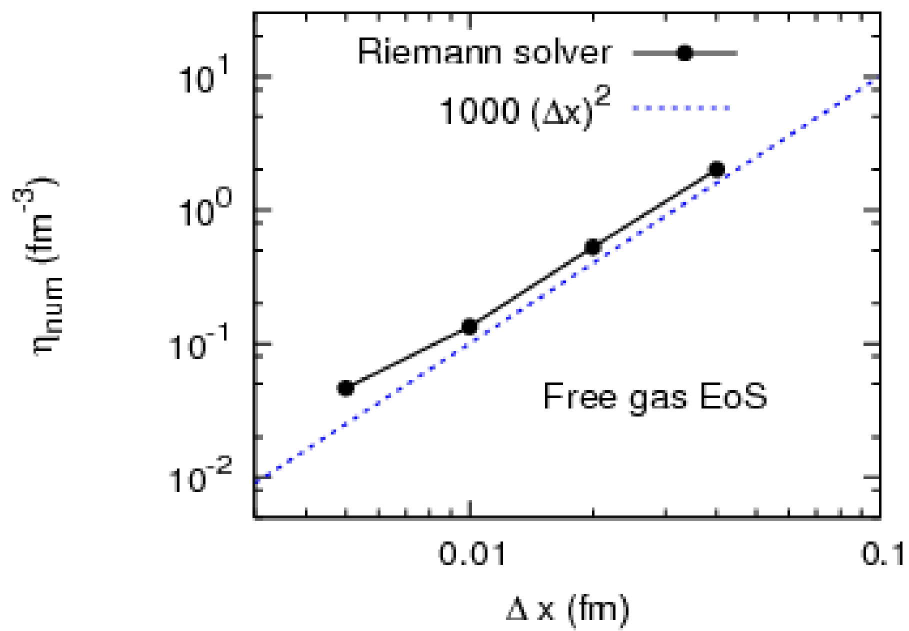

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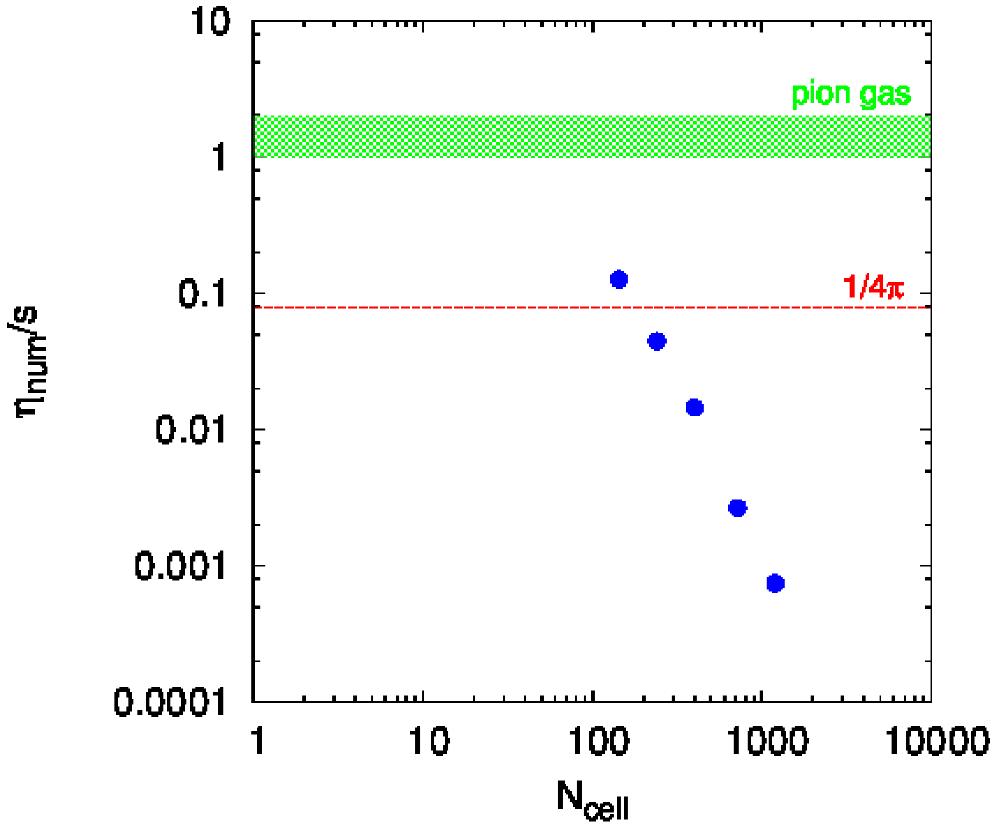
Sound wave propagation



Viscosity per entropy:
Our scheme's numerical viscosity compared to pion gas and minumum viscosity of QGP

Testing the scheme

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-> initial conditions:

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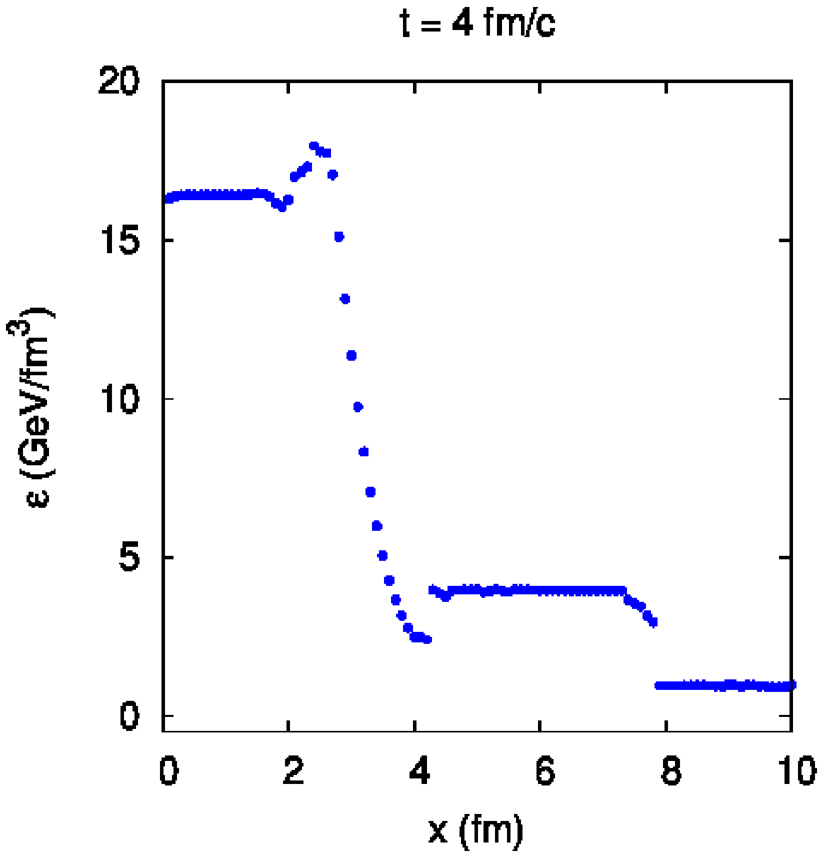
$$p_0=10^3 fm^{-4},\delta p=10^{-1} fm^{-4},\lambda=2 fm$$

• Shock tube problem: response to discontinuity in energy density
-> initial conditions:

$$T_L=400\,MeV,T_R=200\,MeV$$

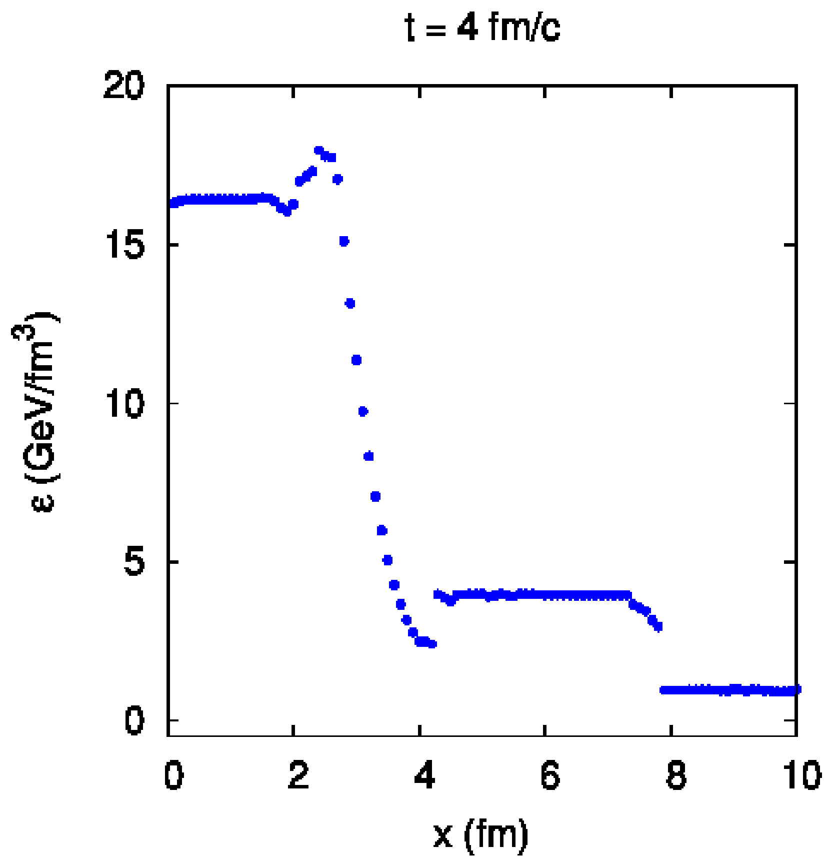
$$\lambda=10\,fm,N_{cell}=100,\Delta t=0.04\,fm.c^{-1}$$

Shock tube problem



Our numerical scheme

Shock tube problem



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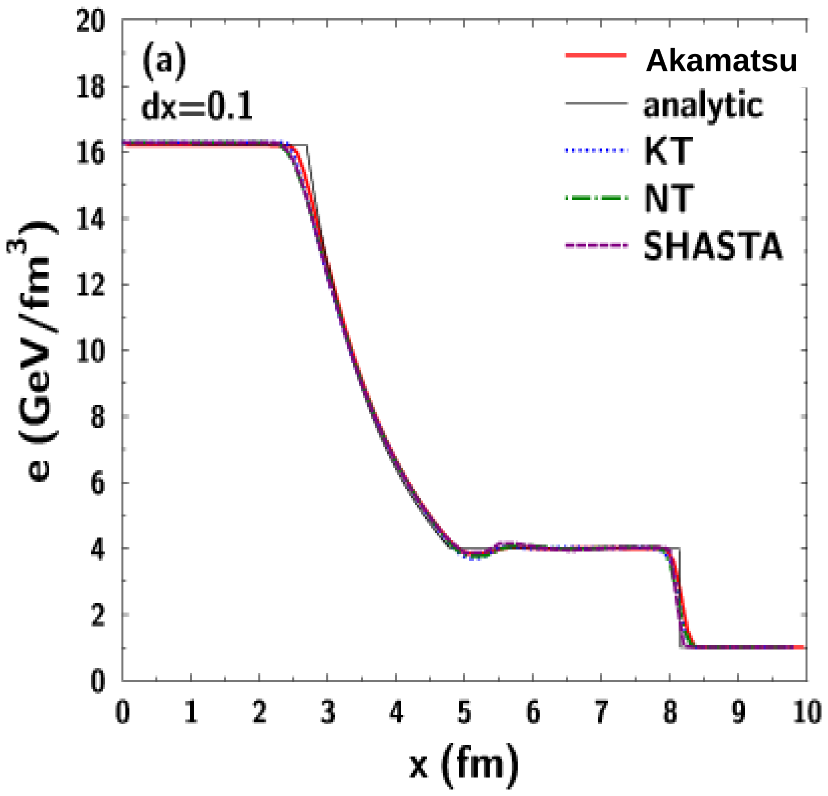

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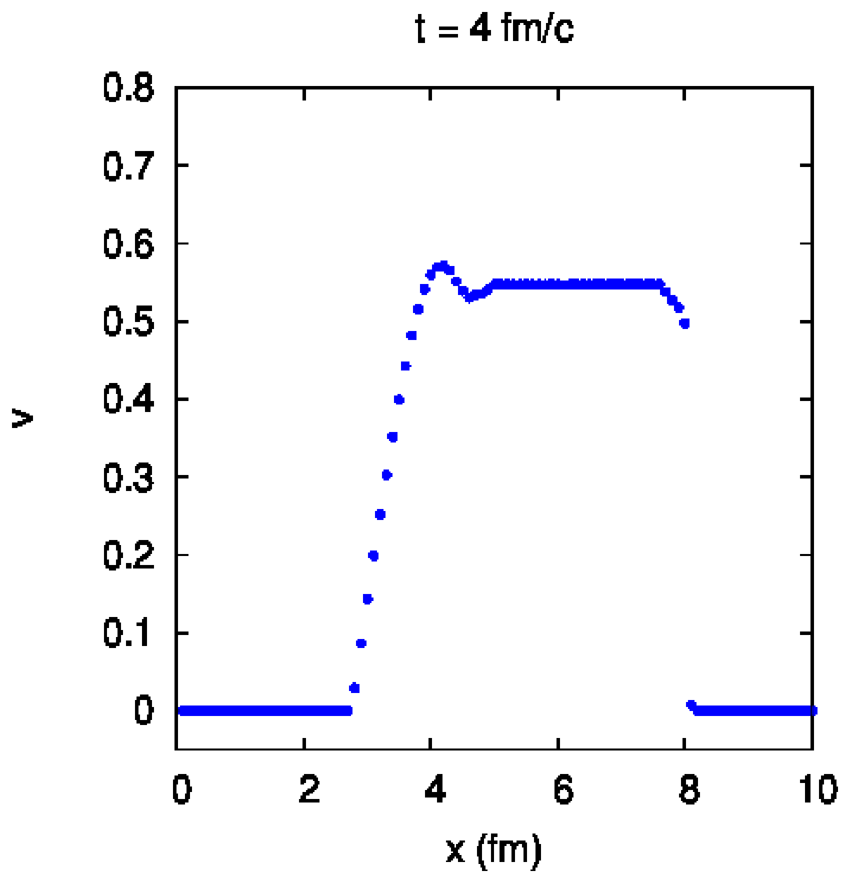
Shock tube problem



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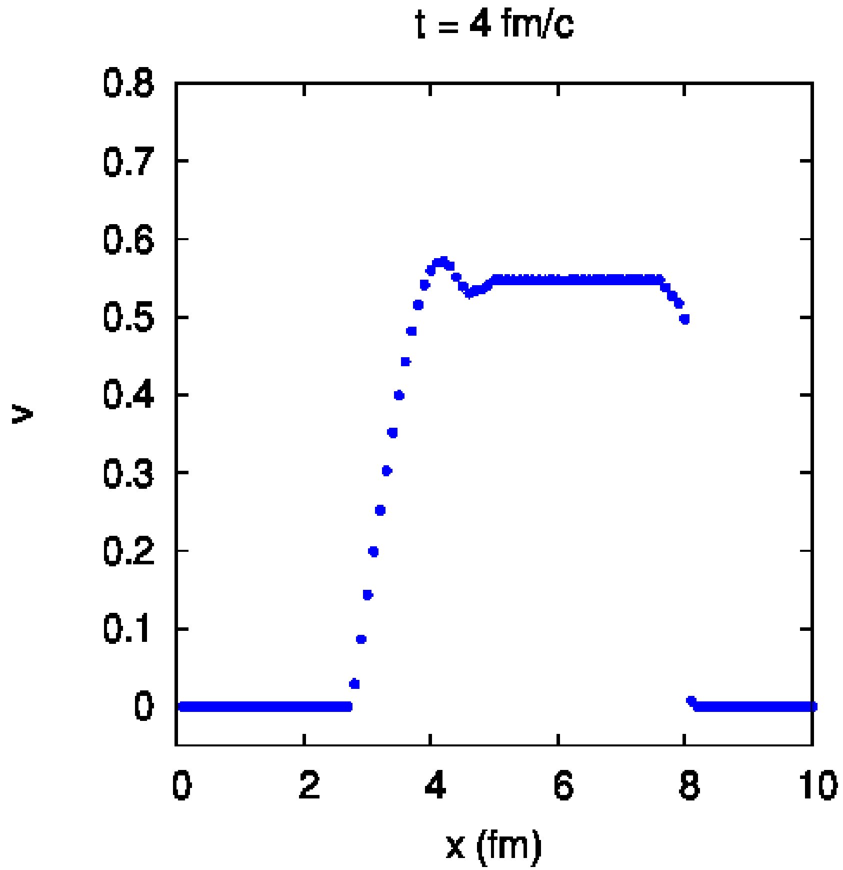
Shock tube problem



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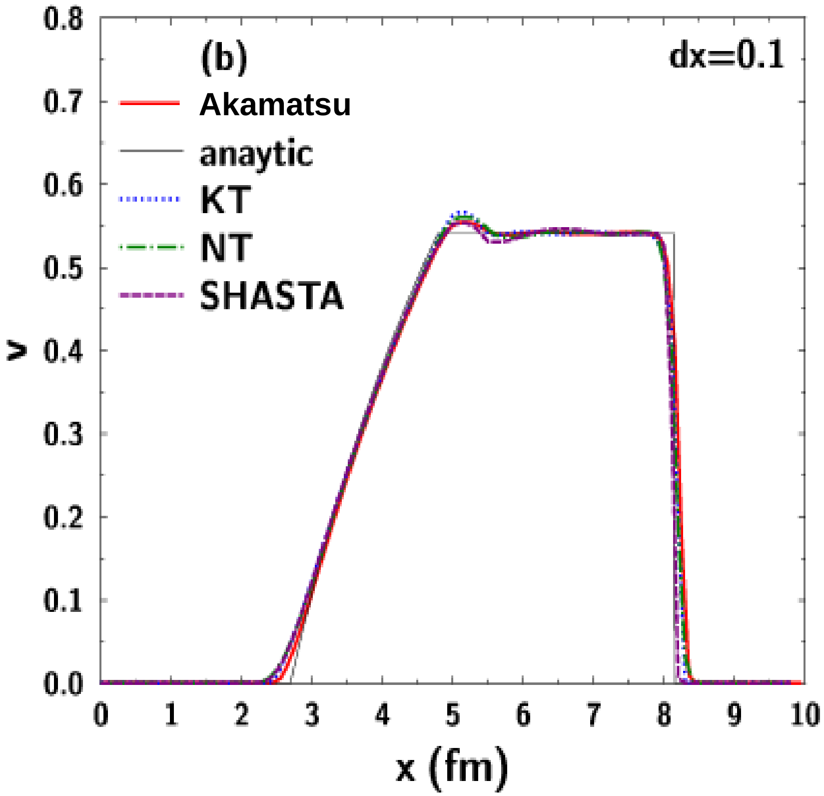

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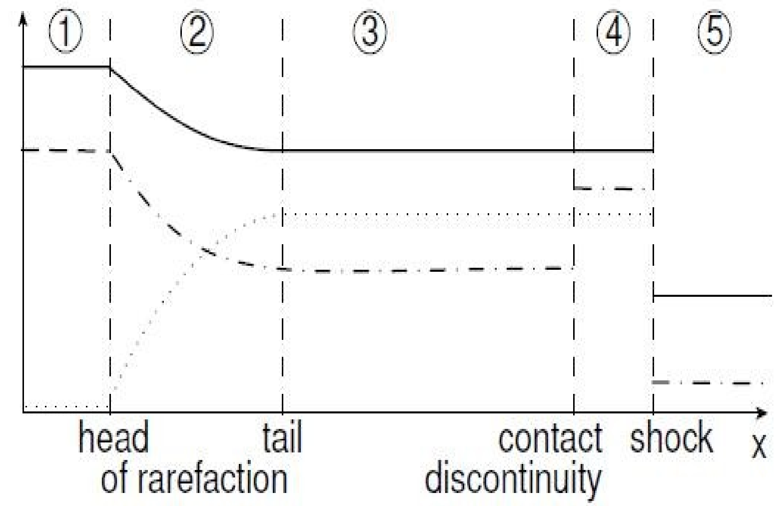
Conclusion

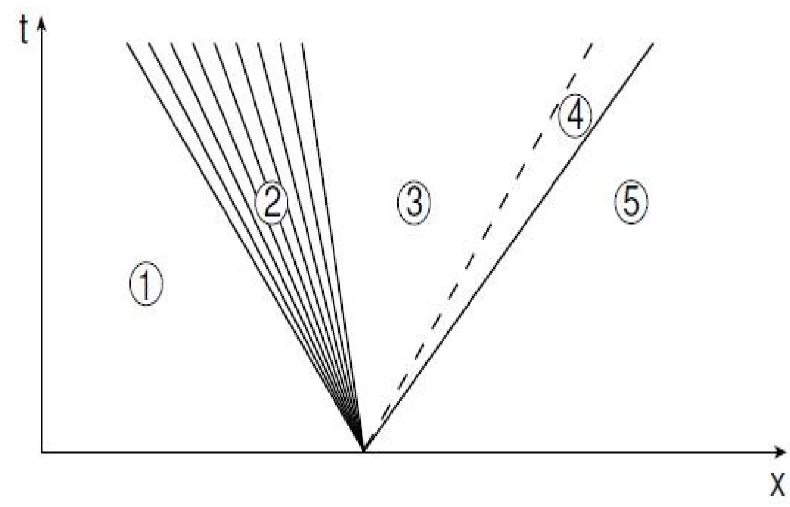
- Ideal hydrodynamics code for quark-gluon plasma modeling
- Successfull testing in 1D
- Future: viscous corrections, 3D and more testing
- Simulating jets penetrating the medium and the response of the medium to the energy deposited

Riemann problem

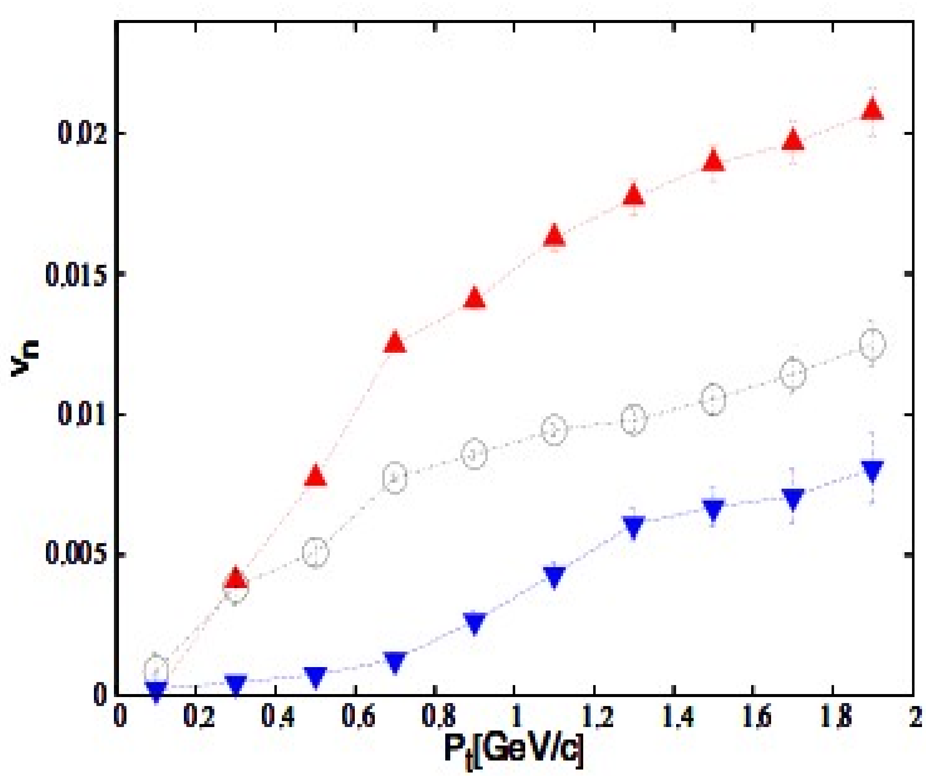
- Initial conditions problem: two constant states
- Exact solution: constructing flows – rarefaction wave, shock wave
- Interface:

$$v_L^x(\epsilon_{new}) = v_R^x(\epsilon_{new})$$





Anisotropic coefficients



M. Schulc, B. Tomasik: arXiv:1409.6116