Higher Order Fluctuations of Conserved Charges in Heavy Ion Collisions

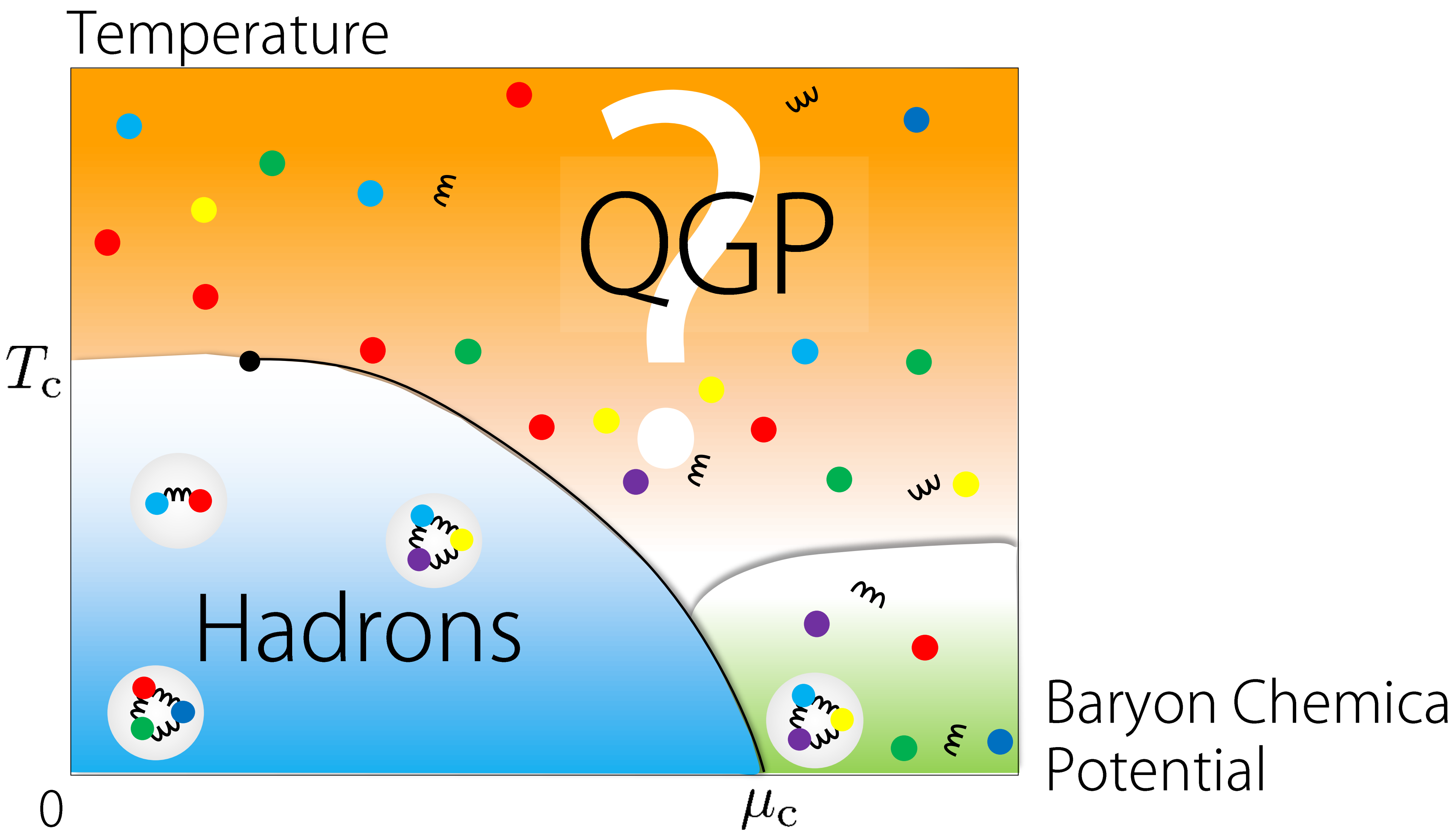
Miki Sakaida (Osaka University)

Masayuki Asakawa, Masakiyo Kitazawa

arXiv:1409.6866[nucl-th] ←New!!

2014.9.25. Hot Quarks2014 @Las Negras, Spain

QCD Phase Diagram



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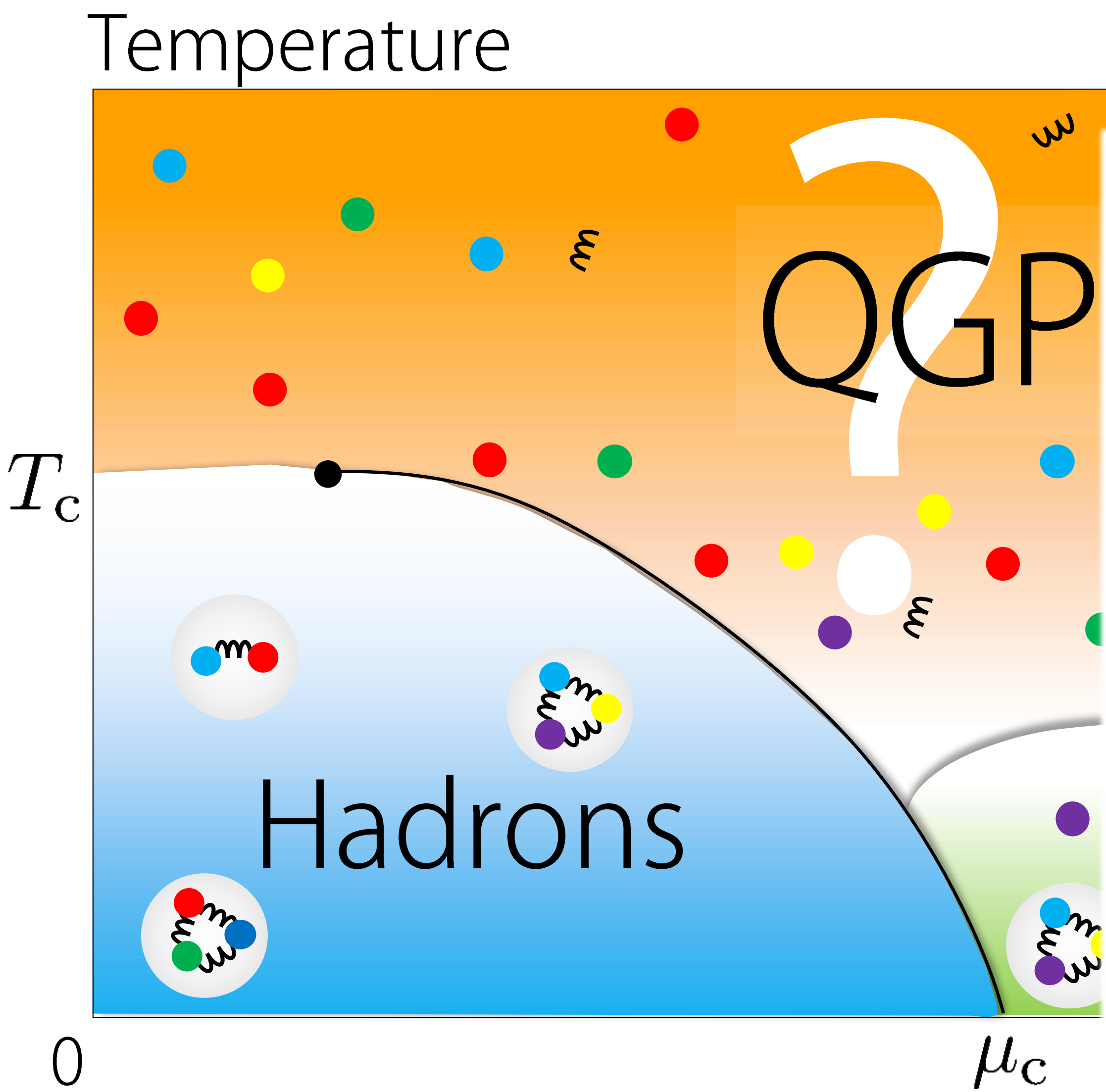
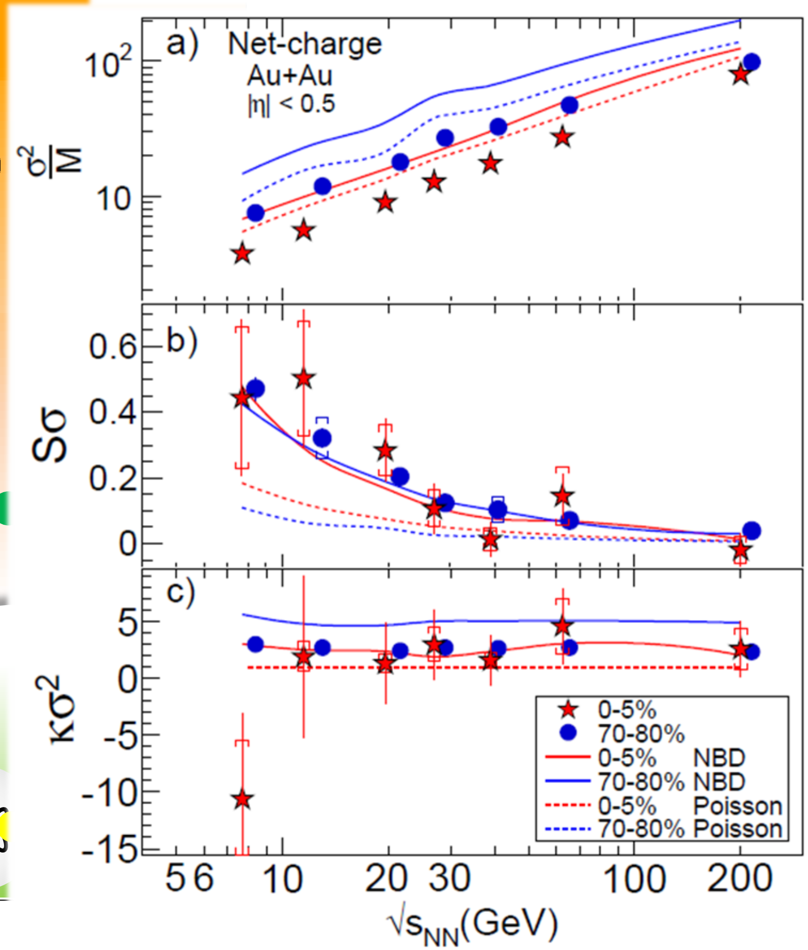
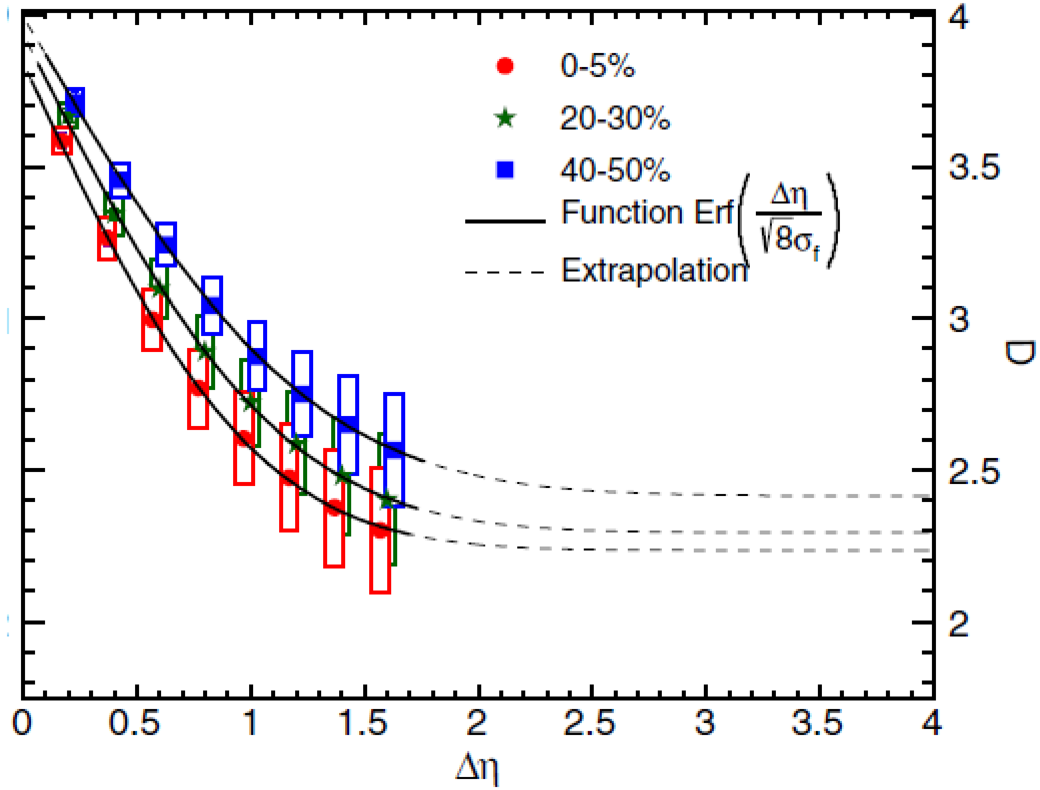


STAR, PRL113, 092301(2014)



QCD Phase Diagram

ALICE, PRL**110**, 152301 (2013)



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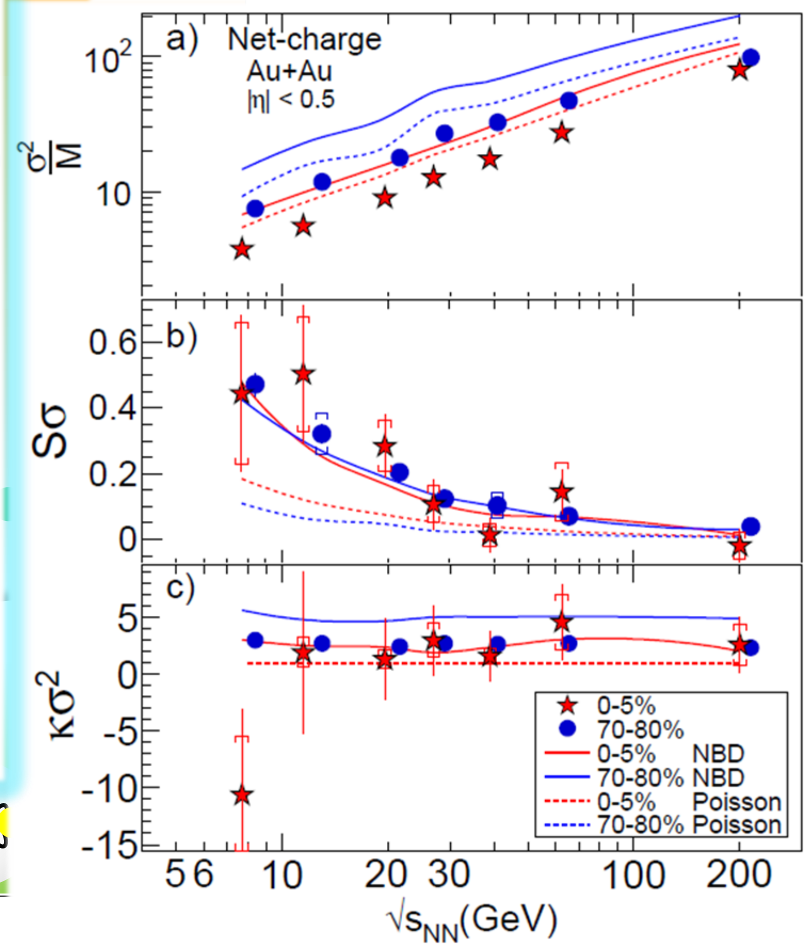
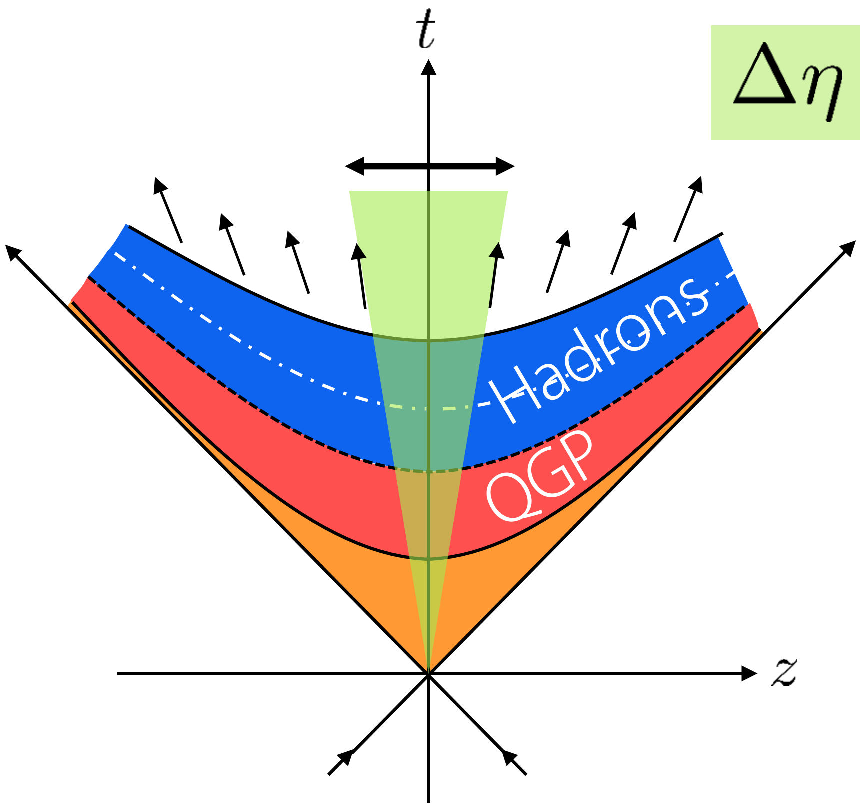


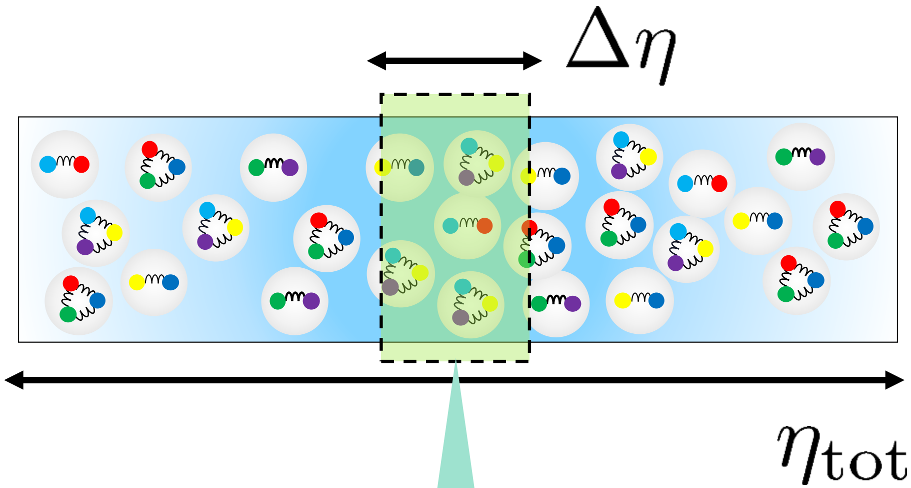
Fluctuations of Conserved Charges
tell us much Information
about the Hot Medium...

Fluctuations of Conserved Charges

Event-by-Event Charge Fluctuations

$\Delta\eta$: Rapidity Window

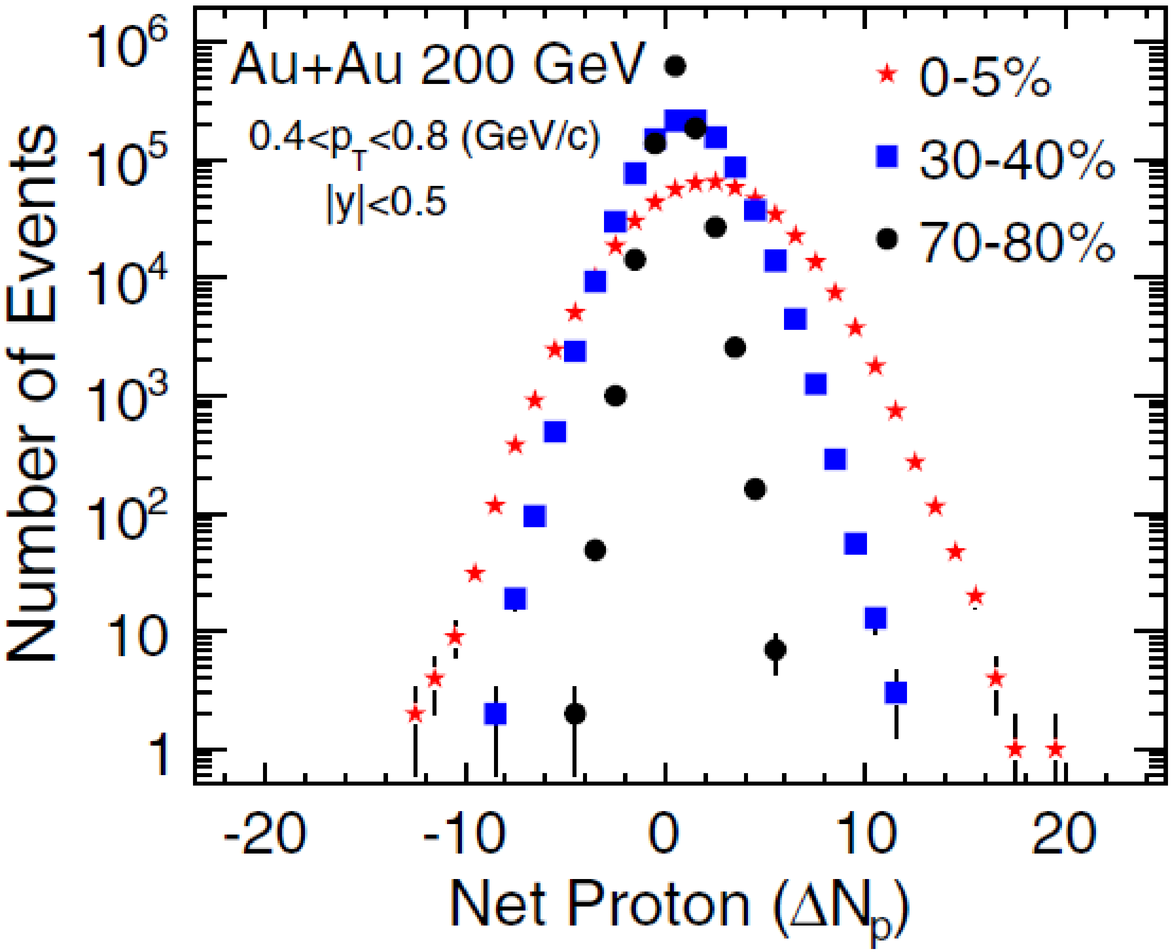




The number of conserved charges are counted in a given $\Delta\eta$ in each event

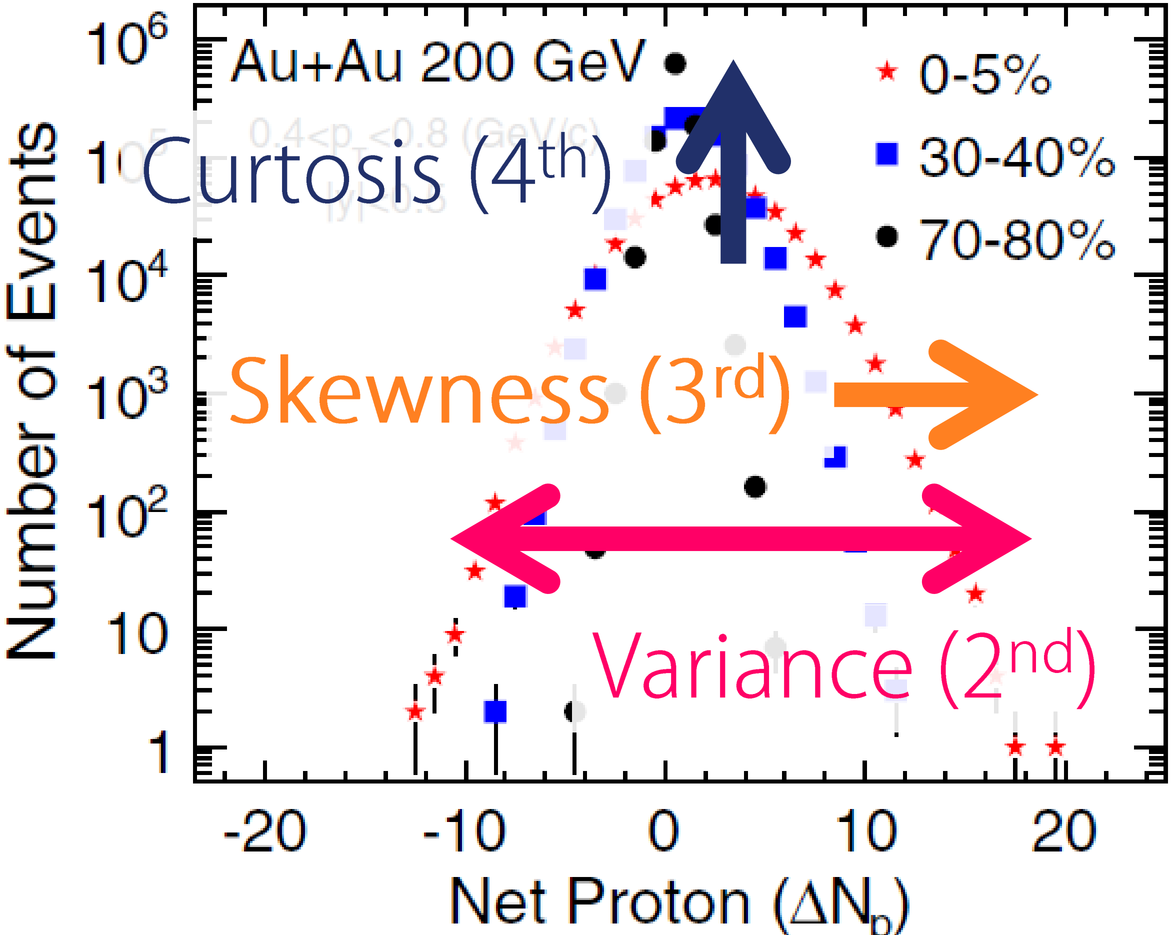
Event-by-Event Charge Fluctuations

STAR, PRL **105** (2010)



Event-by-Event Charge Fluctuations

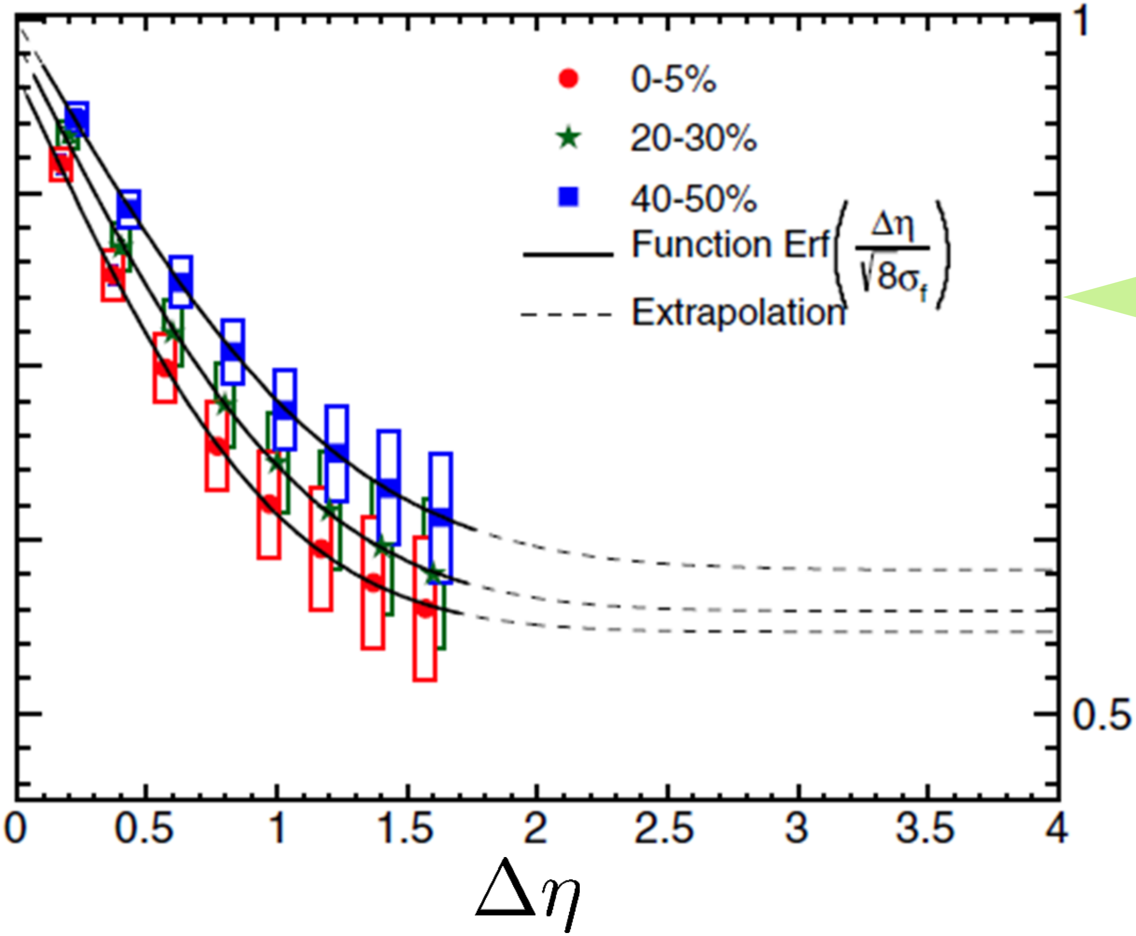
STAR, PRL**105** (2010)



In HICs,
Higher order
Fluctuations
can also
be observed !!

$\Delta\eta$ Dependence of Charge Fluctuation @ ALICE

ALICE, PRL **110**, 152301 (2013)



2nd order Electric Charge Fluctuation

$$\frac{\langle \delta N_Q^{(\mathrm{net})2} \rangle_c}{\langle N_Q^+ + N_Q^- \rangle_c}$$

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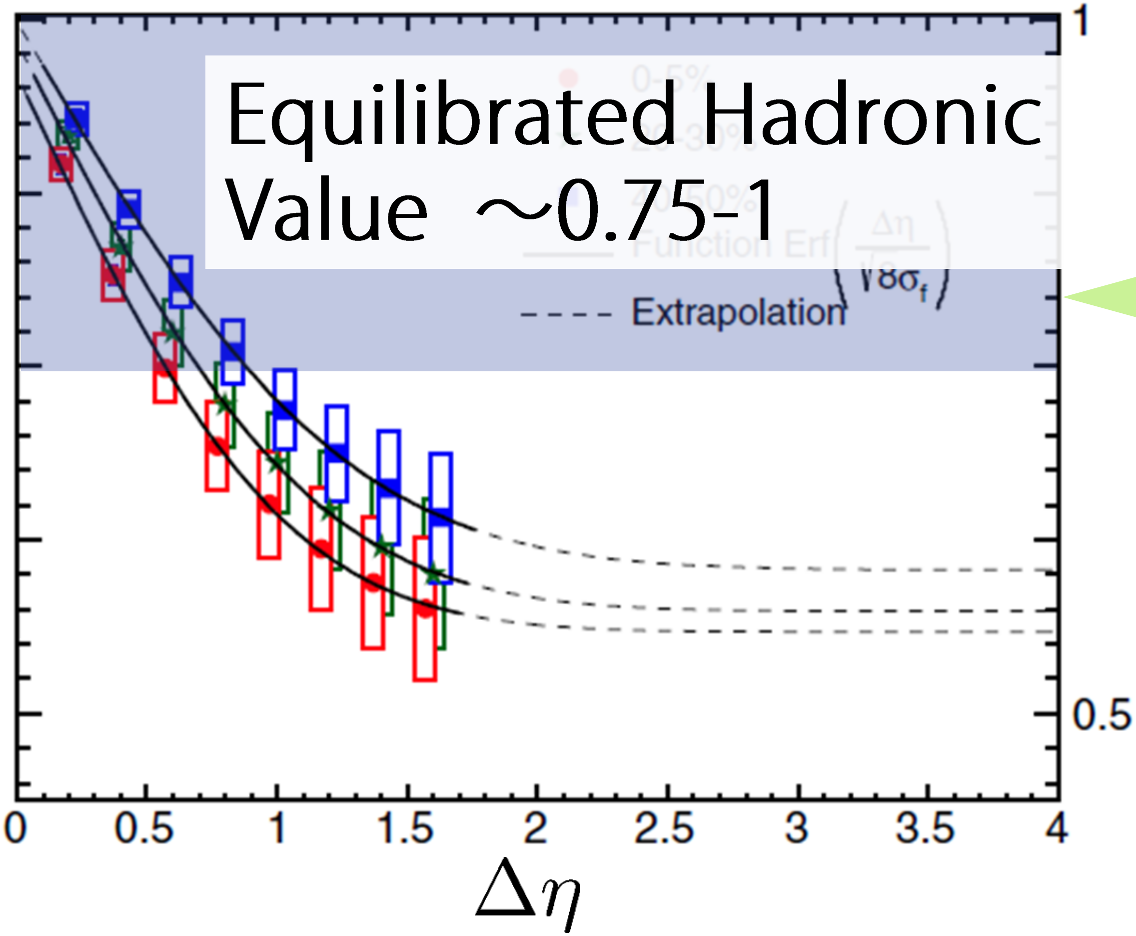

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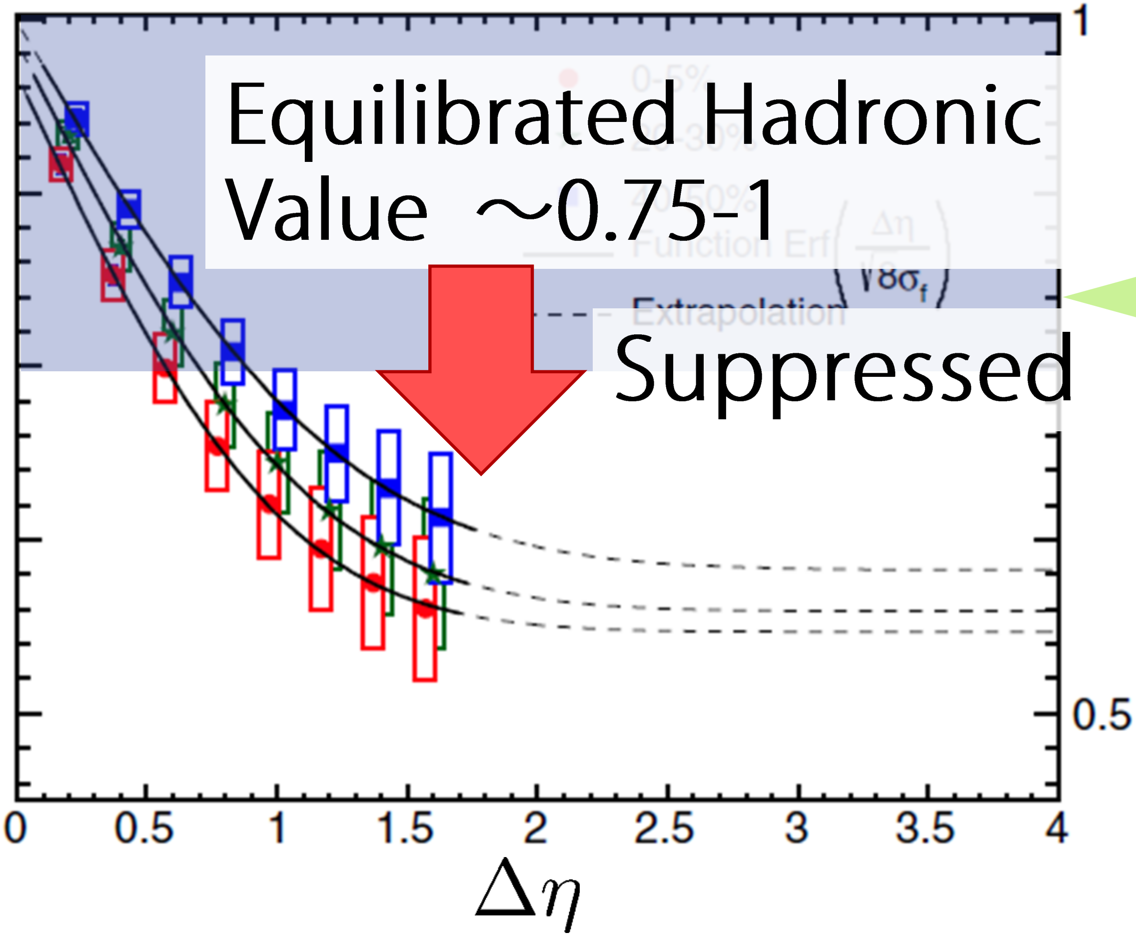
$$\frac{\langle \delta N_{\mathrm{Q}}^{(\mathrm{net})2} \rangle_{\mathrm{c}}}{\langle N_{\mathrm{Q}}^{+} + N_{\mathrm{Q}}^{-} \rangle_{\mathrm{c}}}$$

Asakawa, Heinz, Muller (2000)
Jeon, Koch (2000)

ALICE, PRL **110**, 152301 (2013)



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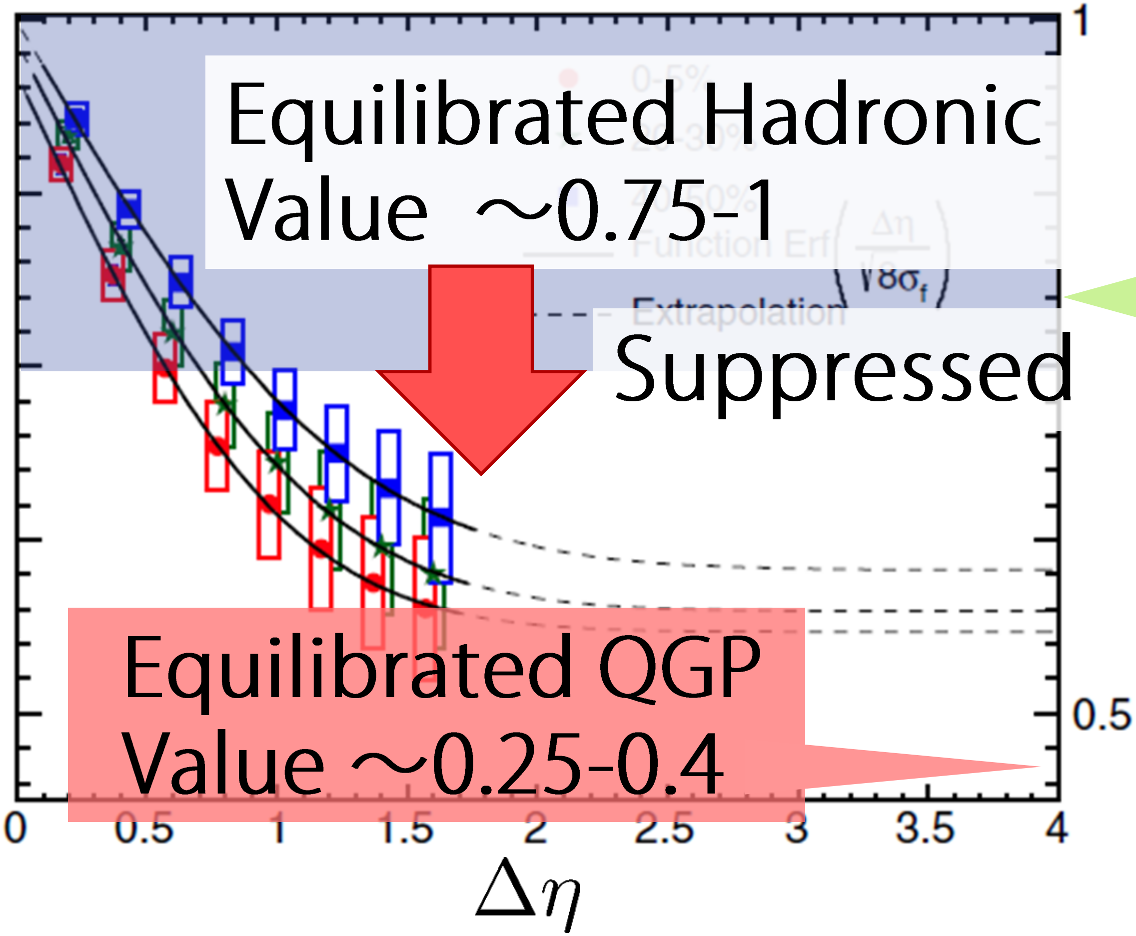

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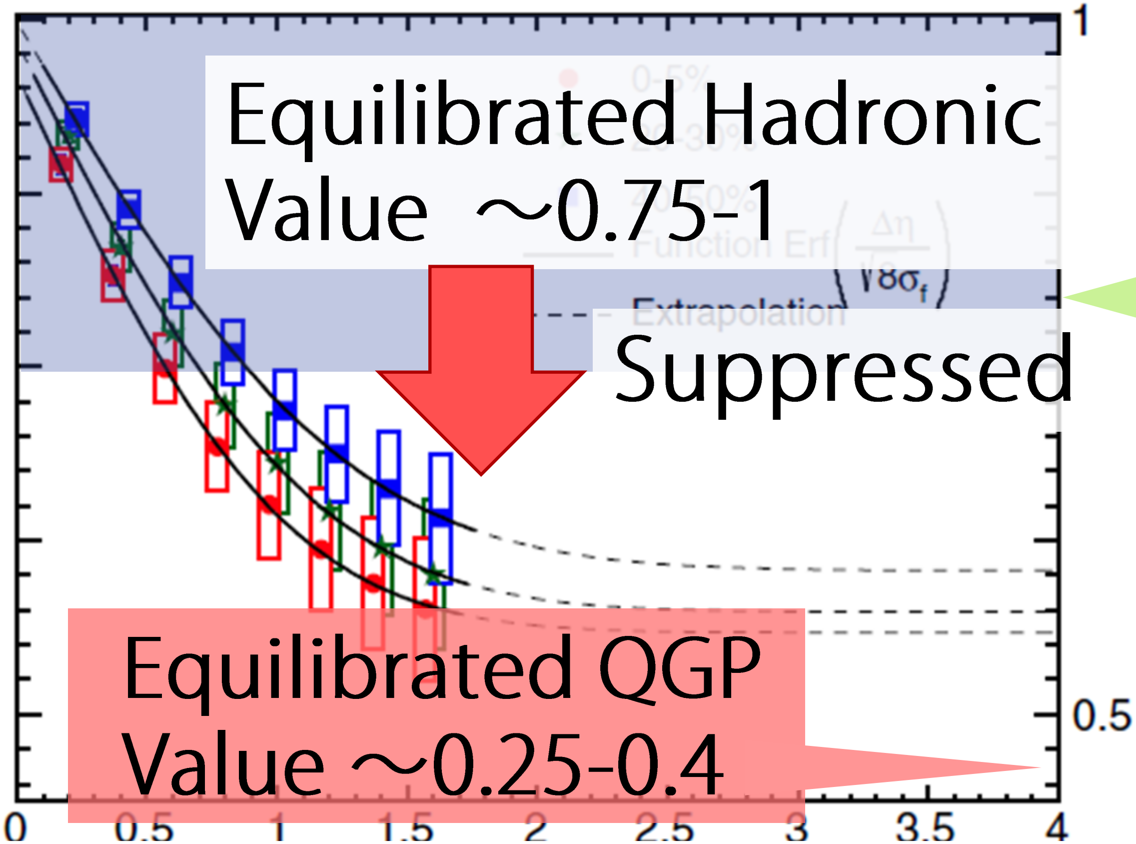


2nd order
Electric Charge
Fluctuation

$$\frac{\langle \delta N_{\mathrm{Q}}^{(\mathrm{net})2} \rangle_{\mathrm{c}}}{\langle N_{\mathrm{Q}}^{+} + N_{\mathrm{Q}}^{-} \rangle_{\mathrm{c}}}$$

Asakawa, Heinz, Muller (2000)

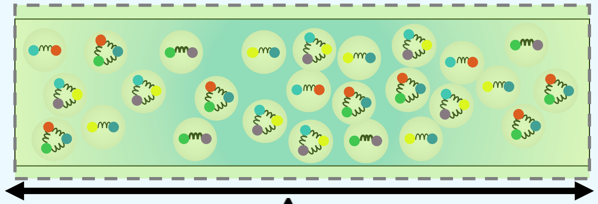
Fluctuations are not Equilibrated at freeze-out (?)

We can observe QGP Fluctuations for Larger $\Delta\eta$ (?)

Problem

Global Charge Conservation (GCC)

If one looks at the Total System,
#Conserved Charge
does NOT Fluctuate !!



$$\Delta\eta = \eta_{\mathrm{tot}}$$

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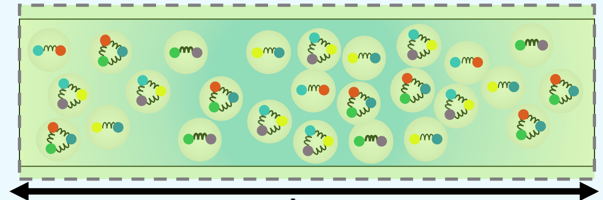

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※GCC Effect also causes Suppression !!

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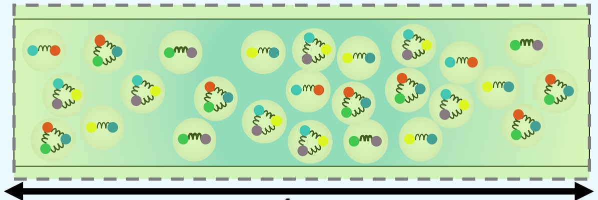
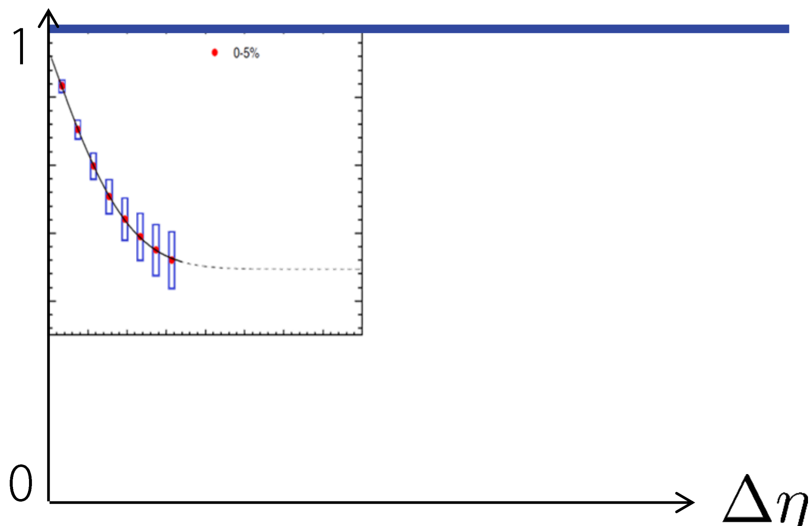


$\Delta\eta = \eta_{\rm tot}$

※GCC Effect also causes Suppression !!

2nd order Fluctuation $\langle Q^2_{(\rm net)}\rangle_c / \langle Q_{(\rm tot)}\rangle_c$



$\Delta\eta$: Rap. Win.

Equilibrated Hadronic Value
(**NOT** Consider GCC Effect)

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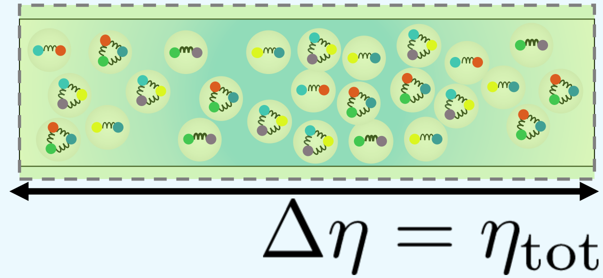

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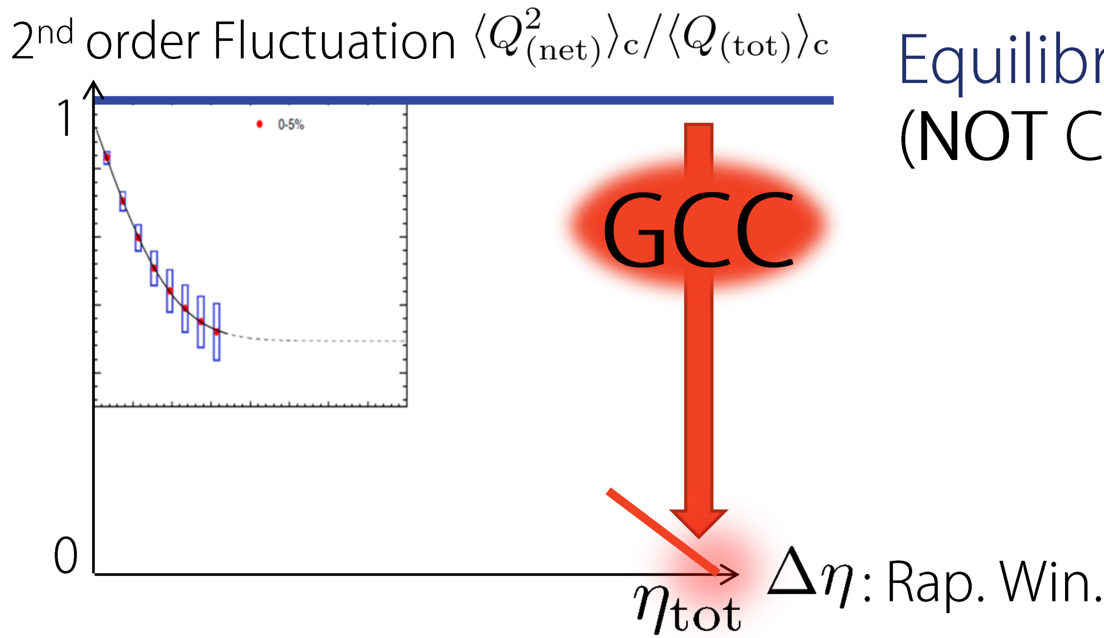

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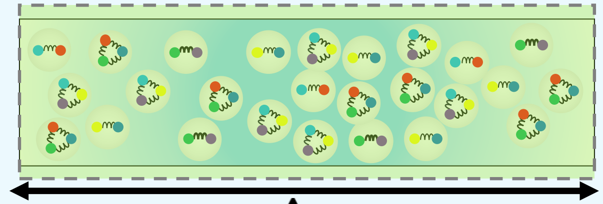

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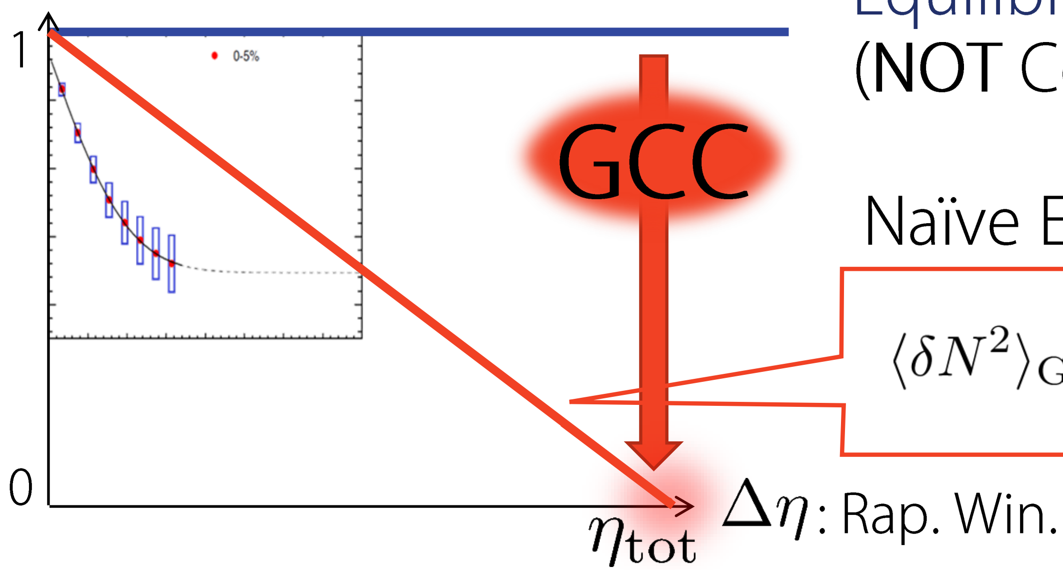


Equilibrated Hadronic Value
(**NOT** Consider GCC Effect)

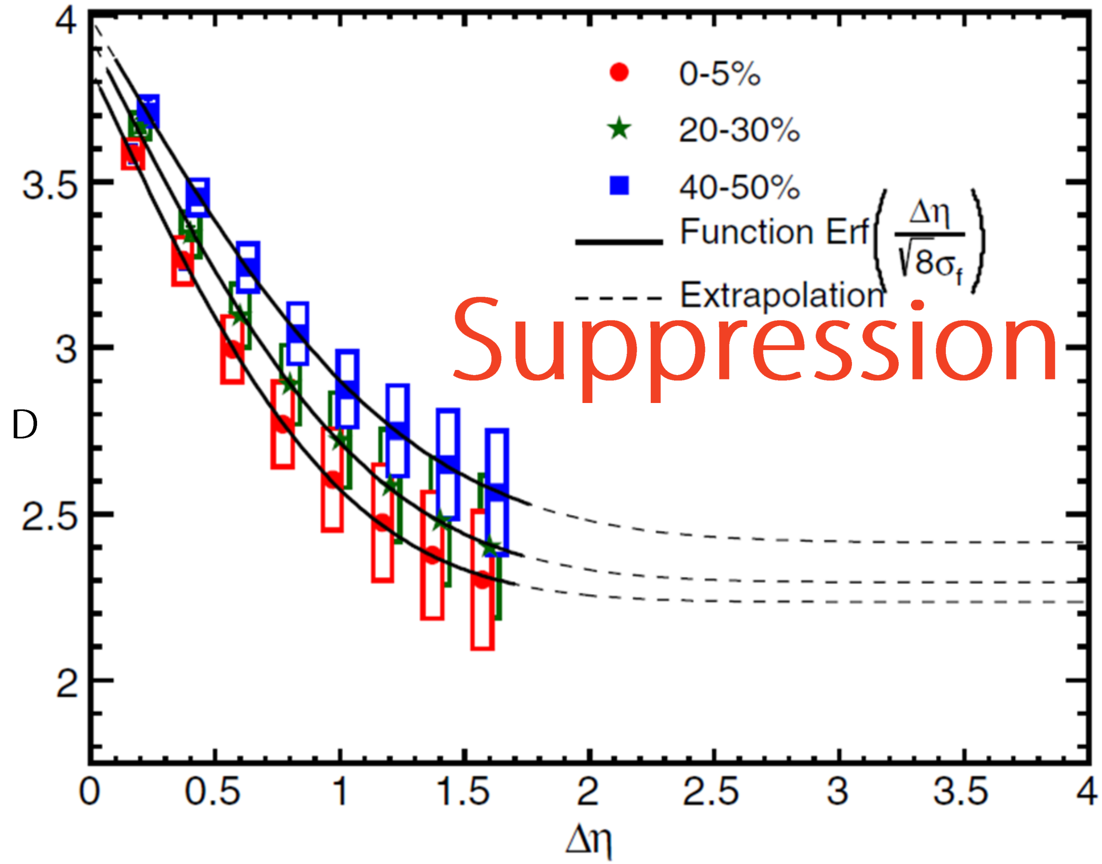
Naïve Estimate of GCC Effect

$$\langle \delta N^2 \rangle_{GCC} = \langle \delta N^2 \rangle_{inf} \times \left(1 - \frac{\Delta\eta}{\eta_{tot}}\right)$$

Bleicher, Jeon, Koch (2000)

What does the Suppression imply ?

Fluctuation is NOT equilibrated??



Global Charge Conservation ??

What does the Suppression imply ?

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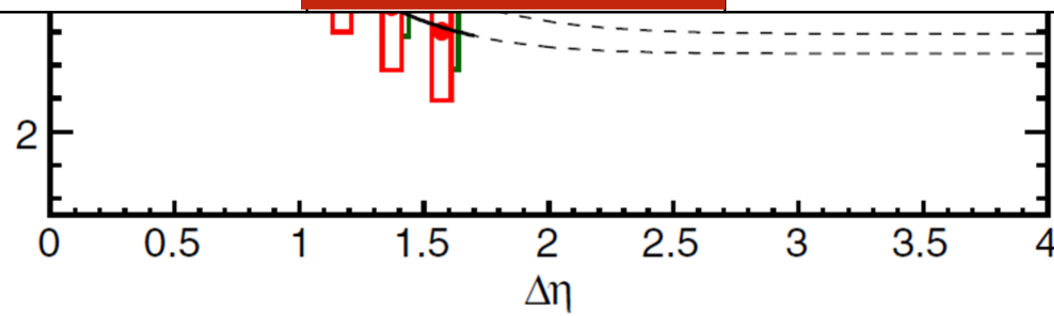
Previous Study	Global Charge Conservation	Time Evolution	Higher Fluctuations
Bleicher, Jeon, Koch (2000)	○	×	×
Shuryak, Stephanov (2001)	×	○	×
Kitazawa, Asakawa, Ono (2013)	×	○	○

?



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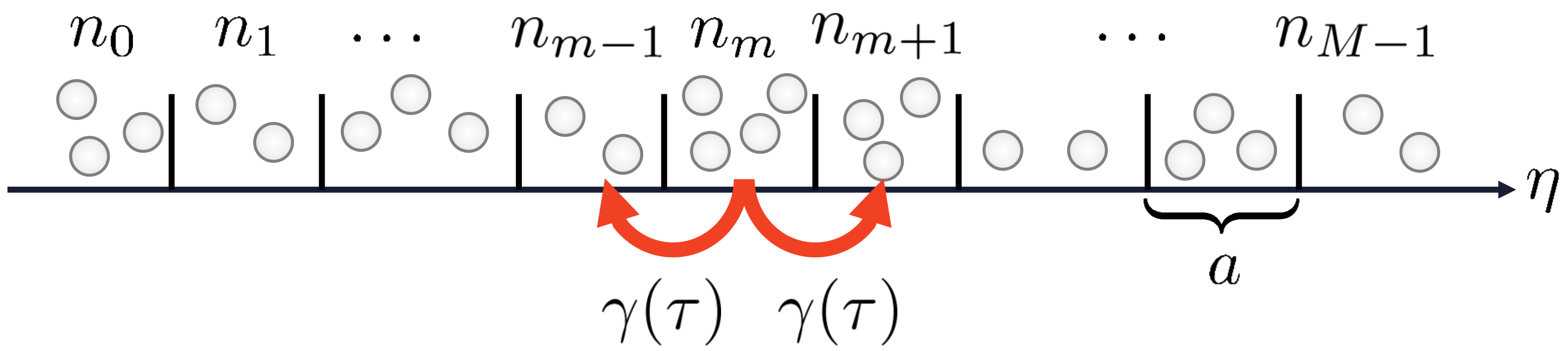
Kitazawa, Asakawa, Ono (2013)	×	○	○
Our Study	○	○	○

GCC Effects on Time Evolution of Fluctuations

Method

Diffusion Model For Hadrons (1D Brownian Motion)

Kitazawa, Asakawa, Ono, Phys. Rev. B**728,** 386 (2013)



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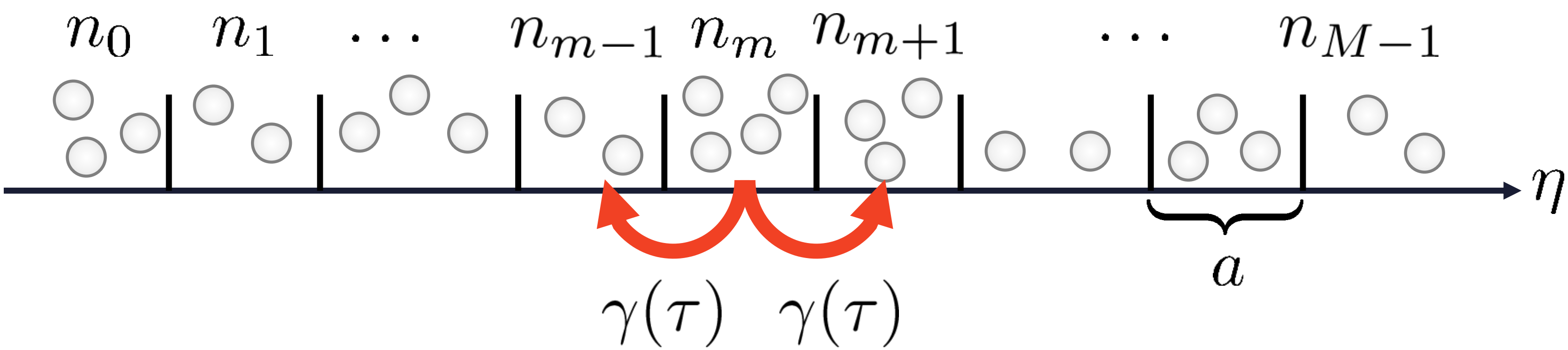
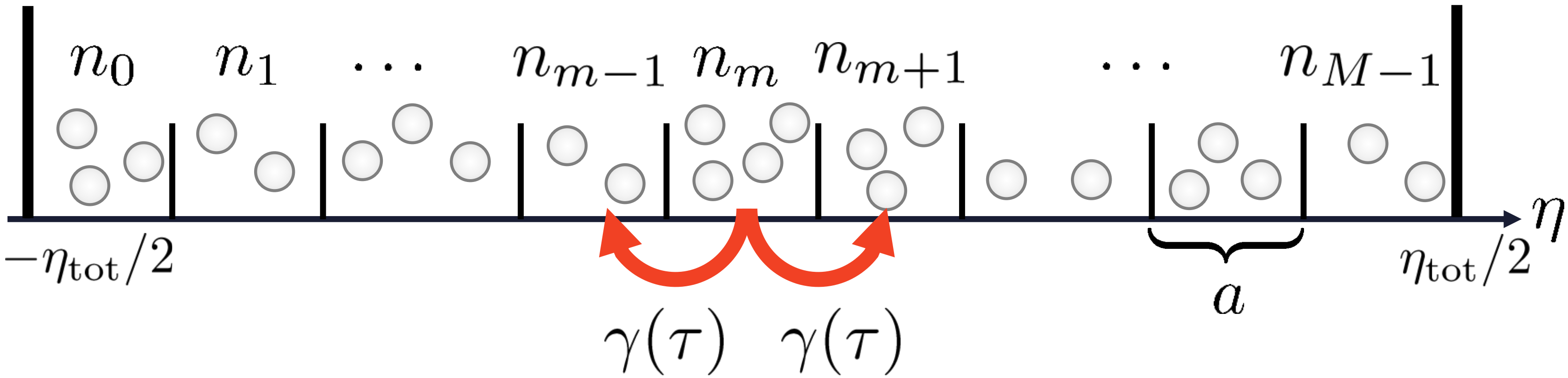


Diffusion Master Equation

→Higher Fluctuation ○

Diffusion Model For Hadrons (1D Brownian Motion)

MS, Asakawa, Kitazawa, arXiv:1409.6866[nucl-th]



Diffusion Master Equation

+

Boundary Condition(GCC Effect)

Particles do NOT flow in/out.

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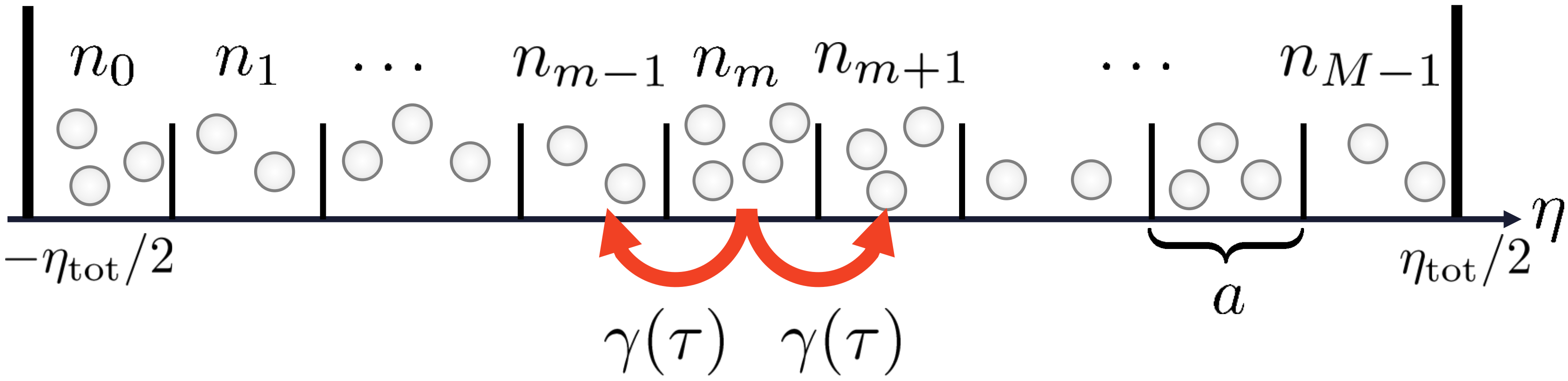

Diffusion Master Equation + **Boundary Condition(GCC Effect)**
Particles do NOT flow in/out.

* Diffusion from Hadronization to Thermal Freeze-out
* Initial Condition : Fluctuations in Thermal QGP

Diffusion Model For Hadrons (1D Brownian Motion)

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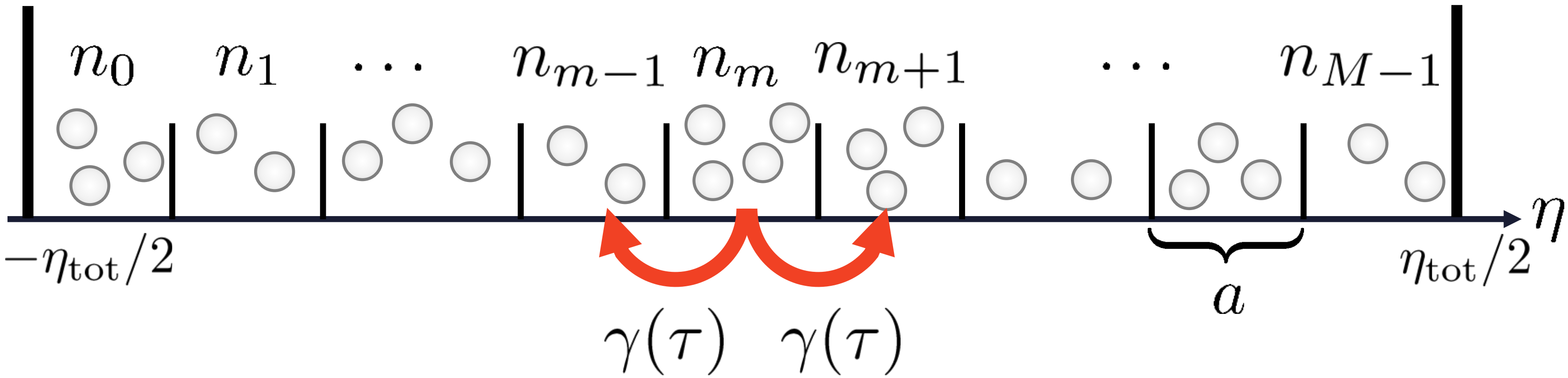

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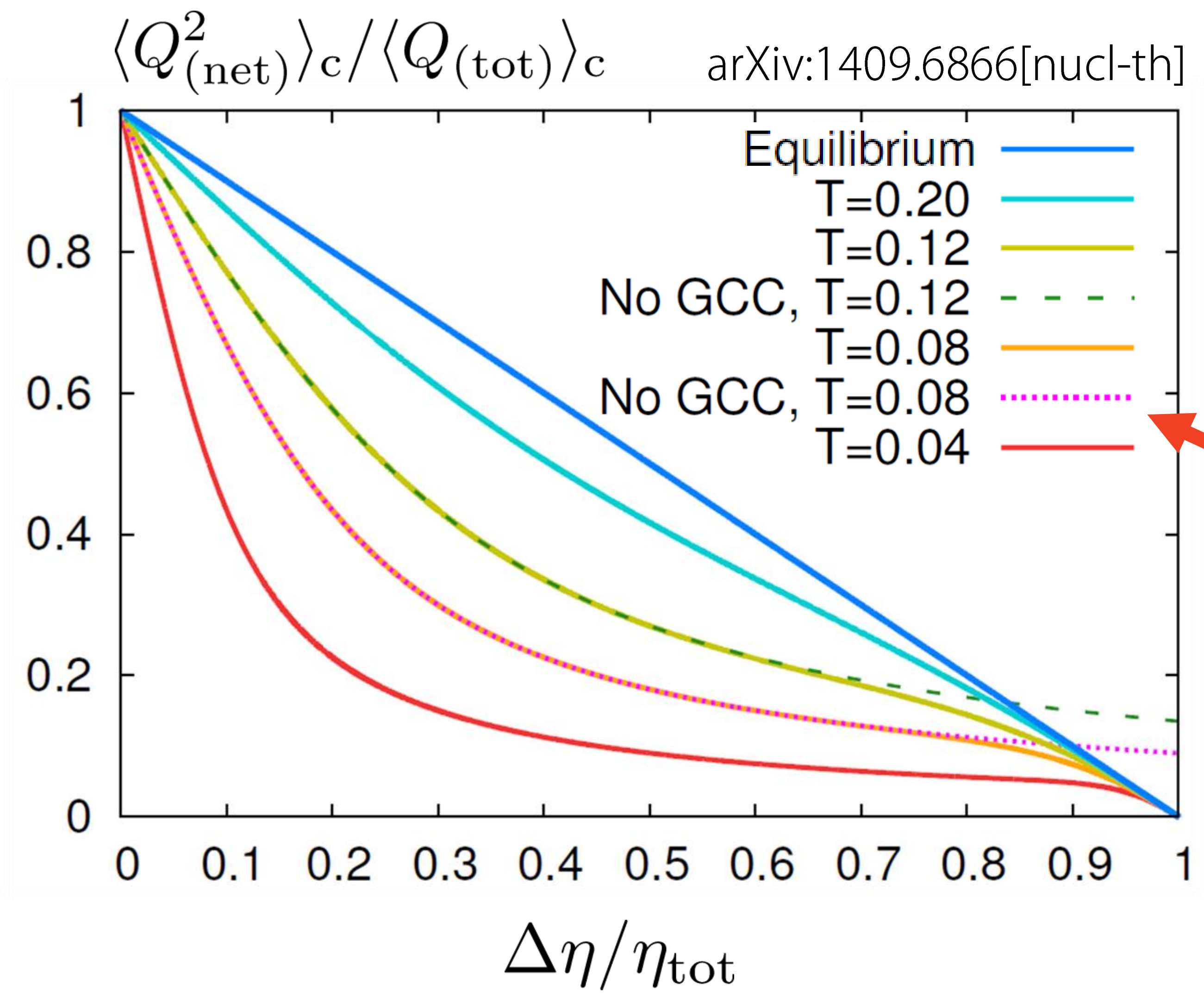
Rapidity Window Dependence of Charge Fluctuations

Results

$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)

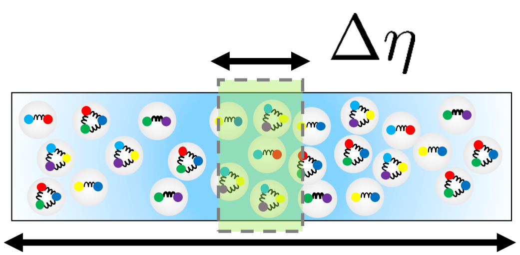


$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Distance

$D(\tau)$: Diffusion Coefficient

$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}$$



η_{tot} : Total Rapidity Length

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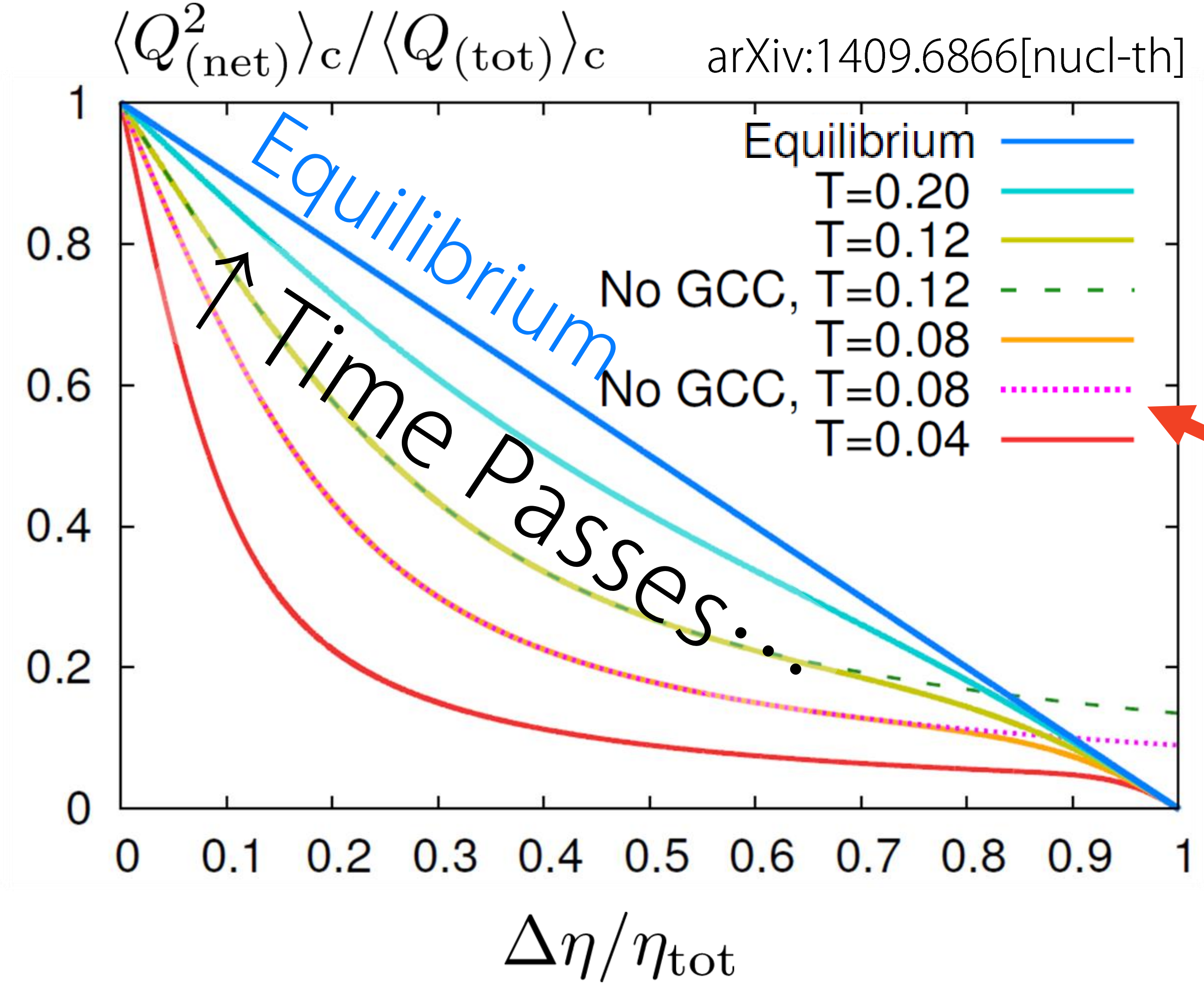

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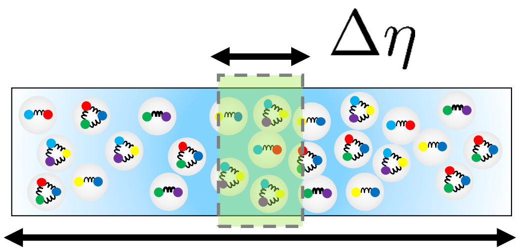

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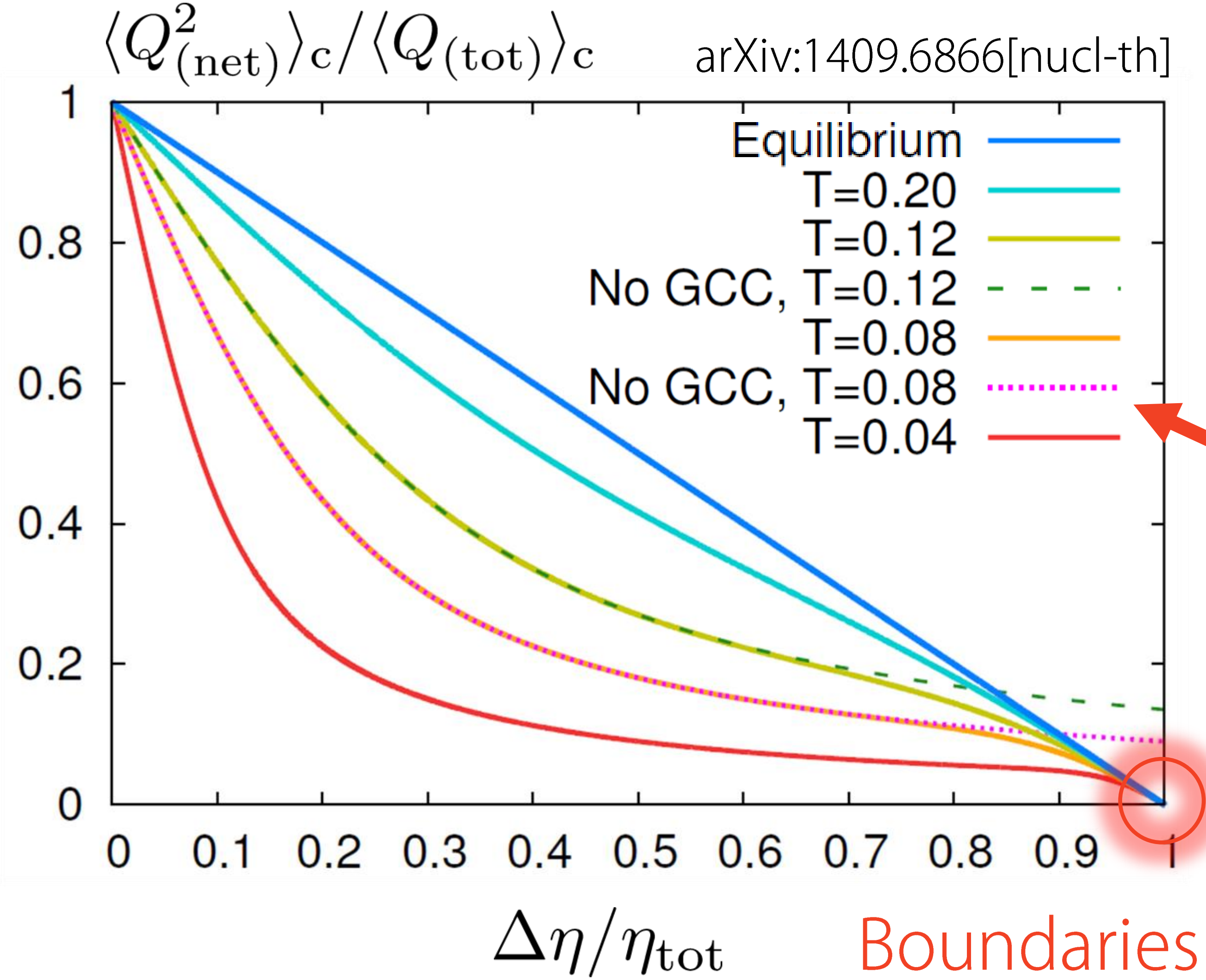

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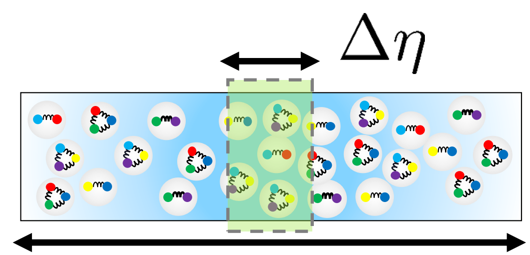

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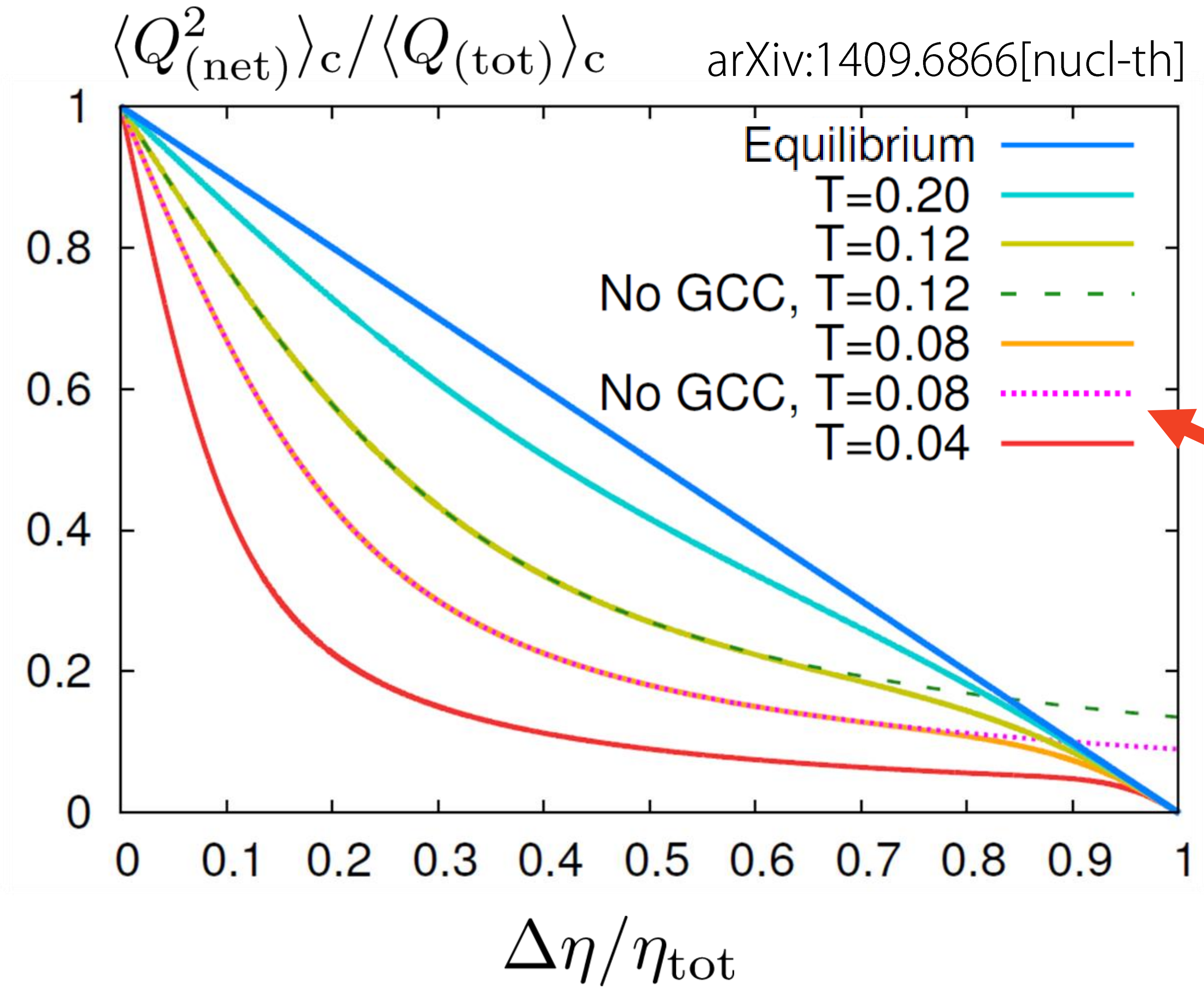

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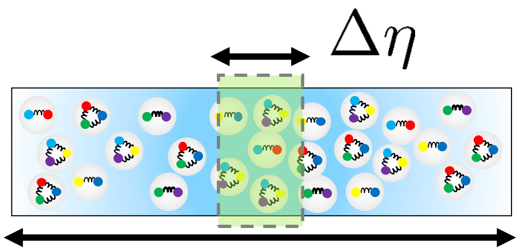

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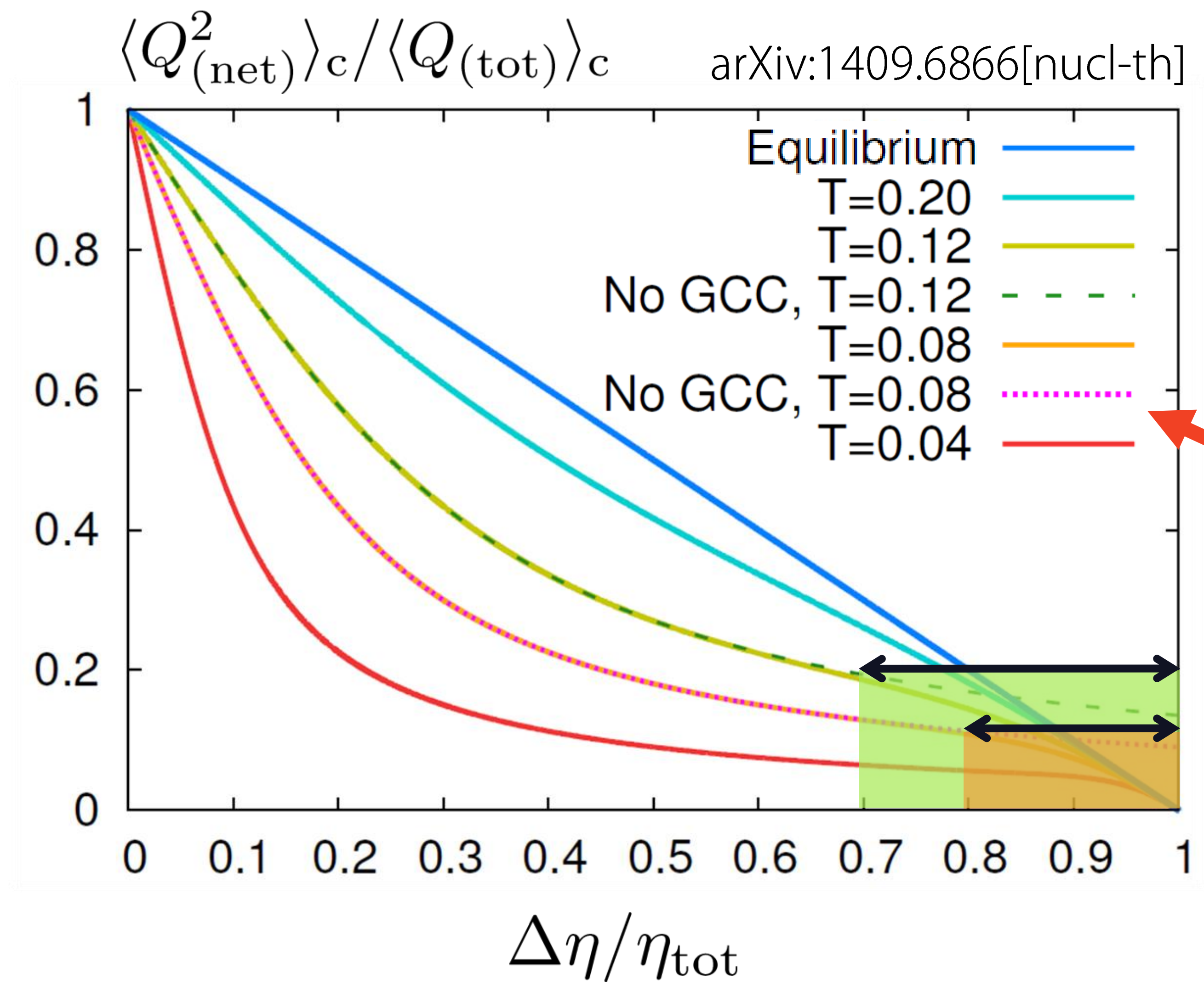

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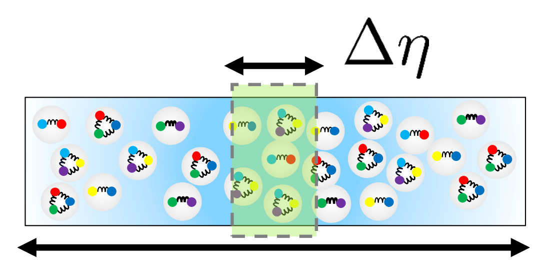

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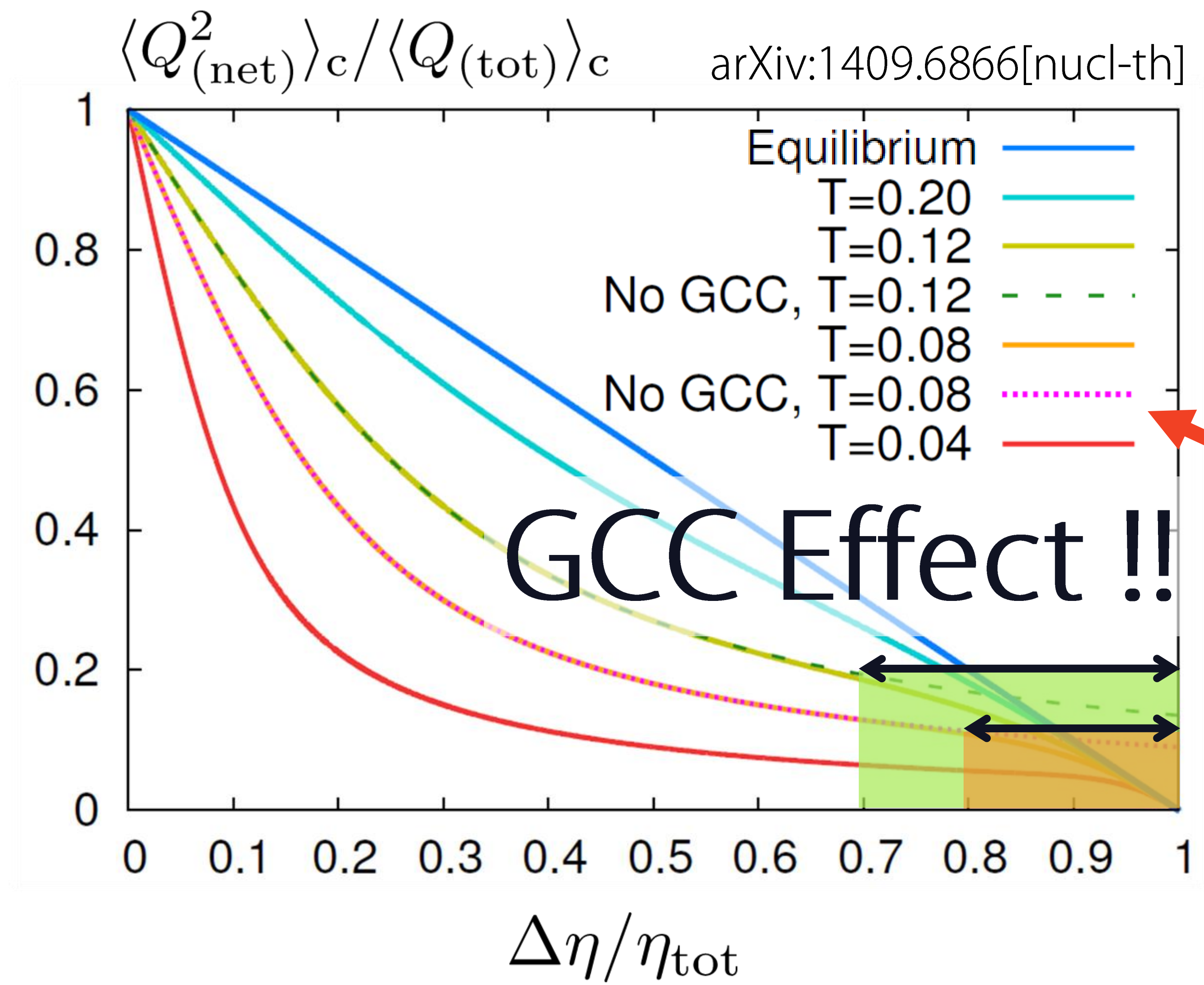

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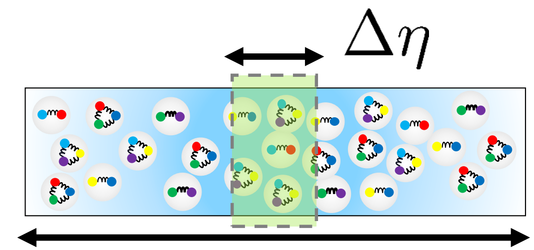
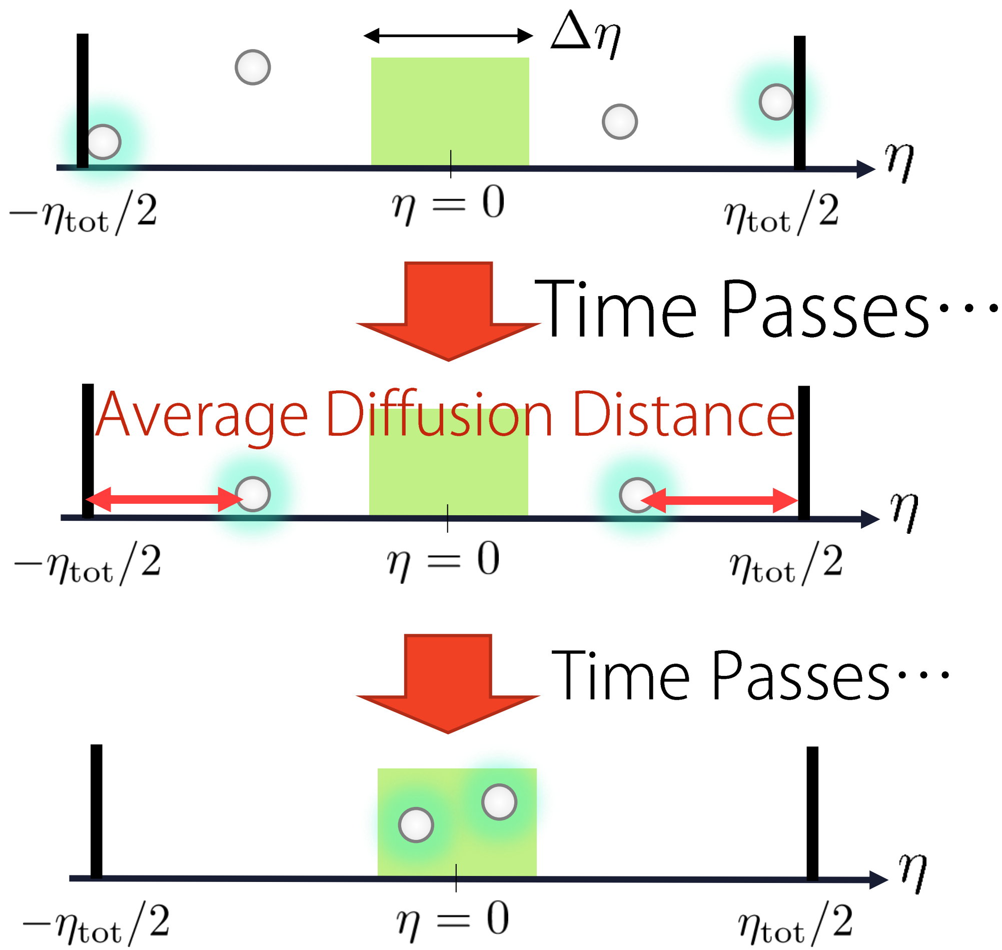


η_{tot} : Total Rapidity Length

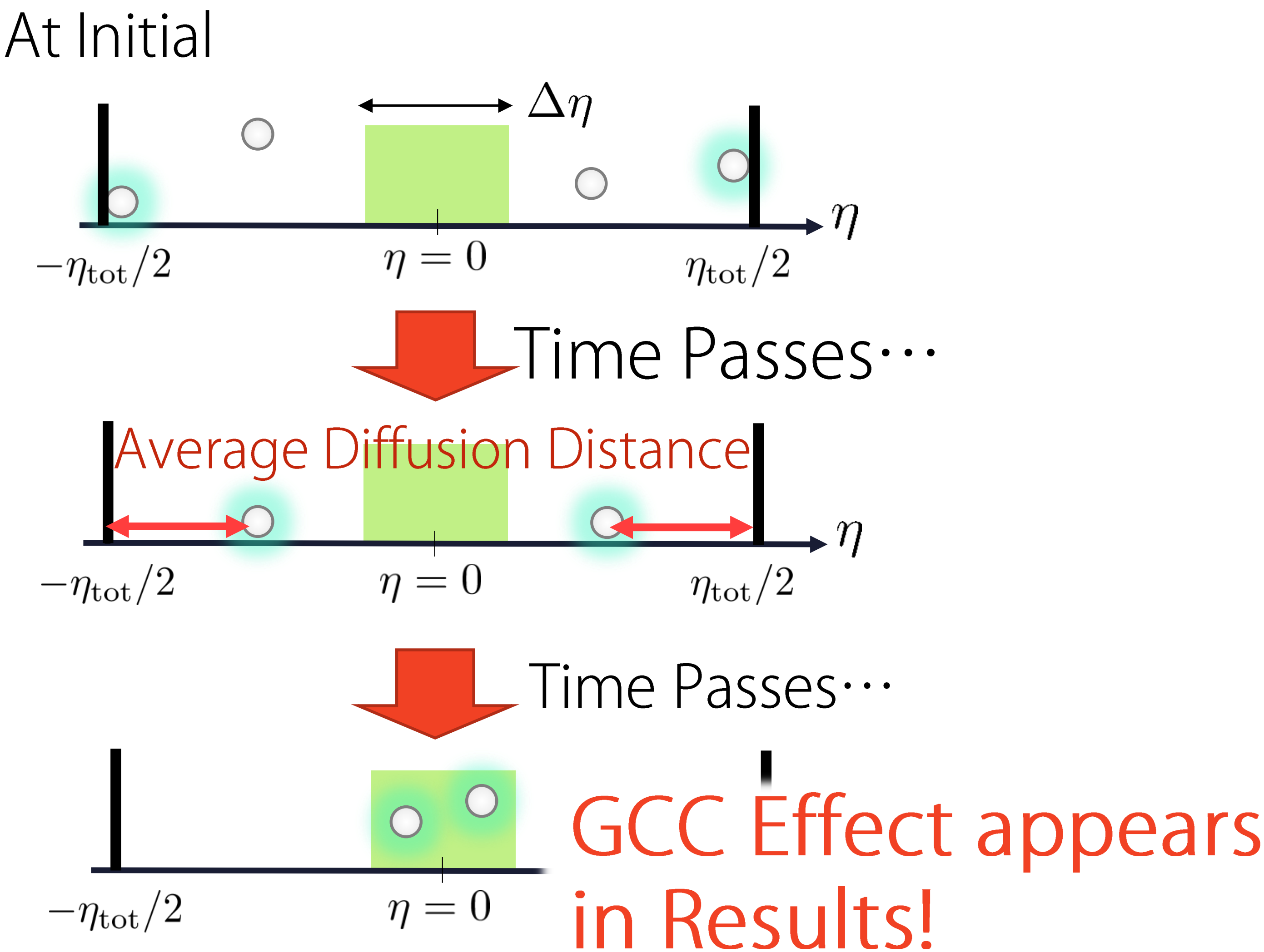
Interpretation of Result for GCC Effect

At Initial



arXiv:1409.6866[nucl-th]

Interpretation of Result for GCC Effect



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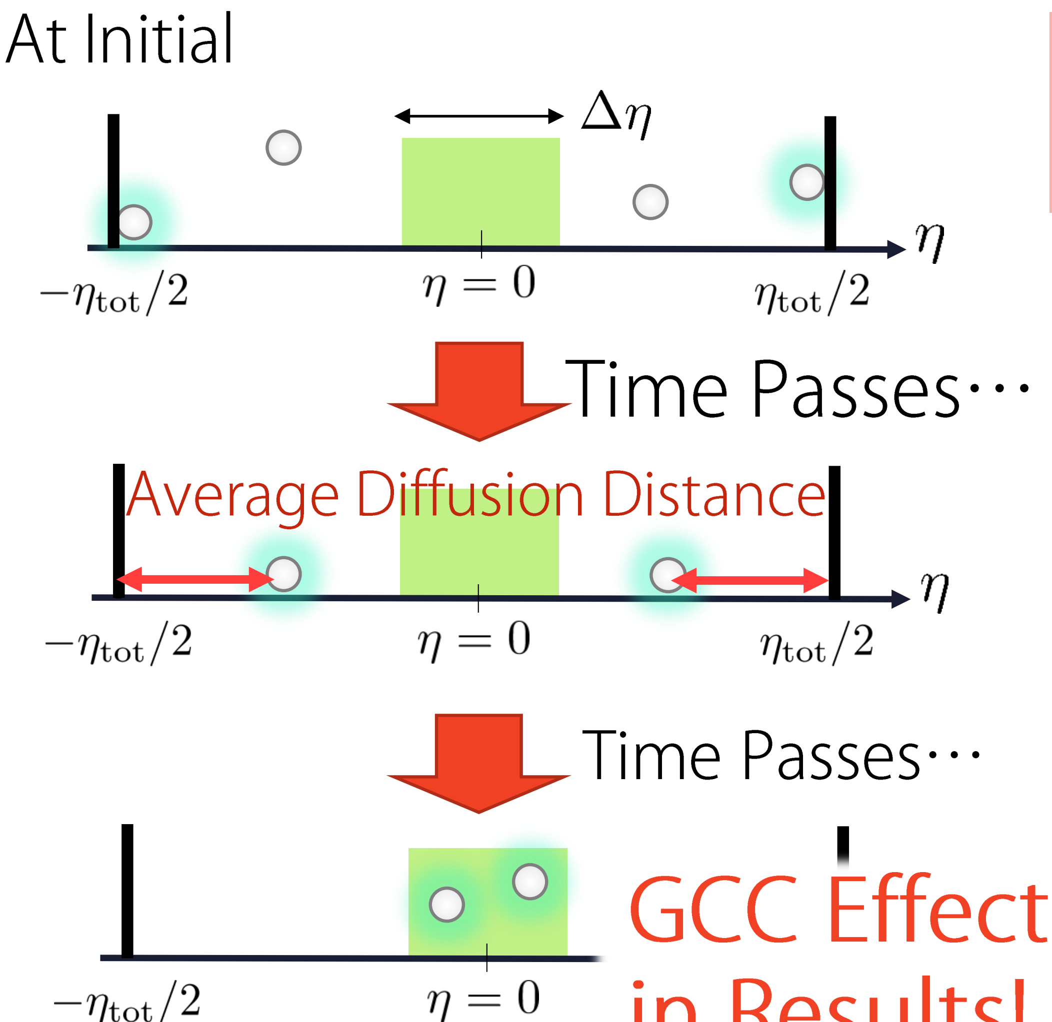
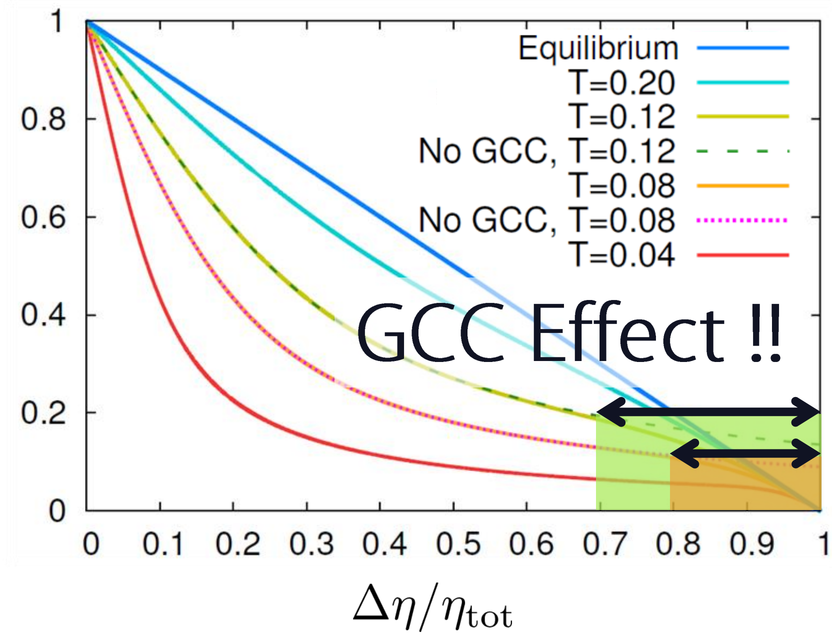


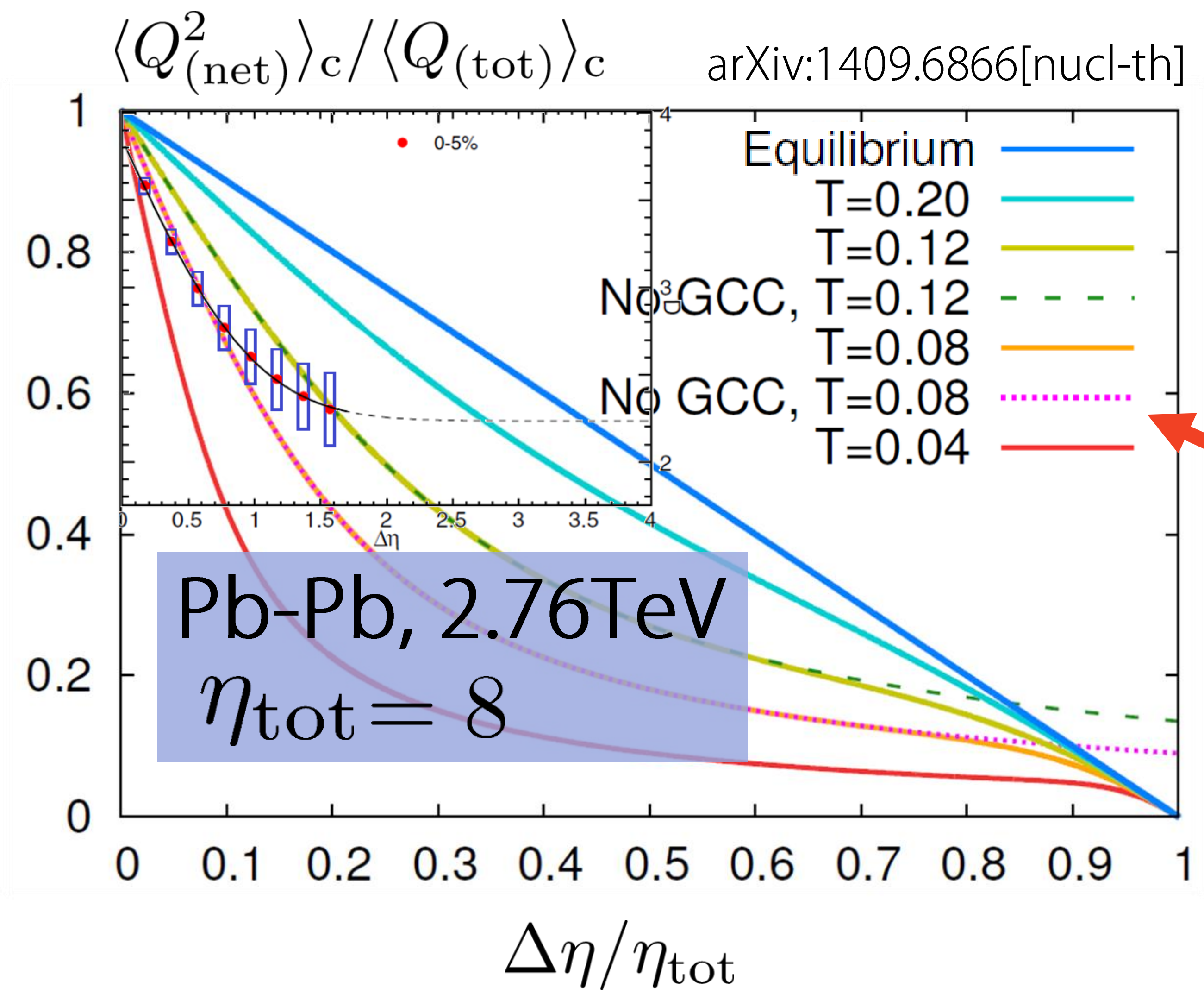
GCC Effect appears ONLY near boundaries!!



GCC Effect appears in Results!

arXiv:1409.6866[nucl-th]

Comparison with Experimental Result @ALICE



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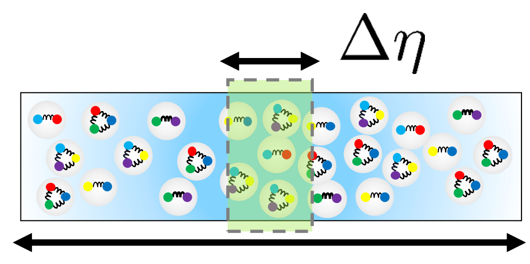

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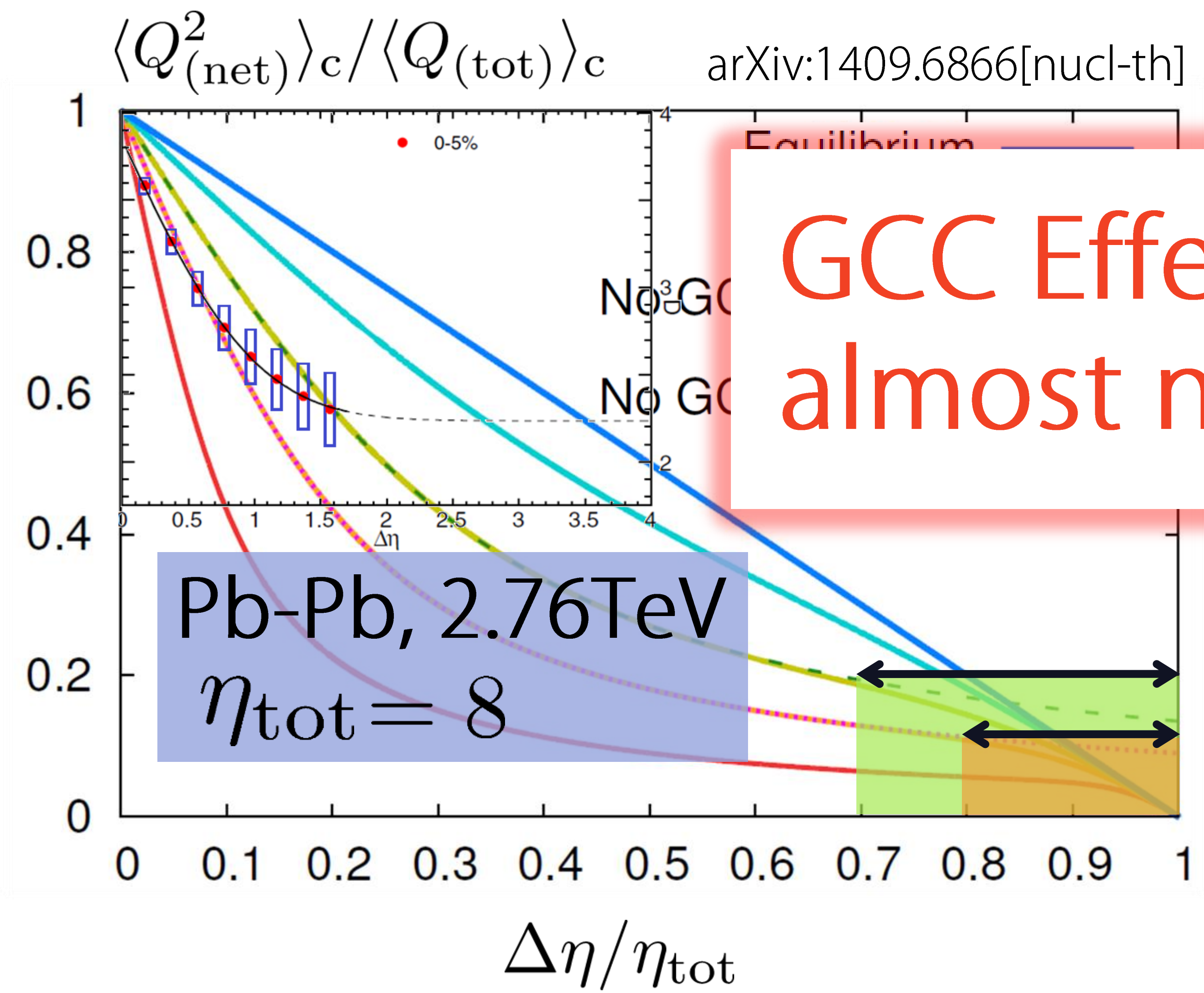

GCC Effect is almost negligible !

$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

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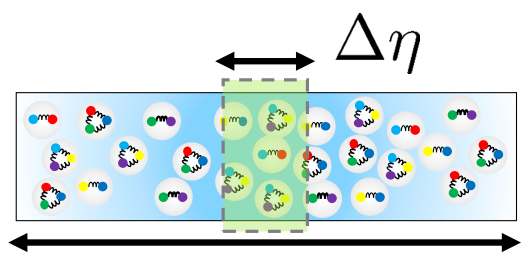

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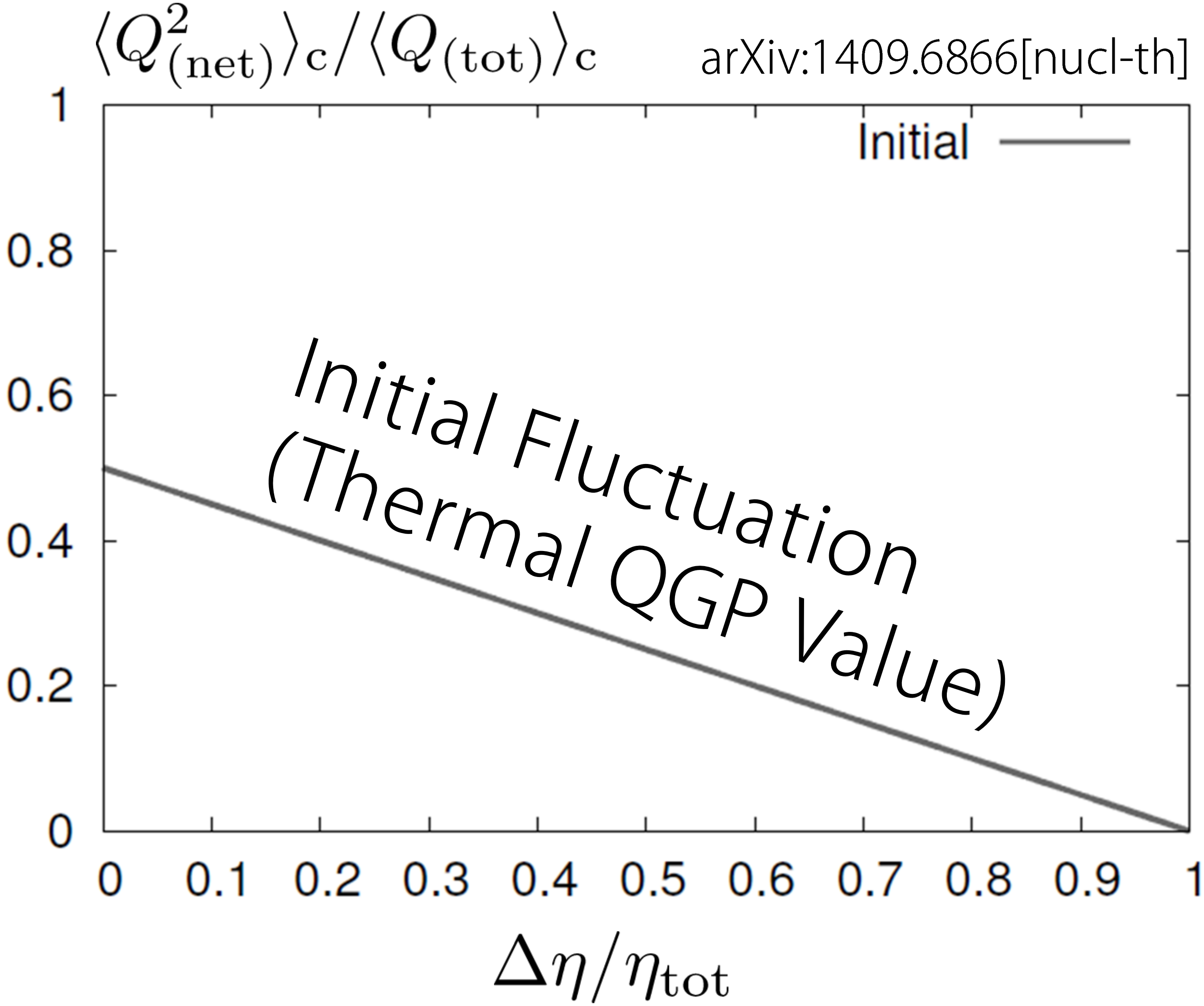
$\Delta\eta$ Dep. of Fluctuations (+ Initial Fluctuation)



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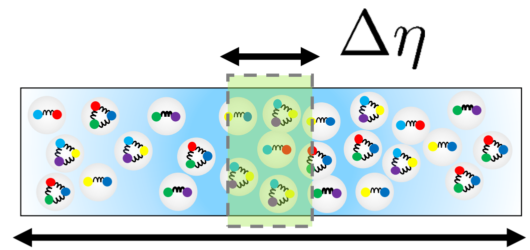
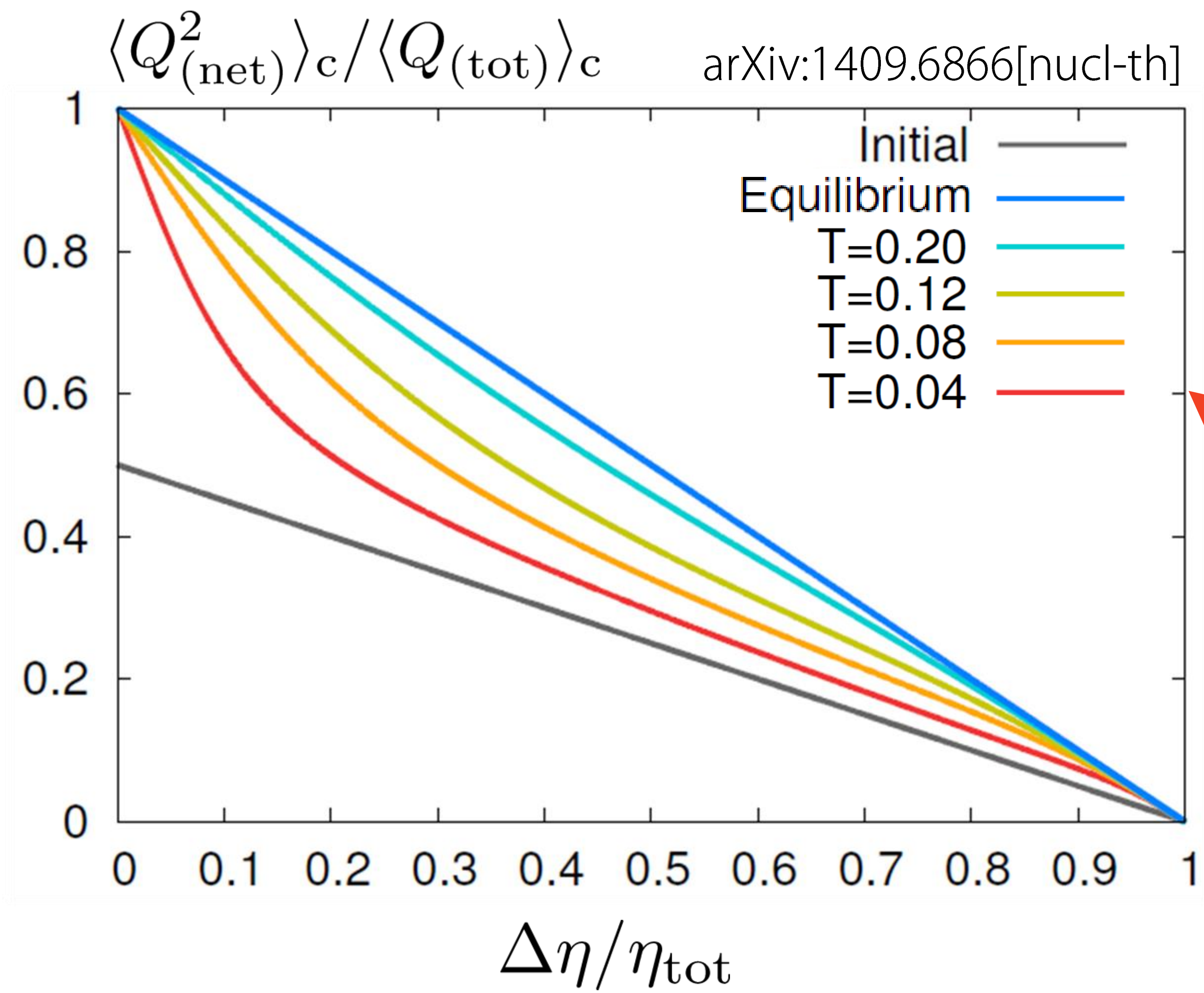
$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}$$



η_{tot} : Total Rapidity Length

$\Delta\eta$ Dep. of Fluctuations (+ Initial Fluctuation)

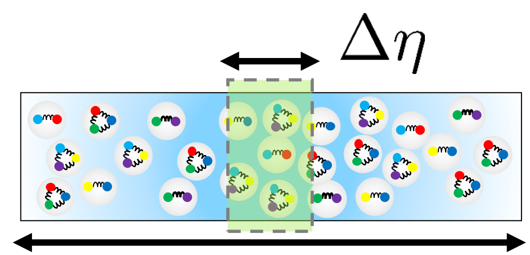


$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Distance

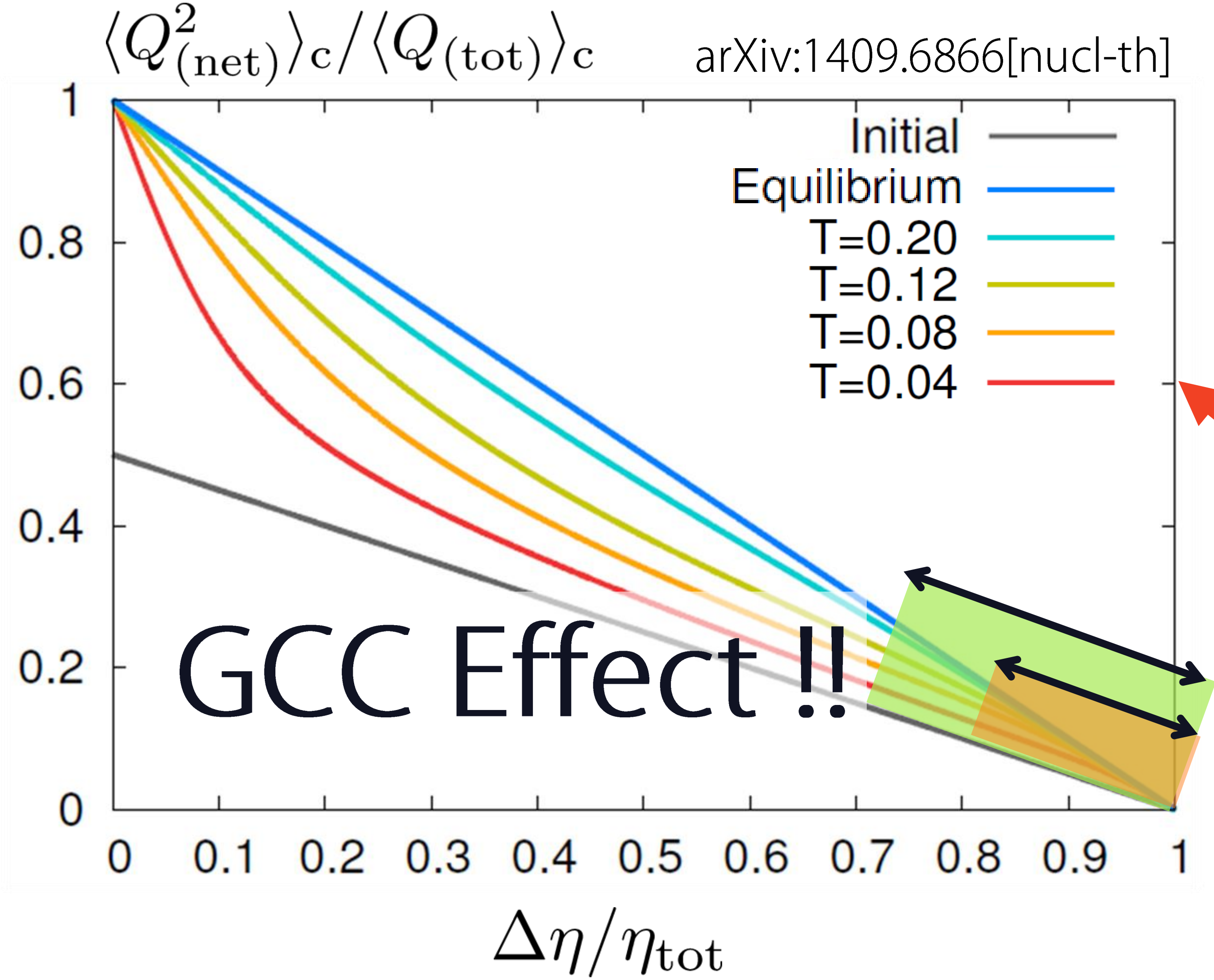
$D(\tau)$: Diffusion Coefficient

$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}$$



η_{tot} : Total Rapidity Length

$\Delta\eta$ Dep. of Fluctuations (+ Initial Fluctuation)

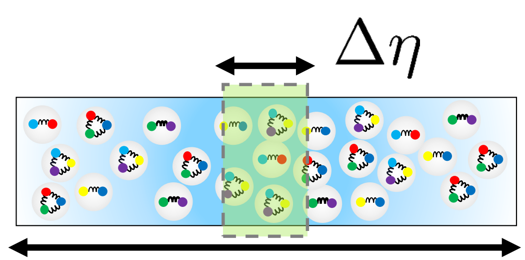


$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Distance

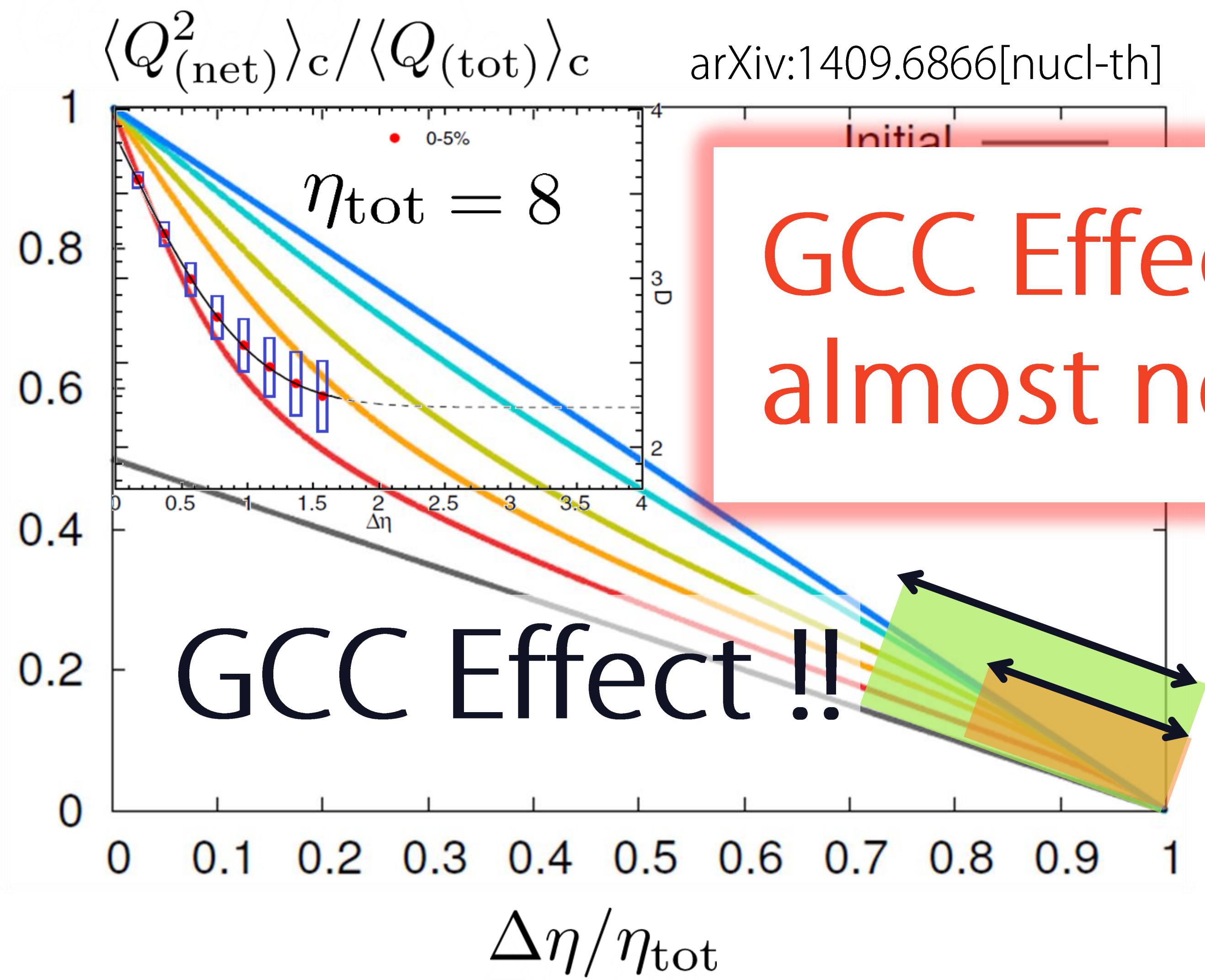
$D(\tau)$: Diffusion Coefficient

$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}$$



η_{tot} : Total Rapidity Length

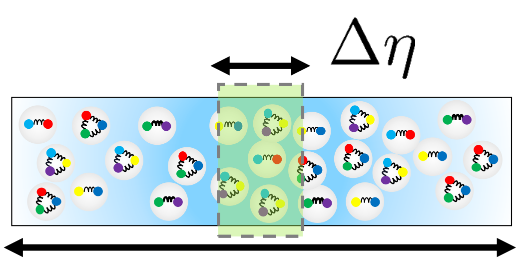
Comparison with Experiment (+ Initial Fluctuation)



GCC Effect is almost negligible !

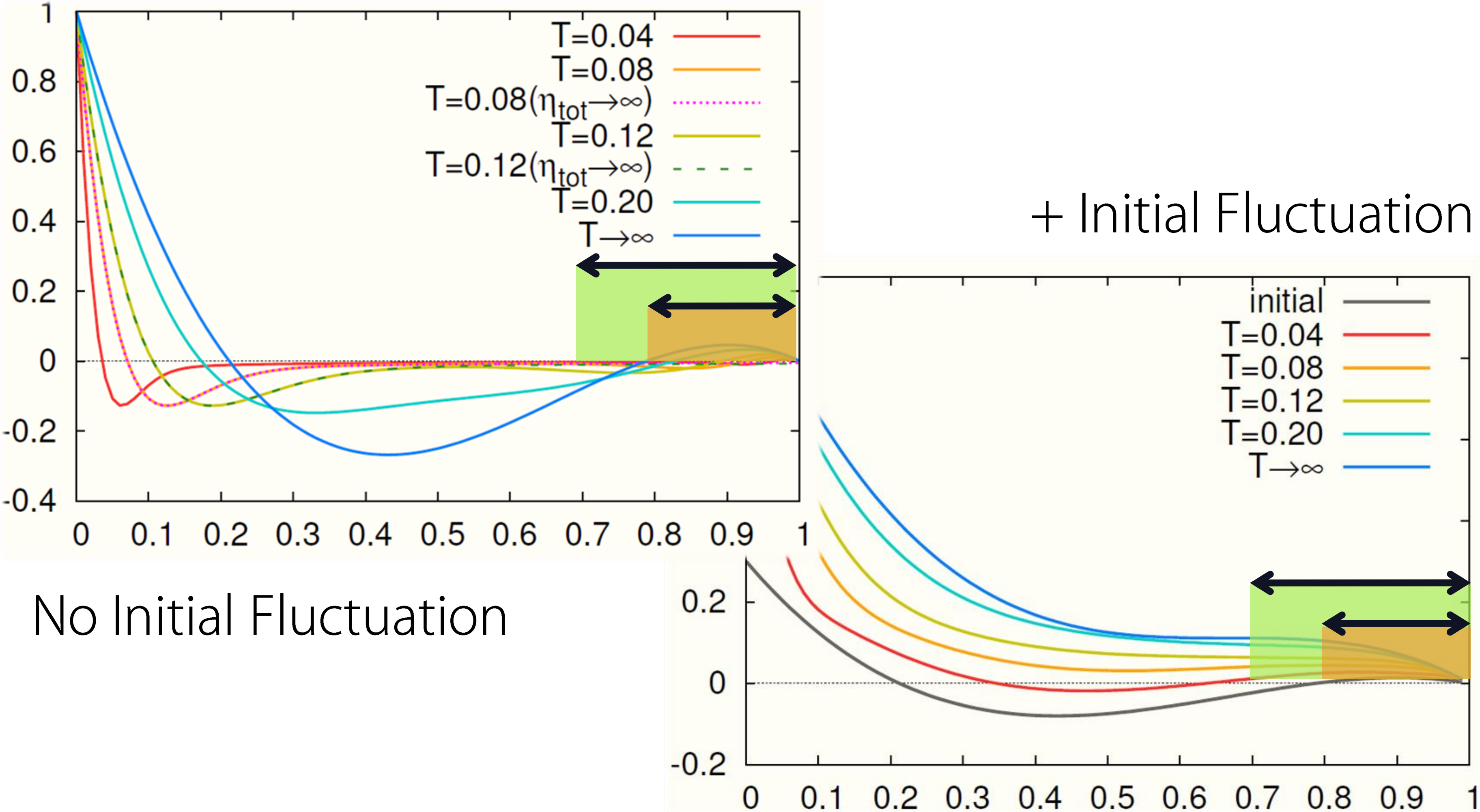
$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}$$



η_{tot} : Total Rapidity Length

4th order Fluctuations



+ Initial Fluctuation

No Initial Fluctuation

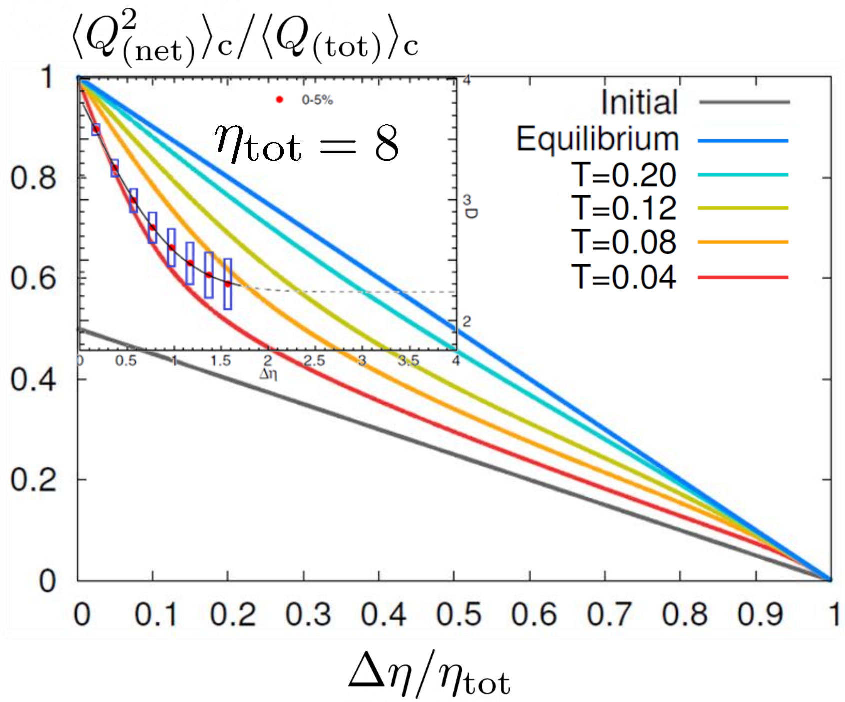
GCC Effect appears ONLY near Boundaries !

What Information can we obtain?

Suppression of Charge Fluctuation observed @ALICE

→ ~~Global Charge Conservation~~

Fluctuations are NOT Equilibrated!!



We can know about

* Fluctuation in QGP
* Time Evolution
* Diffusion Coefficient

etc…

arXiv:1409.6866[nucl-th]

Summary

* We investigated the Rapidity Window Dependence of Fluctuations of Conserved Charges and GCC Effect.

* GCC Effect appears ONLY near boundaries.

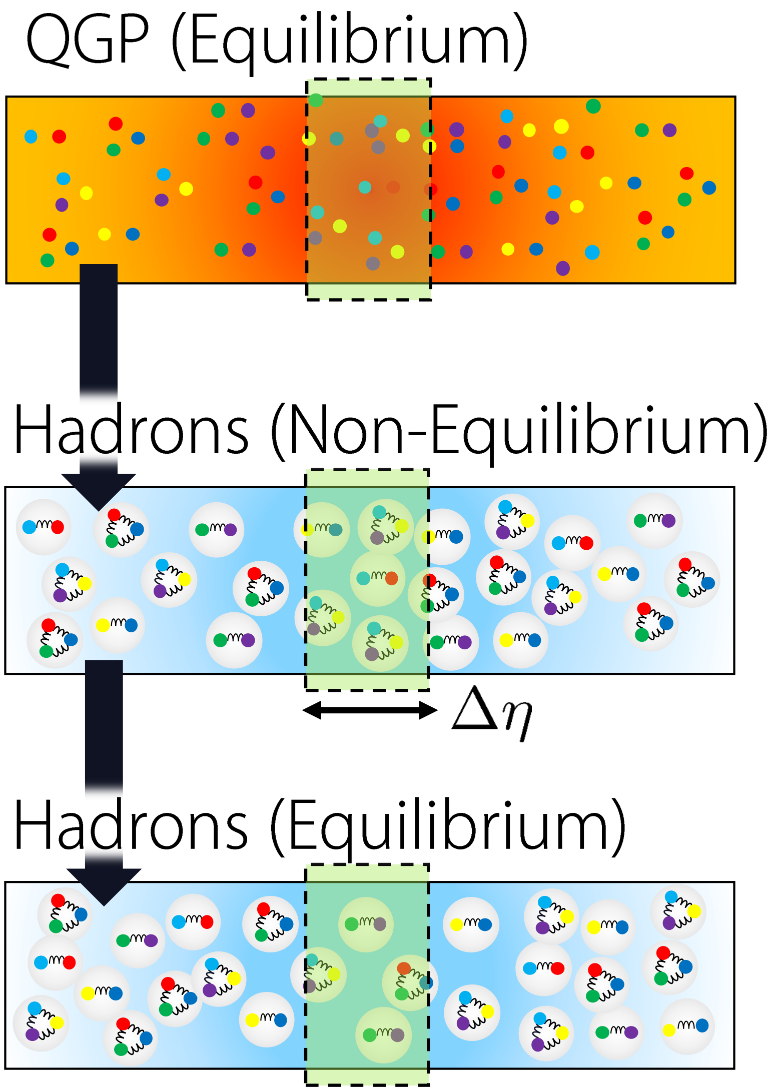
* GCC Effects is almost negligible in Results at ALICE.

Fluctuations of Conserved Charges tell us Much Information about the Hot Medium.

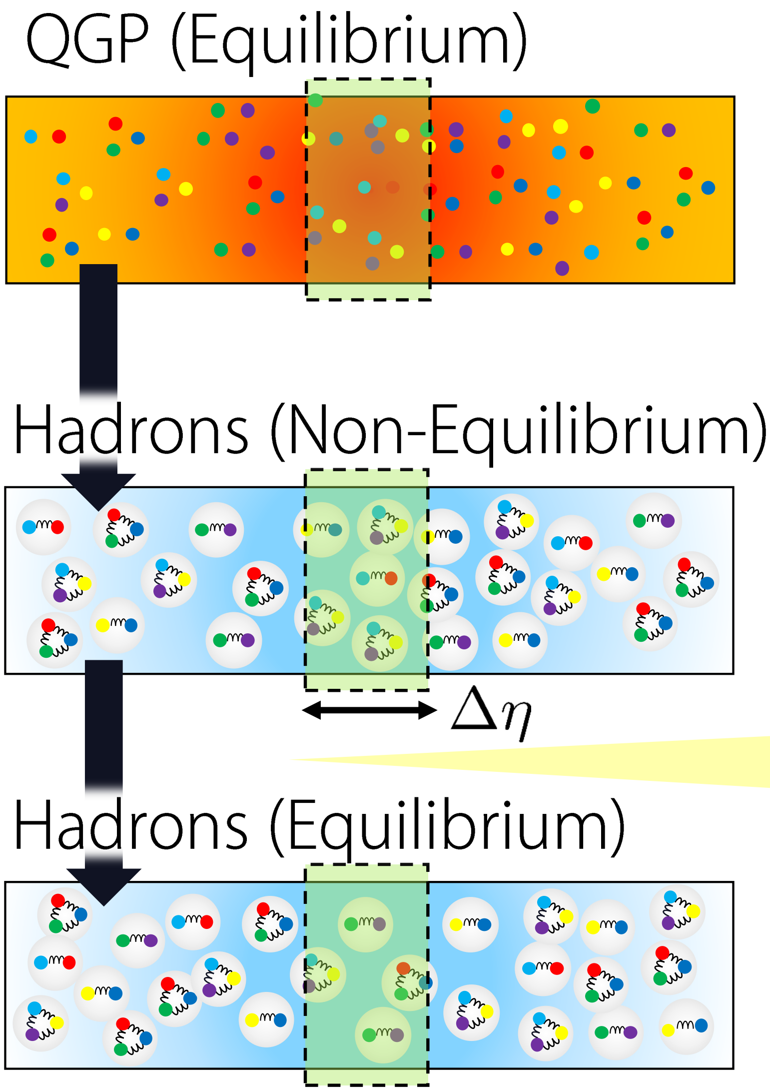
Fluctuations in QGP, Time Evolution of Hot medium,
Diffusion Coefficient, Hadronization Mechanism, etc···

Back Up

Time Evolution of Fluctuations

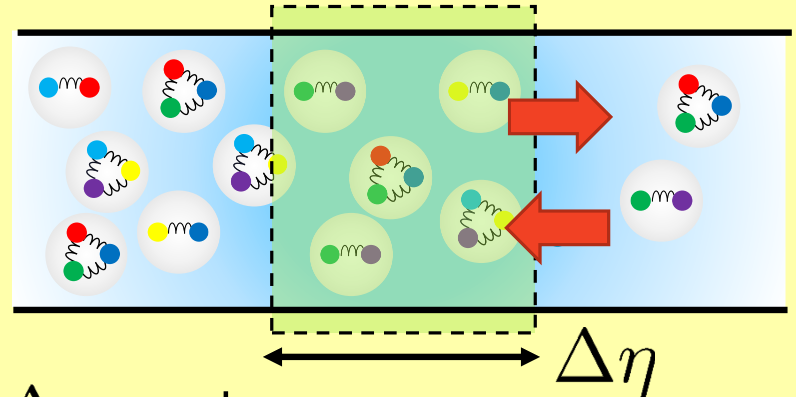


Time Evolution of Fluctuations



Relaxation can **only** proceed by **Diffusion of Charges** !!

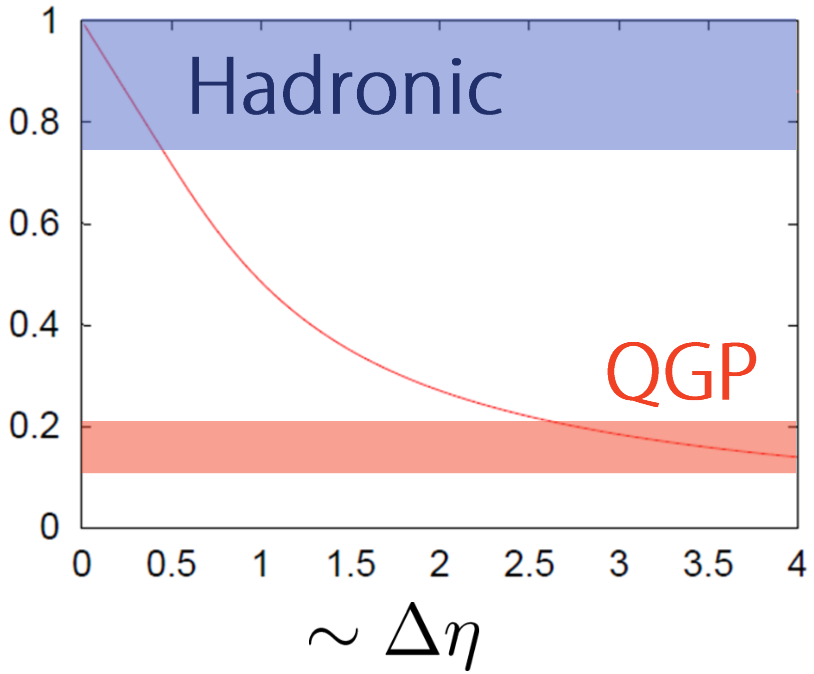
Shuryak, Stephanov (2001)



$\Delta\eta \rightarrow$ Larger

➡ Relaxation Time→Longer (More the QGP Value)

Time Evolution of Fluctuations

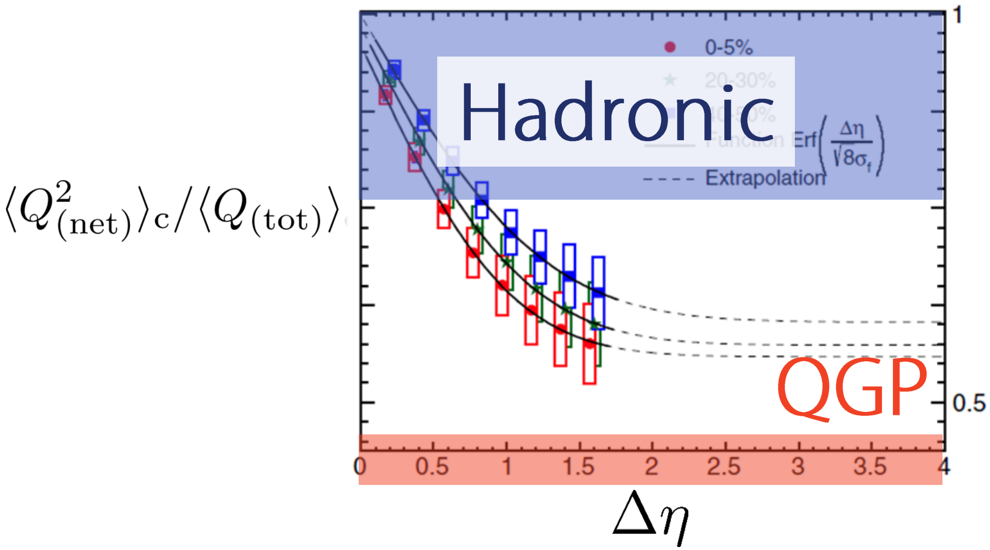


Fluctuating Hydro.

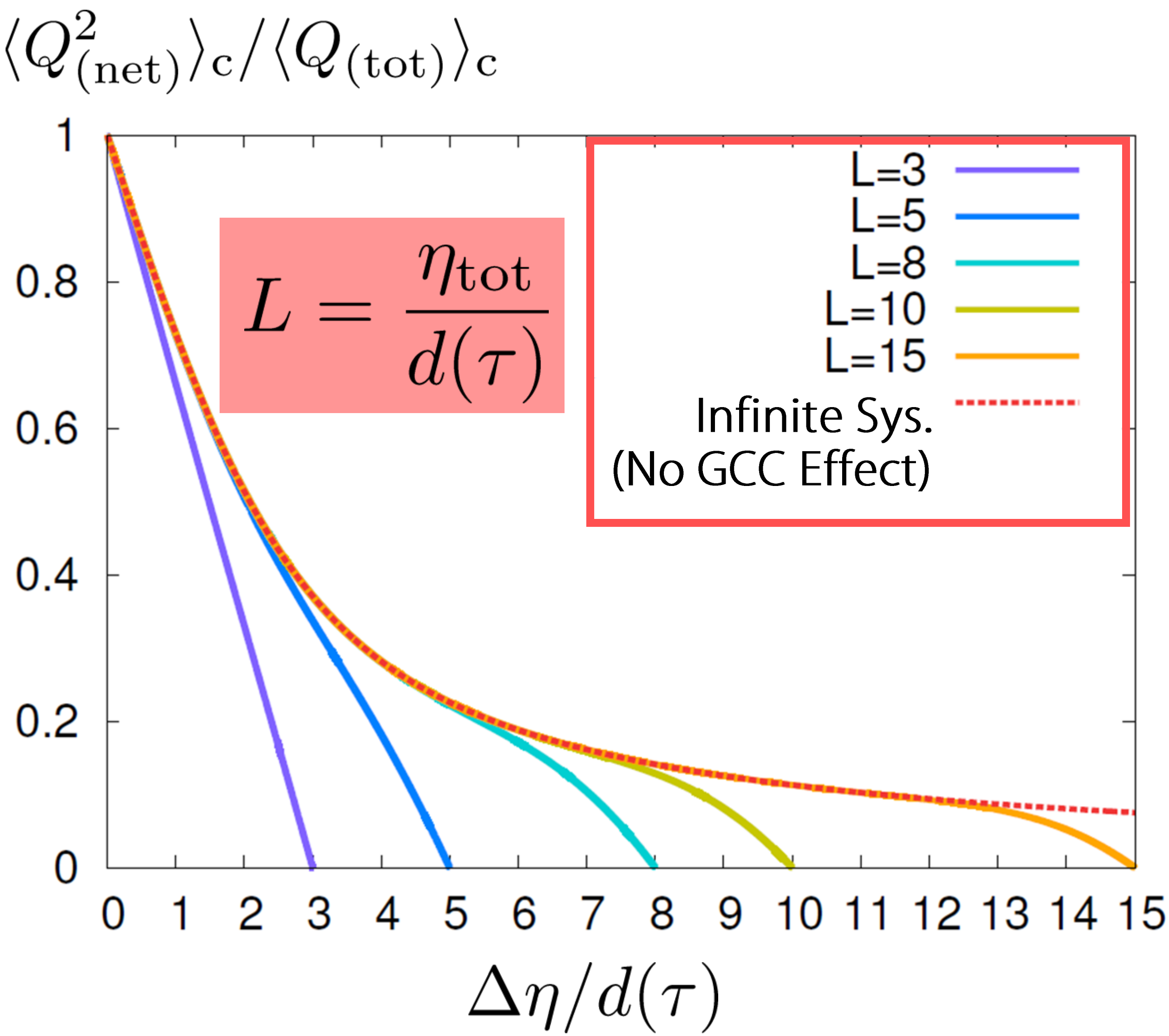
(Stochastic Diffusion Eq.)

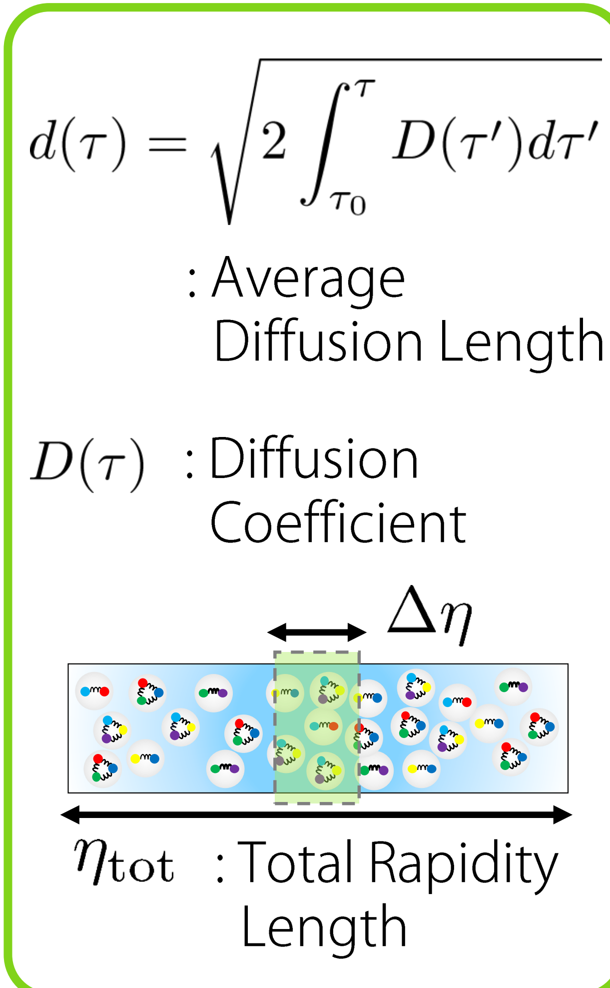
Shuryak, Stephanov (2001)

ALICE, PRL **110**, 152301 (2013)

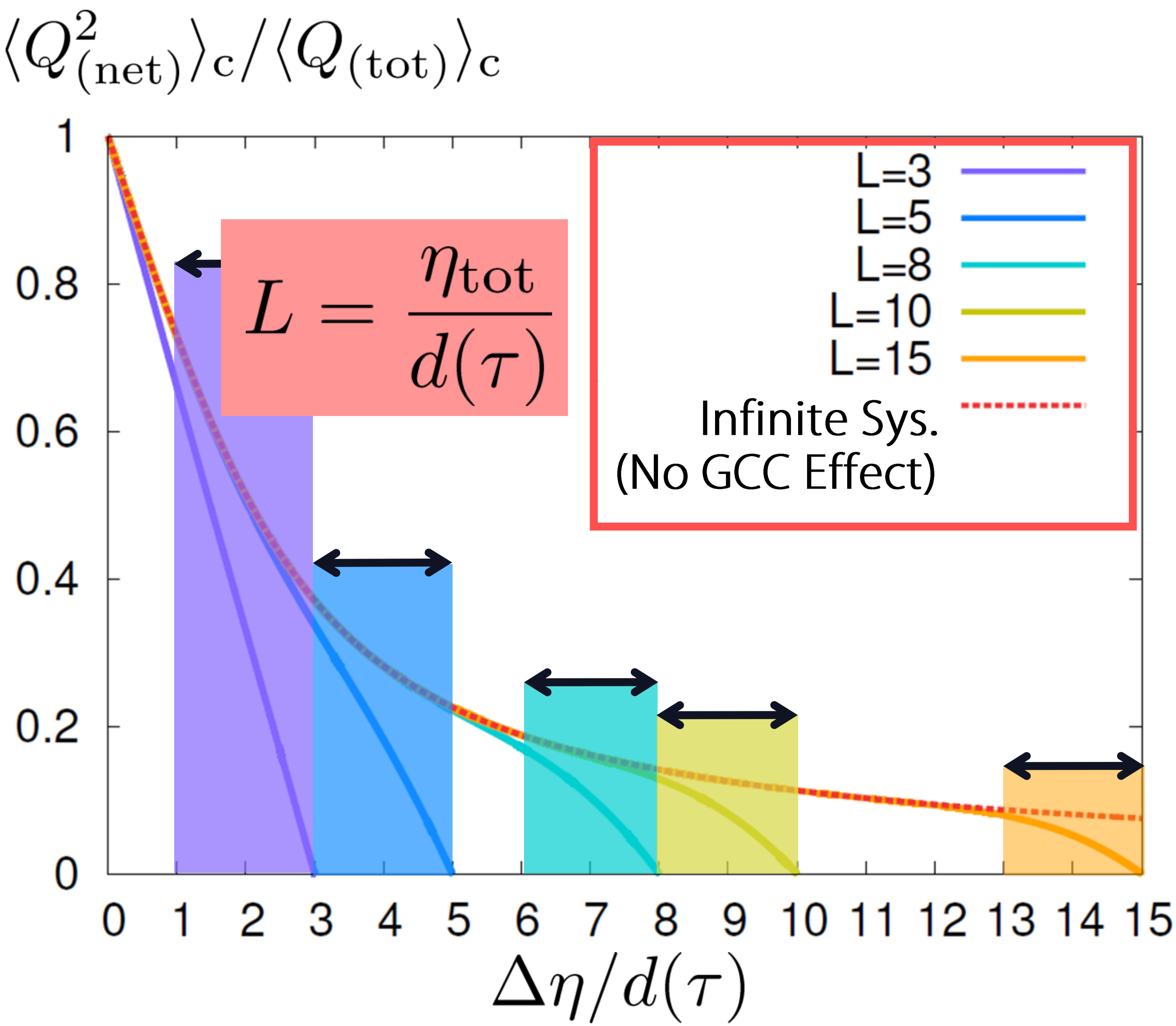


$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)





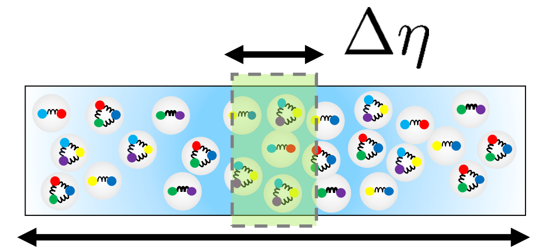
$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)



$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

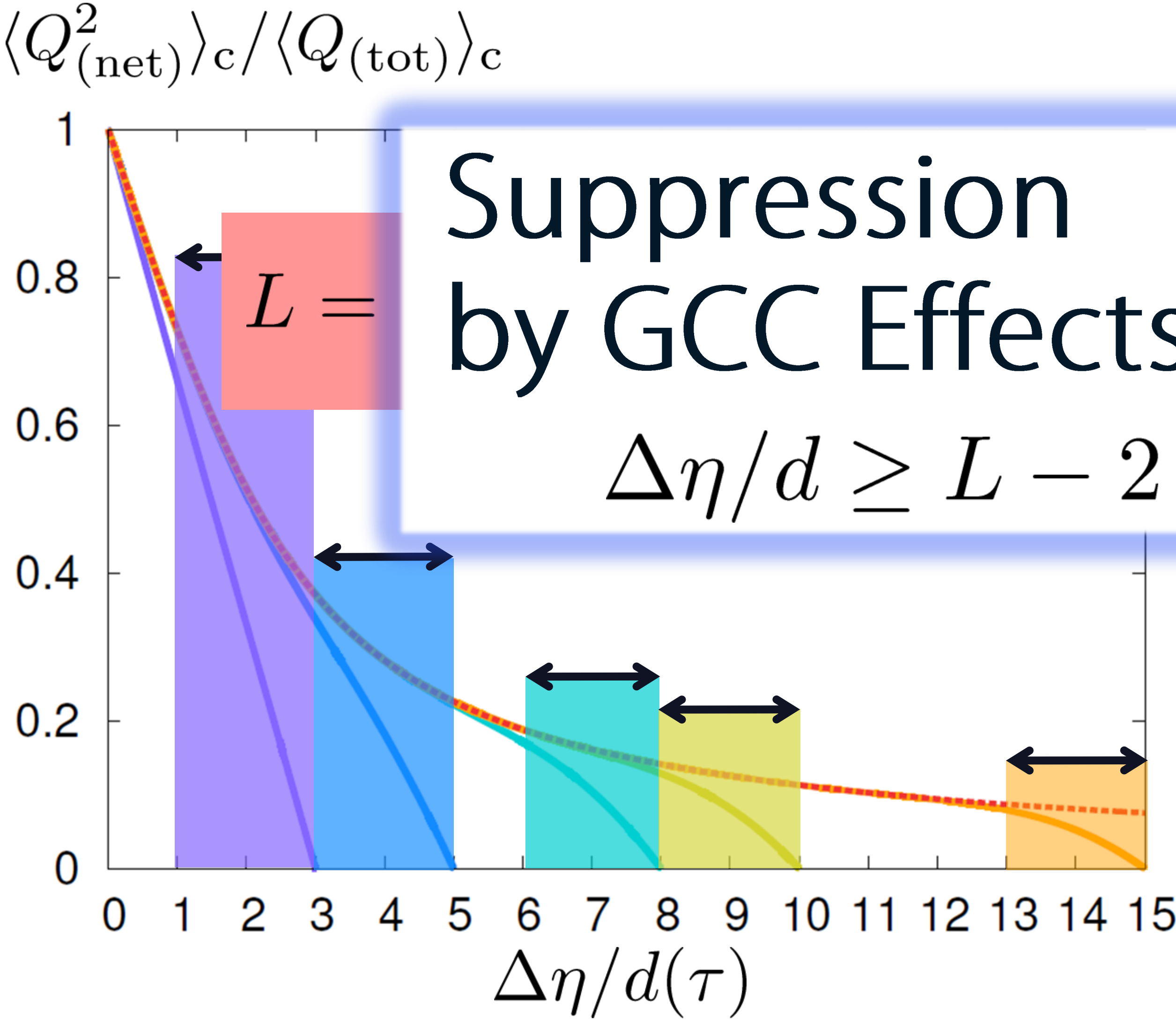
: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient



η_{tot} : Total Rapidity Length

$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)



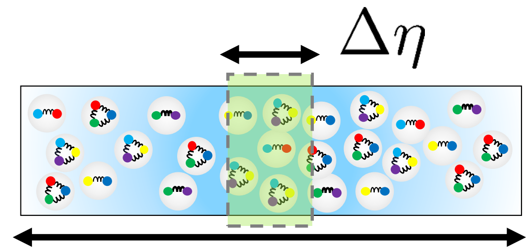
Suppression by GCC Effects

$$\Delta\eta / d \geq L - 2$$

$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

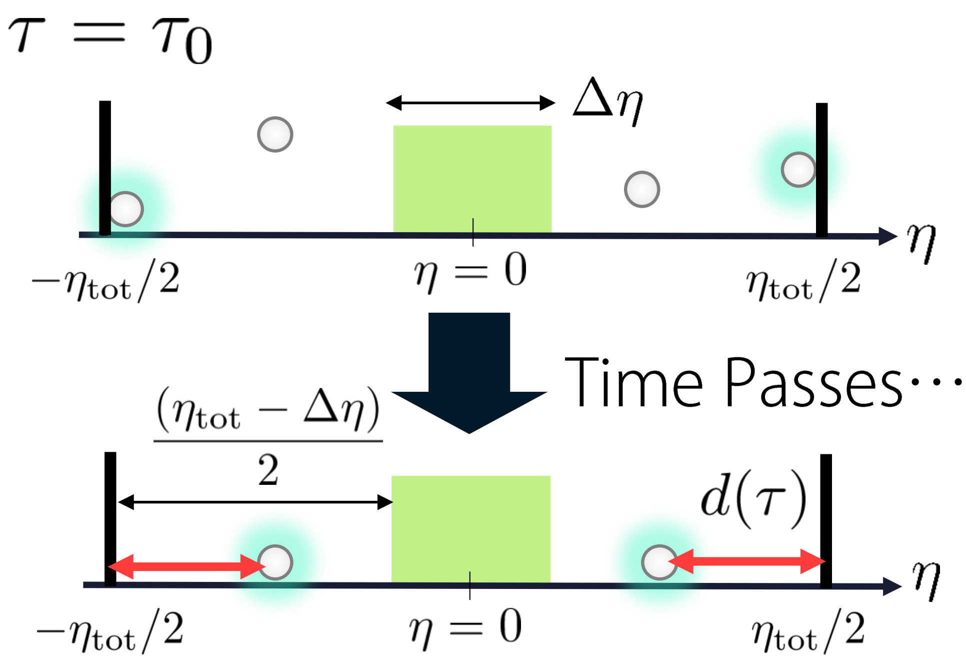
: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient



η_{tot} : Total Rapidity Length

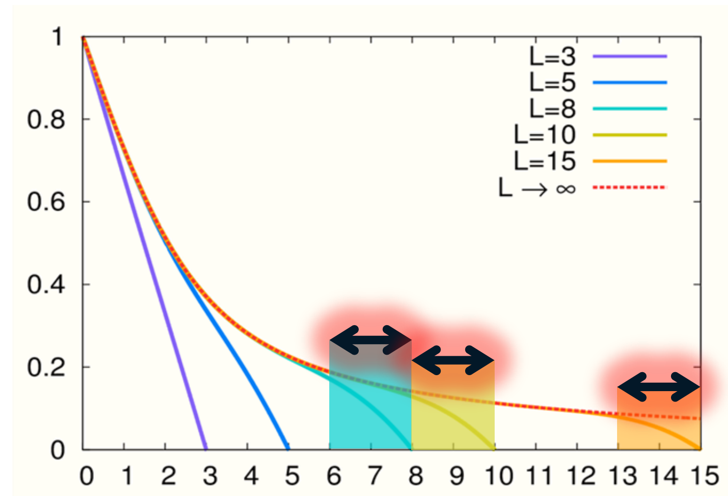
Interpretation of Results for GCC Effect



$d(\tau)$: Averaged Diffusion Distance

$D(\tau)$: Diffusion Coefficient

η_{tot} : Total Length of Matter



Condition for effects of the GCC

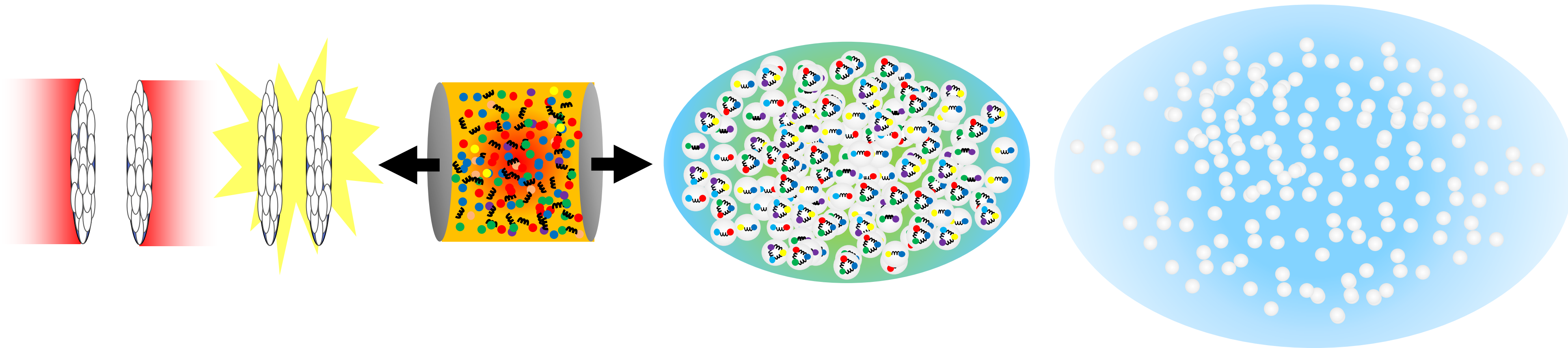
$$\Delta\eta/d \geq L - 2 \Leftrightarrow \frac{\eta_{tot} - \Delta\eta}{2} \leq d$$

Effects of the GCC appear only near the boundaries.

Time Evolution of Heavy Ion Collisions

Hadronization

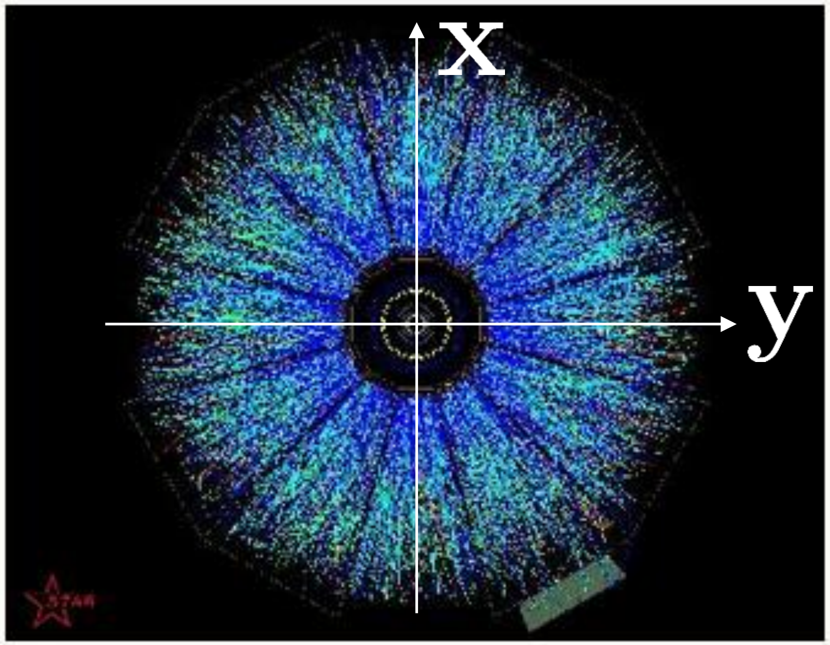
Accelerate Collide QGP → Interaction with Hadrons → Detected



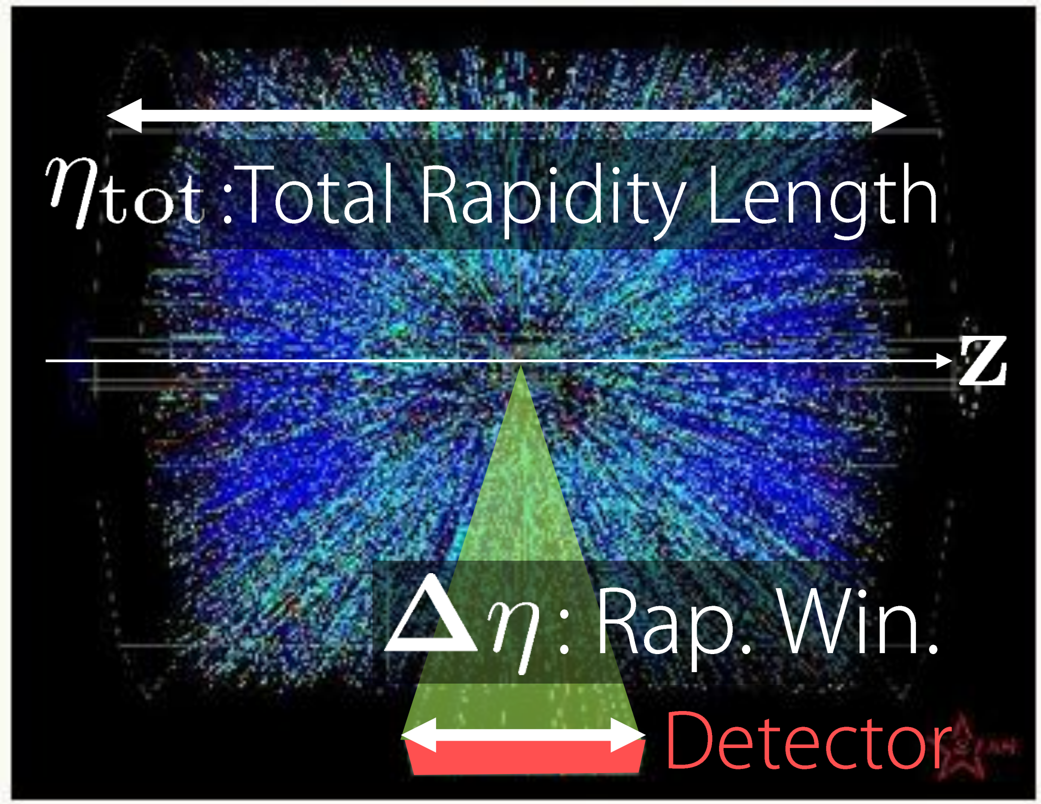
＊Detected Particles：Hadrons

＊After Phase Transition, Interaction → **Lose Information About QGP**

Event-by-Event Fluctuations

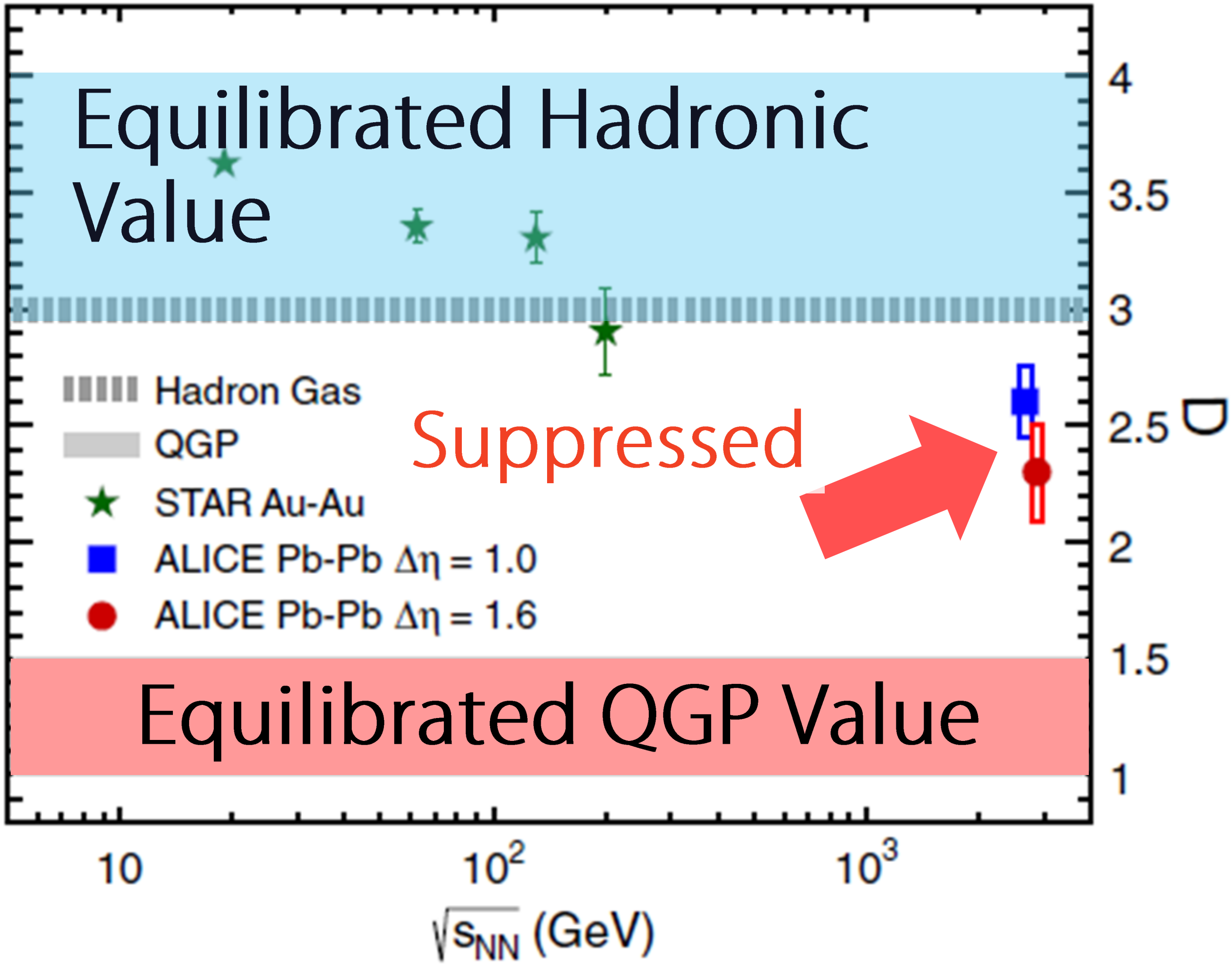


Integrated



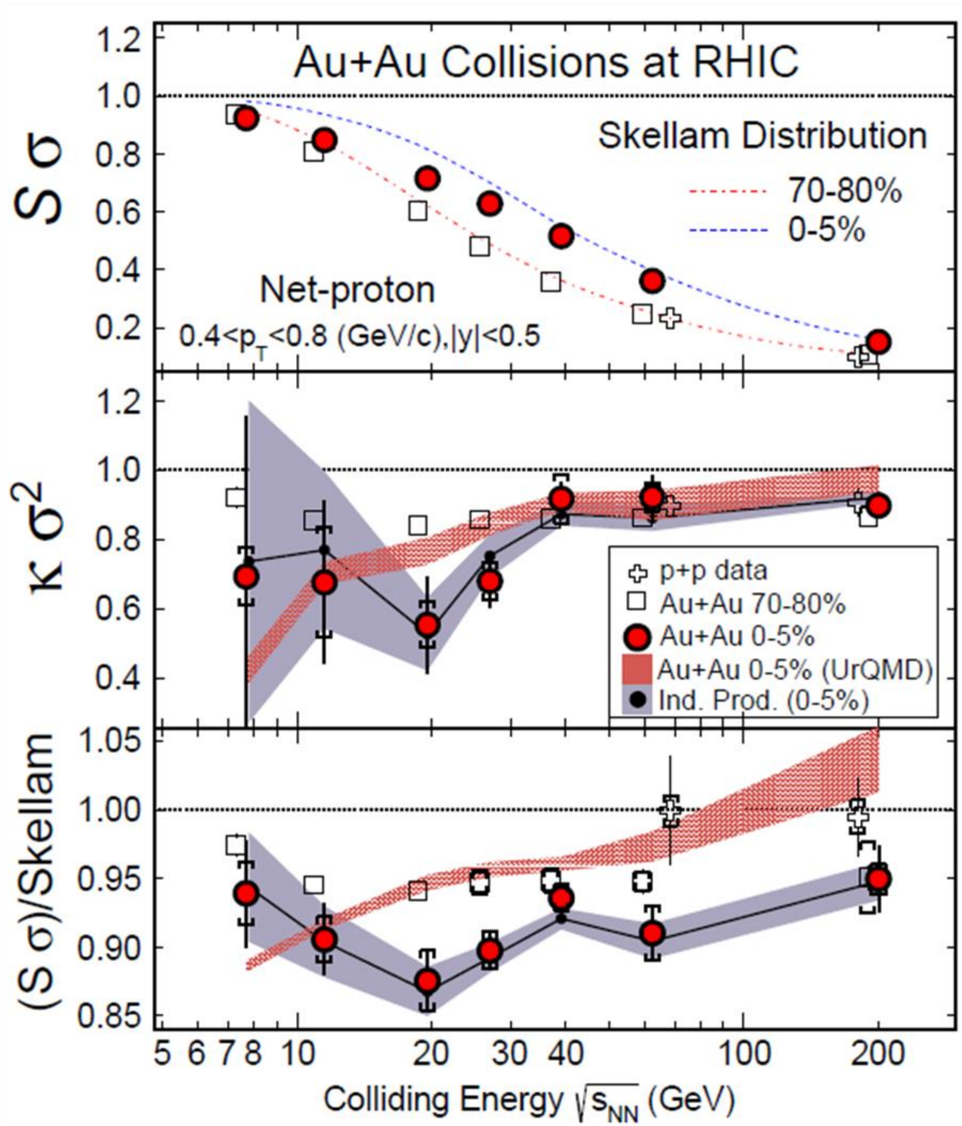
Net Electric Charge Fluctuation @ ALICE

ALICE, PRL **110**, 152301 (2013)



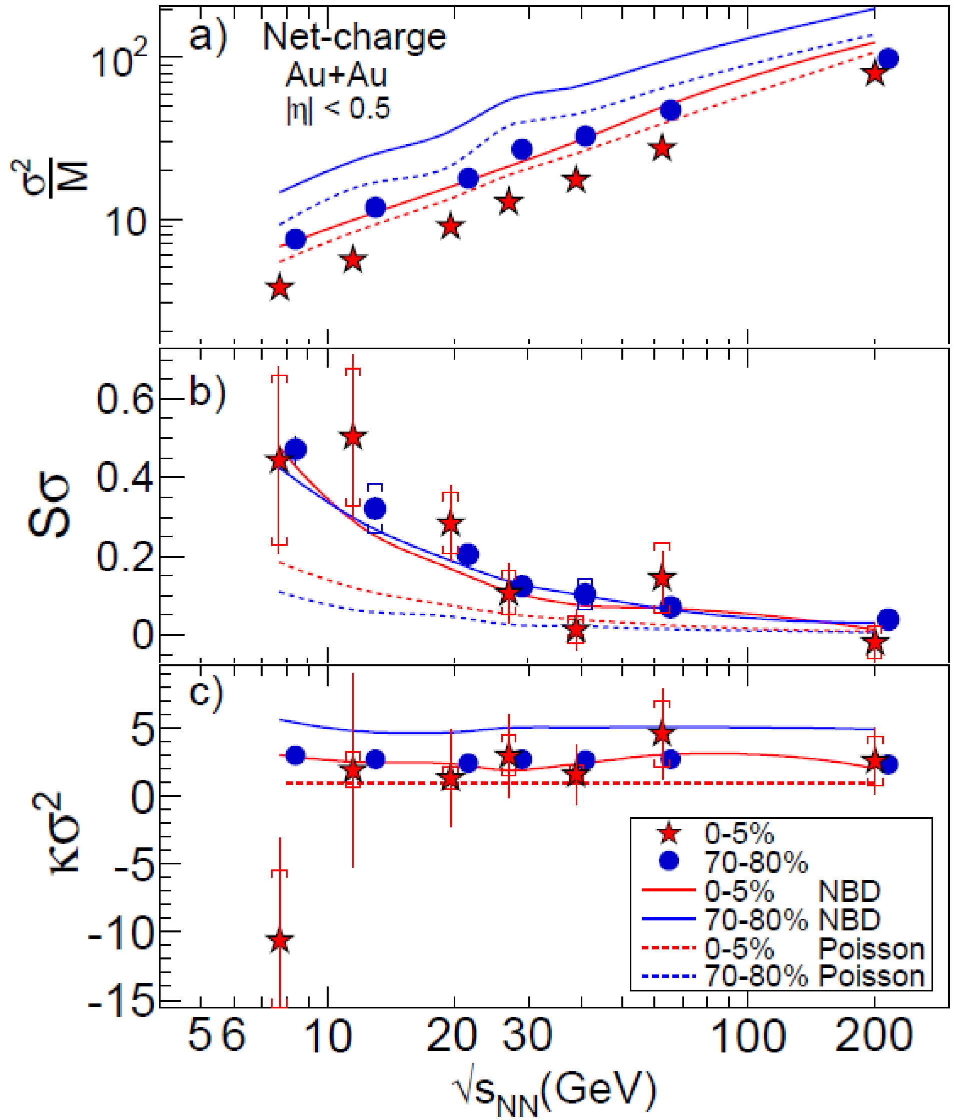
$$D = \frac{4\langle \delta N_{\mathrm{Q}}^{(\mathrm{net})2} \rangle_{\mathrm{c}}}{\langle N_{\mathrm{Q}}^{+} + N_{\mathrm{Q}}^{-} \rangle_{\mathrm{c}}}$$

Net Proton Number Fluctuation @ BES



RHIC (2014)

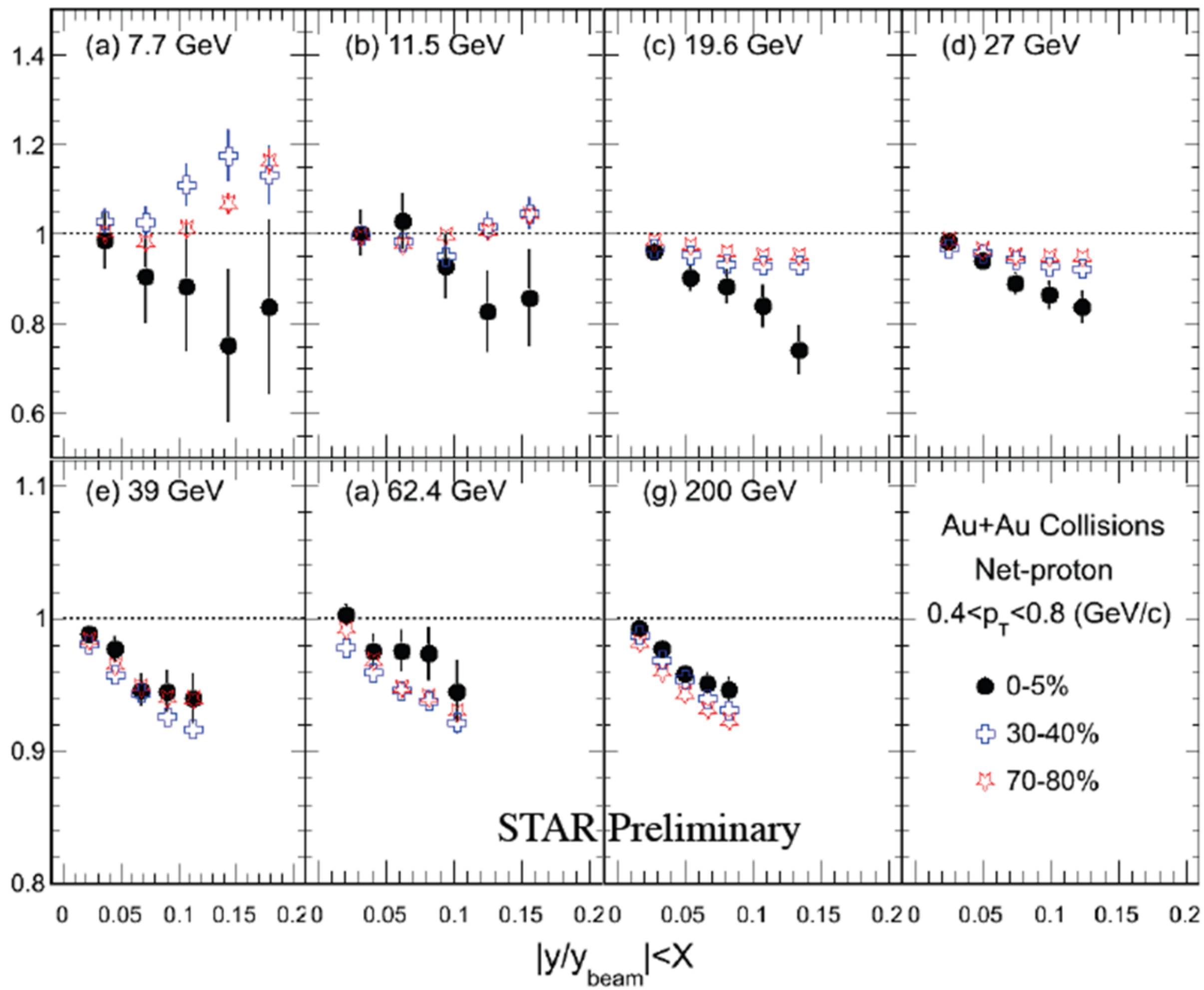
Net Electric Charge Fluctuation @ BES



RHIC (2014)

Rapidity Window Dependence of Net Proton Number Fluctuation @ STAR

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$



Binomial and Poisson Distribution

Binomial Distribution

$$B_p(n;N) = p^n(1-p)^{N-n}\ {}_nC_N$$

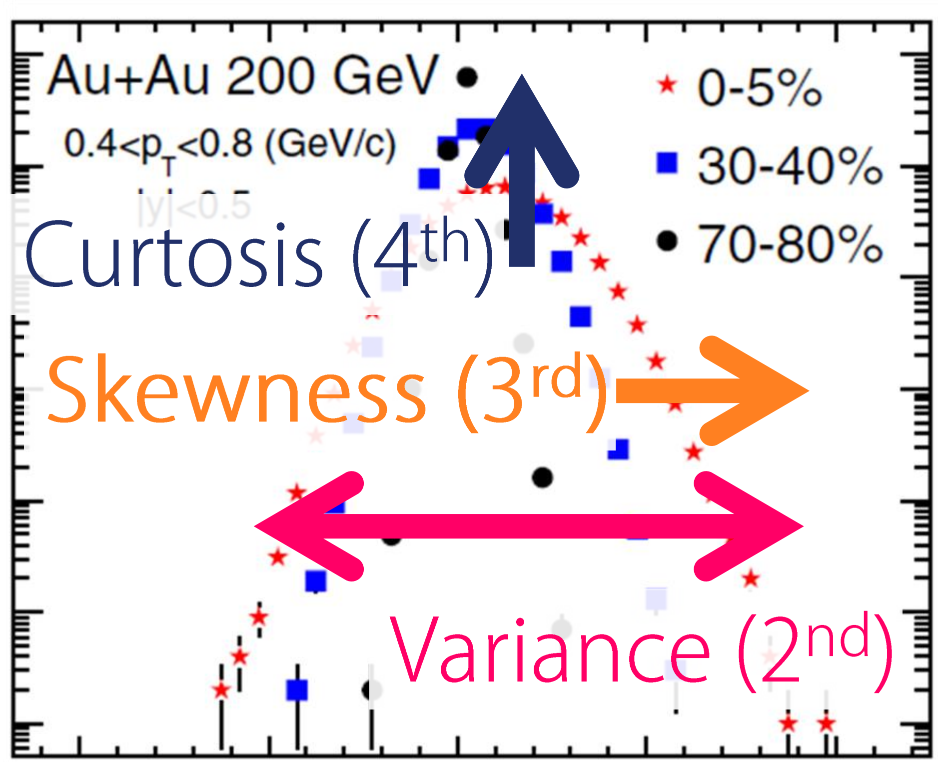
$$p \rightarrow 0$$

Poisson Distribution

$$P_\lambda(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

Cumulants $\langle \delta N^n \rangle_c$

Quanta characterizing
the Probability Distribution



$$\langle (\delta N)^2 \rangle_c = \langle (\delta N)^2 \rangle = \sigma^2$$

$$\langle (\delta N)^3 \rangle_c = \langle (\delta N)^3 \rangle = S\sigma^3$$

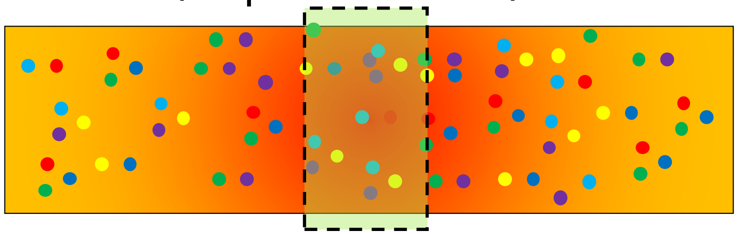
$$\langle (\delta N)^4 \rangle_c = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2 = \kappa\sigma^2$$

Baryon Number Fluctuation

Classical Free Gas in Equilibrated Infinite System

→Poisson $\langle \delta N^n \rangle_c = \langle N \rangle$

QGP (Equilibrium)

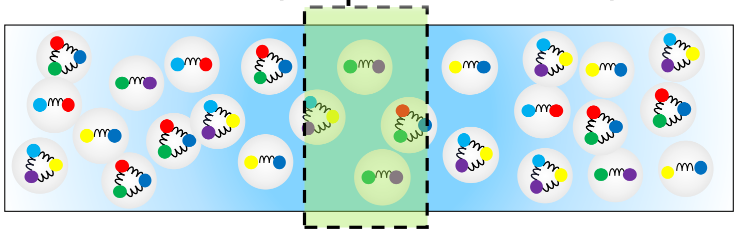


$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$3N_B = N_q$$

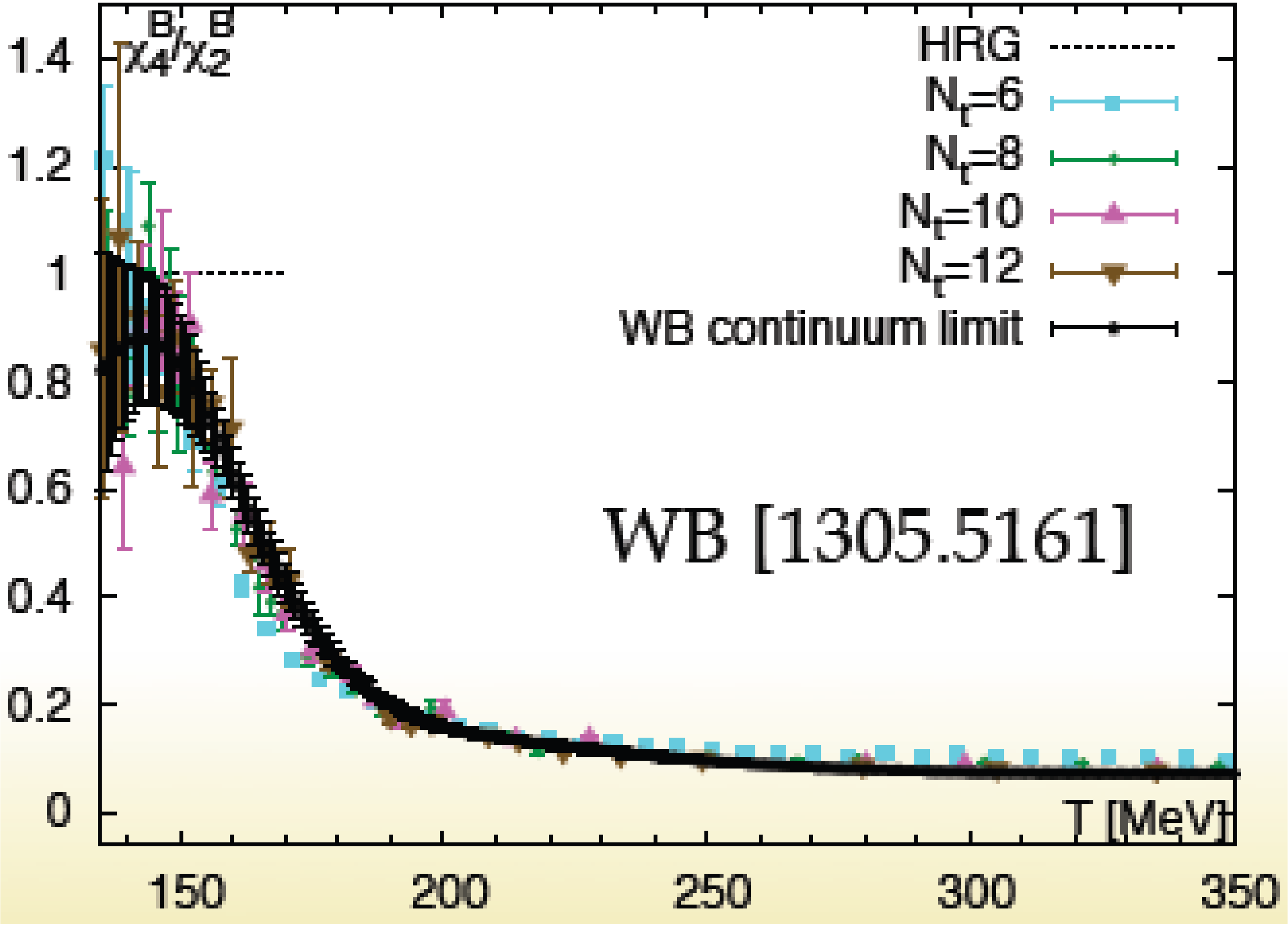
$$\langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

Hadrons (Equilibrium)



$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

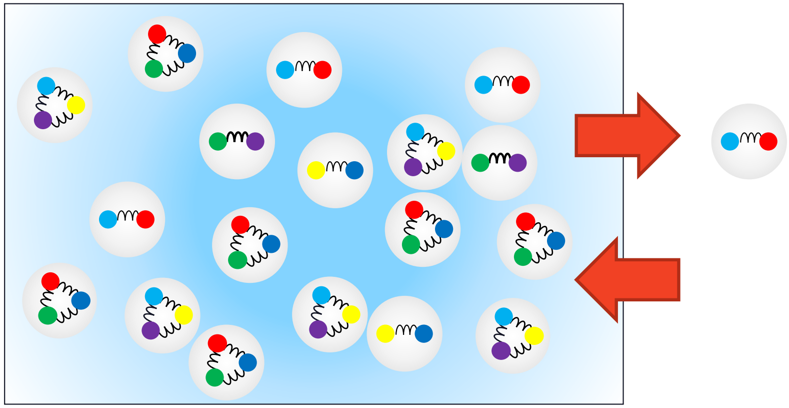
Baryon Number Fluctuation



Conserved VS Non-Conserved Charges

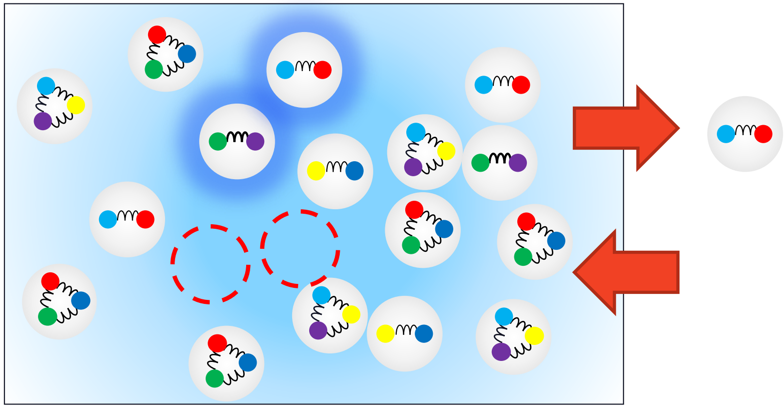
Relaxation time are larger than typical time scale(?)

Conserved Charges



ONLY by Diffusion of Charges

Non-Conserved Charges



Can change everywhere

Information about QGP is easy to survive !!

Asakawa,Heinz,Muller,2000;Jeon,Koch,2000

Stochastic Diffusion Eq. and DME

Stochastic Diffusion Eq.

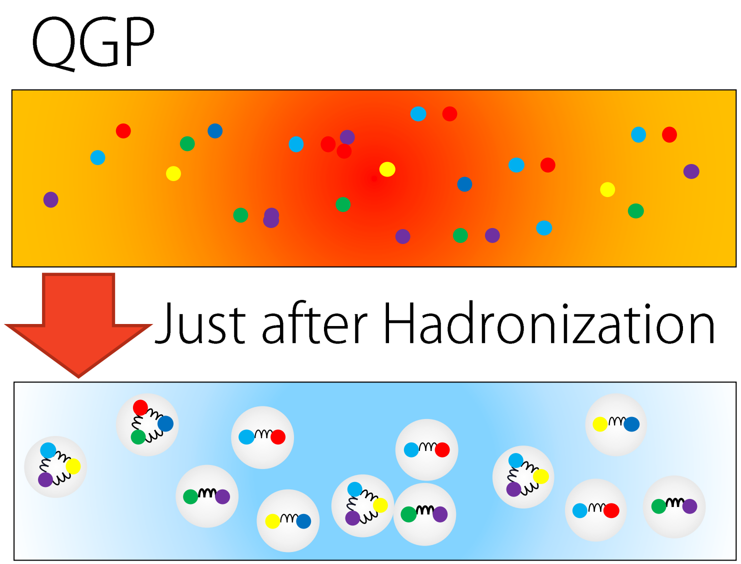
$$\partial_\tau n = D(\tau)\partial_\eta^2 n + \partial_\eta \xi$$

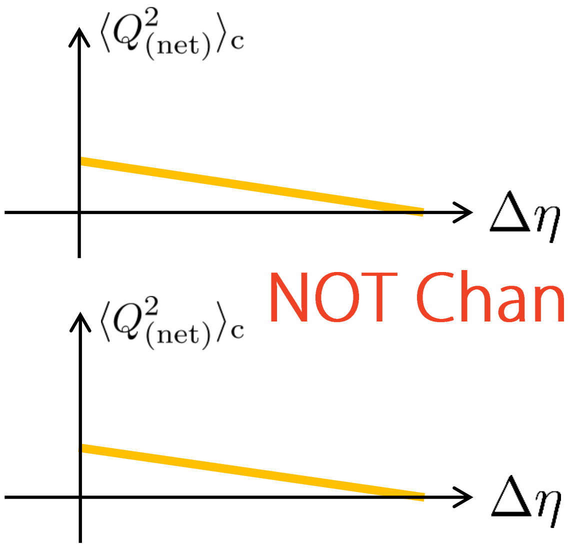
$$\gamma(\tau)a^2 = D(\tau)$$

Diffusion Master Eq.

$$\partial_t P(\boldsymbol{n},\tau) = \gamma(\tau)\sum_m [(n_m+1)\{P(\boldsymbol{n}+\boldsymbol{e}_m-\boldsymbol{e}_{m+1},\tau) + P(\boldsymbol{n}+\boldsymbol{e}_m-\boldsymbol{e}_{m-1},\tau)\} - 2n_m P(\boldsymbol{n},\tau)]$$

Initial Fluctuations





NOT Change!!

Assumption

- Fluctuation does not change just after hadronization
- Thermal Value in the final QGP State

Parameters

Initial Fluctuation

$$\langle Q^2_{(\mathrm{net})}\rangle_c \quad \langle Q^4_{(\mathrm{net})}\rangle_c \quad \langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c$$

Small compared with Equilibrated Hadronic Value

$$\langle Q^2_{(\mathrm{tot})}\rangle_c$$

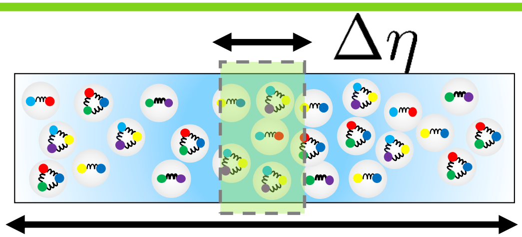
Large?

Free Parameter

$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}, L = \frac{\eta_{\mathrm{tot}}}{d(\tau)}$$

$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

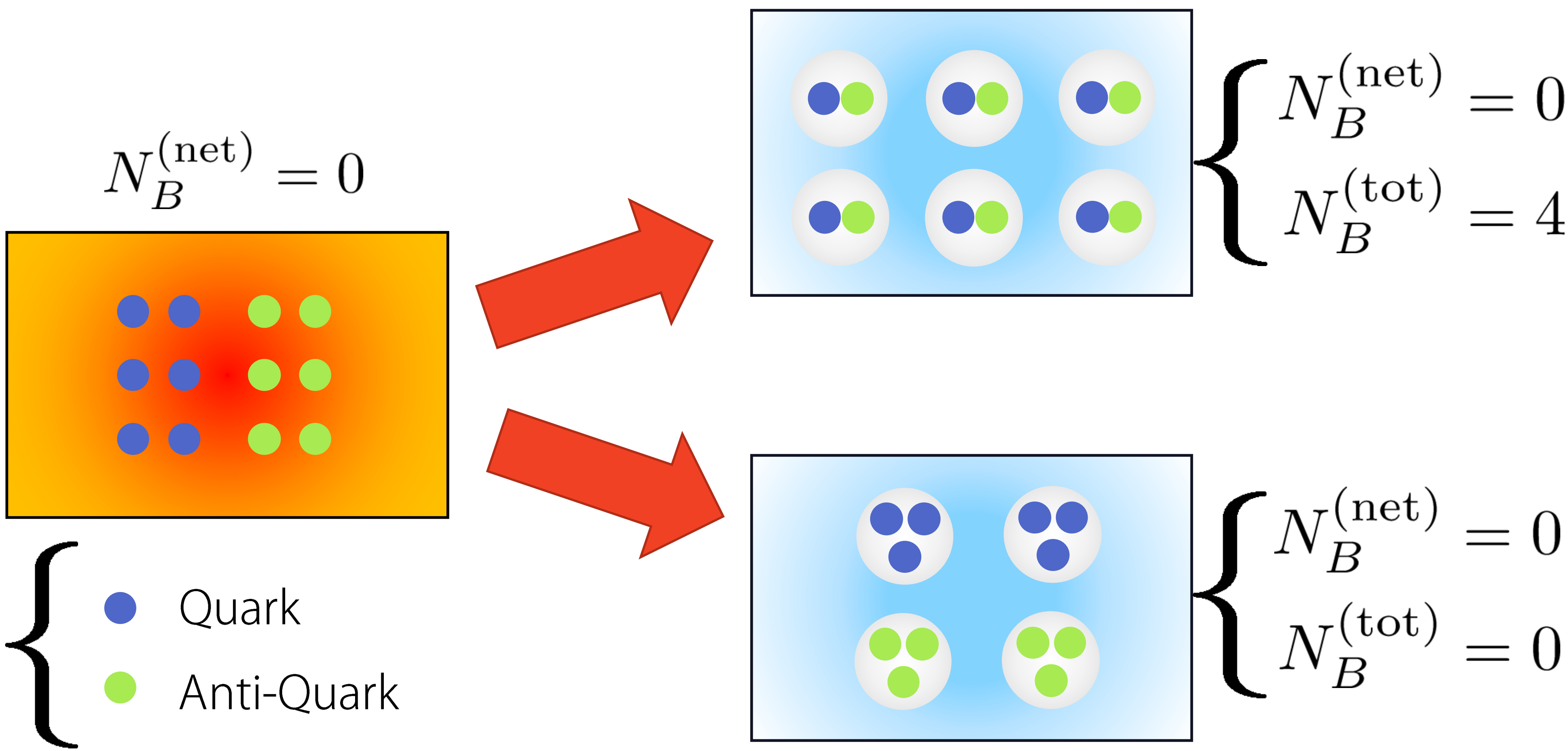
: Average Diffusion Distance



$D(\tau)$: Diffusion Coefficient

η_{tot} : Total Rapidity Length

$\langle Q^2_{(\mathrm{tot})} \rangle_c$

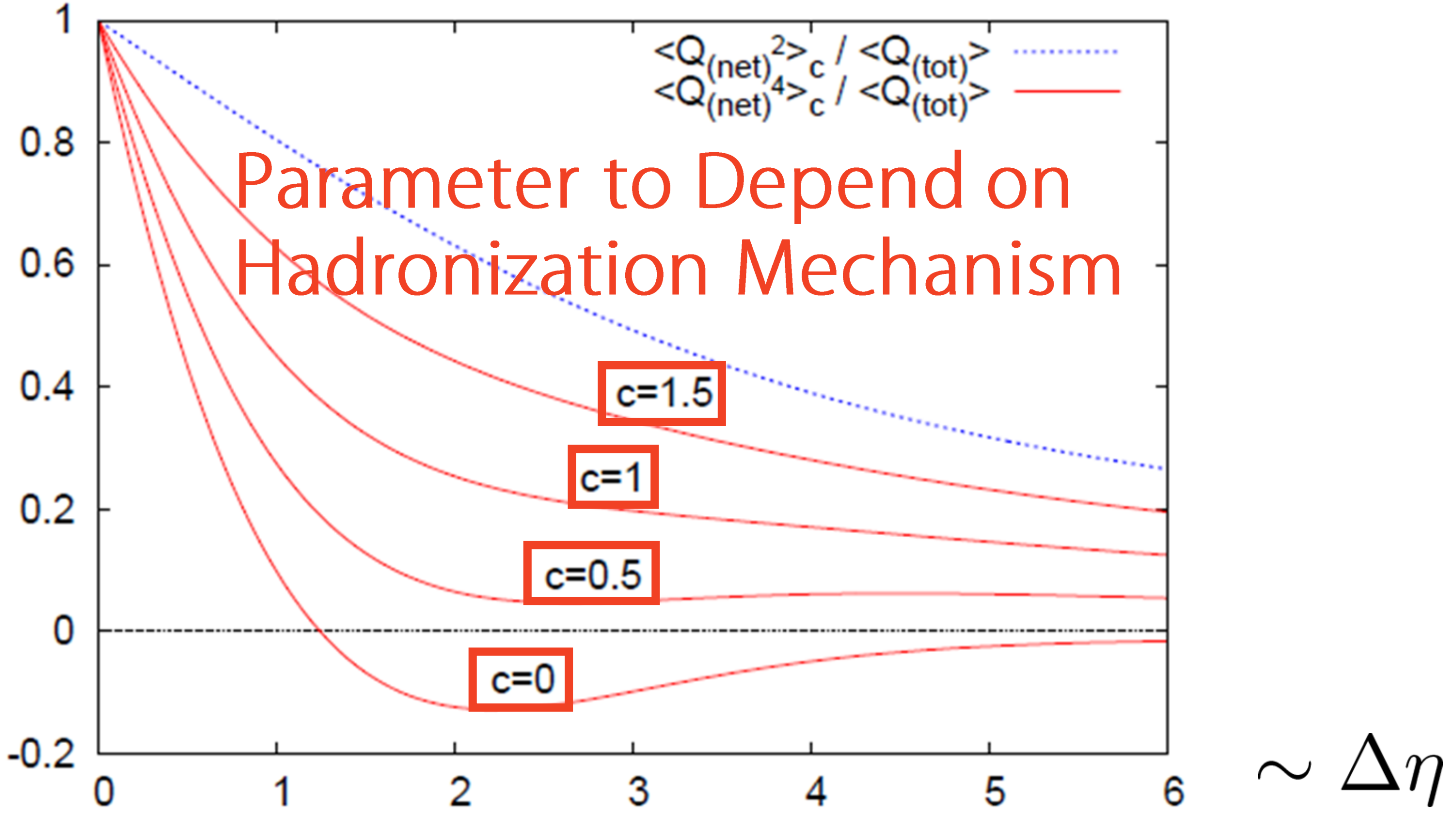


$\langle Q^2_{(\mathrm{tot})} \rangle_c$ strongly depends on Hadronization Mechanism.

Why Higher Order?

Kitazawa, Asakawa, Ono, Phys. Rev. B**728**, 386 (2013)
Sakaida, Asakawa, Kitazawa,

4th order
→Strongly Depend on Hadronization Mechanism!!



Diffusion Constant

Assuming $D(\tau) = D_{\mathrm{H}}\tau^{-2}$,

Const.

In Cartesian Coordinate, $D_{\mathrm{H}} = \frac{T^2}{2}\eta_{\mathrm{tot}}^2 \left[\frac{1}{\tau_0} - \frac{1}{\tau_{\mathrm{fo}}}\right]^{-1}$

Total Rap. Length:	$\eta_{\mathrm{tot}} = 8$
Hadronization Time:	$\tau_0 \sim 8 - 12[\mathrm{fm}]$
Freeze-out Time:	$\tau_{\mathrm{fo}} \sim 20 - 30[\mathrm{fm}]$
Comparison with data→	$T \sim 0.04 - 0.06$

We can estimate Diffusion Const.!! $D_{\mathrm{H}} = 0.6 - 3.5[\mathrm{fm}]$

Charge Density Distribution @ALICE Pb-Pb 2.76TeV

ALICE, PRB 276 (2013) 610-622