Chromo-Weibel instabilities in classical Yang-Mills evolution

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Abstract. The standard model of relativistic heavy-ion collisions describes the hadronic collision in four distinct stages: the initial Color glass condensate, the glasma, the quark gluon plasma and finally the hot hadron gas. Exploring the transition from at first Lorentz contracted sheets of hadrons, the color glass condensate, to thermal QCD matter is the core of this work. The study of anisotropically expanding plasmas shows that non-Abelian plasmas isotropize fast enough thanks to the chromo-Weibel instability. In recent work we find filamentation instabilities in SU(3) Yang-Mills plasma. Real time simulations are a highly promising approach to shed light on the fast thermalization of non-equilibrium matter. This involves to further develop both the theoretical foundations of the description of initial-state fluctuations and the computational framework for the turbulent dynamics of nuclear matter at extreme energy densities.

1. Introduction

A considerable amount of work on the study of the early soft fields [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] that are generated by two ultra-relativistic colliding color sheets has been done from a weakly coupled field-theory perspective in order to better understand the early fast thermalization. In the context of collective excitations Chromo-Weibel instabilities [11, 12, 13, 14, 15, 16, 17, 18, 19, 20] are a subject of intense research.

2. Expanding Vlasov-Yang-Mills

The dynamics of the color glass condensate, the initial effective field theory description, gives rise to very interesting dynamics. In particular there are strong chromo-electric and chromo-magnetic fields that shape longitudinal flux tubes. As a consequence, one has to deal with a very anisotropic momentum distribution in the plasma that leads to the chromo-Weibel instability also called filamentation instability. The instability causes soft modes, the expected chromo-magnetic and chromo-electric quantum fluctuations, to exponentially grow and is characterized by a final turbulent cascade.
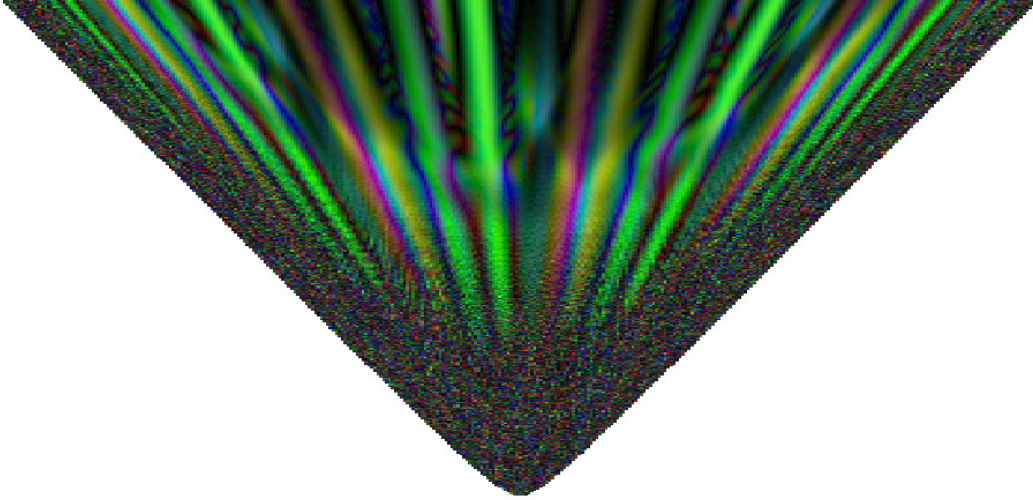
Figure 1. Visualization of a single space-time development of color correlations in a non-Abelian plasma in Bjorken expansion.

Fig. 1 shows a single run real-time evolution depicting the Weibel instability in the forward lightcone. See [4] for details on the Vlasov-Yang-Mills setup. The SU(2) chromo-color is mapped to the RGB color space. Of course on average over several runs one would only see white noise, but the single run nicely depicts the filaments that can exponentially grow from small initial random fluctuations. If one looks carefully one can also find depictions of the three vertex splitting (which is the most efficient process in energy transport from small to higher modes). Note that the most unstable mode is responsible for the creation of the filaments. This mode with the highest growth rate has almost the same wavelength for different anisotropies. In the expanding case the anisotropy of this free streaming setup grows with proper time.

3. SU(3) Yang-Mills box
The transverse/longitudinal electric and magnetic components of the field energy density are given by

$$\mathcal{E} = \mathcal{E}_T + \mathcal{E}_L \;\; = \;\; \mathcal{H}_T^B + \mathcal{H}_T^E + \mathcal{H}_L^B + \mathcal{H}_L^E$$
$$= \;\; \mathrm{tr}\left[\left(F_{zx}^2 + F_{zy}^2\right) + \left(\Pi_x^2 + \Pi_y^2\right) + F_{xy}^2 + \Pi_z^2\right], \tag{1}$$

where the gauge-covariant field strength tensor is defined as $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - ig[A_\alpha, A_\beta]$ and the canonical conjugate field momenta as $\Pi_\alpha(x) = \partial_0 A_\alpha(x)$.

Fig. 2 shows the local transversal energy densities averaged over 30 runs in a SU(3) Yang-Mills evolution [21]. It was found that the chromo-Weibel instability is at first stronger in chromo-magnetic fields. Hence we show its behavior in the xz plane. In Fig. 2 the perpendicular axis is in the longitudinal direction (along the beam axes) and the horizontal axis is in the transversal direction. Each box is for a given time step. The chosen runs have 120 sites in the longitudinal direction and 40 in the transverse. The saturation scale $Q_s = g^2\mu$ is set to $Q_s = 2GeV$ for typical values for ultrarelativistic collisions at the Large Hadron Collider (hence each upper time step value is to be divided by 100 to be expressed in fm/c).
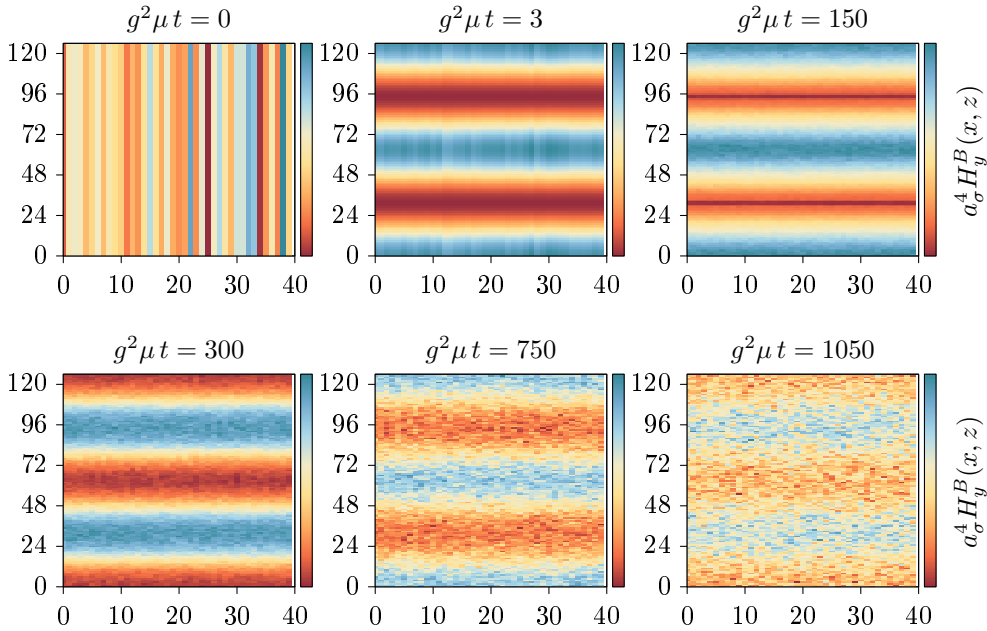
Figure 2. Visualization of the local transversal chromo-magnetic energy densities in a fixed box $SU(3)$ Yang-Mills evolution in the yz plane for several timesteps.

At startup of the evolution there are only small random fluctuations in the chromo-magnetic fields that get rapidly ordered in the next timestep perpendicular to the beam direction. As the initial condition initializes the chromo-electric fields with the strong leading terms the Weibel instabilities are not immediately visible in its local transversal energy component, but they turn out equally strong after some evolution. The time it takes for this to happen of course depends on the initial amplitude of the initialized fluctuations.

As expected the check plots in Fig. 3 show that there are no filamentation instabilities happening in the transversal plane for both the local longitudinal chromo-electric and chromo-magnetic energy components. The chosen timesteps (startup plus two additional) for this test case do not matter, as any of them will show a random value outcome. At any timestep of the evolution the Gauss law constraint is respected, and the fields are Coulomb gauge fixed.

4. Conclusions

$SU(3)$ Yang-Mills evolution with anisotropic initial conditions exhibits filamentation instabilities in the transverse components of the local energy densities. We found evidence for the emergence of the chromo-Weibel instability that isotropizes the soft field pressures. We are planning to improve our numerical results by adding fermionic degrees of freedom and also studying pure Yang-Mills dynamics in an longitudinally expanding geometry. Preliminary results of this upcoming work with fermionic degrees of freedom shown at the HotQuarks 2014 conference plot a fast fermionic energy rise, which would lead to a strong initial entropy production. In the future it would be interesting to compare the momentum-space anisotropy dependence of many important heavy-ion collision observables with experimental results.
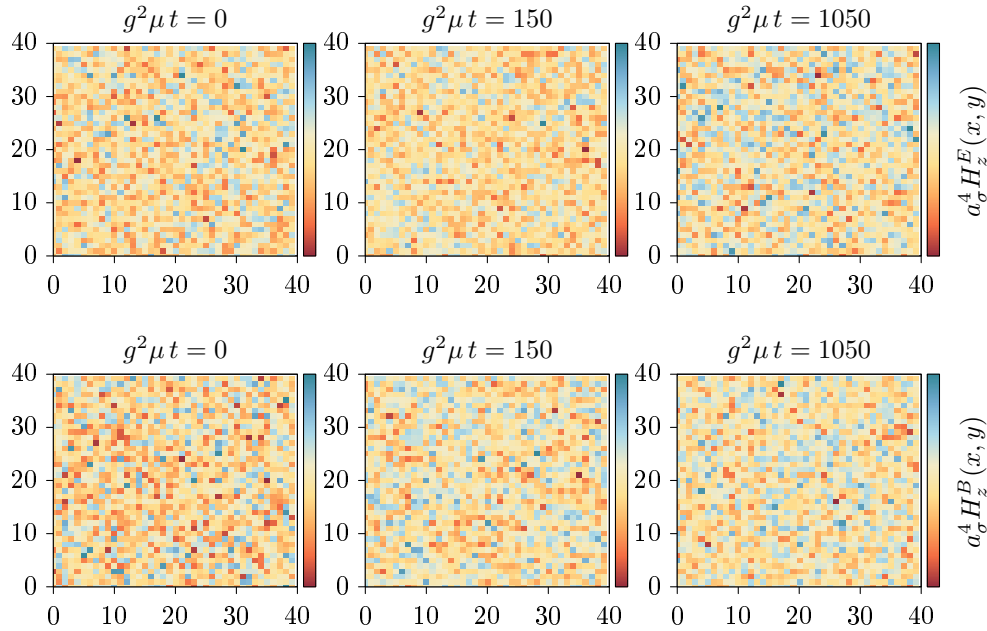
Figure 3. Visualization of the local longitudinal chromo-electric and chromo-magnetic energy density in a fixed box SU(3) Yang-Mills evolution in the transversal xy plane for three different timesteps.

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