Electric conductivity of the quark-gluon plasma investigated using Boltzmann transport theory

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Abstract. The electric conductivity of a hot quark-gluon plasma is obtained numerically by solving the relativistic Boltzmann equation. We use a relativistic, semi-classical partonic cascade including screened binary and inelastic, radiative $2 \leftrightarrow 3$ perturbative QCD scattering. We employ the Green-Kubo formula and, independently, evaluate the static electric current established by the influence of an external electric field, in order to extract the conductivity. Both numerical methods give the same result. By using only constant, isotropic cross sections, we are able to compare the numerical results and analytic formulas with excellent agreement. Using pQCD scattering with running coupling in the transport code allows us to contrast our results with recent lattice QCD data.

1. Introduction

The quark-gluon plasma (QGP), generated temporarily in ultrarelativisitic collisions of heavy nuclei, is a poorly understood state of matter, governed by quantum chromodynamics (QCD). Relativistic transport coefficients, although not directly measurable, provide a meaningful way by which the QGP can be characterized.

The precise calculation of transport coefficients in a realistic medium requires numerically solvable theories, such as the relativistic Boltzmann transport theory. In the past, the shear viscosity over entropy density ratio η/s has been evaluated from the numerical solution of the Boltzmann equation [1–3], as well as the heat conductivity coefficient κ [4].

The longitudinal static electric conductivity σ_{el} relates the response of the charged particle diffusion current density $\vec{j}$ to an externally applied static electric field $\vec{E}$, $\vec{j} = \sigma_{\mathrm{el}}\vec{E}$. Recently, several scientific groups focused on the electric conductivity [5–17]. It is related to the soft dilepton production rate [18] and the diffusion of magnetic fields in the medium [19–21]. Results for the conductivity in different theoretical frameworks provide us furthermore with the possibility to compare effective cross sections of charged particles.

Here we present a systematic study of the electric conductivity coefficient for a hot QCD plasma [22], applying the microscopic relativistic transport code *Boltzmann Approach to Multi-Parton Scatterings* (BAMPS) [23].

2. Numerical methods

We solve the relativistic 3+1-dimensional Boltzmann equation,

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{E}\cdot\vec{\bigtriangledown}\right)f_k(x,t)=\mathcal{C}_k^{2\to 2}\left[f\right]+\mathcal{C}_k^{2\leftrightarrow 3}\left[f\right], \tag{1}$$

using the BAMPS framework, developed and previously employed in Refs. [1, 2, 4, 23–32]. On-shell particles (3 quark flavors and gluons) propagate freely but collide within the stochastic interpretation of transition rates, including all leading-order $2\to 2$ and $2\leftrightarrow 3$ pQCD processes, or alternatively, using a fixed, isotropic cross section. The inelastic cross sections are obtained through the Gunion-Bertsch matrix element [33], which was further improved [30] and applied within BAMPS computations in Ref. [34]. Within BAMPS, the running of the strong coupling α_s is evaluated explicitly at the microscopic scale of the momentum transfer of the respective channel [28, 29, 34]. The Landau-Pomeranchuk-Migdal (LPM) is effectively modeled, see Ref. [34]. To extract the electric conductivity, we employ the zero-frequency Green-Kubo [35, 36] formula for an equilibrated system with volume V and temperature $T=\beta^{-1}$,

$$\sigma_{\mathrm{el}}=\beta V\int\limits_0^\infty \langle j^x(0)j^x(t)\rangle\,\mathrm{d}t, \tag{2}$$

with the total electric current density

$$j^\mu=\sum_{k=1}^{M}q_k V_k^\mu,\;\; N_k^\mu=n_k u^\mu+V_k^\mu, \tag{3}$$

where the sum runs over all particle species k with density n_k and charge q_k. The particle flow $N_k^\mu(x)$ can be decomposed with respect to the timelike total four velocity u^μ and the spacelike part V_k^μ. The electric current is extracted from BAMPS in equilibrium box setups, for different temperatures. Separately, we apply an external static electric field in x direction, $\vec{E}=E_x\vec{e}_x$. Because of momentum transfer due to collisions a finite electric current will establish in the simulation after a sufficiently large time. The average magnitude of this static current is proportional to the electric field, and defines the zero-frequency electric conductivity σ_{el} directly, $\langle j^x(t)\rangle_{\mathrm{static}}=\sigma_{\mathrm{el}}E^x$. It can be seen that the results of both methods are equal within the numerical uncertainties [22]. For constant, isotropic cross sections, results from BAMPS could be precisely reproduced analytically by solving the linearized Boltzmann equation within kinetic theory. Using the relaxation time approximation of the collision term, the result for the conductivity $\sigma_{\mathrm{el}}=1/(3T)\sum_{k=1}^{M}q_k^2 n_k\tau$ deviates by about 10% from BAMPS, using the transport relaxation time $\tau^{-1}=n_{\mathrm{total}}\sigma_{\mathrm{transport\ cross\ section}}$.

3. pQCD-based cross sections

The methods to obtain the electric conductivity presented in Sec. 2 can be readily applied to more realistic scenarios, where leading-order pQCD-based cross sections [23,30,34] are employed within BAMPS. Figure 1 depicts our results for the electric conductivity using pQCD cross sections.

The filled red squares show results for a scenario using only elastic $2\leftrightarrow 2$ pQCD interactions when the strong coupling constant is fixed to $\alpha_s=0.3$. The ratio σ_{el}/T is constant within the uncertainties. This is expected by dimensional analysis, as $\sigma_{\mathrm{el}}/T\sim T^{-2}\sigma_{pQCD}^{-1}$ and leading-order pQCD cross sections $\sigma_{\mathrm{pQCD}}\sim T^{-2}$. In Ref. [5], the authors predict the same temperature dependence.

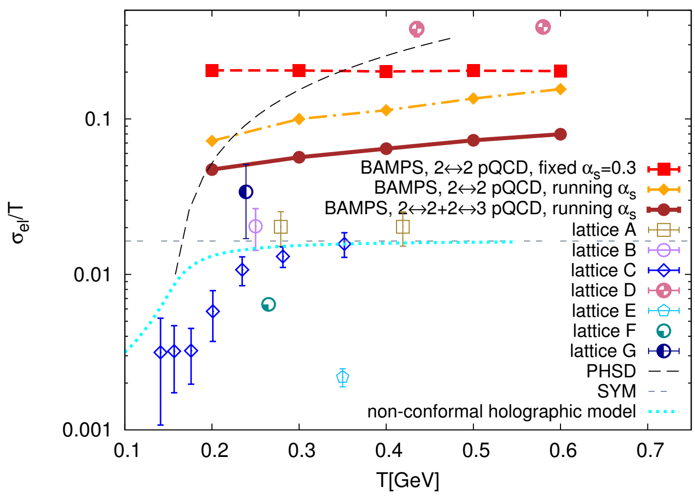

Figure 1. Numerical results for the electric conductivity (filled symbols) compared to recent results from literature [22]. The open symbols represent results from lattice QCD. PHSD: [14], SYM: [37], nonconformal holographic model: [17], lattice A: [8], lattice B: [12], lattice C: [13], lattice D: [7], lattice E: [9], lattice F: [11], lattice G: [10]. The electric charge is explicitly multiplied out, $e^2 = 4\pi/137$.

The filled yellow diamonds depict the electric conductivity over temperature for a running coupling constant α_s [28]. The QCD running coupling is evaluated at the momentum transfer of each microscopic interaction, and leads to a temperature dependence of the average coupling [34], and hence a qualitatively different temperature dependence of the electric conductivity is obtained. The interaction strength decreases with increasing temperature, and the effective cross section decreases accordingly.

The filled dark red circles are results for the most realistic scenario. Here we employ elastic $2 \leftrightarrow 2$ and inelastic $2 \leftrightarrow 3$ scatterings, and running coupling. The LPM effect is modeled as described in Ref. [34]. The inclusion of inelastic collisions accounts for an overall higher effective cross section than in the elastic scenarios. Therefore, the electric conductivity decreases by about 40%, whereas the temperature dependence seems to be dominated by the running of α_s.

In Fig. 1, we contrast the electric conductivity obtained using BAMPS with recent lQCD results, the transport model PHSD, a conformal, and a nonconformal holographic computation.

This study allows us in a unique way to study the overall effective scattering rates for a hot QCD plasma microscopically, including all leading-order elastic and inelastic processes. The electric conductivity reflects in a profound way the effect of inelastic pQCD scattering and the running of α_s. We believe that this is an important result of pQCD, and comparisons with other theories are reasonable.

4. Conclusion and Outlook

In this work we extracted the electric conductivity coefficient for a dilute gas of massless and classical particles numerically by solving the relativistic Boltzmann equation. We use two independent methods to extract the transport coefficient and compare with analytic results whenever possible. Running coupling is explicitly seen in a rise of $\sigma_{\rm el}/T$ with temperature. Inclusion of inelastic collisions decreases the conductivity. We see a similar functional dependence of the electric conductivity on temperature compared to recent lQCD calculations. Interaction properties of the QGP are reflected in the electric conductivity. More refined analytic and lQCD calculations, compared with microscopic transport simulations, will further restrain the value of the conductivity and shed light upon the microscopic interactions inside the QGP.

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