Electric Conductivity of the QGP

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Abstract. The transport coefficients of strongly interacting matter have attracted a great interest in the field of Quark-Gluon Plasma (QGP). In this work we compute electric conductivity σ_{el} solving numerically the Relativistic Boltzmann Transport (RBT) equation in a uniform box with periodic boundary conditions considering 2−body scatterings. We compare numerical results obtained using two methods, Green-Kubo correlator and E-field method, with analytic formulas in Relaxation Time Approximation (RTA). We present results for the realistic case of the QGP system considering both a quasi-particle model tuned to lattice QCD thermodynamics as well as the case of a pQCD gas with a running coupling constant. Calculations based on RTA underestimate σ_{el} of about 60%.

1. Introduction

Relativistic Heavy Ion Collisions (HICs) experiments performed by Relativistic Heavy Ion Collider (RHIC) at BNL and by Large Hadron Collider (LHC) at CERN have reached the same conditions of temperature and energy density as they have existed in the early universe. A system of strongly interacting particles above the critical temperature $T_c \sim 160$ MeV [1, 2] is expected to undergo to a phase transition from hadron matter to Quark-Gluon Plasma (QGP) [3]. The phenomenological studies by viscous hydrodynamics [4, 5, 6] and parton transport [7, 8, 9] about the collective behaviour of such matter have shown that the QGP has a very small shear viscosity to entropy density ratio η/s, quite close to the conjectured lower bound for a strongly interacting system in the limit of infinite coupling $\eta/s = 1/4\pi$ [10].

As the hot QCD matter is a plasma, another key transport coefficient, yet much less studied, is the electric conductivity σ_{el}. This coefficient represents the linear response of the system to an applied external electric field. HICs are expected to generate very high electric and magnetic field ($eE \simeq eB \simeq m_\pi^2$) in the very early stage of the collisions. A large value of σ_{el} would lead to a relaxation time for electromagnetic field of the order of $1-2$ fm/c [11, 12], which would be of fundamental importance for the strength of the Chiral-Magnetic Effect [13]. Also in mass asymmetric collisions, like $Cu + Au$, the electric field directed from Au to Cu induces currents resulting in charge asymmetric collective flow directly related to σ_{el} [12]. Furthermore σ_{el} can be directly related to the emission rate of soft photons [14]. Despite its relevance there is yet only a poor theoretical and phenomenological knowledge of σ_{el} and its temperature dependece. In this work we compute the electric conductivity σ_{el} solving numerically the Relativistic Boltzmann Transport (RBT) equation considering two body elastic cross section.

2. Electric conductivity from the Relativistic Boltzmann Equation

The electric conductivity σ_{el} is the transport coefficient that characterizes the response of a system to an external electric field: Ohm's law $\vec{j} = \sigma_{el}\vec{E}$ defines the electric conductivity in linear response theory. The starting point of our computation is the Relativistic Boltzmann Transport (RBT) equation that in the presence of an external field can be written as [15]:

$$p^\mu \partial_\mu f(x,p) + qF^{\alpha\beta} p_\beta \frac{\partial}{\partial p^\alpha} f(x,p) = \mathcal{C}_{22}(x,p) = \int_{2,1',2'} (f_{1'}f_{2'} - f_1 f_2)\, |\mathcal{M}_{1'2'\to 12}| \delta^4(p_1+p_2-p'_1-p'_2) \tag{1}$$

where $f(x,p)$ is the distribution function, $F^{\alpha\beta}$ is the electromagnetic field strength tensor, q is the charge. $\mathcal{C}_{22}[f]$ is the collision integral considering only $2 \to 2$ scatterings: $\mathcal{M}$ is the transition matrix for the elastic process linked to the differential cross section $|\mathcal{M}|^2 = 16\pi s(s-4m^2)d\sigma/dt$ with s and t the Mandelstam variables and m the mass of particles. As the Boltzmann equation is an integro-differential equation for the function $f(x,p)$, it is necessary to approximate the right hand side of Eq. (1) in order to obtain an analytical solution. The most simple scheme is the Relaxation Time Approximation (RTA) which simplifies the collision integral as $\mathcal{C}[f] \simeq -\frac{p^\mu u_\mu}{\tau}(f - f_{eq})$ where τ is the so-called relaxation time which represents the time scale for the system to relax toward the equilibrium state characterized by f_{eq}. Following simple mathematical steps, one obtains the following analytical formula [16, 17]:

$$\sigma_{el} = \frac{e^2}{3T} \sum_{j=q,\bar{q}} q_j^2 \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{E^2} \tau_j f_{eq} = \frac{e_\star^2}{3T} \left\langle \frac{\vec{p}^2}{E^2} \right\rangle \tau_q \rho_q \tag{2}$$

where q_j is the quarks charge $(\pm 1/3, \pm 2/3)$, ρ_q is the quark density, $e_\star^2 = e^2 \sum_{j=\bar{q},q} q_j^2 = 4e^2/3$. τ_q is the quark transport relaxation time which for a quark of species i can be written as $\tau_{tr,i}^{-1} = \sum_{j=q,\bar{q},g} \langle \rho_j v_{rel}^{ij} \sigma_{tr}^{ij} \rangle$ where v_{rel} is the relative velocity of the two scattering particles and σ_{tr}^{ij} is the transport cross section.

We have computed the electric conductivity σ_{el} solving numerically the RBT equation (see Ref. [18] for details) in a box with periodic boundary conditions using two methods: *Green-Kubo correlator* and *E-field* methods. The first one, in the framework of Linear Response Theory, relates transport coefficients to correlation functions of the corresponding flux or tensor in thermal equilibrium. In this approach one obtains $\sigma_{el} = \frac{V}{T}\int_0^\infty dt\, \langle j_z(t) j_z(0) \rangle = \frac{V}{T} \langle j_z(0) j_z(0) \rangle \tau_{\sigma_{el}}$ where j_z is the z component of the electric current, $\langle \ldots \rangle$ is the thermal average at equilibrium, i.e. without any external electric field. The second method used is suggested by the definition itself of $\sigma_{el} = J/E$: taking the ratio between the electric current measured and the electric field applied one obtains σ_{el}. To simulate a constant electric field $\vec{E}$ in the box, it is sufficient to solve the following equation of motions for particles $dp_z^i/dt = q_i e E_z$ (assuming $\vec{E}$ along the z direction).

In Fig. 1 we show the Green-Kubo correlator as a function of time for a system of massless particles interacting via an isotropic cross-section $\sigma_{tot} = 3$ mb for different temperatures $T = 0.1 - 0.6$ GeV: the behaviour of such correlators is clearly an exponential function $\exp(-t/\tau_{\sigma_{el}})$. In Fig. 2 we plot x,y and z components of electric current $\vec{j}$ as a function of time for a system of particles with $m = 0.4$ GeV, $T = 0.2$ GeV, $\sigma_{tot} = 10$ mb and an external electric field $eE = 0.05$ GeV/fm along z direction: j_x and j_y fluctuate around zero while j_z reaches a saturation value proportional to E.

3. Electric conductivity of the QGP

In this section we consider the more realistic case of quarks, antiquarks and gluons interacting via different anisotropic and energy dependent cross section according to the pQCD-like scheme with a screening mass m_D as arising from HTL approach: $m_D \sim g(T)T$. The total cross section

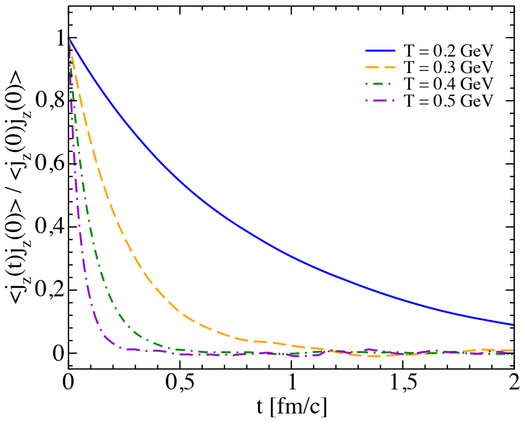

Figure 1. Green-Kubo correlators as a function of time for different temperatures for a system of massless particles interacting via isotropic scatterings with $\sigma_{tot} = 3$ mb.

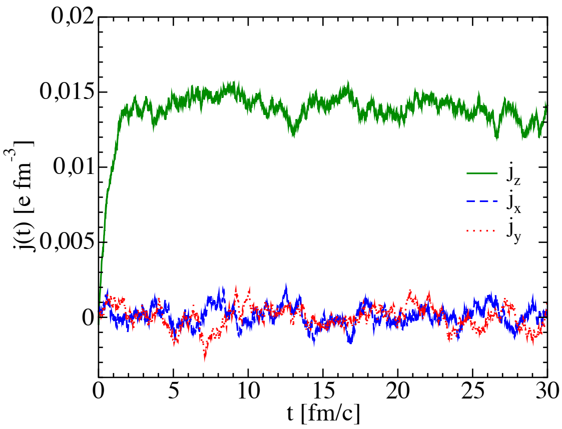

Figure 2. x, y and z components of electric current $\vec{j}$ for a system of massive particles ($m = 0.4$ GeV) at $T = 0.2$ GeV with an applied electric field $eE = 0.05$ GeV/fm.

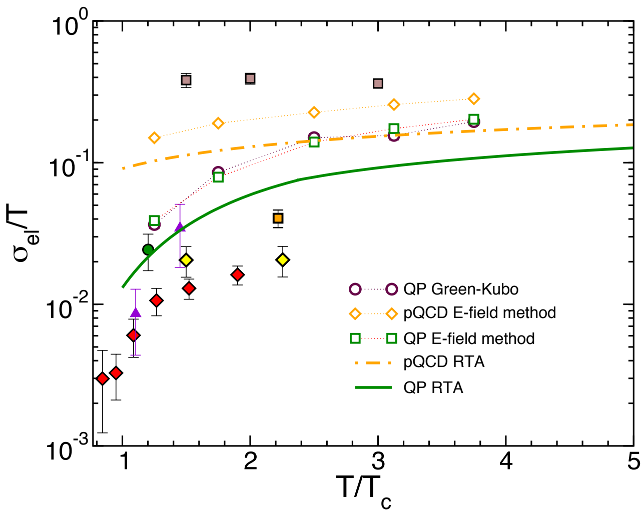

Figure 3. Electric conductivity σ_{el}/T as a function of T/T_c. Open circles are Green-Kubo results for QP model, green open squares are QP model results computed with E-field method, orange diamonds are pQCD results with E-filed method; green line and orange dot-dashed line are RTA for the QP model and pQCD case. Symbols are Lattice data (see text for details).

used has the following form: $\sigma_{tot}^{ij} = \beta^{ij}\sigma(s) = \beta^{ij}\frac{\pi\alpha_s^2}{m_D^2}\frac{s}{s+m_D^2}$ where $\alpha_s = g^2/(4\pi)$ and β^{ij} depends on the species of interacting particles: $\beta^{qq} = 16/9$, $\beta^{qq'} = 8/9$, $\beta^{qg} = 2$ and $\beta^{gg} = 9$.

In order to take into account the thermodynamics from lattice QCD computations, we employ the quasi-particle (QP) model. The aim of a QP model is to describe a strongly interacting system in terms of weakly interacting particles whose masses are generated by the non-perturbative effects. In this model the running coupling $g(T)$ can be parametrized by $g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$ with $\lambda = 2.6$, $T_s/T_c = 0.57$, $T_c = 0.16$ GeV as in [19] similarly to [20]. Quarks' and gluons' masses are given by $m_q^2 = g^2T^2/3$ and $m_g^2 = 3g^2T^2/4$. We also study the behaviour of electric conductivity using the pQCD running coupling $g_{pQCD} = \frac{8\pi}{9}\ln^{-1}\left(\frac{2\pi T}{\Lambda_{QCD}}\right)$ considering massless particles: even if this case does not describe the phase transition, it is interesting to study the σ_{el} dependence on a different running coupling and also the asymptotic limit valid for $T \gg T_c$.

In Fig. 3 we show $\sigma_{el/T}$ as a function of T/T_c. Open circles are computed using Green-Kubo correlator, green squares with the E-field method (applying $eE = 0.02 \div 0.05$ GeV/fm to guarantee the saturation of electric current) for the quasi particle model, orange open diamonds represent result for the massless pQCD case computed only with the E-field method. Green line is RTA for QP model, orange dot-dashed line is RTA for the massless pQCD case. Symbols denotes Lattice data: grey squares [21], violet triangles [22], green circles [23], yellow diamonds [24], red diamonds [25] and orange square [26]. Green-Kubo results are in agreement with the E-field method. Numerical results predicted by the QP model are about a factor of 4 greater then recent Lattice QCD calculations [25]. Calculations based on RTA underestimate σ_{el}/T for both QP model and pQCD case of about a factor 1.7.

4. Conclusions

Transport coefficients characterize the response of a system to different kind of perturbations and determine the dynamics of the system toward the equilibrium state through dissipation. As the QGP created in Heavy Ions Collisions is a system far from equilibrium, the study of transport coefficients is mandatory. In this work we have computed σ_{el} solving numerically the RBT equation using two methods: the Electric field method, suggested by the definition itself ($J = \sigma_{el} E$), and Green-Kubo correlator. We find that the two methods are in very good agreement. Furthermore we find that RTA underestimates σ_{el} by about a factor of 1.7. Our results are quite general but they have been found considering only $2 \leftrightarrow 2$ collisions. It is very interesting to have similar information also when $2 \leftrightarrow 3$ collisions are included as in BAMPS [27].

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