

Multi-Reggeon exchanges at high pt from gluon saturation

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based on arXiv:1407.3080

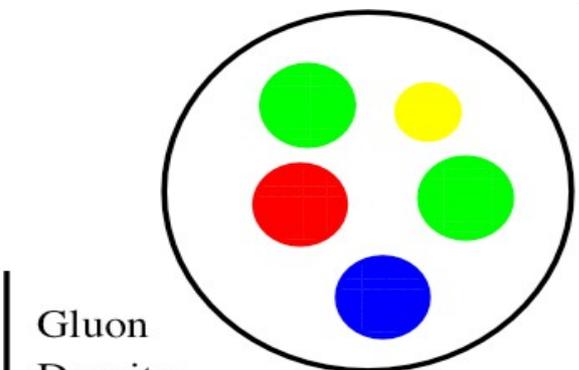
A. Ayala, E. Cazaroto, L. Hernandez, J. Jalilian-Marian, M.E. Tejeda-Yeomans

**10th international workshop on high pt Physics in the RHIC/LHC era September 9-12, 2014
Nantes, France**

Gluon saturation/CGC $S \rightarrow \infty, Q^2 \text{ fixed}, x_{Bj} \equiv \frac{Q^2}{S} \rightarrow 0$

*Gribov-Levin-Ryskin
Mueller-Qiu*

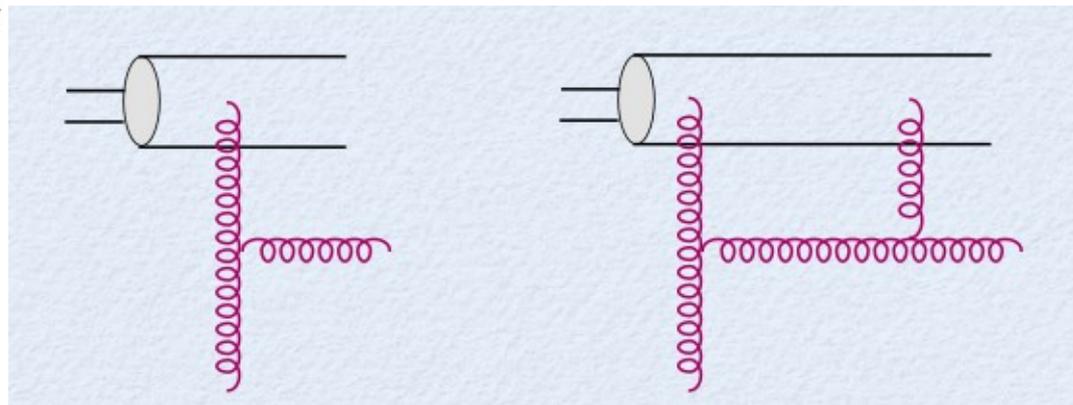
**“attractive” bremsstrahlung
vs. “repulsive” recombination**



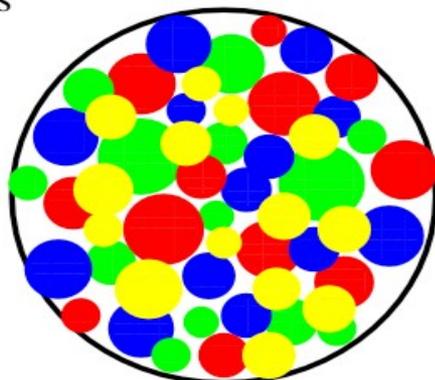
Gluon
Density
Grows

Low Energy

$\frac{1}{x}$ ↓



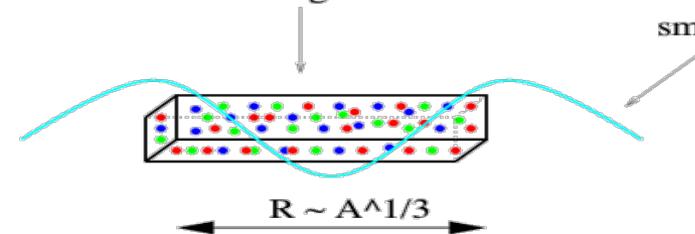
High Energy



color charges
at large x

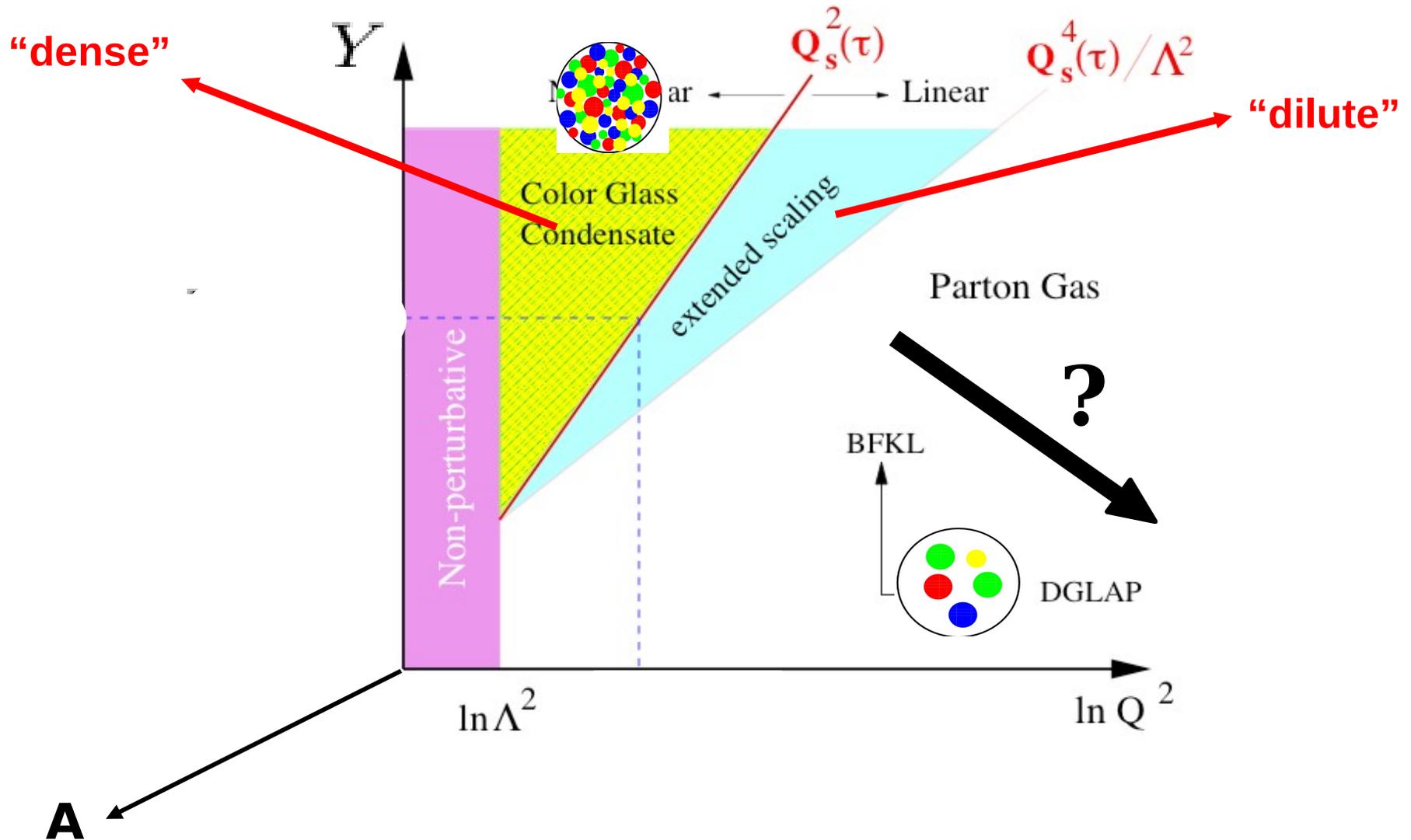
small x gluon

$$\frac{\alpha_s x G(x, b_t, Q^2)}{S_{\perp} Q^2} \sim 1$$



$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

Road Map of QCD Phase Space



QCD at low x : CGC

(a high gluon density environment)

two main effects:

“multiple scatterings”

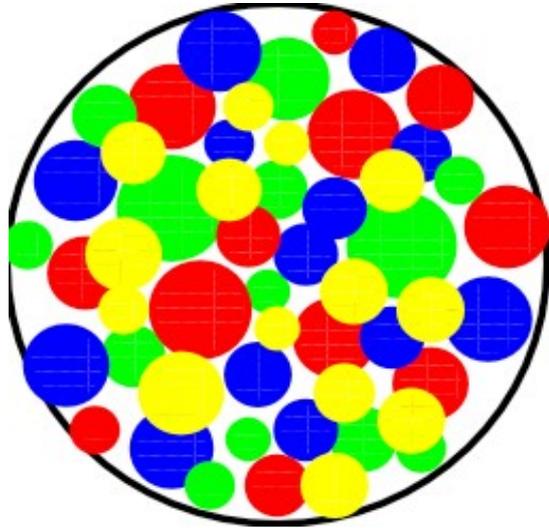
evolution with $\ln(1/x)$

pQCD with collinear factorization:

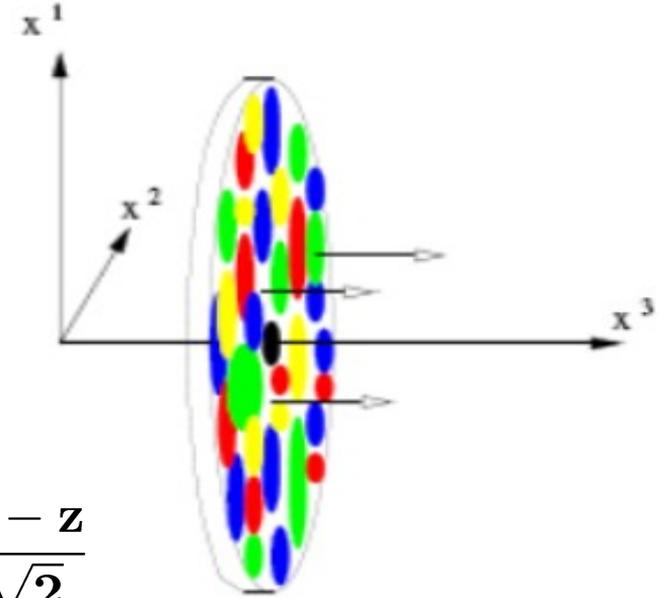
single scattering

evolution with $\ln Q^2$

MV model of nuclei at high energy



boost



$$x^+ \equiv \frac{t + z}{\sqrt{2}}$$

$$x^- \equiv \frac{t - z}{\sqrt{2}}$$

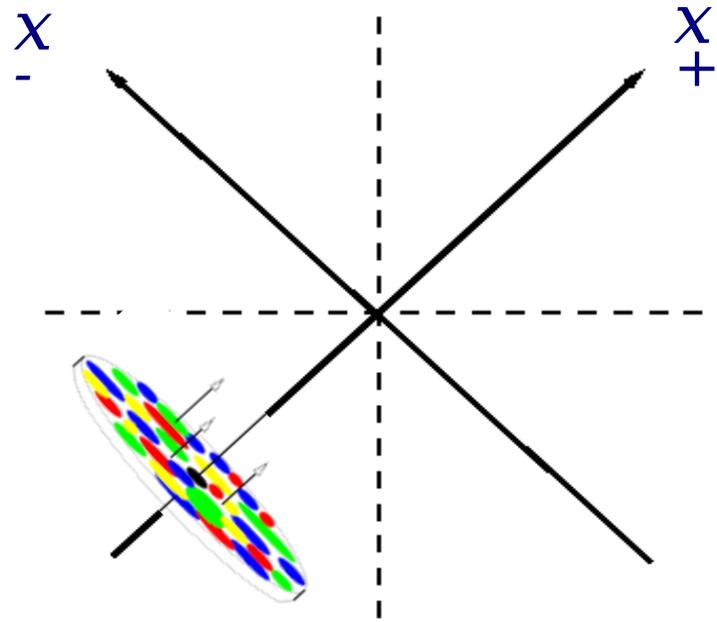
sheet of color charge moving along x^+ and sitting at $x^- = 0$

$$\mathbf{J}_a^\mu(\mathbf{x}) \equiv \delta^{\mu+} \delta(\mathbf{x}^-) \rho_a(\mathbf{x}_t)$$

color current

color charge

current has only one large component



Classical Field: $\alpha^{\mathbf{a}}(\mathbf{k}_t)$

$$\mathbf{A}_{\mathbf{a}}^{\mu}(\mathbf{x}_t, \mathbf{x}^-) \sim \delta^{\mu+} \delta(\mathbf{x}^-) \alpha_{\mathbf{a}}(\mathbf{x}_t)$$

where the classical field α and the color charge density ρ are related via equations of motion

$$\alpha^{\mathbf{a}}(\mathbf{k}_t) = \mathbf{g} \rho^{\mathbf{a}}(\mathbf{k}_t) / \mathbf{k}_t^2$$

unintegrated gluon distribution: $\phi(\mathbf{x}, \mathbf{k}_t^2) \sim \langle \rho_{\mathbf{a}}^*(\mathbf{k}_t) \rho_{\mathbf{a}}(\mathbf{k}_t) \rangle$

gluon distribution: $\mathbf{xG}(\mathbf{x}, \mathbf{Q}^2) \sim \int^{\mathbf{Q}^2} \frac{d^2 \mathbf{k}_t}{\mathbf{k}_t^2} \phi(\mathbf{x}, \mathbf{k}_t)$

Quantum corrections: *non-linear evolution equation*

$$\frac{d}{d \ln 1/x} \langle O \rangle = \frac{1}{2} \left\langle \int d^2 x d^2 y \frac{\delta}{\delta \alpha_x^b} \eta_{xy}^{bd} \frac{\delta}{\delta \alpha_y^d} O \right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi} \int \frac{d^2 z}{(2\pi)^2} \frac{(x-z) \cdot (y-z)}{(x-z)^2 (y-z)^2} \underbrace{\left[1 + U_x^\dagger U_y \right]}_{\text{virtual}} \underbrace{\left[- U_x^\dagger U_z - U_z^\dagger U_y \right]}_{\text{real}}^{bd}$$

U is a Wilson line in adjoint representation

Evolution (energy dependence) of the 2-point function (**dipole**): DIS, single inclusive production

BK equation (known to NLO)

$$\begin{aligned}
 \frac{d}{dy} \text{Tr} \mathbf{V}(\mathbf{r}) \mathbf{V}^\dagger(\bar{\mathbf{r}}) &= \text{Tr} \mathbf{V}(\mathbf{r}) \mathbf{V}^\dagger(\bar{\mathbf{r}}) \\
 &+ \frac{1}{(\mathbf{r} - \mathbf{z})^2} \text{Tr} \mathbf{V}(\mathbf{r}) \mathbf{V}^\dagger(\mathbf{z}) \text{Tr} \mathbf{V}(\mathbf{z}) \mathbf{V}^\dagger(\bar{\mathbf{r}}) \\
 &+ \frac{(\mathbf{r} - \mathbf{z}) \cdot (\bar{\mathbf{r}} - \mathbf{z})}{(\mathbf{r} - \mathbf{z})^2 (\bar{\mathbf{r}} - \mathbf{z})^2} \text{Tr} \mathbf{V}(\mathbf{r}) \mathbf{V}^\dagger(\bar{\mathbf{r}}) \text{Tr} \mathbf{V}(\mathbf{z}) \mathbf{V}^\dagger(\mathbf{z})
 \end{aligned}$$

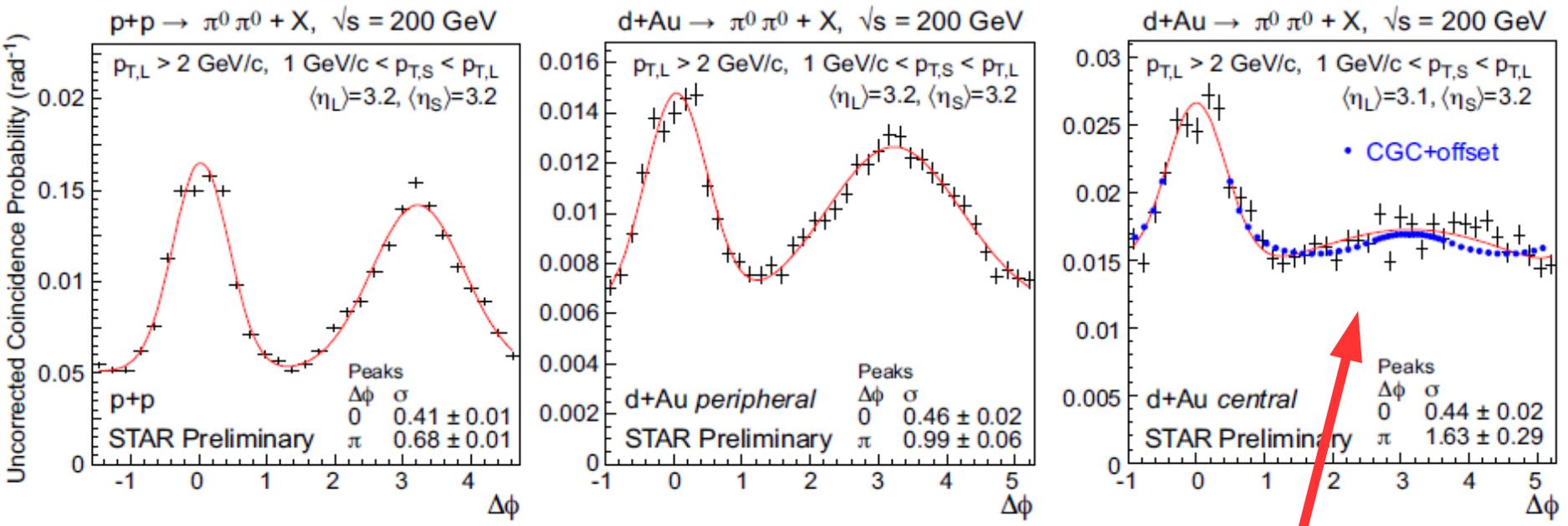
$$\begin{aligned}
 \frac{d}{dy} \mathbf{T}(\mathbf{r} - \bar{\mathbf{r}}) &= \frac{\bar{\alpha}_s}{2\pi} \int d^2 \mathbf{z} \frac{(\mathbf{r} - \bar{\mathbf{r}})^2}{(\mathbf{r} - \mathbf{z})^2 (\bar{\mathbf{r}} - \mathbf{z})^2} \otimes \\
 &\left[\underbrace{\mathbf{T}(\mathbf{r} - \mathbf{z}) + \mathbf{T}(\mathbf{z} - \bar{\mathbf{r}})}_{\text{Linear (BFKL)}} - \mathbf{T}(\mathbf{r} - \bar{\mathbf{r}}) - \mathbf{T}(\mathbf{r} - \mathbf{z}) \mathbf{T}(\bar{\mathbf{r}} - \mathbf{z}) \right]
 \end{aligned}$$

non-linear term

disappearance of back to back hadrons

C. Marquet, NPA796 (2007) 41

Recent STAR measurement (arXiv:1008.3989v1):



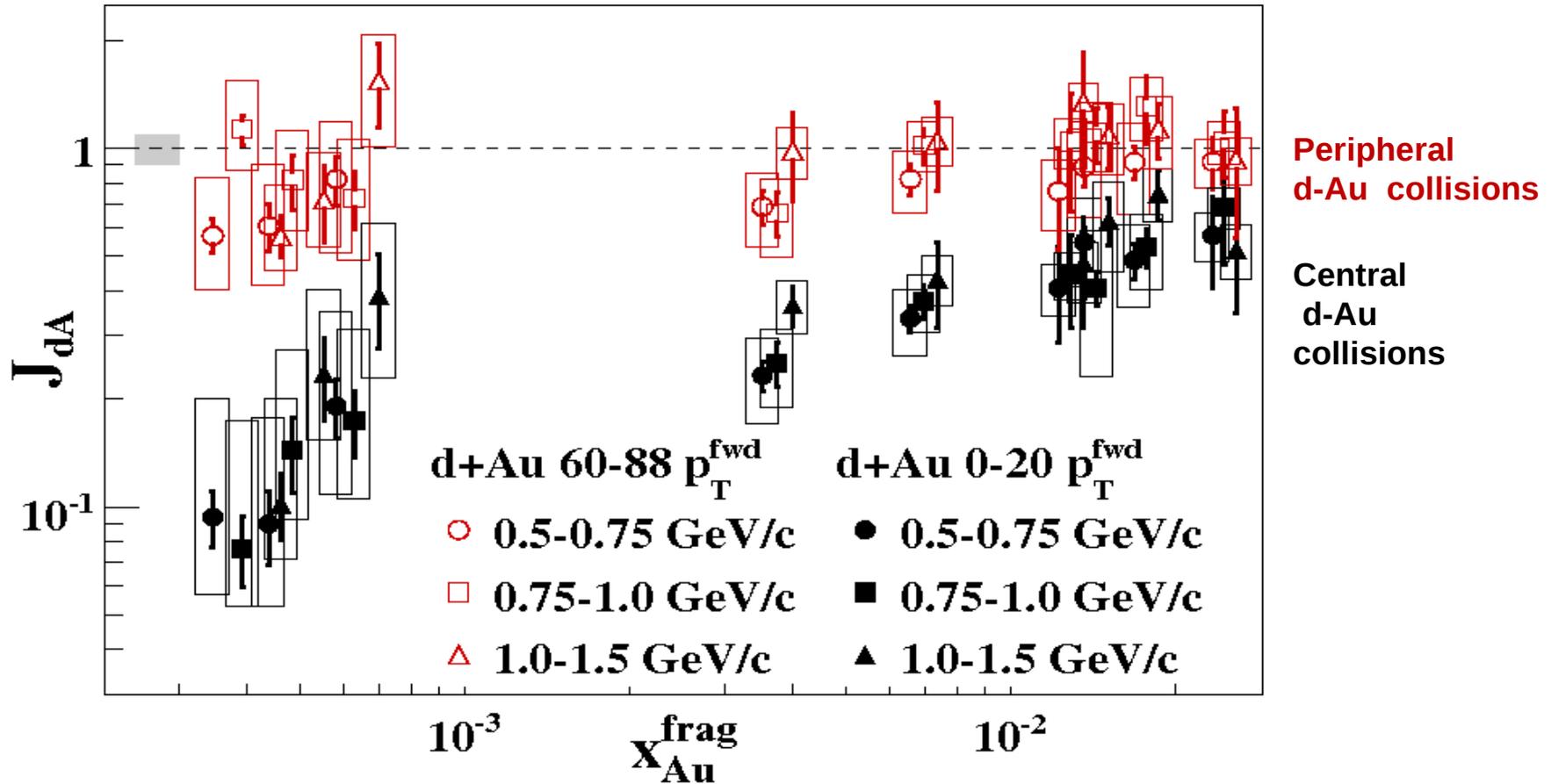
CGC fit from Albacete + Marquet, PRL (2010)
 Tuchin, NPA846 (2010)
 A. Stasto, B-W. Xiao, F. Yuan, PLB716 (2012)
 T. Lappi, H. Mantysaari, NPA908 (2013)

**multiple scatterings
 de-correlate
 the hadrons**

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

disappearance of back to back jets

$$J_{dA} \sim \frac{N_{dAu}^{\text{back-to-back}} / N_{\text{N-N-collision}}}{N_{pp}^{\text{back-to-back}}}$$



QCD at high energy

Two distinct approaches:

1) *CGC*

McLerran-Venugopalan effective action
JIMWLK-BK evolution

2) *Reggeized-gluon exchange*

BJKP equation
triple,... pomeron vertex

***Is there a connection between CGC and BJKP +
multi-pomeron vertices***

arXiv:1407.3080

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CGC vs. BJKP

BJKP equation describes evolution of n-Reggeized gluons (in a singlet state) in t channel

conserves the number of exchanged gluons

CGC: JIMWLK evolution equation for 2n Wilson lines

linearize the equation

expand in powers of α/p_t

the linear regime

$\mathbf{O}(\alpha^2)$: 2-gluon (**BFKL Pomeron**) exchange

$\mathbf{O}(\alpha^3)$: 3-gluon (**odderon**) exchange

dipole odderon: *Kovchegov, Szymanowski, Wallon*

baryonic odderon: *Hatta, Iancu, Itakura, McLerran*

$\mathbf{O}(\alpha^4)$: 4-gluon (**quadrupole**) exchange

Dominguez, Mueller, Munier, Xiao

J. Jalilian-Marian

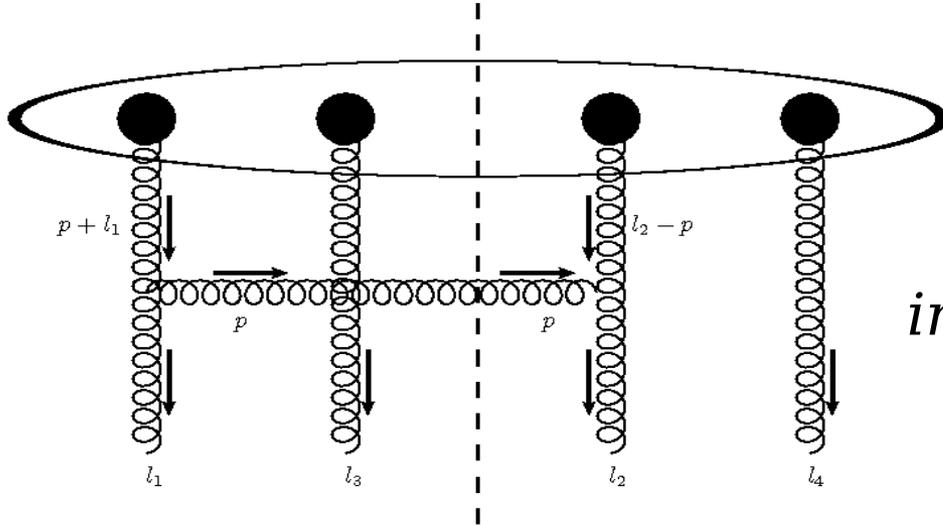
D. Triantafyllopoulos

quadrupole evolution: linear regime

BJKP equation

$\mathcal{O}(\alpha^4)$: 4-gluon exchange

J. Jalilian-Marian, PRD85 (2012) 014037



independent of color averaging

$$\begin{aligned} \frac{d}{dy} \hat{T}_4(l_1, l_2, l_3, l_4) &= \frac{N_c \alpha_s}{\pi^2} \int d^2 p_t \left[\frac{p^i}{p_t^2} - \frac{(p^i - l_1^i)}{(p_t + l_1)^2} \right] \cdot \left[\frac{p^i}{p_t^2} - \frac{(p^i - l_2^i)}{(p_t + l_2)^2} \right] \\ &\quad \hat{T}_4(p_t + l_1, l_2 - p_t, l_3, l_4) + \dots \\ &- \frac{N_c \alpha_s}{(2\pi)^2} \int d^2 p_t \left[\frac{l_1^2}{p_t^2 (l_1 - p_t)^2} + \{l_1 \rightarrow l_2, l_3, l_4\} \right] \hat{T}_4(l_1, l_2, l_3, l_4) \end{aligned}$$

this will de-correlate di-jets at high $p_t > Q_s$

2n-Wilson line evolution: linear regime

$\mathcal{O}(\alpha^{2n})$: 2n-gluon exchange

$$\begin{aligned} \frac{d}{dY} T_{(\prod_{k=1}^{2n} l_k)}^{(2n)} &= -\frac{\bar{\alpha}}{2\pi} \sum_{j=1}^{2n} \int d^2 p_t \left[\frac{l_j^2}{p_t^2 [p_t^2 + (p_t - l_j)^2]} \right] T_{(\prod_{k=1}^{2n} l_k)}^{(2n)} \\ + \frac{\bar{\alpha}}{4\pi} \int d^2 p_t &\left[\frac{l_1^2}{p_t^2 (p_t + l_1)^2} + \frac{l_{2n}^2}{p_t^2 (p_t - l_{2n})^2} - \frac{(l_1 + l_{2n})^2}{(p_t + l_1)^2 (p_t - l_{2n})^2} \right] T_{(l_1+p_t)(\prod_{k=2}^{2n-1} l_k)(l_{2n}-p_t)}^{(2n)} \\ + \frac{\bar{\alpha}}{4\pi} \sum_{j=2}^{2n} \int d^2 p_t &\left[\frac{l_{j-1}^2}{p_t^2 (p_t + l_{j-1})^2} + \frac{l_j^2}{p_t^2 (p_t - l_j)^2} - \frac{(l_{j-1} + l_j)^2}{(p_t + l_{j-1})^2 (p_t - l_j)^2} \right] T_{(\prod_{k=1}^{j-2} l_k)(l_{j-1}+p_t)(l_j-p_t)(\prod_{k=j+1}^{2n} l_k)}^{(2n)}. \end{aligned}$$

A Mathematica program that gives the equation can be downloaded from

faculty.baruch.cuny.edu/naturalscience/physics/Jalilian-Marian/

paginas.fisica.uson.mx/elena.tejeda/code.nb

Non-linear regime: triple pomeron vertex

Chirilli, Szymanowski, Wallon PRD83 (2011) 014020

start with the dipole evolution equation

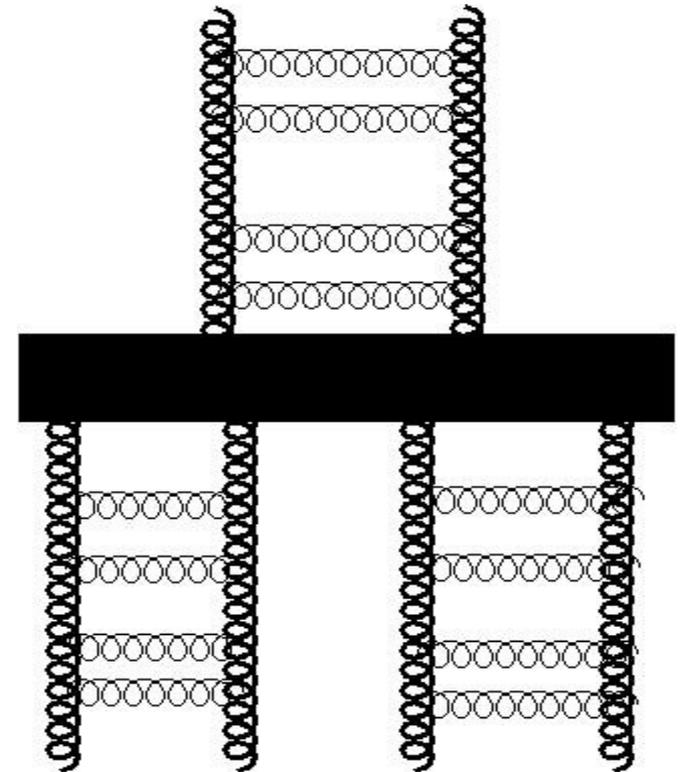
$$\frac{d}{dy} \mathbf{T}(\mathbf{r} - \bar{\mathbf{r}}) = \frac{\bar{\alpha}_s}{2\pi} \int d^2\mathbf{z} \frac{(\mathbf{r} - \bar{\mathbf{r}})^2}{(\mathbf{r} - \mathbf{z})^2 (\bar{\mathbf{r}} - \mathbf{z})^2} \otimes$$

$$\left[\mathbf{T}(\mathbf{r} - \mathbf{z}) + \mathbf{T}(\mathbf{z} - \bar{\mathbf{r}}) - \mathbf{T}(\mathbf{r} - \bar{\mathbf{r}}) - \mathbf{T}(\mathbf{r} - \mathbf{z})\mathbf{T}(\bar{\mathbf{r}} - \mathbf{z}) \right]$$

non-linear term

recall $\mathbf{T}(\mathbf{r} - \bar{\mathbf{r}}) \equiv \mathbf{1} - \frac{1}{N_c} \text{Tr} \mathbf{V}(\mathbf{r}_t) \mathbf{V}^\dagger(\bar{\mathbf{r}}_t)$

with $\mathbf{V}(\mathbf{r}_t) = \hat{\mathbf{P}} \exp \left\{ -ig \int_{-\infty}^{\infty} dz^- \alpha(\mathbf{z}^-, \mathbf{r}_t) \right\}$

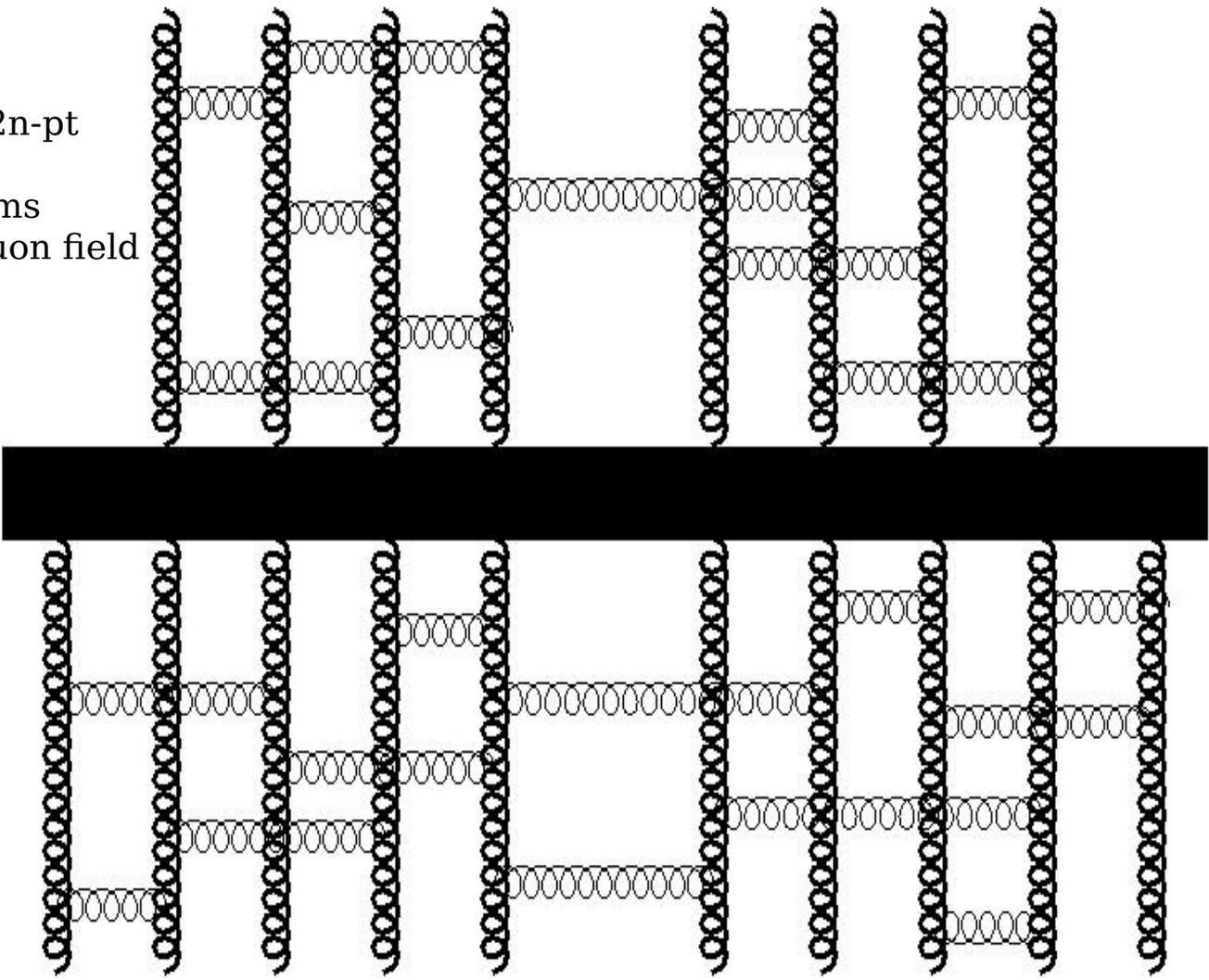


**Triple pomeron-vertex from $\mathcal{O}(\alpha^4)$ terms in the expansion of the non-linear term
agrees with Reggeized gluon exchange results of Bartles et al.**

Non-linear regime: $n \rightarrow n+1$ pomeron vertex

A. Ayala, E. Cazaroto, L. Hernandez, J. Jalilian-Marian, M.E. Tejeda-Yeomans, in progress

- Follow the same strategy:
- Evolution equation for $2n$ -pt function
- Keep the non-linear terms
- Expand in powers of gluon field
- Extract the vertex



First case:
2 \rightarrow 3 vertex
(first high density correction)

SUMMARY

JIMWLK evolution equation for $2n$ Wilson lines in the linear regime is equivalent to the BJKP equation for evolution of a singlet state of $2n$ Reggeized gluons

It appears possible to extract the vertex for any n to $n+1$ "Pomeron" vertex

Can one make a saturation based effective theory of diffraction in proton-proton collisions?