theory of medium-induced gluon radiation: an update

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High-pT Physics in the RHIC/LHC era
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- main focus: *fully coherent* induced radiation

  ✦ crucial for phenomenology
  (J/psi nuclear suppression in pA and AB)

  ✦ interesting theoretically

- talk based on:

Arleo, S.P., Sami 1006.0818 [APS10]

Arleo & S.P. 1212.0434 [AP12]

S.P., Arleo, Kolevatov 1402.1671 [PAK14]

S.P. & Kolevatov 1405.4241 [PK14]

see also: S.P. & Smilga 0810.5702 [PS08]

fully coherent energy loss in phenomenology

J/psi suppression in pA and AB collisions

predictions for J/psi suppression
from fully coherent energy loss
compared to available low pT data

from fixed target to collider energies

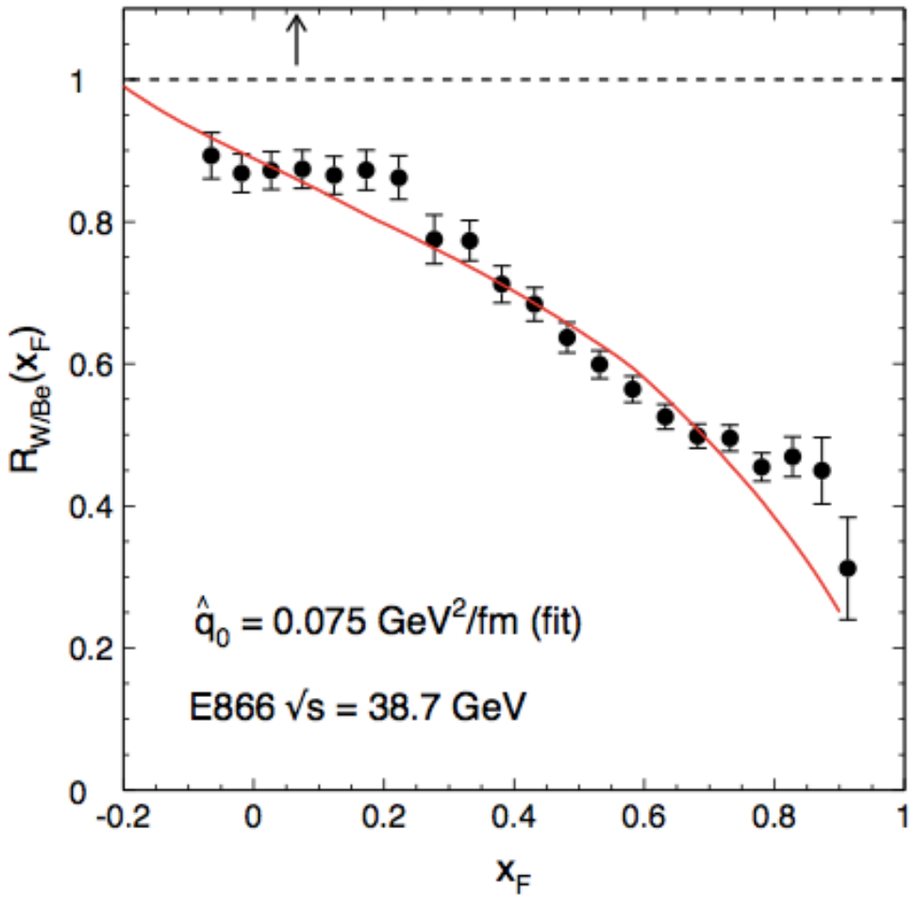
Arleo, S.P., 1204.4609 and 1212.0434 [AP12]

Arleo, Kolevatov, S.P., Rustamova 1304.0901 [AKPR13]

Arleo, S.P. 1407.5054 [AP14]

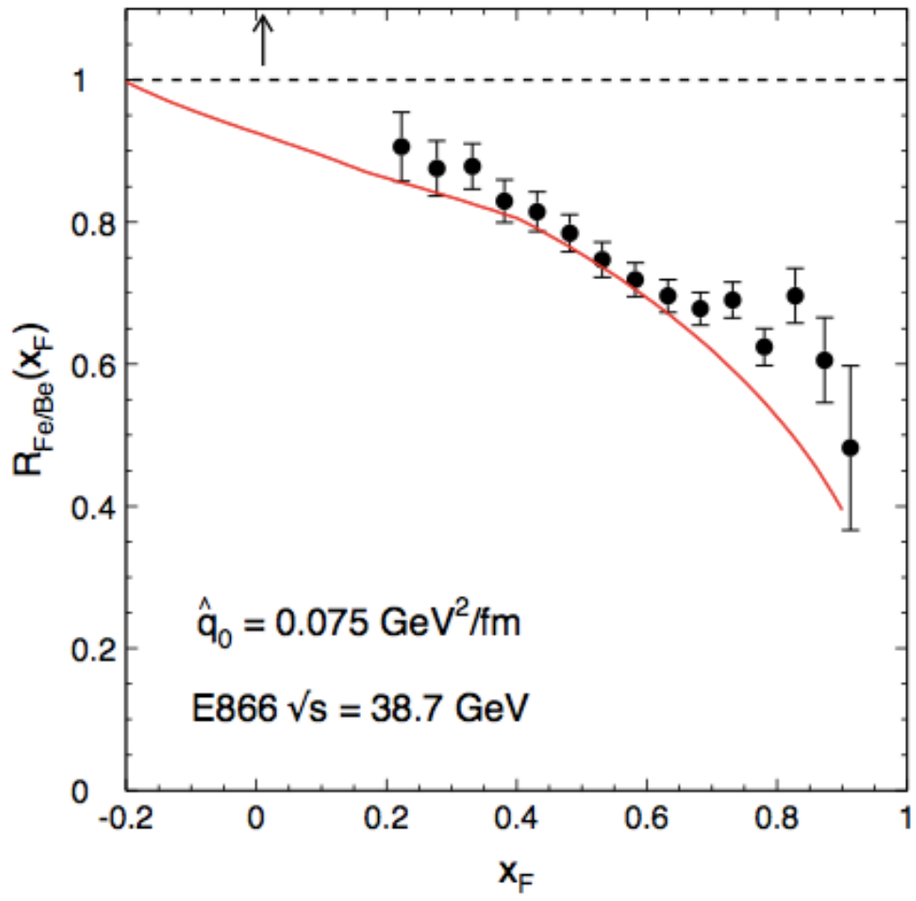
J/psi suppression in pA

$\hat{q}_0$ fixed from W/Be E866
J/ψ suppression data...



...and used to predict
$R_{\mathrm{pA}}^{\mathrm{J}/\psi}$ for other A, $\sqrt{s}$

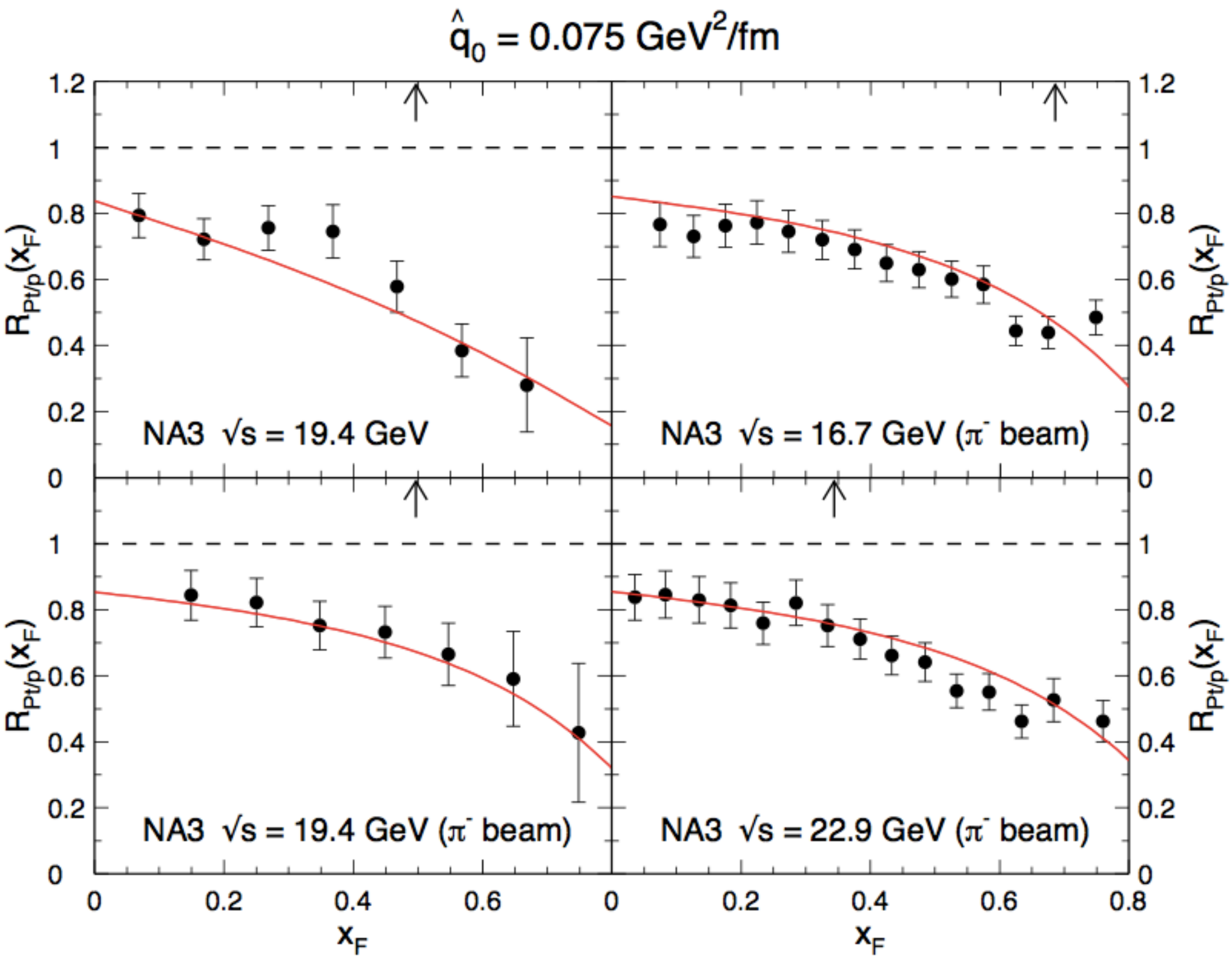
E866 Fe/Be



A–dependence well reproduced

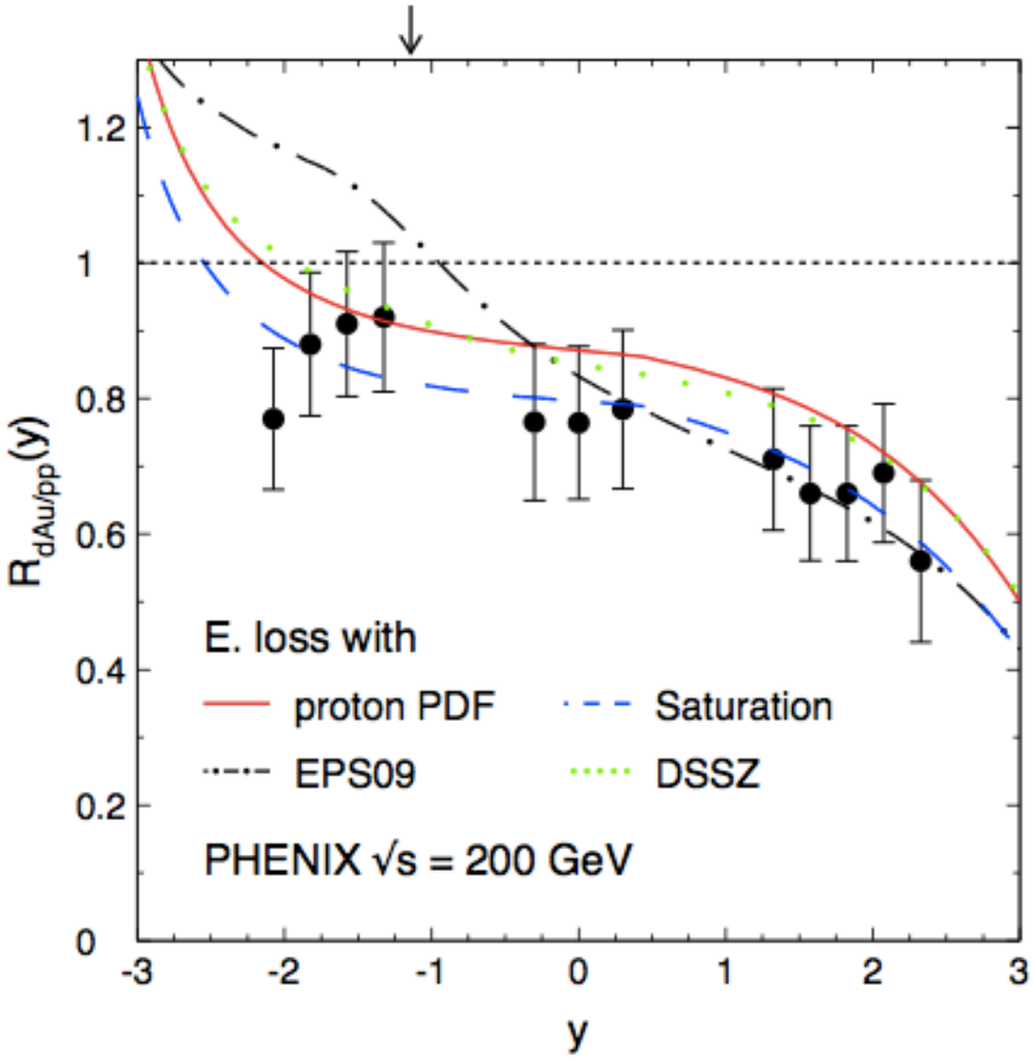
(AP12)

J/ψ NA3 Pt/p

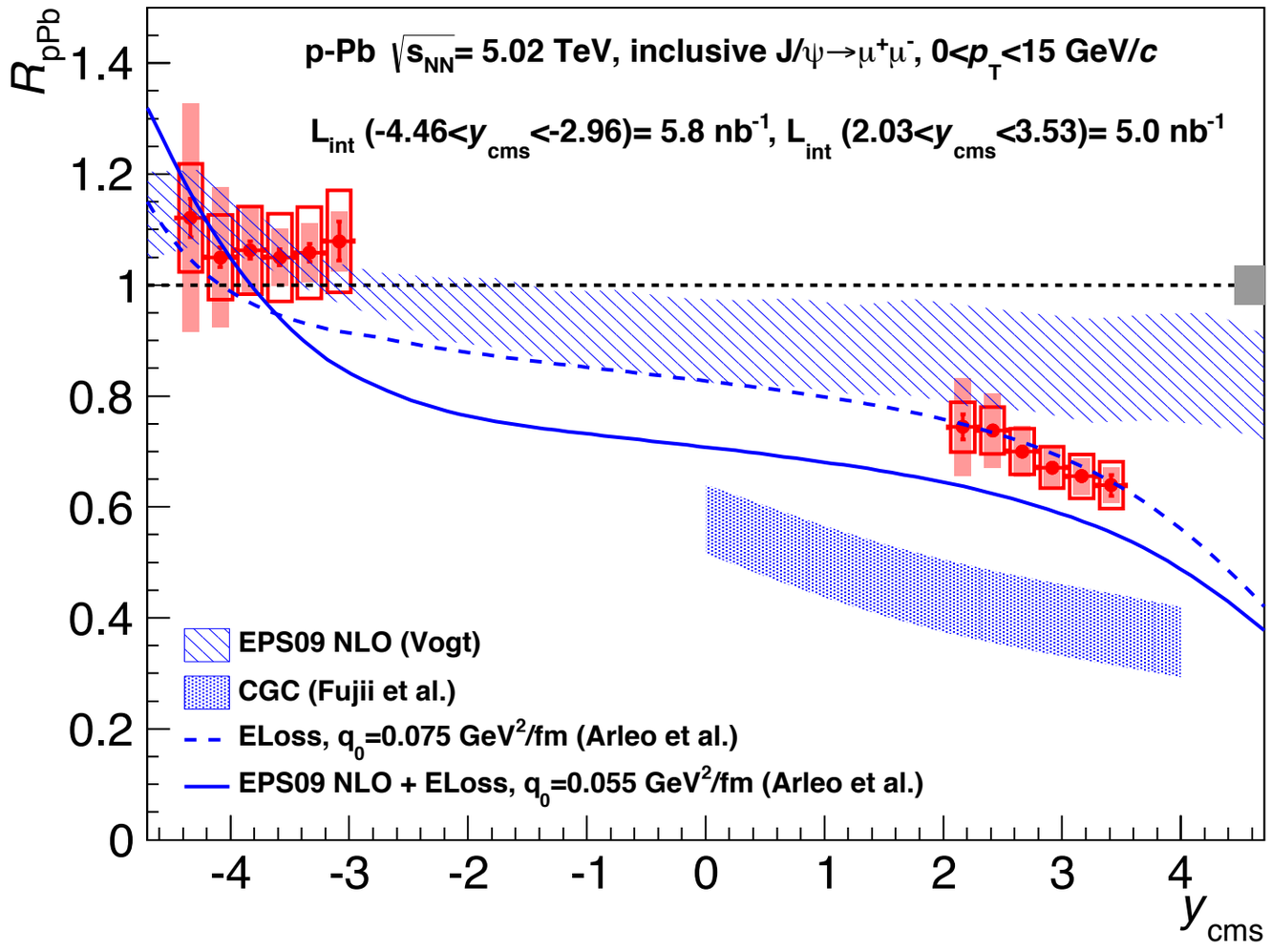


(AP12)

RHIC d-Au (PHENIX)

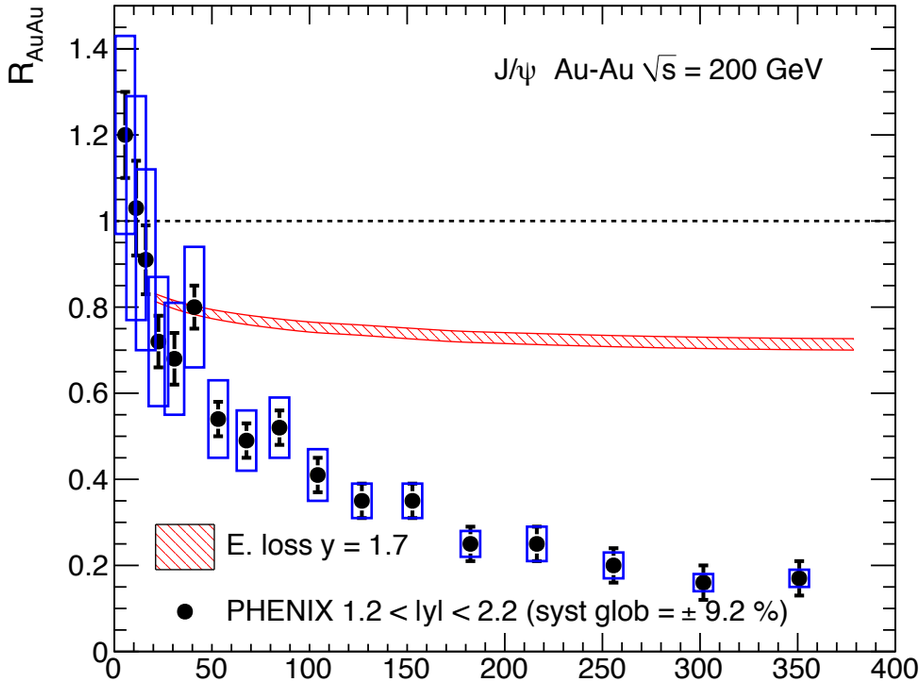


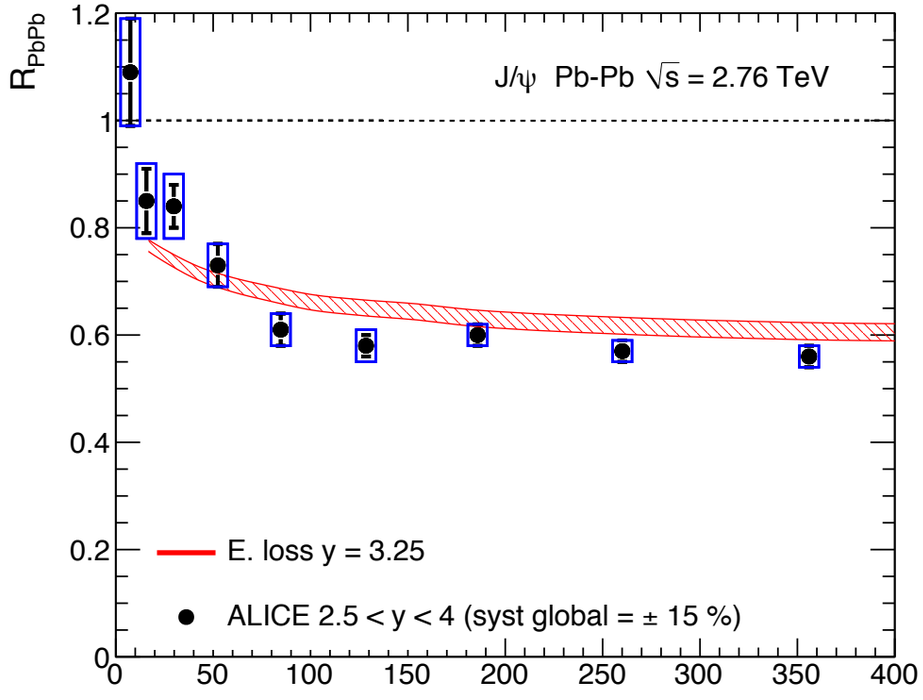
LHC p-Pb (ALICE)



(AP12)

J/psi suppression in AB collisions (AP14)





RHIC

LHC

→ baseline for cold nuclear effects

all those calculations of J/psi suppression

are based on a single quantity:

the fully coherent, medium-induced radiation spectrum

associated to

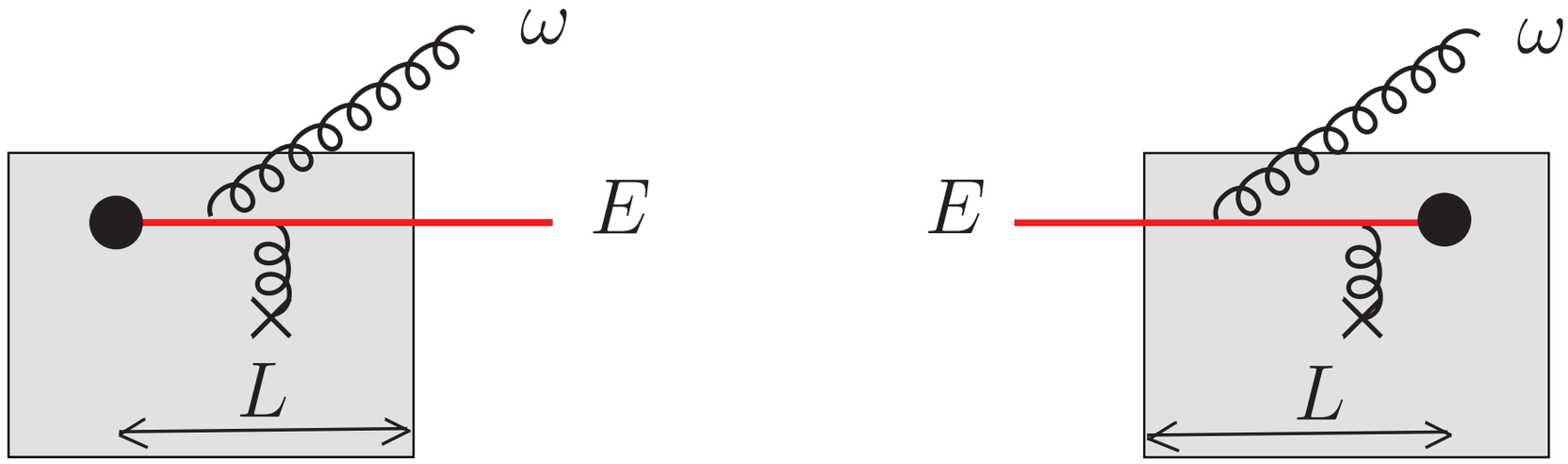
forward scattering of a fast color octet $c\bar{c}$ *pair*

features of coherent medium-induced radiation

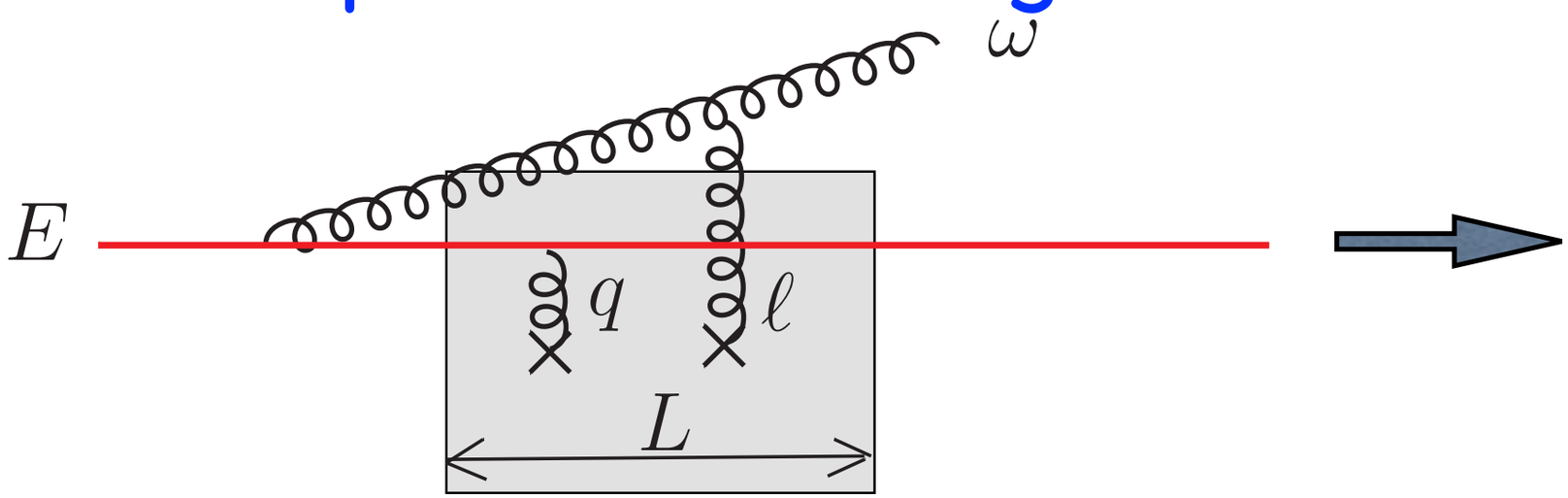
in general, features of medium-induced radiation depend on specific kinematic situation

- two main cases (PS08)

(1) energetic parton suddenly produced or annihilated in nuclear medium

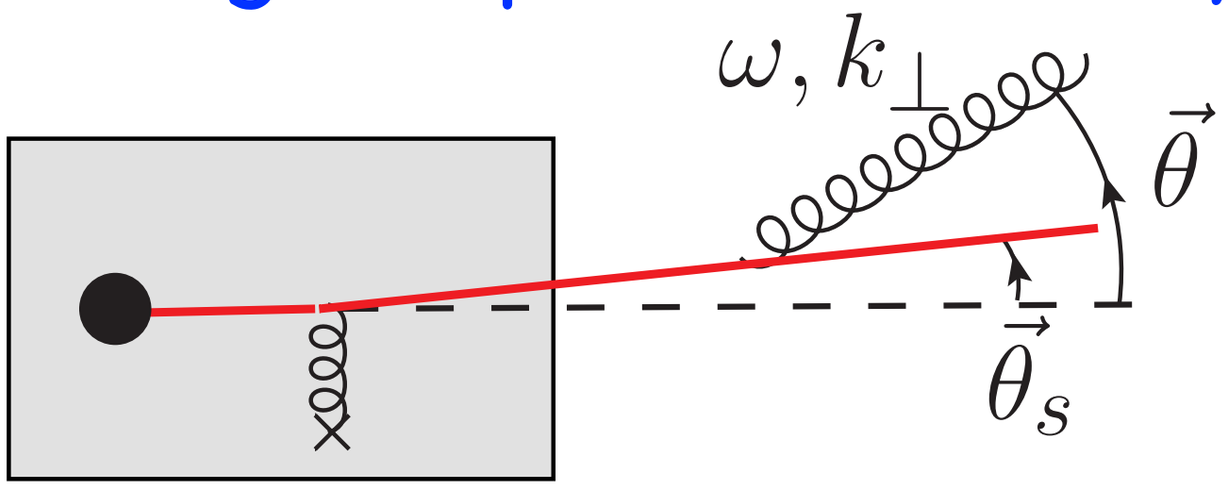


(2) forward scattering of fast 'asymptotic parton' crossing a nuclear medium



fully coherent radiation

(1) energetic parton suddenly produced in medium



- limit $E \to \infty$
- parton color C_R

✦ formation time of induced radiation

$$t_f \sim \frac{\omega}{k_\perp^2} \gg L \;\Rightarrow\; \omega\frac{\mathrm{d}I}{\mathrm{d}\omega}\Big)_L \sim \int \frac{\mathrm{d}^2\vec{\theta}}{(\vec{\theta}-\vec{\theta}_s)^2} \quad L\text{-independent}$$

$$\Rightarrow \text{cancels in } \omega\frac{\mathrm{d}I}{\mathrm{d}\omega}\Big)_{\mathrm{ind}} \equiv \omega\frac{\mathrm{d}I}{\mathrm{d}\omega}\Big)_L - \omega\frac{\mathrm{d}I}{\mathrm{d}\omega}\Big)_{L=0}$$

$$\Rightarrow \boxed{t_f \lesssim L} \Rightarrow \Delta E \sim \alpha_s\langle\omega\rangle \sim \alpha_s L\langle k_\perp^2\rangle \sim \alpha_s L\cdot\hat{q}L$$

$$\Rightarrow \boxed{\Delta E_{\mathrm{LPM}} \sim \alpha_s\hat{q}L^2}$$

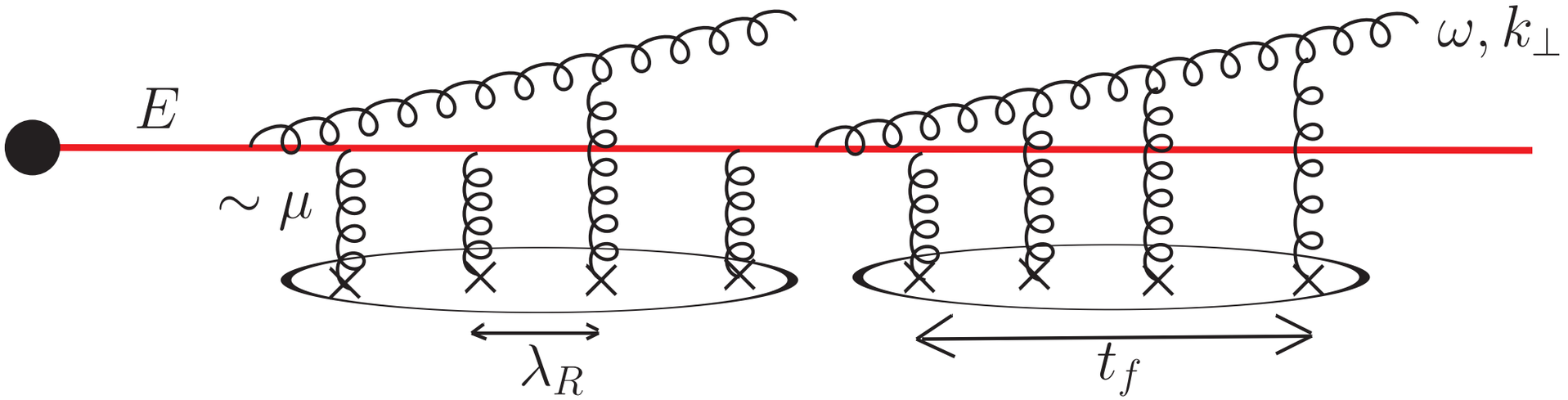
✦ color factor

parton production mechanism is irrelevant

→ ΔE depends only on color charge C_R

$$\Delta E_{\rm LPM} \sim \alpha_s C_R \hat{q} L^2$$

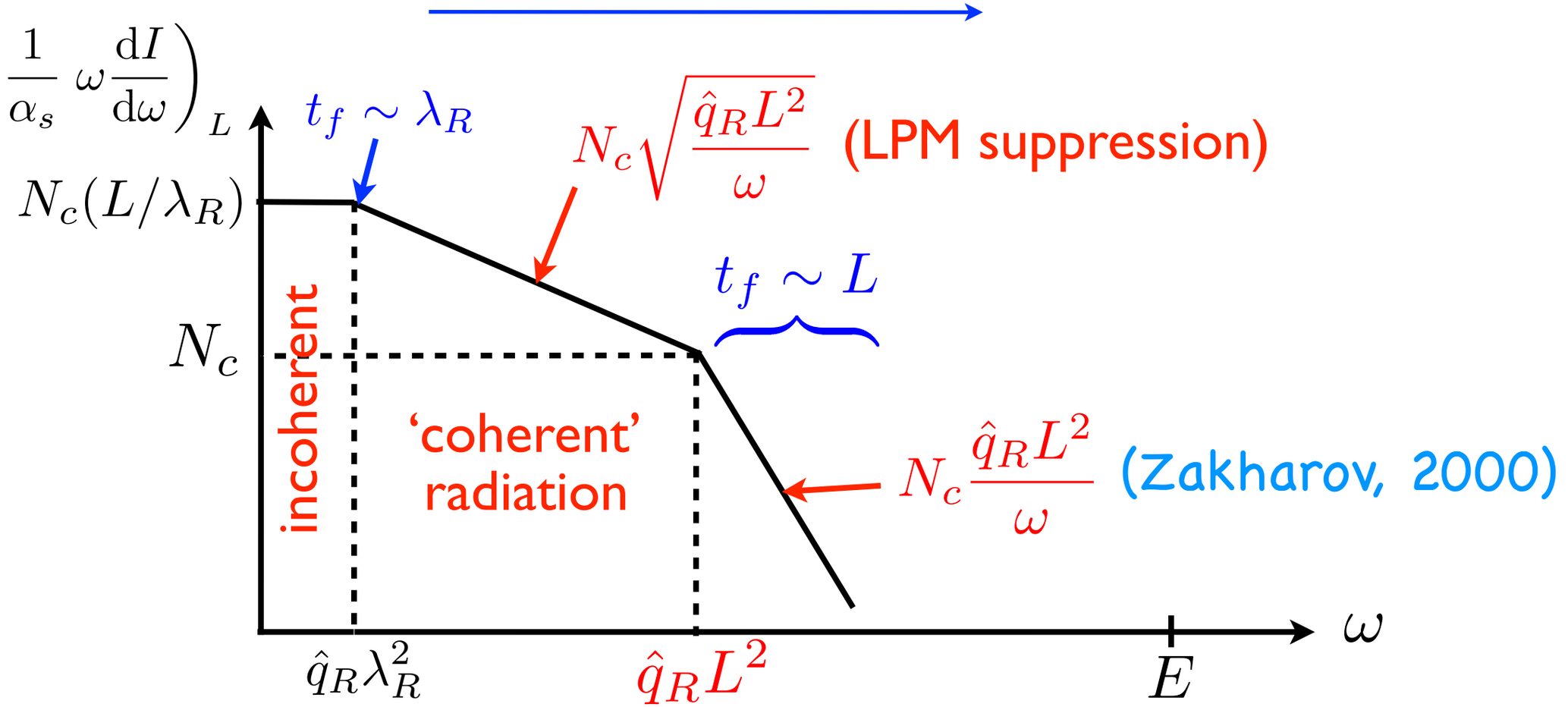
✦ medium-induced energy spectrum



$$\lambda_R < t_f < L \Rightarrow \omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\Big)_L \sim \frac{L}{t_f} \cdot \omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\Big)_1 \sim \frac{L}{t_f} \cdot \alpha_s N_c$$

$$t_f \sim \frac{\omega}{k_\perp^2(t_f)} \sim \frac{\omega}{\mu^2 t_f/\lambda_R} \equiv \frac{\omega}{\hat{q}_R t_f} \Rightarrow \; t_f \sim \sqrt{\omega/\hat{q}_R}$$

$$(\lambda_R < t_f < L \;\; \Leftrightarrow \;\; \hat{q}_R \lambda_R^2 < \omega < \hat{q}_R L^2)$$



when ω exceeds $\hat{q}_R L^2$, t_f saturates at $t_f \sim L$
due to suppression of $t_f \gg L$

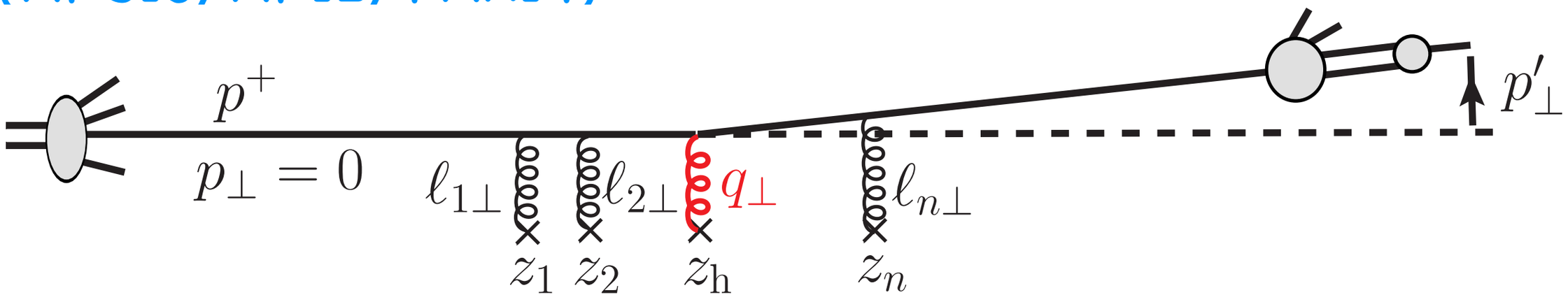
✦ average energy loss

$$\Delta E = \int \mathrm{d}\omega\,\omega\frac{\mathrm{d}I}{\mathrm{d}\omega}\Big)_L \sim \alpha_s N_c \hat{q}_R L^2 \sim \alpha_s C_R \hat{q} L^2 \;\; (\hat{q} \equiv \hat{q}_g)$$

(2) forward scattering of fast 'asymptotic parton' crossing a nuclear medium

setup: high-energy p-A collision in nucleus rest frame

(APS10, AP12, PAK14)



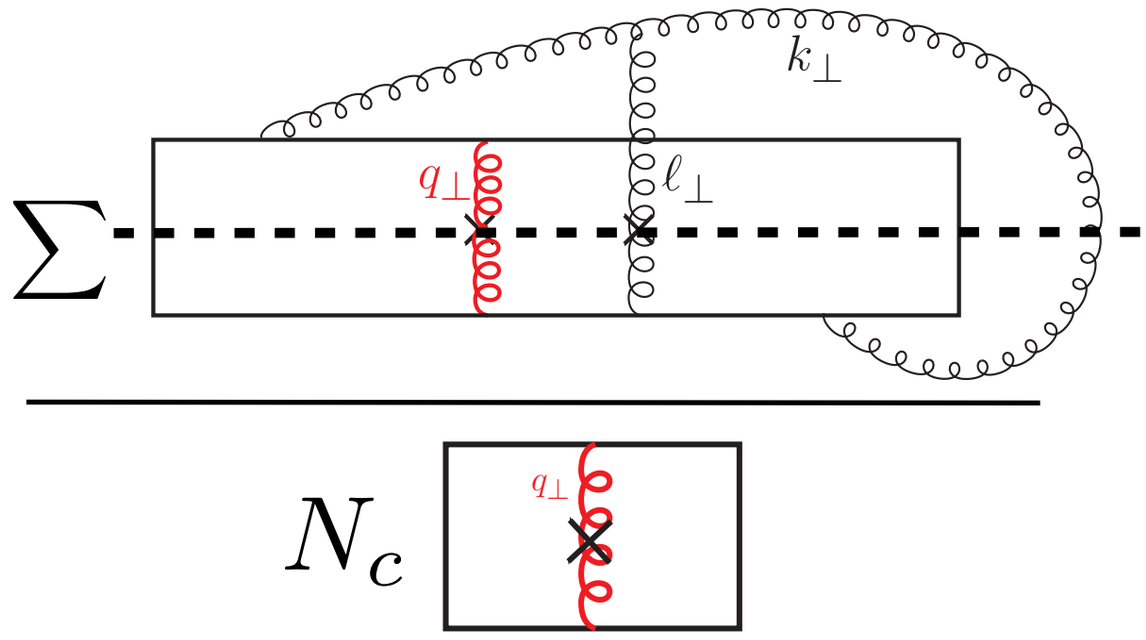
- tag energetic hadron with $p'_\perp|_{\rm hard} \gg \sqrt{\hat{q}L}$
- energetic parent parton suffers:
  - *single* hard exchange $q_\perp \simeq p'_\perp$
  - soft rescatterings $\ell_\perp^2 = (\sum \vec{\ell}_{i\perp})^2 \sim \hat{q}L \sim Q_s^2 \ll q_\perp^2$

two transverse momentum scales: $\ell_\perp \ll q_\perp$

✦ derivation of induced spectrum

- use opacity expansion (Gyulassy, Levai, Vitev 2000)
- look for *soft radiation* with $k_\perp \ll q_\perp$ and $t_f \gg L$

at first order in opacity:



$$x\frac{\mathrm{d}I^{(1)}}{\mathrm{d}x} = \frac{\alpha_s}{\pi^2}\frac{L}{\lambda_g}\int \mathrm{d}^2\boldsymbol{k}\int \mathrm{d}^2\boldsymbol{\ell}\, V(\boldsymbol{\ell})$$

$$V(\boldsymbol{\ell}) \equiv \frac{\mu^2}{\pi(\boldsymbol{\ell}^2+\mu^2)^2} \qquad x \equiv \frac{\omega}{E} \ll 1$$

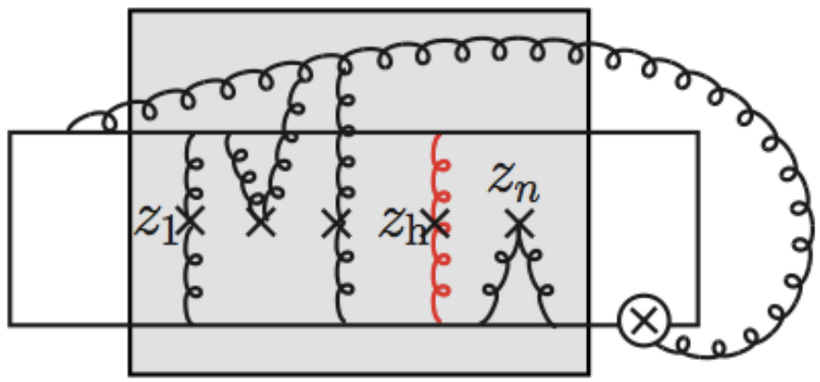
$$\longrightarrow (2C_R - N_c)\left[-\underbrace{\frac{\boldsymbol{k}-\boldsymbol{\ell}}{(\boldsymbol{k}-\boldsymbol{\ell})^2}\cdot\frac{\boldsymbol{k}-x\boldsymbol{q}}{(\boldsymbol{k}-x\boldsymbol{q})^2}}_{\text{interference is } <0} + \frac{\boldsymbol{k}}{\boldsymbol{k}^2}\cdot\frac{\boldsymbol{k}-x\boldsymbol{q}}{(\boldsymbol{k}-x\boldsymbol{q})^2}\right]$$

but magnitude ↘ when $|\vec{\boldsymbol{\ell}}|$ ↗

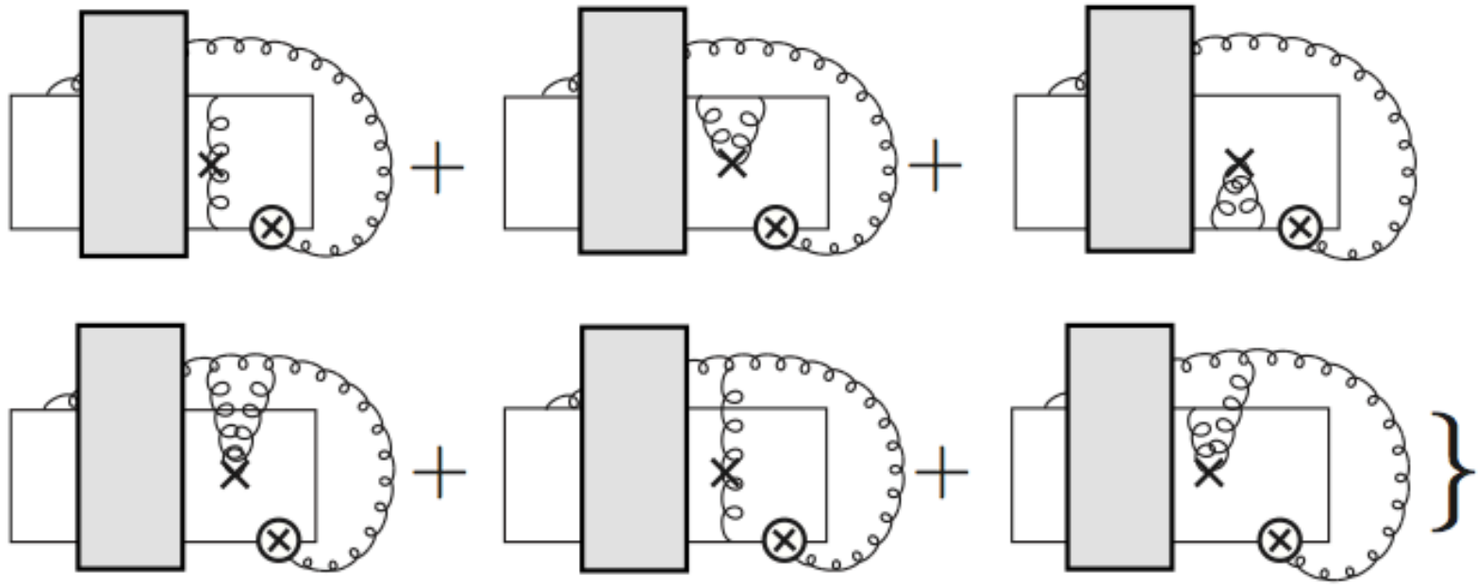
$$\longrightarrow \quad x\frac{\mathrm{d}I^{(1)}}{\mathrm{d}x} = (2C_R - N_c)\,\frac{\alpha_s}{\pi}\,\frac{L}{\lambda_g}\,\log\left(1+\frac{\mu^2}{x^2 q_\perp^2}\right)$$

all orders in opacity (PAK14)

$$C_n(k,\{\ell_i\}) = 2\mathrm{Re}$$



$$C_{n+1} = 2\mathrm{Re}\{$$

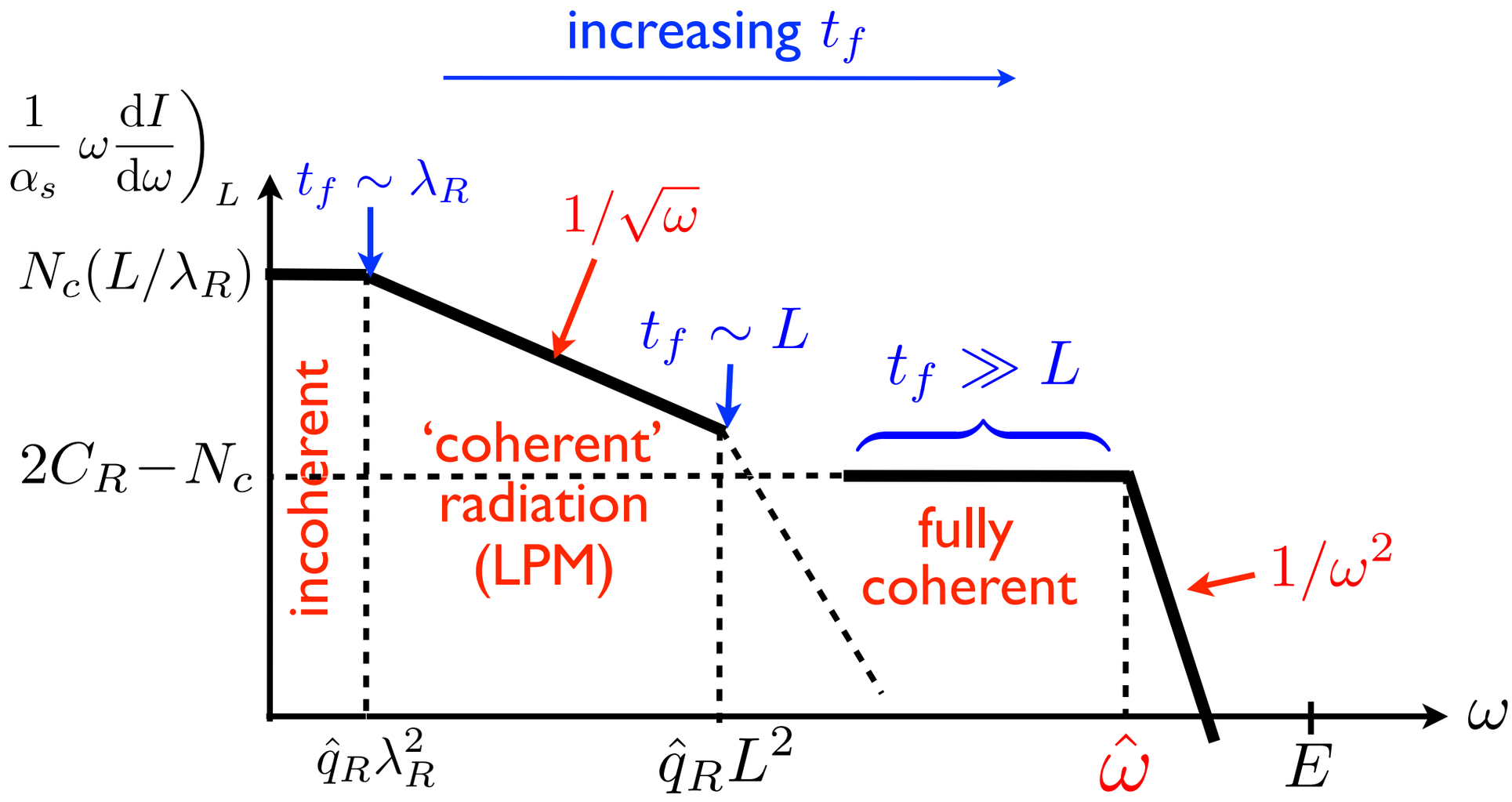


$$\}$$

$$\longrightarrow \quad x\frac{\mathrm{d}I}{\mathrm{d}x} = (2C_R - N_c)\,\frac{\alpha_s}{\pi}\,\log\left(1+\frac{\hat{q}L}{x^2 q_\perp^2}\right)$$

a new scale:

$$\hat{\omega} \equiv \frac{\sqrt{\hat{q}L}}{q_\perp} E \sim \frac{\ell_\perp}{q_\perp} E \gg \hat{q}L^2$$



$$\Delta E_{\rm coh} \sim \alpha_s \hat{\omega} \sim \alpha_s \frac{\sqrt{\hat{q}L}}{Q_{\rm hard}} E \quad (\gg \Delta E_{\rm LPM} \sim \alpha_s \hat{q} L^2)$$

from ***fully coherent*** domain: $t_f \sim \dfrac{\omega}{k_\perp^2} \sim \dfrac{\hat{\omega}}{\hat{q}L} \gg L$

orders of magnitude

p-Pb collision at $\sqrt{s} = 5\,\mathrm{TeV}$

$$\hat{q} = \hat{q}_{\mathrm{cold}} \sim 0.1\,\mathrm{GeV}^2/\mathrm{fm}$$

$$L \sim 10\,\mathrm{fm}$$

light hadron with: $p_T = 3\,\mathrm{GeV}$; $y_{\mathrm{cm}} = 0$

$$\Rightarrow E_{\mathrm{cm}} \simeq 3\,\mathrm{GeV}$$

in nucleus rest frame $E = \sqrt{s}\dfrac{p_T}{2m_p}e^y \sim 7.5\,\mathrm{TeV}$

$$\hat{q}L^2 \simeq 50\,\mathrm{GeV} \ll \hat{\omega} \simeq 2.5\,\mathrm{TeV}$$

✦ physical interpretation

- $k_\perp$ logarithmic interval

when $\hat{q}L^2 \ll \omega \ll \hat{\omega}$ and to log accuracy:

$$x\frac{\mathrm{d}I}{\mathrm{d}x} \propto \alpha_s \log\left(\frac{\hat{q}L}{x^2 q_\perp^2}\right) \quad \text{from} \quad x^2 q_\perp^2 \ll k_\perp^2 \ll \hat{q}L \sim \ell_\perp^2$$

$$\Longrightarrow \; t_f \gg L \text{ and } k_\perp \ll q_\perp \quad \text{(as initially assumed)}$$

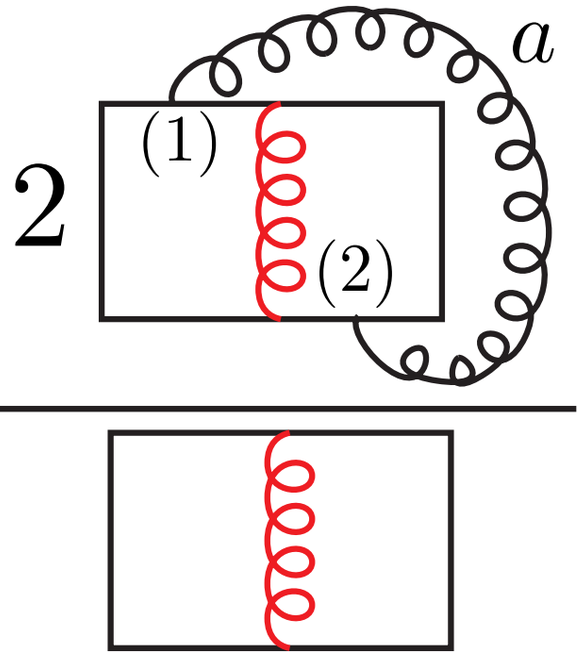
$xq_\perp \ll k_\perp \Leftrightarrow \theta \gg \theta_s$ ⟶ soft gluon does not probe $\Delta\vec{r}$ of core charge compared to propagation in vacuum

$$1/\omega \gg \Delta r_\parallel = |v_{2\parallel} - v_{1\parallel}|\, t_f \sim \frac{q_\perp^2}{E^2}\frac{\omega}{k_\perp^2} \quad and \quad 1/k_\perp \gg \Delta r_\perp = v_{2\perp}\, t_f \sim \frac{q_\perp}{E}\frac{\omega}{k_\perp^2}$$

(see Y. Dokshitzer, 'perturbative QCD for beginners')

$k_\perp \ll \ell_\perp$ ⟶ radiation induced by additional $\ell_\perp$ must be *shaken off* by $\ell_\perp$

- color factor



$$= 2\, T^a_{(1)}\, T^a_{(2)} = (T^a_{(1)})^2 + (T^a_{(2)})^2 - (\underbrace{T^a_{(1)} - T^a_{(2)}}_{T^a(\mathbf{8})})^2$$

$$= C_R + C_R - N_c$$

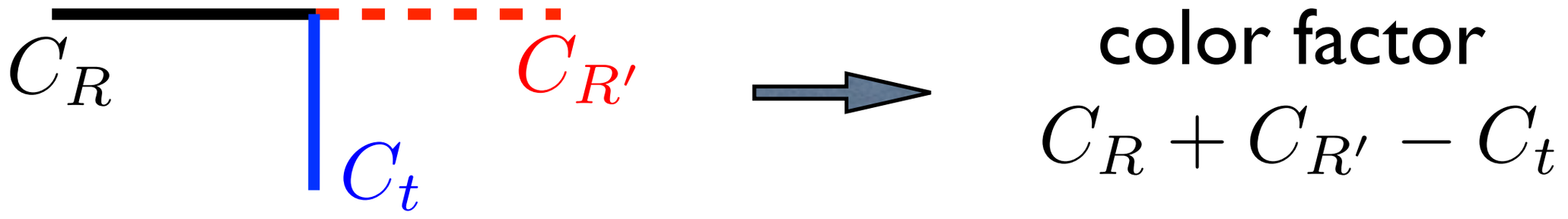
for pictorial representation of color factors, see e.g.
Y. Dokshitzer, 'PQCD (and Beyond)', 1995

✦ remarks (PAK14)

- when fast parton is massive, same results, up to:

$$Q_{\rm hard} = q_\perp \longrightarrow Q_{\rm hard} = M_\perp \equiv \sqrt{M^2 + q_\perp^2}$$

- $1 \to 1$ forward scattering with $C_R \neq C_{R'}$



- fully coherent radiation spectrum has a universal log but is process-dependent

- $\Longrightarrow$ coherent radiation $\notin$ target/proj. wavefunction

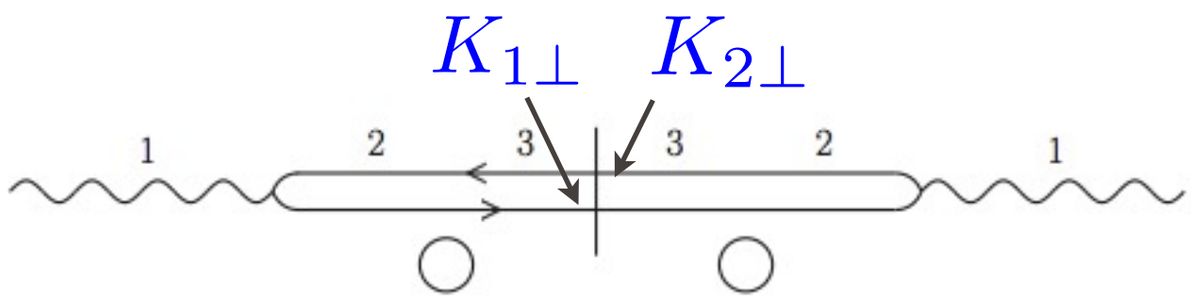
generalization to $1 \to 2$ hard forward processes

see Liou & Mueller, 1402.1647 [LM14] and PK14

- LM14: different formalism and setup: *forward symmetric dijet*

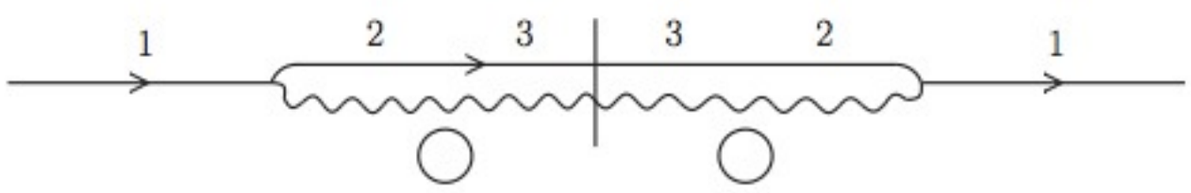
$$K_{1\perp}, K_{2\perp} \gg \ell_\perp \sim Q_s \;\; \text{but} \;\; |\vec{K}_{1\perp} + \vec{K}_{2\perp}| \lesssim Q_s$$

$g \to q\bar{q}$



$$\Rightarrow x\frac{\mathrm{d}I}{\mathrm{d}x} = x\frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{g\to g}$$

$q \to qg$

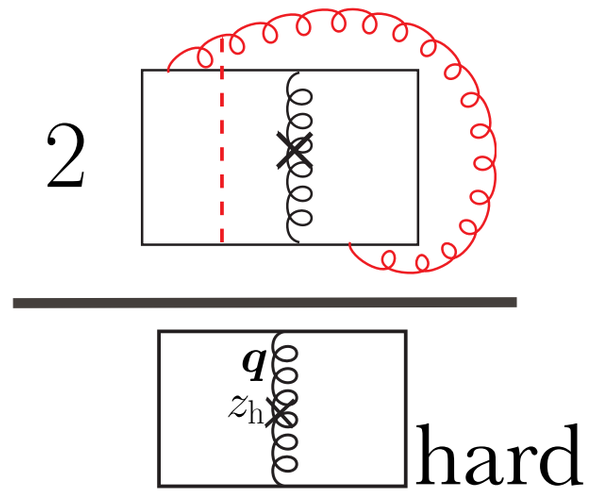


$$\Rightarrow x\frac{\mathrm{d}I}{\mathrm{d}x} = \frac{4}{5}\, x\frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{g\to g}$$

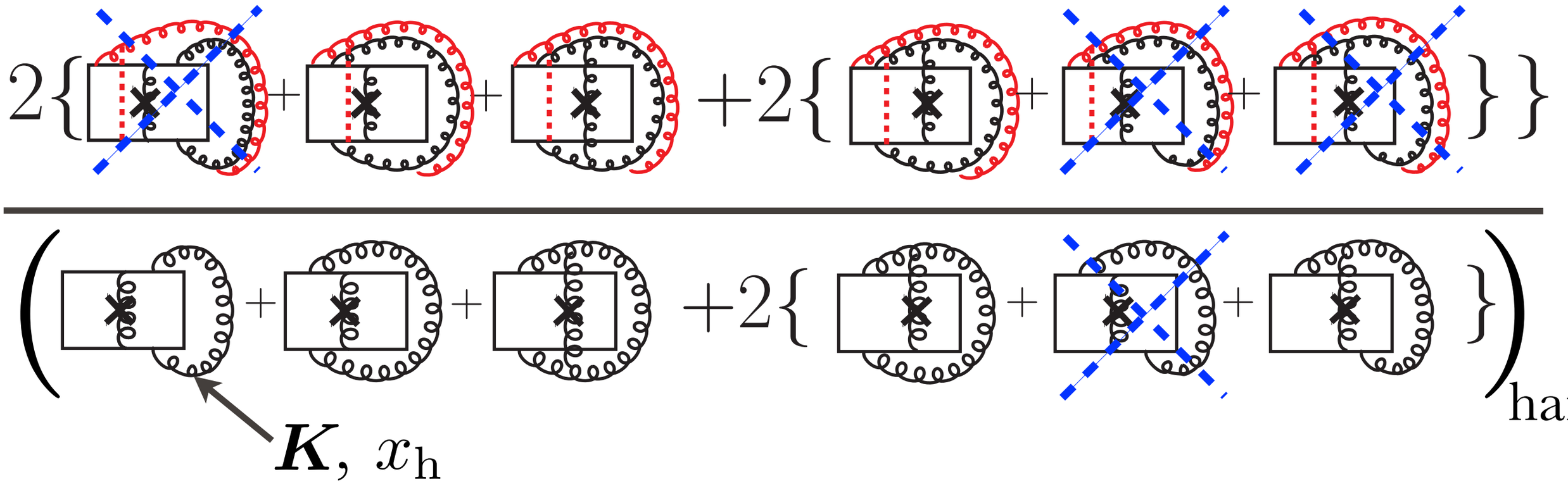
how can we understand similarity between $1 \to 1$ and $1 \to 2$ processes, and interpret strange factor 4/5 ?

derive spectrum associated to $q \to qg$ in opacity expansion and large N_c limit (PK14)

for $g \to g$, hard process is trivial



for $q \to qg$, hard process is less trivial:



- explicit calculation

$$x\frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{q\to qg} = \kappa_{q\to qg}\cdot N_c\cdot\frac{\alpha_s}{\pi}\,\log\left(\frac{\Delta q_\perp^2(L)}{x^2\boldsymbol{K}^2}\right)$$

$$\kappa_{q\to qg} \equiv \frac{(\boldsymbol{K}-x_{\rm h}\boldsymbol{q})^2}{(\boldsymbol{K}-x_{\rm h}\boldsymbol{q})^2+(1-x_{\rm h})^2\boldsymbol{K}^2}$$

log arises from $\quad x^2K_\perp^2 \ll k_\perp^2 \ll \hat{q}L\sim\ell_\perp^2$

at time t_f, radiated gluon does not probe size $\Delta r_\perp$ of qg pair ⟶ $qg\sim$ pointlike

⟶ effectively the same as for $1\to 1$ processes

- interpretation of factor $\kappa_{q\to qg}$

✳ final pointlike qg pair can be in 3 color representations

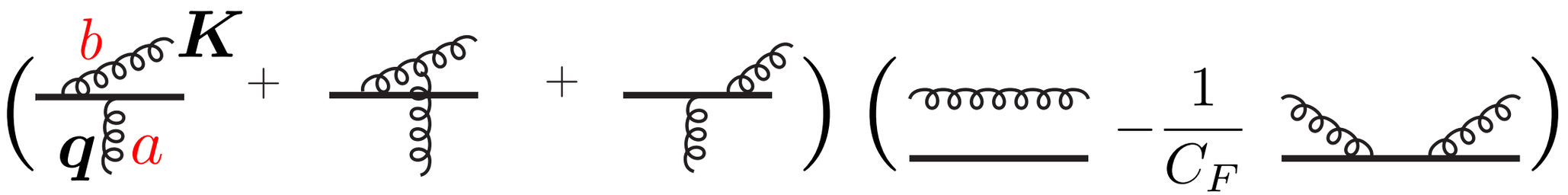
$$\mathbf{3}\otimes\mathbf{8}=\mathbf{3}\oplus\bar{\mathbf{6}}\oplus\mathbf{15}$$

✳ diagrams suppressed when $N_c\gg 1$ are those where final qg pair is in $\mathbf{3}$

(recall $q\to q$ has color factor $2C_F-N_c\;\longrightarrow\;0$)

$N_c\to\infty$

⟶ same result can be obtained by removing 'triplet' component of final qg in production amplitude



$$=\mathcal{M}^{\bar{\mathbf{6}}\oplus\mathbf{15}}_{\rm hard}\;=\;T^aT^b\left(\frac{\boldsymbol{K}}{\boldsymbol{K}^2}-\frac{\boldsymbol{K}-\boldsymbol{q}}{(\boldsymbol{K}-\boldsymbol{q})^2}\right)\Rightarrow\frac{|\mathcal{M}^{\bar{\mathbf{6}}\oplus\mathbf{15}}_{\rm hard}|^2}{|\mathcal{M}_{\rm hard}|^2}=\;\cdots\;=\kappa_{q\to qg}$$

$\kappa_{q\to qg} =$ proba to produce qg pair in $\bar{\mathbf{6}} \oplus \mathbf{15}$ subspace

$$= \frac{(\boldsymbol{K} - x_{\rm h}\boldsymbol{q})^2}{(\boldsymbol{K} - x_{\rm h}\boldsymbol{q})^2 + (1-x_{\rm h})^2\boldsymbol{K}^2} \xrightarrow[q_\perp \ll K_\perp]{x_{\rm h}=1/2} \frac{4}{5}$$

- color factor

$qg \sim$ pointlike $\Longrightarrow$ result must depend only on total color charge, and be given by same rule as for $1 \to 1$

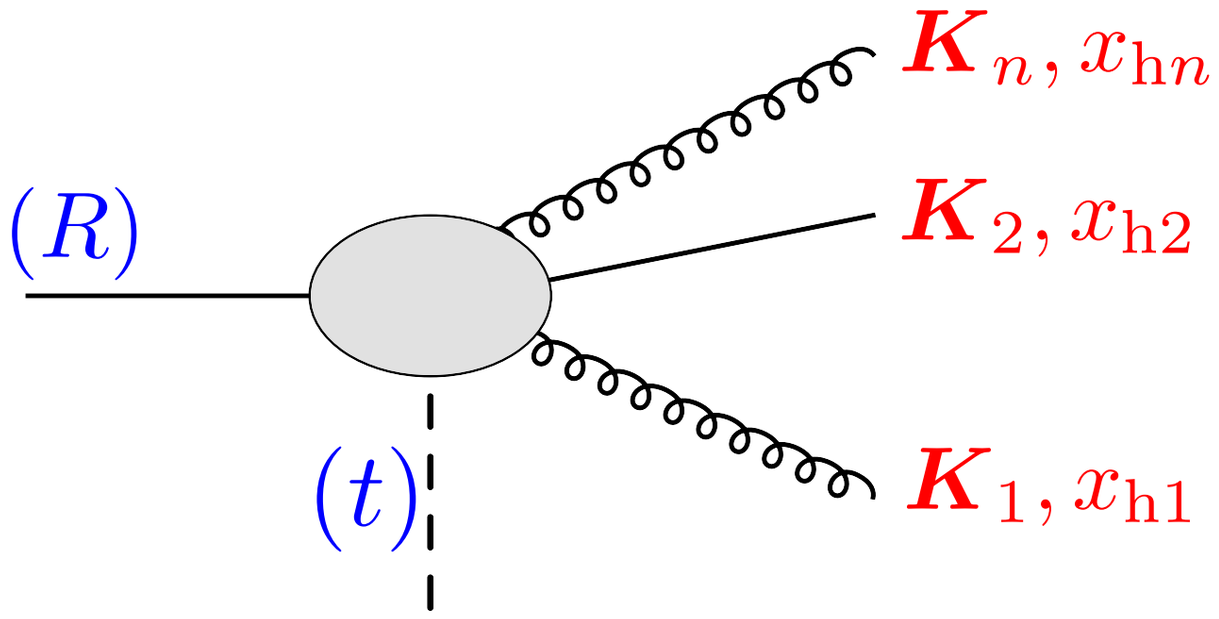
qg is in $R' = \bar{\mathbf{6}}$ or $\mathbf{15}$

$C_{\bar{\mathbf{6}}} \simeq C_{\mathbf{15}} \simeq \frac{3}{2}N_c$ $\Big\} \Longrightarrow$

$$2\,T^a_{\mathbf{3}}T^a_{R'} = C_{\mathbf{3}} + C_{R'} - C_{\mathbf{8}}$$
$$\simeq \frac{N_c}{2} + \frac{3}{2}N_c - N_c$$
$$= N_c$$

conjecture for $1 \to n$ hard forward processes

(PK14)



$$x_{\mathrm{h}i} \sim \mathcal{O}(1)$$

$$K_{i\perp} \sim \mathcal{O}(Q_{\mathrm{hard}})$$

$$x\frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{1\to n} = \sum_{R'} P_{R'} \underbrace{(C_R + C_{R'} - C_t)\frac{\alpha_s}{\pi}\,\log\left(\frac{\Delta q_\perp^2(L)}{x^2 Q_{\mathrm{hard}}^2}\right)}_{\text{same as for } 1\to 1}$$

proba for n-parton state to be produced in color rep (R')

conjecture explicitly checked for: all $1 \to 1$; $q \to qg$

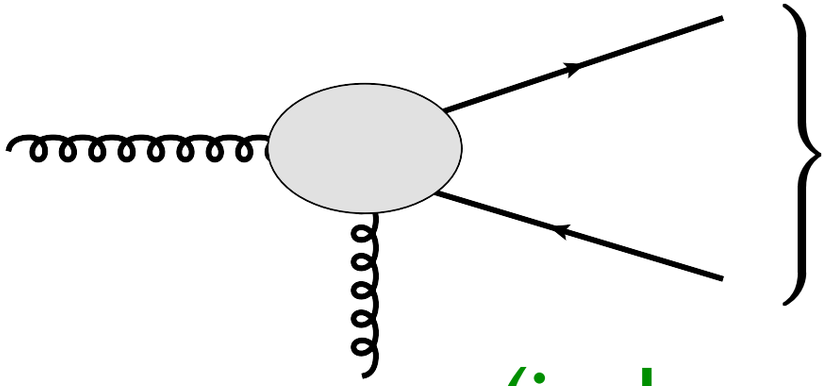
two non-trivial checks:

$$\boxed{1)\ g \to q\bar{q}}$$

explicit calculation:

$$x\frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{g\to q\bar{q}} = N_c\frac{\alpha_s}{\pi}\,\log\left(\frac{\Delta q_\perp^2(L)}{x^2 Q^2_{\mathrm{hard}}}\right) \qquad \text{(LM14)}$$

using conjecture:



dominantly in $R' = \mathbf{8}$

when $N_c \gg 1$ $(P_{\mathbf{8}} = 1)$

(independently of x_{h})

$$\Longrightarrow \quad N_c + N_c - N_c = N_c$$

$q\bar{q}$ pointlike and octet $\Longrightarrow$ same result as $g \to g$

(APS10, PAK14)

2) $g \to gg$

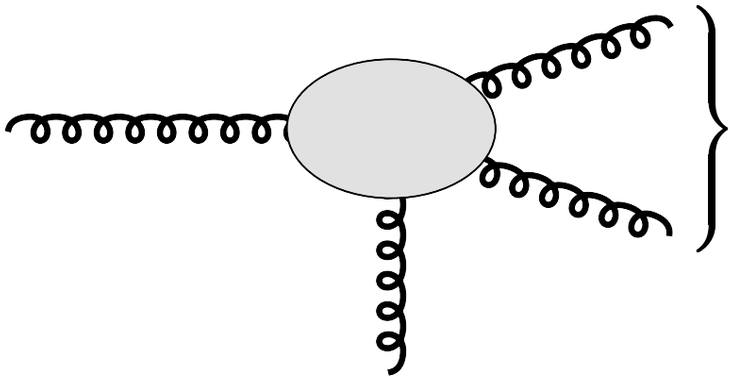
explicit calculation (PK14)

$$x\frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{g\to gg} = \kappa_{g\to gg} N_c \,\frac{\alpha_s}{\pi}\,\log\left(\frac{\Delta q_\perp^2(L)}{x^2 Q^2_{\mathrm{hard}}}\right)$$

$$\kappa_{g\to gg} \equiv 1 + \frac{(\boldsymbol{K}-x_{\mathrm{h}}\boldsymbol{q})^2}{(\boldsymbol{K}-x_{\mathrm{h}}\boldsymbol{q})^2 + x_{\mathrm{h}}^2(\boldsymbol{K}-\boldsymbol{q})^2 + (1-x_{\mathrm{h}})^2\boldsymbol{K}^2}$$

remark: $\kappa_{g\to gg} = 5/3$ when $x_{\mathrm{h}} = 1/2$ and $q_\perp \ll K_\perp$

using conjecture:

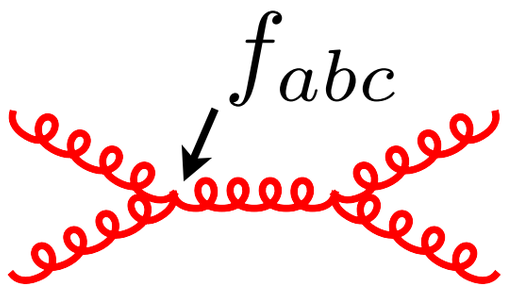


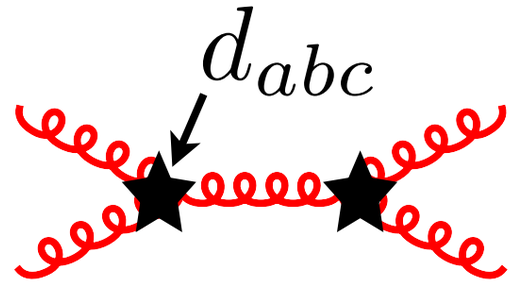
$$\mathbf{8}\otimes\mathbf{8} = \mathbf{1}\oplus\mathbf{8_s}\oplus\mathbf{8_a}\oplus\underbrace{(\mathbf{10}\oplus\overline{\mathbf{10}})}\oplus\mathbf{27}\oplus\mathbf{0}$$

$$C_{R'}\simeq 0; \quad N_c; \quad N_c; \qquad 2N_c; \qquad 2N_c; \quad 2N_c$$

$$\Longrightarrow \sum_{R'} P_{R'}(C_R + C_{R'} - C_t) = \sum_{R'} P_{R'} C_{R'} = (2 - P_{\mathbf{8_a}} - P_{\mathbf{8_s}})\, N_c$$

$P_{\mathbf{8_a}}$ and $P_{\mathbf{8_s}}$ can be simply calculated using pictorial rules for color projection operators

$\mathcal{P}_{\mathbf{8_a}} = \frac{1}{N_c}$ 

$\mathcal{P}_{\mathbf{8_s}} = \frac{N_c}{N_c^2 - 4}$ 

Y. Dokshitzer, 'PQCD (and Beyond)', 1995

$$\longrightarrow \quad (2 - P_{\mathbf{8_a}} - P_{\mathbf{8_s}})\, N_c = \quad \ldots \quad = \kappa_{g\to gg}\, N_c$$

(remark: $P_{\mathbf{8_s}}$ and $P_{\mathbf{8_a}}$ depend on x_{h})

summary and final remarks

- *fully coherent radiation expected in all hard forward processes with color in both initial and final states*

$$(\Delta E_{coh} \gg \Delta E_{\rm LPM})$$

→ *di-jet, light hadron... nuclear suppression in pA vs pp*

(Kolevatov, Arleo, S.P., work in progress)

- *fully coherent radiation derived in specific setup: tagged hadron* $p'_\perp|_{\rm hard} \gg \Delta q_\perp(L) \sim \sqrt{\hat{q}L}$

- *within this setup, features of coherent radiation are the same for* $1 \to 1$ *and* $1 \to 2$ *processes:*

universal log but process-dependent prefactor

- *radiation sees final parton system as pointlike*

  ⟶ *conjecture for all* $1 \to n$ *processes*

- *results for fully coherent spectra follow from first principles of QCD radiation and color algebra, and might be obtained in different formalisms*

- *results of Liou & Mueller (dipole formalism) and PAK14, PK14 (opacity expansion) coincide where they can be compared (symmetric dijet)*

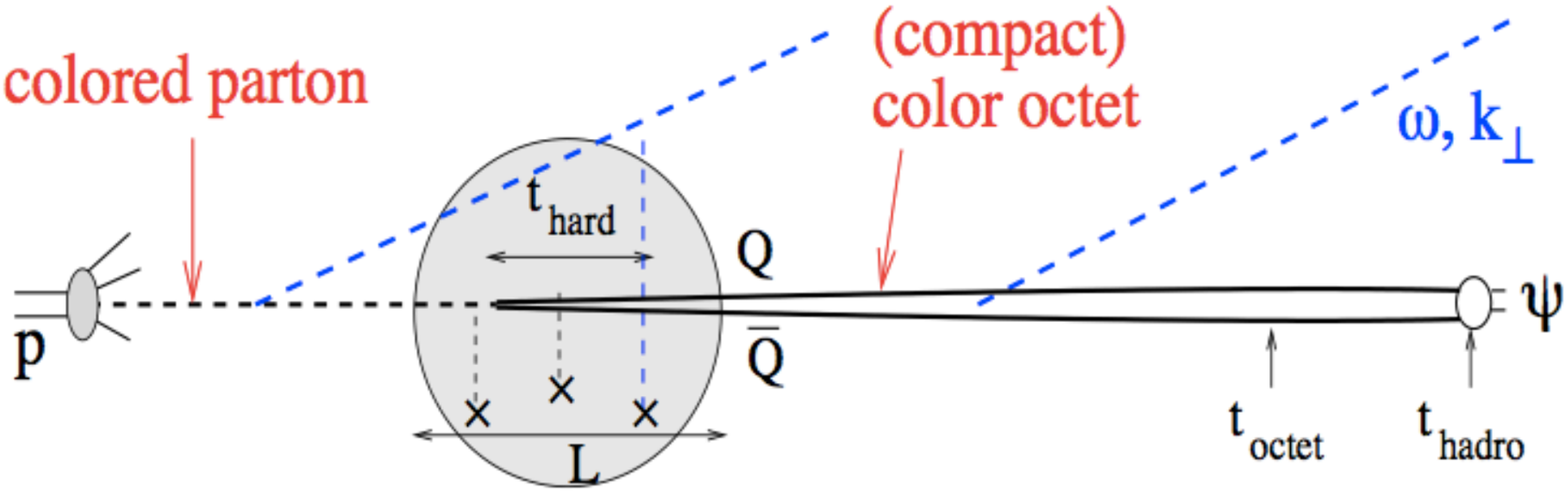
THANK YOU !

Back-up

model for quarkonium pA suppression

Arleo, S.P., JHEP 1303 (2013) 122

$x_{\rm F} > x_{\rm F}^{critical}$



⟶ coherent radiation

- arises from $t_{hard}, L \ll t_f \ll t_{octet}$
- depends on L via $\Delta q_\perp^2 = \hat{q}\, L \ll M$

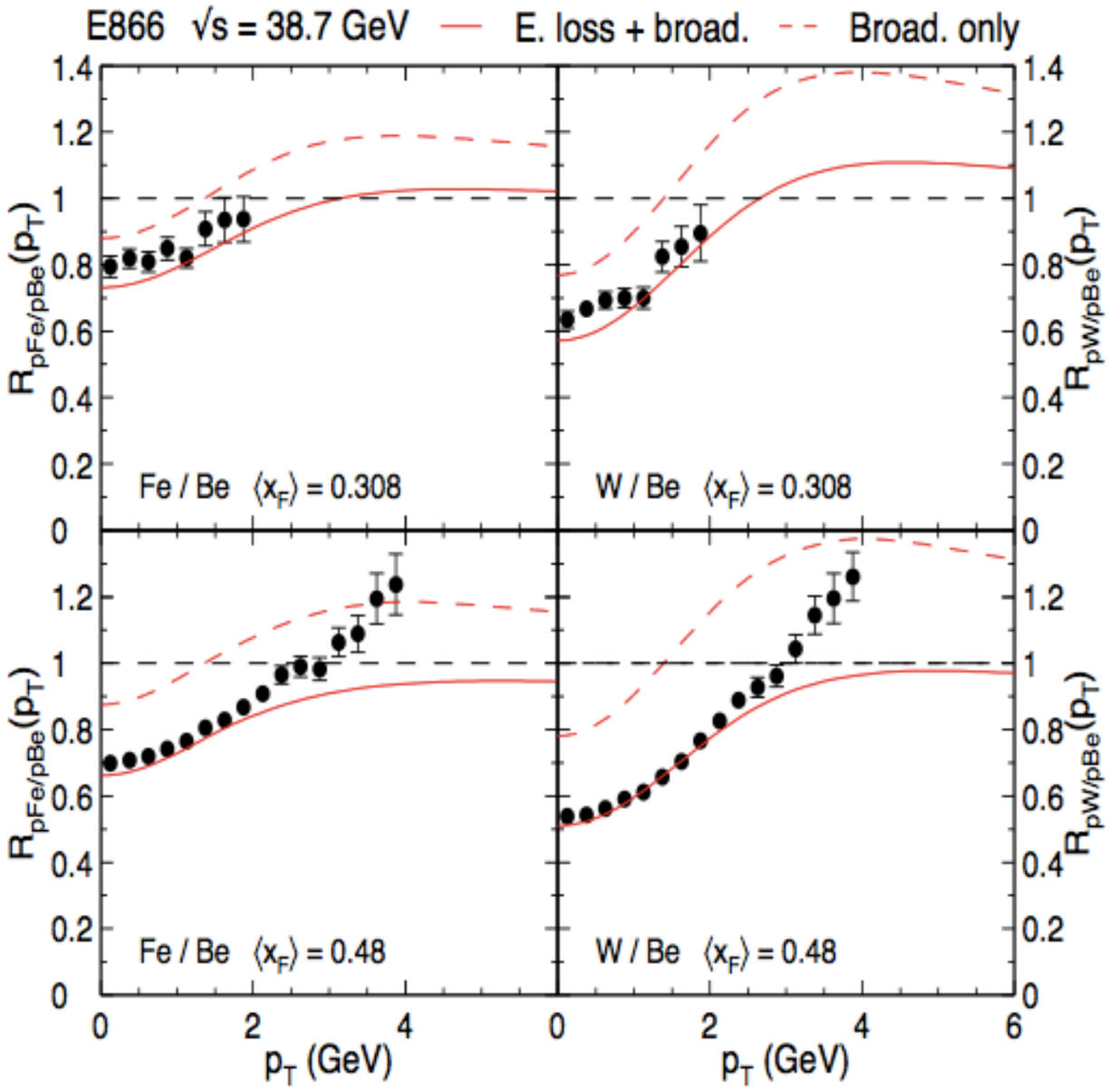
use standard way to implement energy loss:

$$\frac{1}{A}\frac{{\rm d}\sigma_{\rm pA}^{\psi}}{{\rm d}E}\,(E) = \int_0^{\varepsilon^{\rm max}} {\rm d}\varepsilon\, \mathcal{P}(\varepsilon, E, \ell_{\rm A}^2)\, \frac{{\rm d}\sigma_{\rm pp}^{\psi}}{{\rm d}E}\,(E+\varepsilon)$$

$p_\perp$-dependence (AKPR13)

- energy loss + $p_\perp$-broadening of pointlike $c\bar{c}$:

$$\frac{1}{A}\frac{\mathrm{d}\sigma^{\psi}_{\mathrm{pA}}}{\mathrm{d}E\,\mathrm{d}^2\vec{p}_\perp} = \int_\varphi \int_\varepsilon \mathcal{P}(\varepsilon)\,\frac{\mathrm{d}\sigma^{\psi}_{\mathrm{pp}}}{\mathrm{d}E\,\mathrm{d}^2\vec{p}_\perp}\,(E+\varepsilon, \vec{p}_\perp - \Delta\vec{p}_\perp)$$



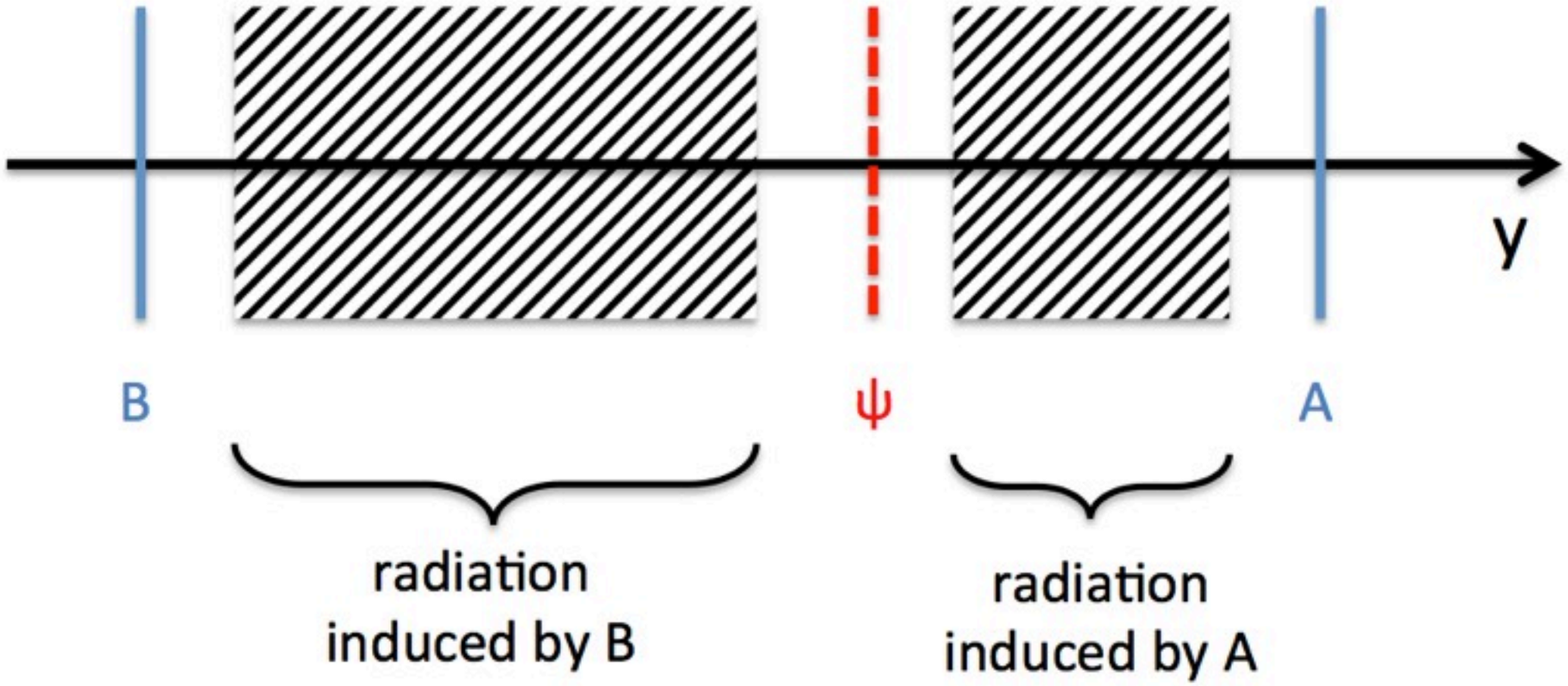
no free parameter:

$\Delta p_\perp$ induces ΔE

$\Delta p_\perp \longrightarrow$
Cronin effect

energy loss $\longrightarrow$
normalization

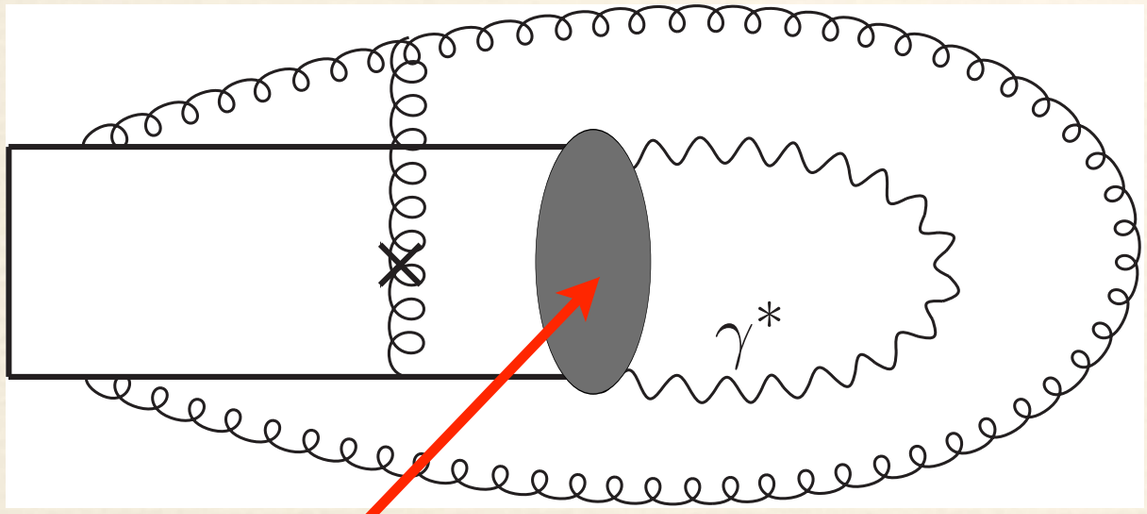
quarkonium suppression in AB collisions (AP14)



$$\frac{1}{AB}\frac{\mathrm{d}\sigma^{\psi}_{\mathrm{AB}}}{\mathrm{d}y}(y) = \int_0^{\delta y^{\max}(y)} \mathrm{d}\delta y_{\mathrm{B}}\, \hat{\mathcal{P}}_{\mathrm{B}}(\varepsilon_{\mathrm{B}}) \int_0^{\delta y^{\max}(-y)} \mathrm{d}\delta y_{\mathrm{A}}\, \hat{\mathcal{P}}_{\mathrm{A}}(\varepsilon_{\mathrm{A}})\, \frac{\mathrm{d}\sigma^{\psi}_{\mathrm{pp}}}{\mathrm{d}y}(y + \delta y_{\mathrm{B}} - \delta y_{\mathrm{A}})$$

✦ coherent loss is «higher-twist»

- $\Delta E_{\rm coh}$ is suppressed by a power of $Q_{\rm hard}$
- $\Delta E_{\rm coh}$ is process-dependent
  - for a fast quark, color factor $\propto 1/N_c$
  - effect is absent when no color charge is produced in the hard process:



in this case radiation is purely from initial state, and $t_f \gg L$ *cancels out, giving*

$$\Delta E_{\rm initial} \sim L^2$$