Atomic Quantum Simulation of Abelian and non-Abelian Gauge Theories

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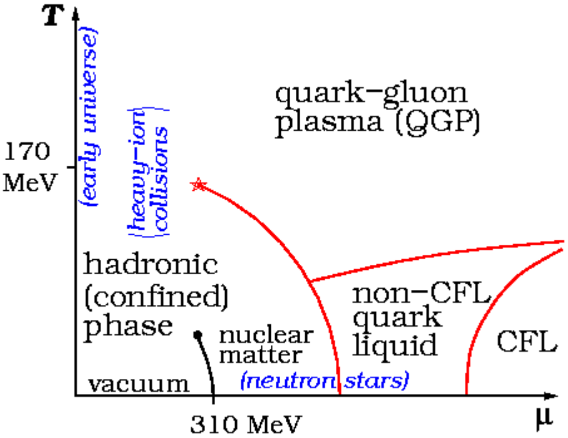
Outline

Outline

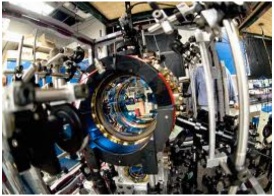
Conjectured QCD Phase Diagram

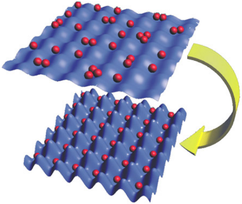


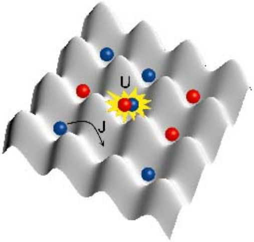
Great (and perhaps insurmountable) challenges for traditional lattice QCD simulations: Sign and complex action problems at non-zero baryon chemical potential and for real-time evolution.

Outline

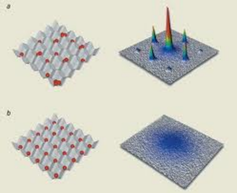
Ultra-cold atoms in optical lattices as analog quantum simulators





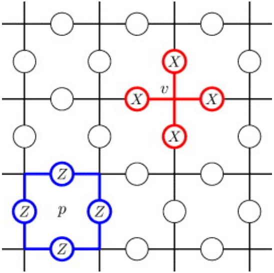


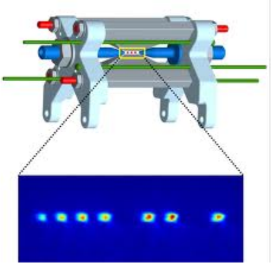
Superfluid-Mott insulator transition in the bosonic Hubbard model



M. Greiner, O. Mandel, T. Esslinger, T. Hänsch, I. Bloch, Nature 415 (2002) 39.

Digital quantum simulation of Kitaev's toric code (a $\mathbb{Z}(2)$ lattice gauge theory) with trapped ions

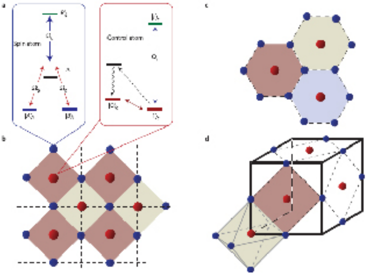


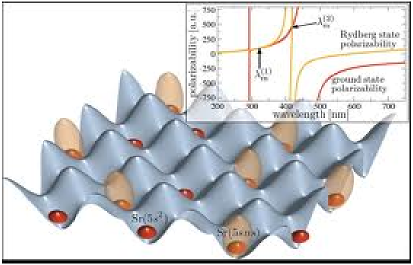


- Precisely controllable many-body quantum device, which can execute a prescribed sequence of quantum gate operations.
- State of simulated system encoded as quantum information.
- Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

A. Y. Kitaev, Ann. Phys. 303 (2003) 2.
B. P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J. T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, C. F. Roos, Science 334 (2011) 6052.

Quantum spin liquids ($U(1)$ gauge theories) to be simulated with Rydberg atoms in an optical lattice





- Lasers can excite atoms to high-lying Rydberg states.
- Rydberg atoms are large and have collective interactions.
- Ensemble Rydberg atoms represent qubits at link centers.
- Control atoms at lattice sites ensure the Gauss' law.

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502.
H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H. P. Büchler, Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

Analog quantum simulators

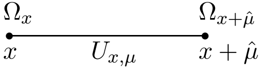
- Time evolution proceeds continuously, not using discrete quantum gates.
- Limited to simpler interactions, but more easily scalable.

Proposals for analog quantum simulators for Abelian and non-Abelian gauge theories with and without matter

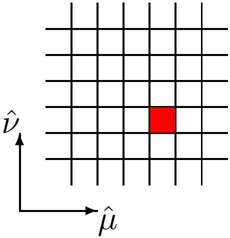
H. P. Büchler, M. Hermele, S. D. Huber, M. P. A. Fisher, P. Zoller, Phys. Rev. Lett. 95 (2005) 040402.
E. Kapit, E. Mueller, Phys. Rev. A83 (2011) 033625.
E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. Lett. 109 (2012) 125302.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. Lett. 110 (2013) 055302.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. Lett. 110 (2013) 125304.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. A 88 (2013) 023617.
D. Banerjee, M. Dalmote, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.
D. Banerjee, M. Bögli, M. Dalmote, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303.
UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

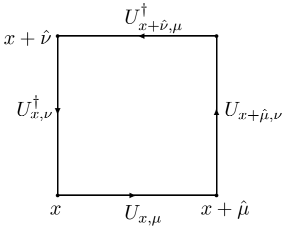
Outline

Wilson's classical link variables



$${}^{\Omega}U_{x,\mu} = \Omega_x U_{x,\mu} \Omega^{\dagger}_{x+\hat{\mu}}, \quad U_{x,\mu}, \Omega_x \in G = SU(N), SO(N), Sp(N)$$





Gauge invariant plaquette action

$$S[U] = -\frac{1}{2g^2} \sum_{x,\mu\neq\nu} \mathrm{Tr}(U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu} + \mathrm{h.c.})$$

Functional integral using Haar measure

$$Z = \prod_{x,\mu} \int_G dU_{x,\mu} \exp(-S[U])$$

defines a quantum field theory using continuous classical field variables as fundamental degrees of freedom. Wilson (1974)

Hamiltonian formulation of $U(1)$ lattice gauge theory

$$U = \exp(i\varphi), \ U^\dagger = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_\varphi, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 0$$

Generator of $U(1)$ gauge transformations

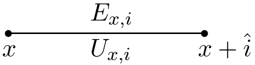
$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in an infinite-dimensional Hilbert space per link

$U(1)$ quantum link model



$$U = S_1 + iS_2 = S_+, \; U^\dagger = S_1 - iS_2 = S_-$$

Electric flux operator E

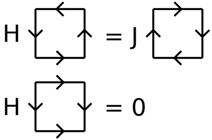
$$E = S_3, \; [E, U] = U, \; [E, U^\dagger] = -U^\dagger, \; [U, U^\dagger] = 2E$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

Gauge invariant Hamiltonian for $S = \frac{1}{2}$

$$H = -J \sum_\square (U_\square + U_\square^\dagger)$$



defines a gauge theory with a 2-d Hilbert space per link

D. Horn, Phys. Lett. B100 (1981) 149

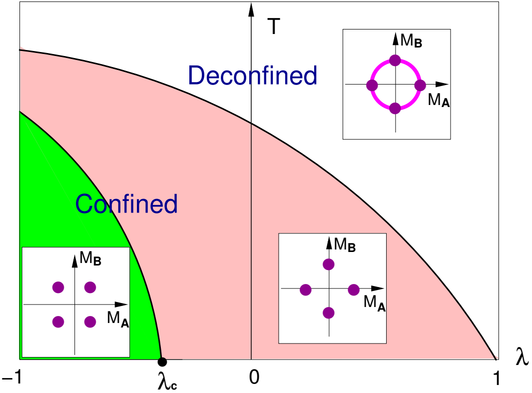
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

Outline

Hamiltonian with Rokhsar-Kivelson term

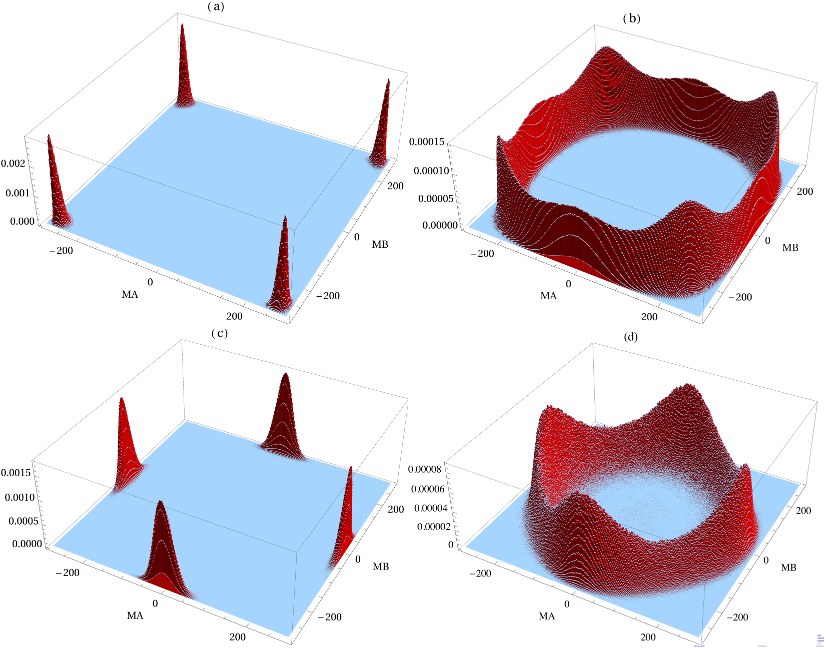
$$H = -J \left[\sum_{\square} (U_\square + U_\square^\dagger) - \lambda \sum_{\square} (U_\square + U_\square^\dagger)^2 \right]$$

Phase diagram



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, arXiv:1303.6858, to appear in JSTAT.

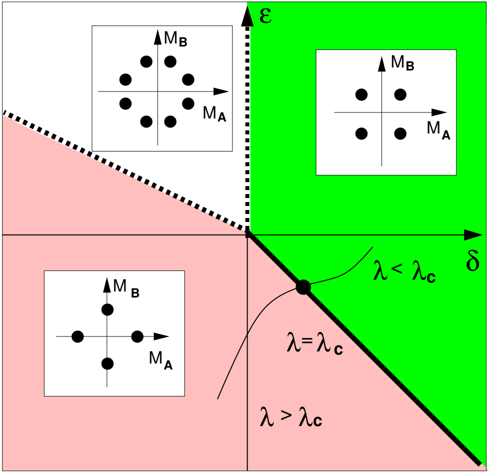
Probability Distribution of the Order Parameters



Low-Energy $\mathbb{R}P(1)$ Effective Field Theory

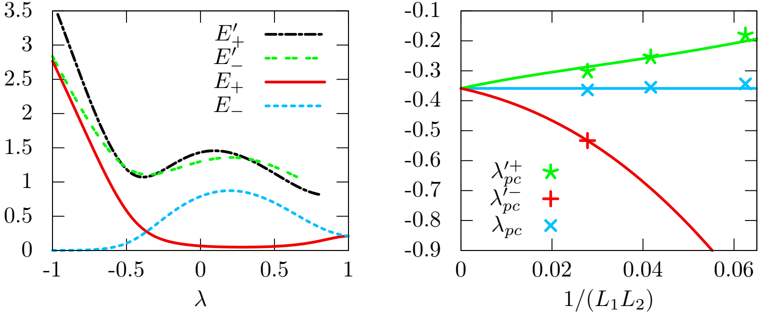
$$S[\varphi] = \int d^3x \frac{1}{c} \left[\frac{\rho}{2} \partial_\mu \varphi \partial_\mu \varphi + \delta \cos^2(2\varphi) + \varepsilon \cos^4(2\varphi) \right]$$

Phase diagram



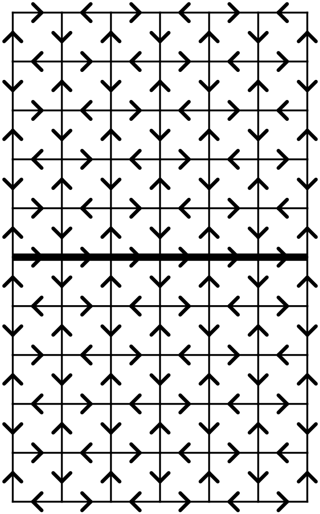
Determination of the Low-Energy Parameters from the Finite-Volume Energy Spectrum

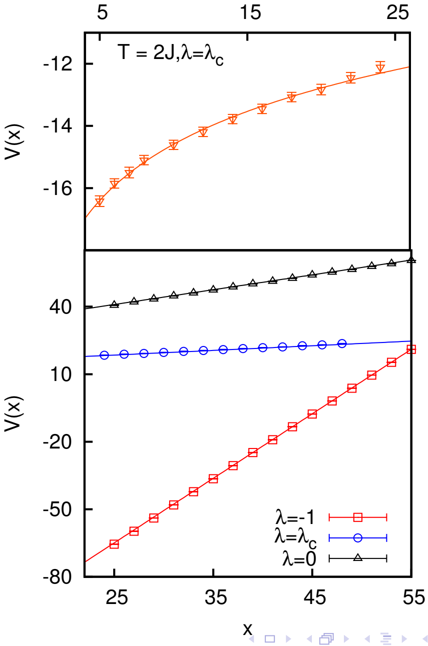
$$S[\varphi] = \int d^3x \frac{1}{c} \left[\frac{\rho}{2} \partial_\mu \varphi \partial_\mu \varphi + \delta \cos^2(2\varphi) + \varepsilon \cos^4(2\varphi) \right]$$



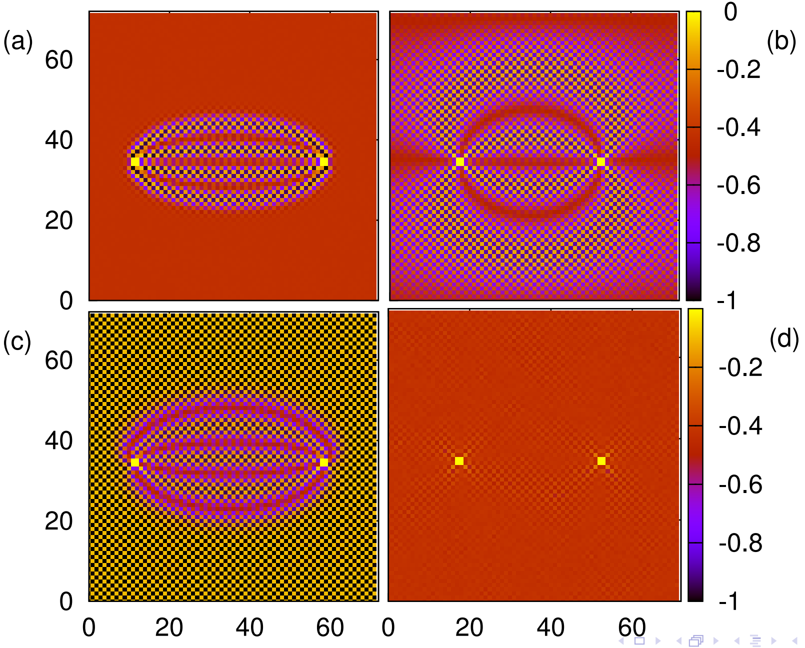
$$\lambda_c = -0.359(5), \rho = 0.45(3)J, c = 1.5(1)Ja, \delta_c = -\varepsilon_c = 0.01(1)J/a^2$$

Confinement versus Deconfinement





Energy density of charge-anti-charge pair $Q = \pm 2$



Outline

Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{h.c.}\right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

Bosonic rishon representation of the quantum links

$$U_{x,x+1} = b_x b_{x+1}^\dagger, \; E_{x,x+1} = \frac{1}{2}\left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x\right)$$

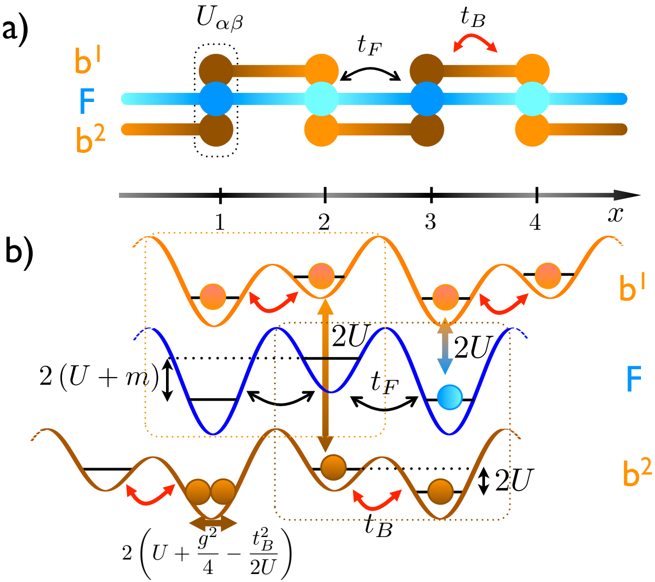
Gauge generator

$$\widetilde{G}_x = n_x^F + n_x^1 + n_x^2 - 2S + \frac{1}{2}\left[(-1)^x - 1\right]$$

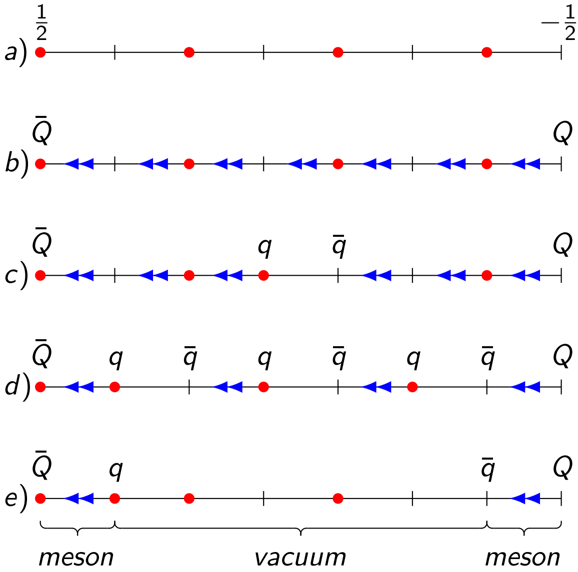
Microscopic Hubbard model Hamiltonian

$$\begin{aligned} \widetilde{H} &= \sum_x h_{x,x+1}^B + \sum_x h_{x,x+1}^F + m \sum_x (-1)^x n_x^F + U \sum_x \widetilde{G}_x^2 \\ &= -t_B \sum_{x \text{ odd}} b_x^{1\dagger} b_{x+1}^1 - t_B \sum_{x \text{ even}} b_x^{2\dagger} b_{x+1}^2 - t_F \sum_x \psi_x^\dagger \psi_{x+1} + \mathrm{h.c.} \\ &+ \sum_{x,\alpha,\beta} n_x^\alpha U_{\alpha\beta} n_x^\beta + \sum_{x,\alpha} (-1)^x U_\alpha n_x^\alpha \end{aligned}$$

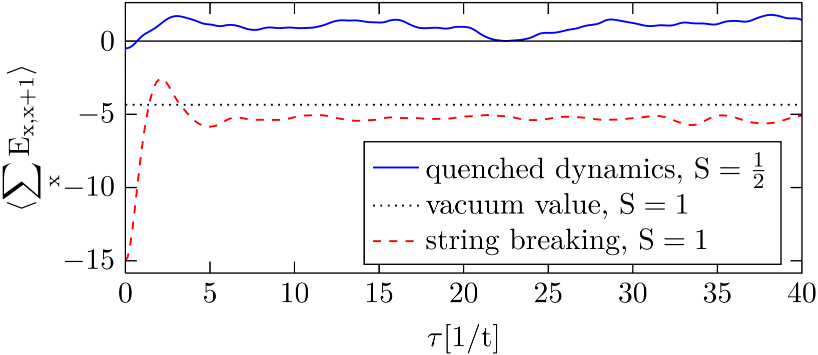
Optical lattice with Bose-Fermi mixture of ultra-cold atoms



Schematic illustration of string breaking in real time in the 1-d $S = 1$ $U(1)$ quantum link model



Quantum simulation of the real-time evolution of string breaking



D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

Outline

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij}, \; U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \; i,j \in \{1,2,\dots,N\}, \; [U^{ij}, (U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = i f_{abc} L^c, \; [R^a, R^b] = i f_{abc} R^c, \; a,b,c \in \{1,2,\dots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U, \; [R^a, U] = U\lambda^a, \; [E, U] = U$$

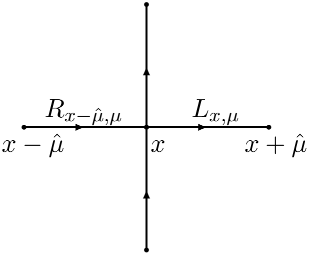
Algebraic structures of different quantum link models

$U(N)$: $U^{ij}, \; L^a, \; R^a, E, \; 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

$SO(N)$: $O^{ij}, \; L^a, \; R^a, \; N^2 + 2\frac{N(N-1)}{2} = N(2N-1)$ $SO(2N)$ generators

$Sp(N)$: $U^{ij}, \; L^a, \; R^a, \; 4N^2 + 2N(2N+1) = 2N(4N+1)$ $Sp(2N)$ generators

R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502



Generator of $SU(N)$ gauge transformations

$$G_x^a = \sum_\mu (R^a_{x-\hat{\mu},\mu} + L^a_{x,\mu})$$

$U(N)$-invariant Hamiltonian "action" operator

$$H = -J \sum_{x,\mu<\nu} \mathrm{Tr}(U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu} + \mathrm{h.c.}), \ [H, G_x^a] = 0$$

Functional integral of a quantum link model

$$Z = \mathrm{Tr} \exp(-\beta H)$$

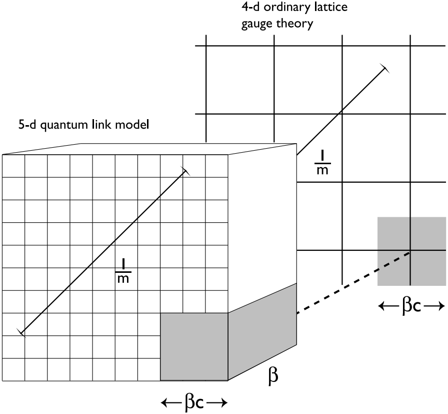
defines a quantum field theory using discrete variables

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr}\ \partial_5 G_\mu \partial_5 G_\mu \right), \ G_5 = 0$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

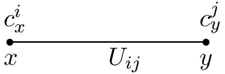
$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu}, \ \frac{1}{g^2} = \frac{\beta}{e^2}, \ \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11Ne^2}\right)$$



Fermionic rishons at the two ends of a link

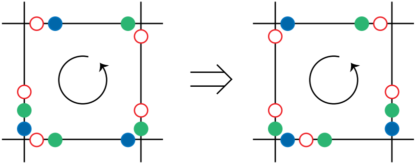
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra

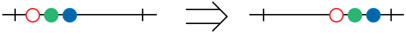


$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a “rishon abacus” implemented with ultra-cold atoms be used as a quantum simulator?



Tr Up



det $U_{x,\mu}$

d-dimensional $SU(N)$ gauge theory with staggered fermions

$$
\begin{aligned}
H &= -t\sum_{\langle xy\rangle}\left(s_{xy}\psi_x^{i\dagger}U_{xy}^{ij}\psi_y^j + \mathrm{h.c.}\right) + m\sum_x(-1)^x\psi_x^{i\dagger}\psi_x^i \\
&+ \frac{g^2}{2}\sum_{\langle xy\rangle}\left(L_{xy}^a L_{xy}^a + R_{xy}^a R_{xy}^a\right) + \frac{g'^2}{2}\sum_{\langle xy\rangle}E_{xy}^2 \\
&- \frac{1}{4g^2}\sum_{\langle wxyz\rangle}\left(U_{wx}U_{xy}U_{yz}U_{zw} + \mathrm{h.c.}\right) - \gamma\sum_{\langle xy\rangle}\left(\det U_{xy} + \mathrm{h.c.}\right)
\end{aligned}
$$

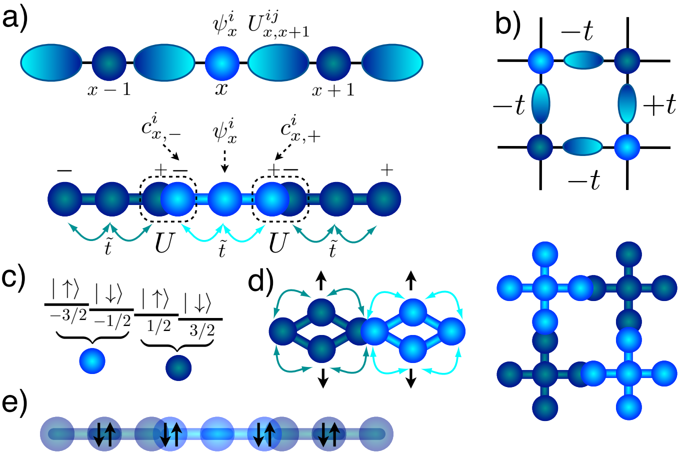
Meson, constituent quark, and glueball operators

$$
M_x = \psi_x^{i\dagger}\psi_x^i, \quad Q_{x,\pm k} = c_{x,\pm k}^{i\dagger}\psi_x^i, \quad \Phi_{x,\pm k,\pm l} = c_{x,\pm k}^{i\dagger}c_{x,\pm l}^i
$$

form a local $U(2d+1)$ algebra at each site x, thus providing a formulation in terms of manifestly $SU(N)$ gauge invariant objects. However, the conserved rishon number gives rise to a $U(1)$ gauge symmetry on the links

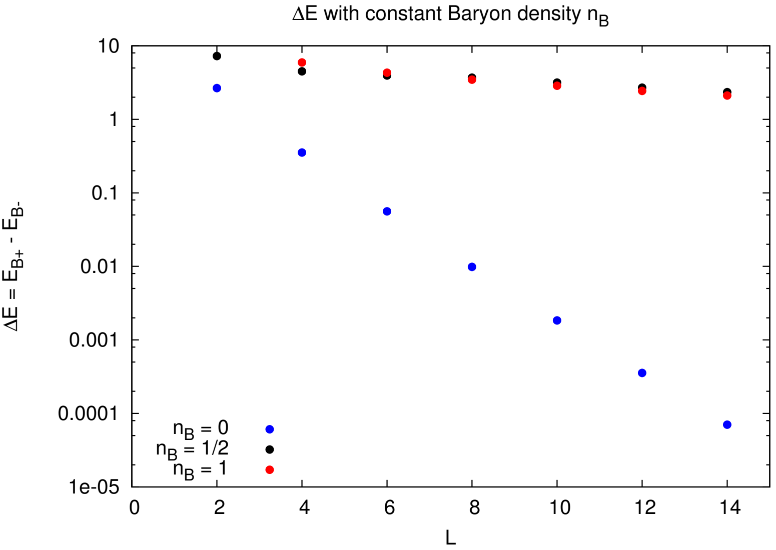
$$
N_{xy} = c_x^{i\dagger}c_x^i + c_y^{i\dagger}c_y^i, \quad [N_{xy}, H] = 0
$$

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin

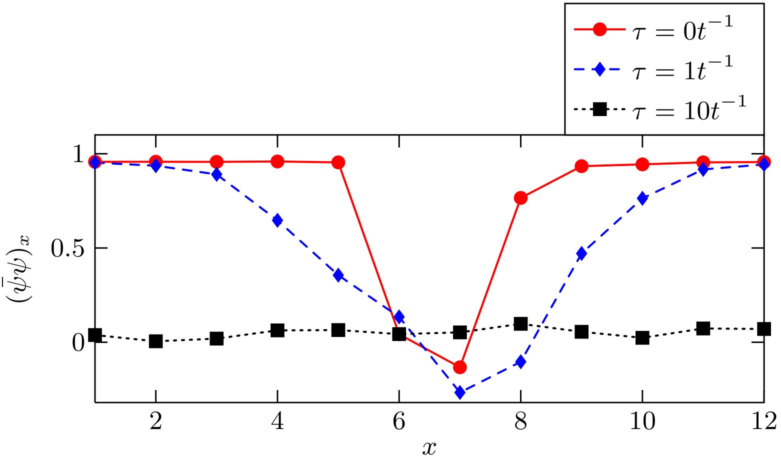


D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$



Expansion of a “fireball” mimicking a hot quark-gluon plasma



Outline

Conclusions

• If quantum link models can be implemented with ultra-cold atoms, such systems can be used as quantum simulators for dynamical Abelian and non-Abelian gauge theories, which can be validated in efficient classical cluster algorithm simulations, at least in the Abelian case.

• Quantum simulator constructions have already been presented for the $U(1)$ quantum link model as well as for $U(N)$ and $SU(N)$ quantum link models with fermionic matter.

• This would allow the quantum simulation of the real-time evolution of string breaking as well as the quantum simulation of "nuclear" physics and dense "quark" matter, at least in qualitative toy models for QCD.

• Accessible effects may include chiral symmetry restoration, baryon superfluidity, or color superconductivity at high baryon density, as well as the quantum simulation of "nuclear" collisions.

• The path towards quantum simulation of QCD will be a long one. However, with a lot of interesting physics along the way.