Recent Progress in Strongly Coupled CFT

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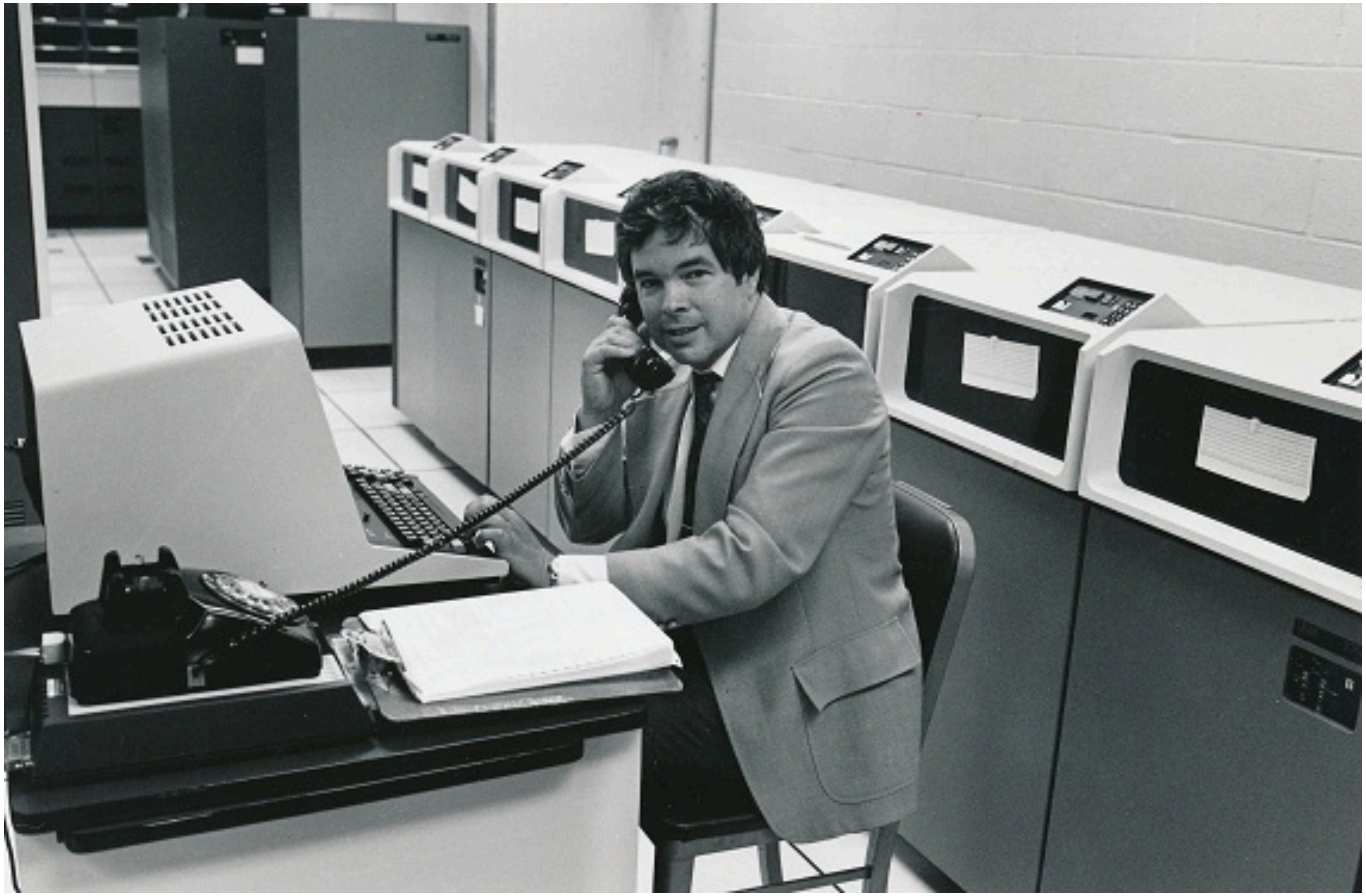
CERN, 26/03/2014

Ken Wilson



Unsolved problems - I

- where Laws of Nature are not known/hypothetical
  - high T_c superconductivity
  - quantum gravity
  - Beyond SM...

Unsolved problems - 2

- where Laws of Nature are relatively certain, but extracting predictions remains hard*
  - turbulence
  - strongly coupled QFT (QCD + many condensed matter examples)

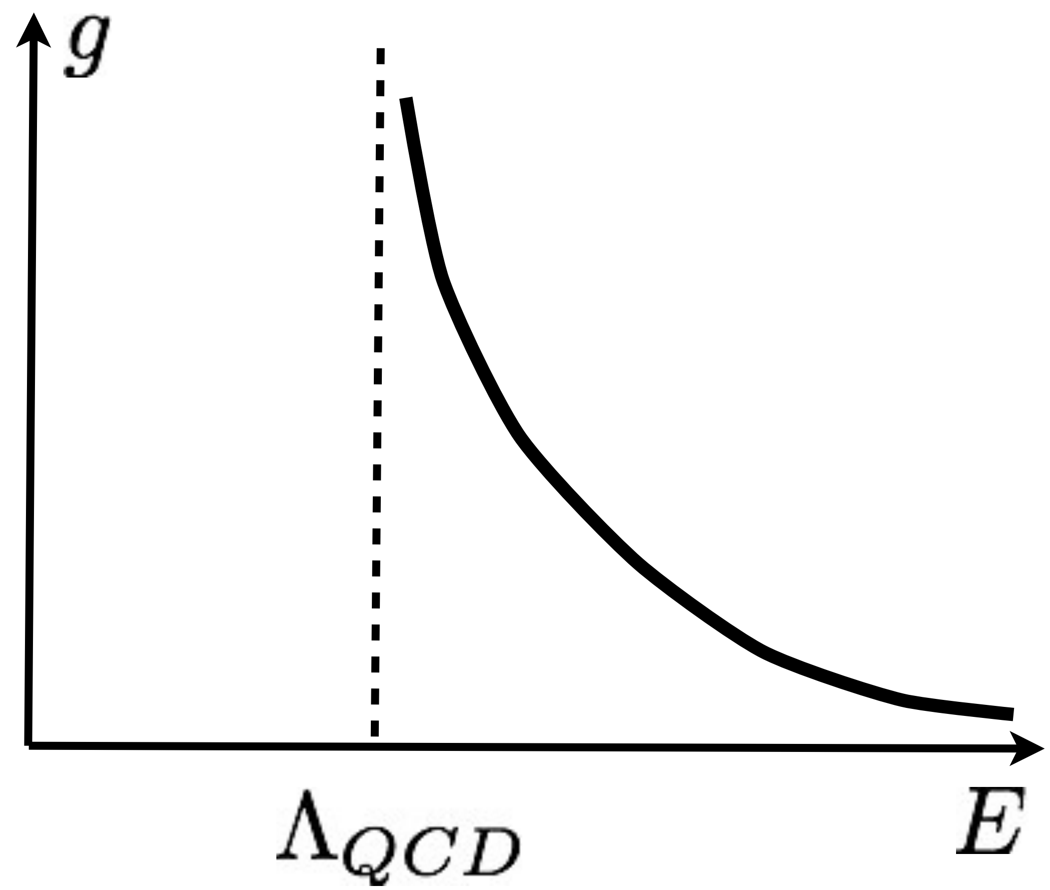
Computational complexity:
can we find better algorithms?

*Hard = nontrivial even with supercomputers

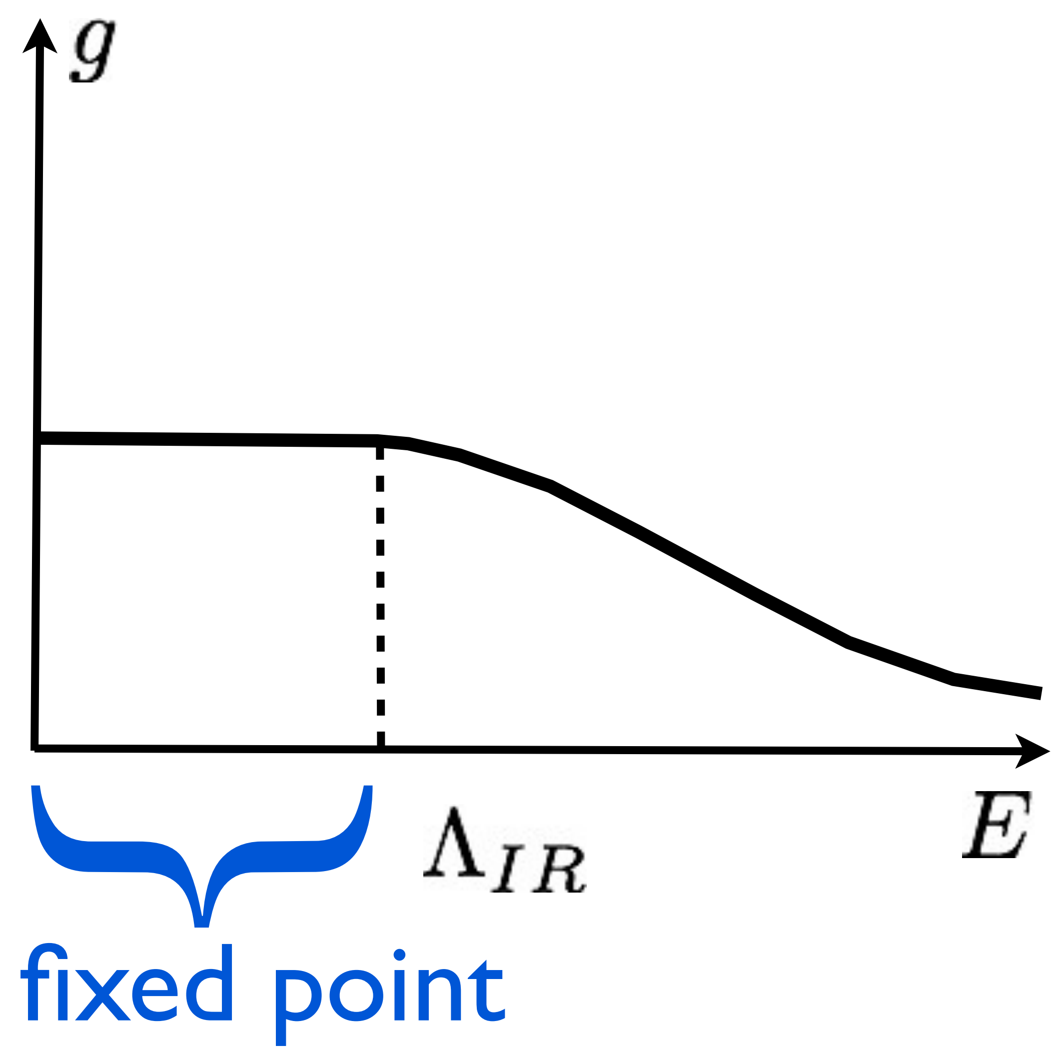
Conformal Field Theories

Running coupling in QCD:

$N_c = 3, \quad N_f = 3$



$N_c = 3, \quad 12 \lesssim N_f \leq 16$



IR fixed points are

- scale invariant (by definition)
- conformally invariant (generically, and perhaps always in unitary theories)

Condition for conformal invariance: $T^{\mu}_{\mu} = 0 \quad (1)$

Condition for scale invariance: $T^{\mu}_{\mu} = \partial_{\lambda} V^{\lambda} \quad (2)$

Generically (2)⇒(1)

Scale invariance

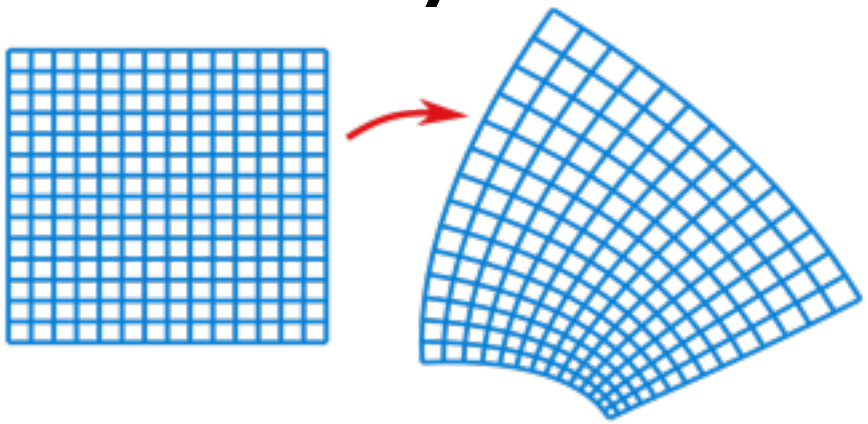
$$x \to \lambda x$$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \propto \frac{1}{|x-y|^{2\Delta_{\mathcal{O}}}}$$

scaling dimension

Conformal invariance

$x \to x' = f(x)$ s.t. locally rotation + dilatation



Physically...

Theorem [Liouville, 1850]

In d≥3 the group of conformal transformations is

- finite-dimensional
- isomorphic to SO(d+1,1)
- generated by $M_{\mu\nu}$, P_μ, D, and K_μ

$$K_\mu = I \circ P_\mu \circ I = 2x_\mu (x\partial) - x^2 \partial_\mu$$

special conformal transformation

What does it buy you?

For starters, also 3-point functions are fixed:

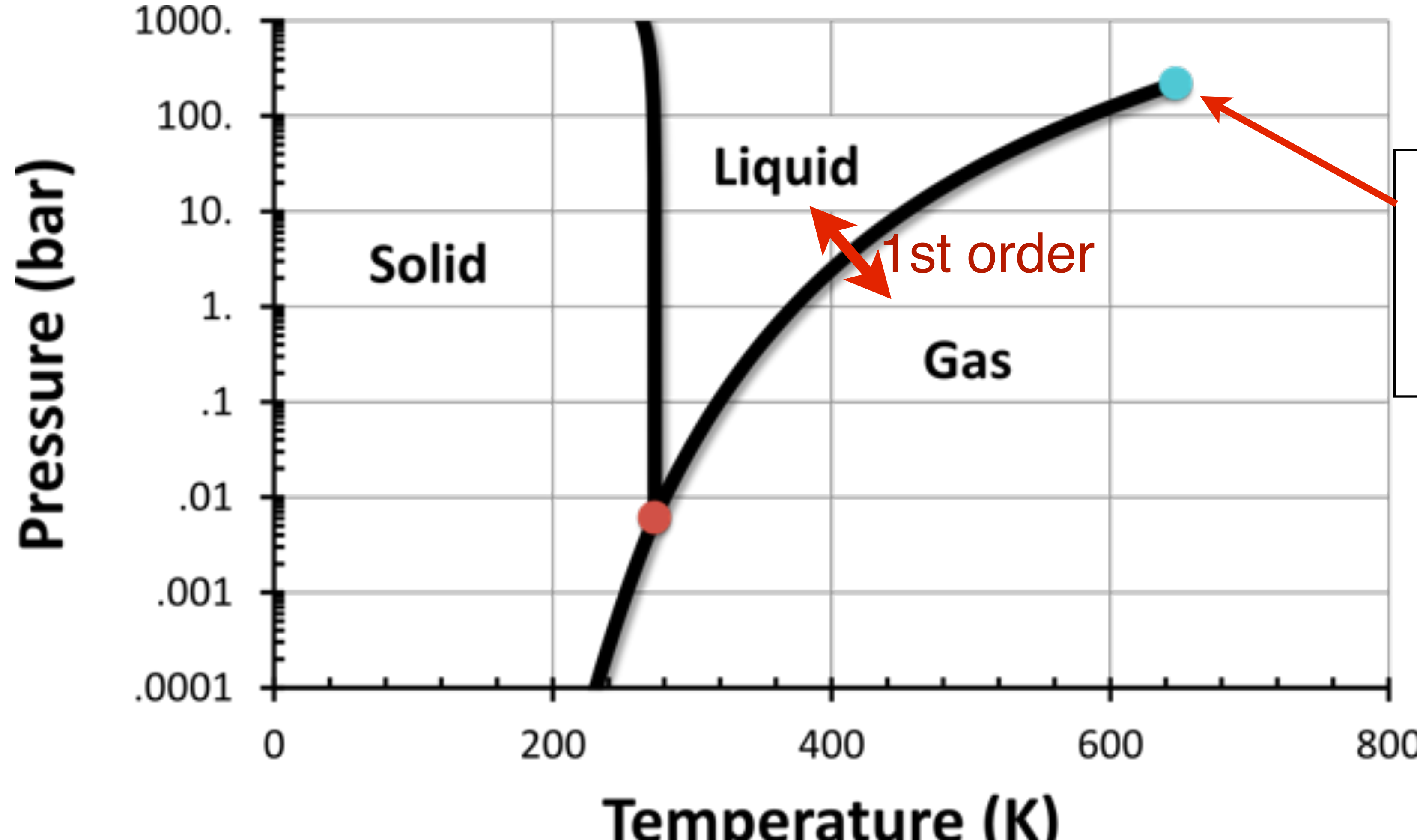
$$\langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z)\rangle \propto \frac{1}{|x-y|^{\Delta_{\mathcal{O}}}|x-z|^{\Delta_{\mathcal{O}}}|y-z|^{\Delta_{\mathcal{O}}}}$$

[Polyakov 1971]

CFTs in the real world

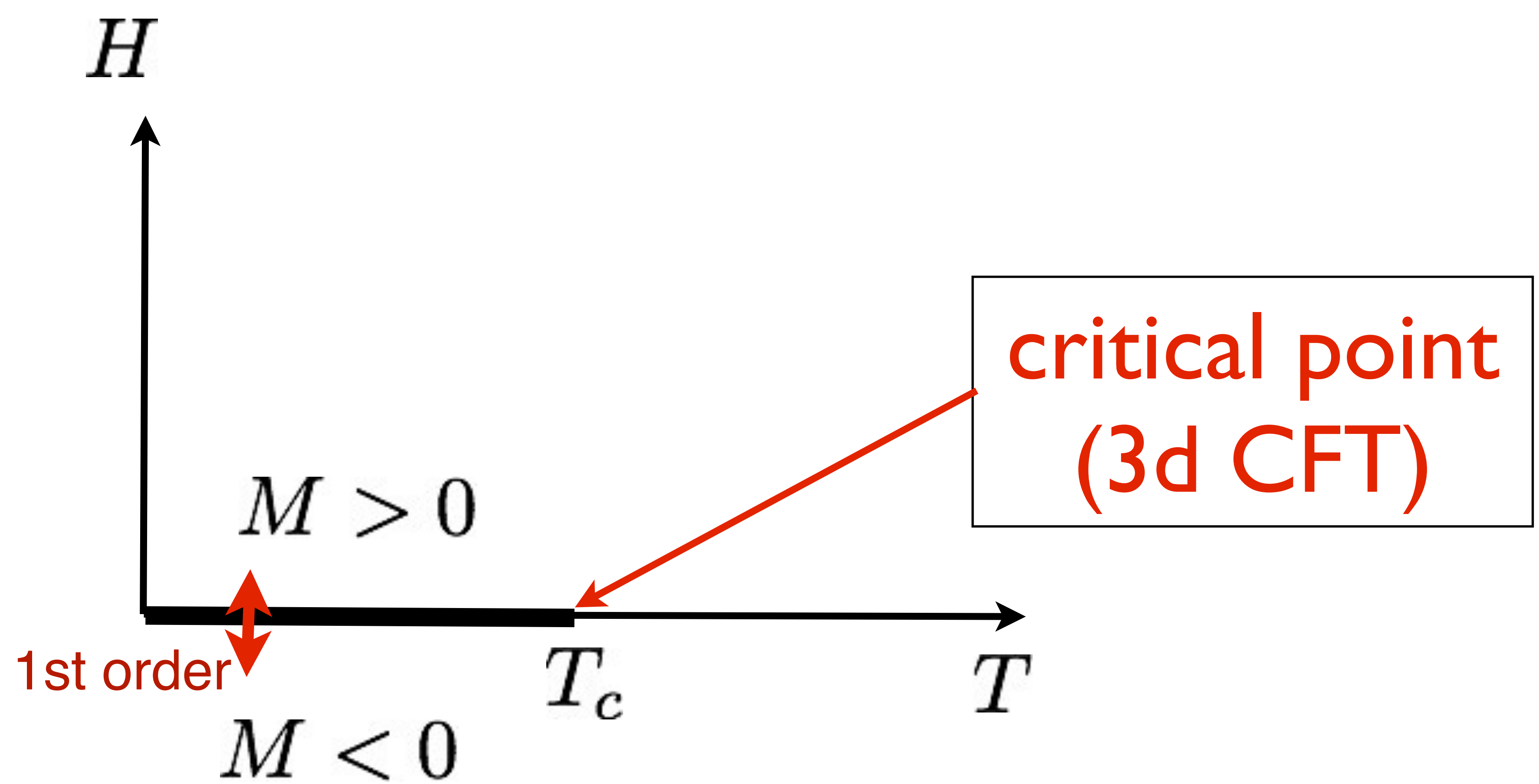
Rich source: condensed matter and statistical physics (2nd order phase transitions)

Example A: liquid-vapor transitions

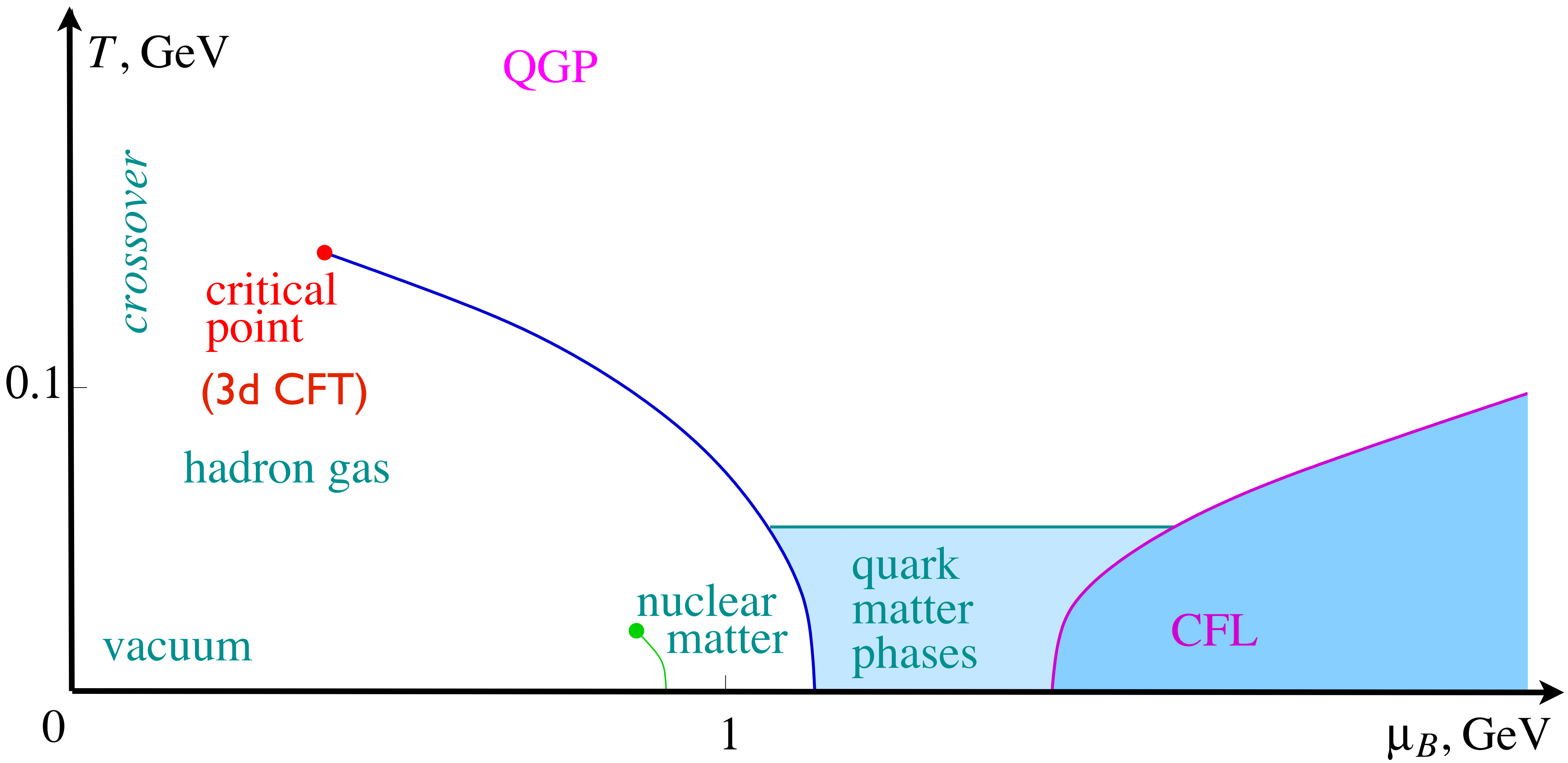


critical point (3d CFT)

Example B: uniaxial ferromagnets



Example C: QCD matter



from a talk by Stephanov'2004

Universality

Critical points fall into large
"universality classes", described by the same CFT.

[Justification: RG theory of phase transitions]

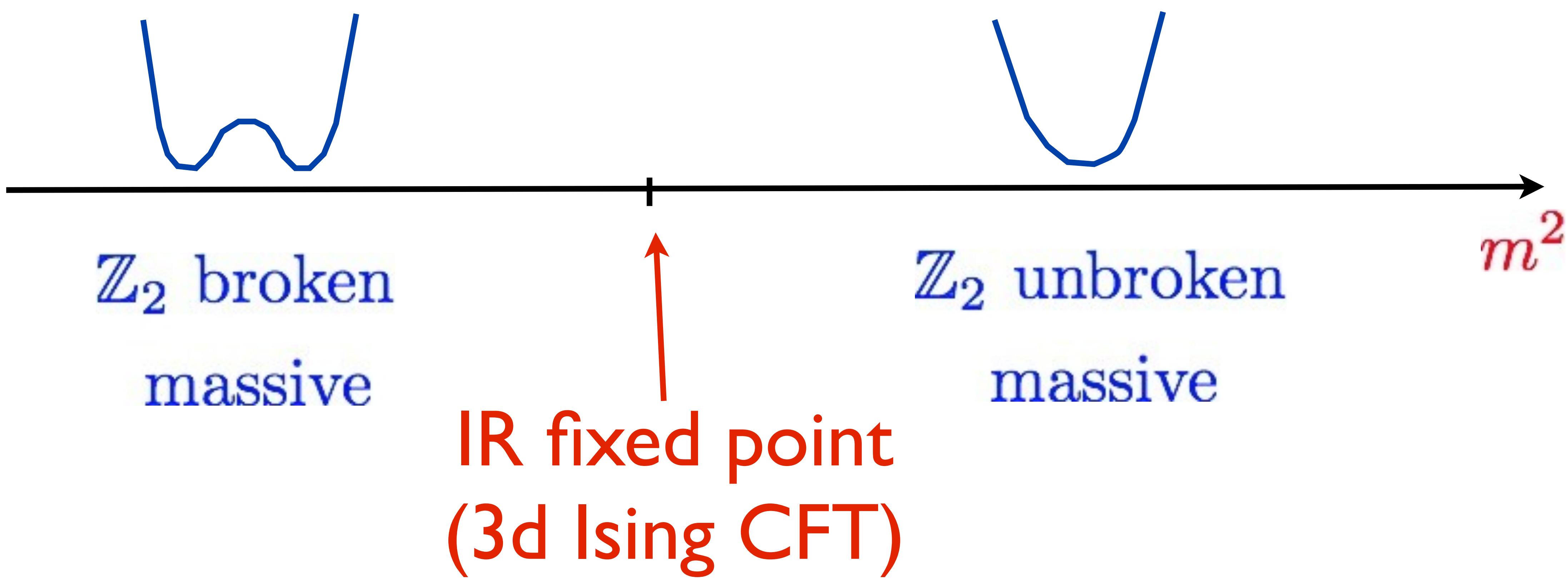
Critical points A,B,C are all described by the same CFT
(have the same operator spectrum and correlation functions)

"3d Ising model CFT"

3d Ising CFT: RG definition

$$S = \int d^d x \left\{ \frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \lambda\phi^4 \right\}$$

3d IR physics is different from 4d,
because [φ]=1/2 and λ is relevant, [λ]=1



What do we need to find-1

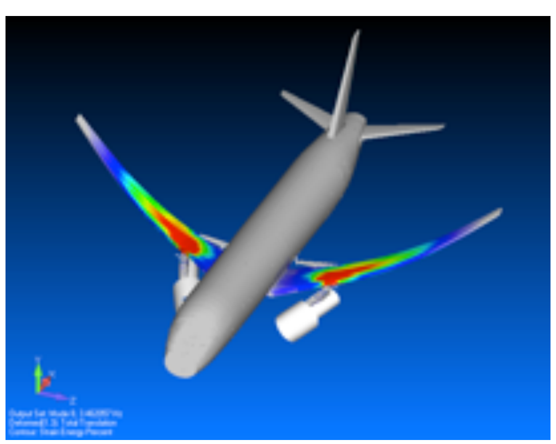
- Scaling dimensions of operators Δ_i

normalization

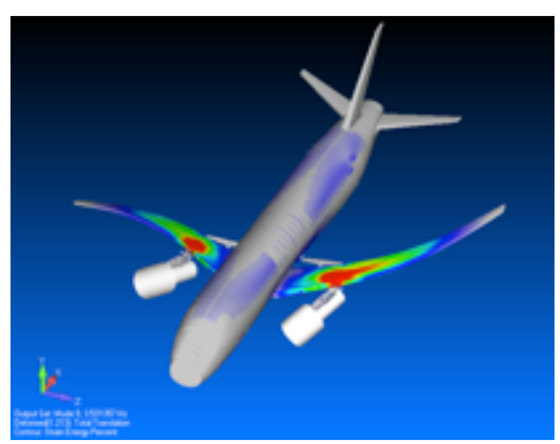
$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$$

there are ∞ many operators of each spin

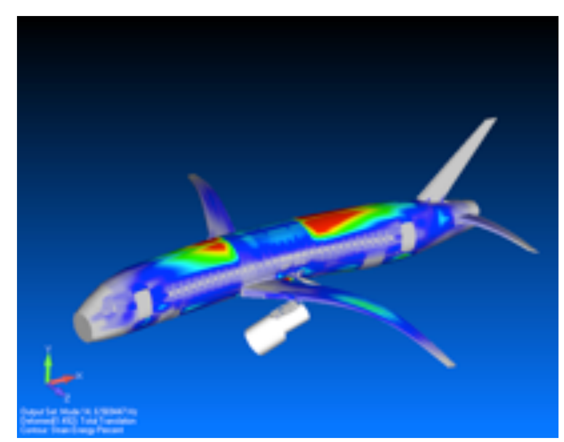
analogy: eigenmodes in mechanics:



ω_1



ω_2



ω_3

What do we need to find-2

- 3-point function couplings f_{ijk}:

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle = \frac{f_{123}}{|x-y|^{h_{123}}|x-z|^{h_{132}}|y-z|^{h_{231}}}$$

$$h_{ijk} = \Delta_i + \Delta_j - \Delta_k$$

Unlike in engineering, eigenmodes interact nonlinearly

Δ_i's and f_{ijk}'s determine everything

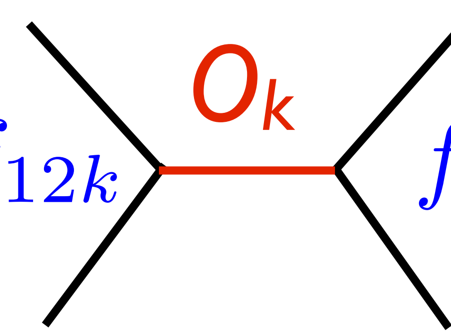
This follows from Operator Product Expansion:

$$O_i(x) \times O_j(0) = \sum \frac{f_{ijk}}{|x|^{\Delta_i+\Delta_j-\Delta_k}} O_k(0) + \text{derivatives}$$

In CFT's, OPE has finite radius of convergence and can be used to reconstruct higher point functions, e.g.:

$$\langle \underbrace{O_1(x_1)O_2(x_2)}\underbrace{O_3(x_3)O_4(x_4)} \rangle = \sum f_{12k} f_{34k} \times (\text{2-point fns})$$

$$= \sum_k \quad f_{12k}$$



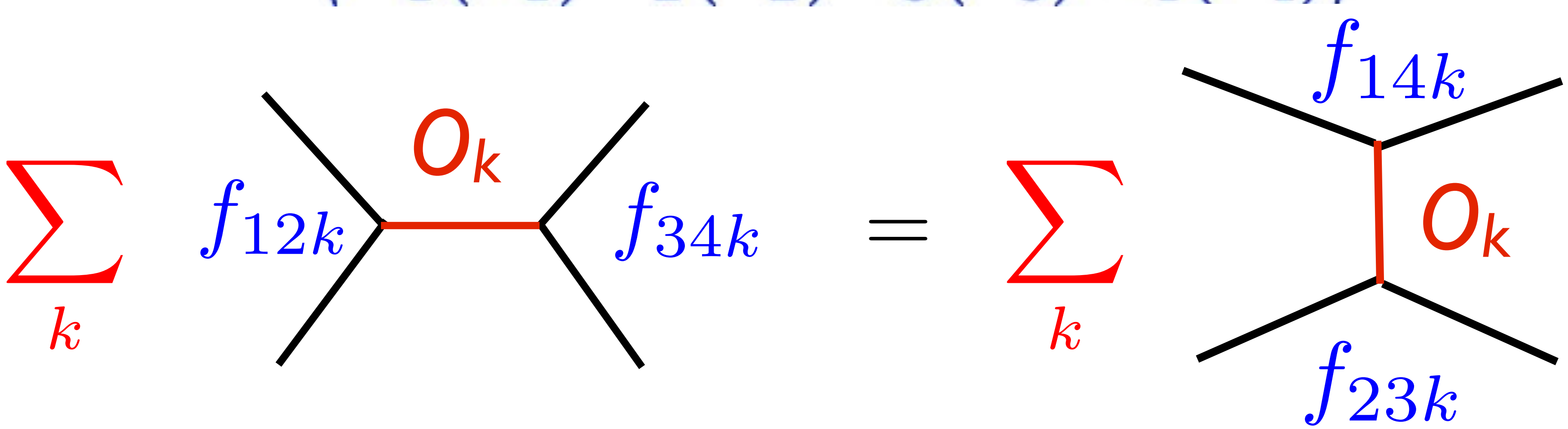
$$f_{34k}$$

Bootstrap idea

Ferrara, Gatto, Grillo 1973; Polyakov 1974

To fix Δ_i's and f_{ijk}'s impose that 4pt functions be unique:

$$\langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4)\rangle$$



Was shown to work in d=2 (Belavin, Polyakov, Zamolodchikov 1984)

Work in d≥3 restarted only recently

(Rattazzi, S.R., Tonni, Vichi 2008)

d=2 vs d≥3

* conformal algebra is infinite dimensional (Virasoro)

* conformal algebra is finite dimensional, SO(d+1,1)

* there are interesting CFTs (e.g. 2d Ising model) with a **finite** # of primary operators of **known** dimension

* any CFT contains **infinitely** many primaries; their dimensions are **unknown**

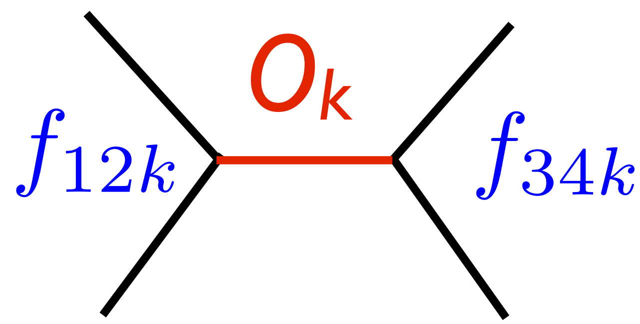
* bootstrap needs to fix f_{ijk}'s only: a problem of **fin-dim** linear algebra

* bootstrap is a problem of **infinite-dimensional** analysis

Exponential decoupling thm

(Pappadopulo, S.R, Espin, Rattazzi 2012)

$$\left| \sum_{\substack{k \\ \Delta_k \geq \Delta_*}} f_{12k} \, f_{34k} \right| \lesssim \exp(-C\Delta_*)$$



(valid for any unitary CFT, and any d)

⇒ makes bootstrap numerically tractable

O(100) papers since 2008

Red = to do

$2<d\leq 4$ and $d=2$ SL(2,C)

- bounds (dims, ope coeffs, central charges)
- extremal spectrum studies
- numerical techniques (simplex method, SDPA, dual/ direct) ellipsoid method minor method by Gliozzi
- global syms
- impact of SUSY
- large N $\leftrightarrow$ AdS
- lightcone results: large spin, small twist numerical impact?
- several correlators
- basis optimization
- external states with spin (T, J)

Conformal blocks

- -exact expressions
- -power series expansions
- -recursions
- -for ops with spin
- -large d limit

CFT_d with bdry

Conformal defects $\leftrightarrow$ d=1 bootstrap $d \rightarrow 1$ limit

[Study of 2d CFT torus partition functions]

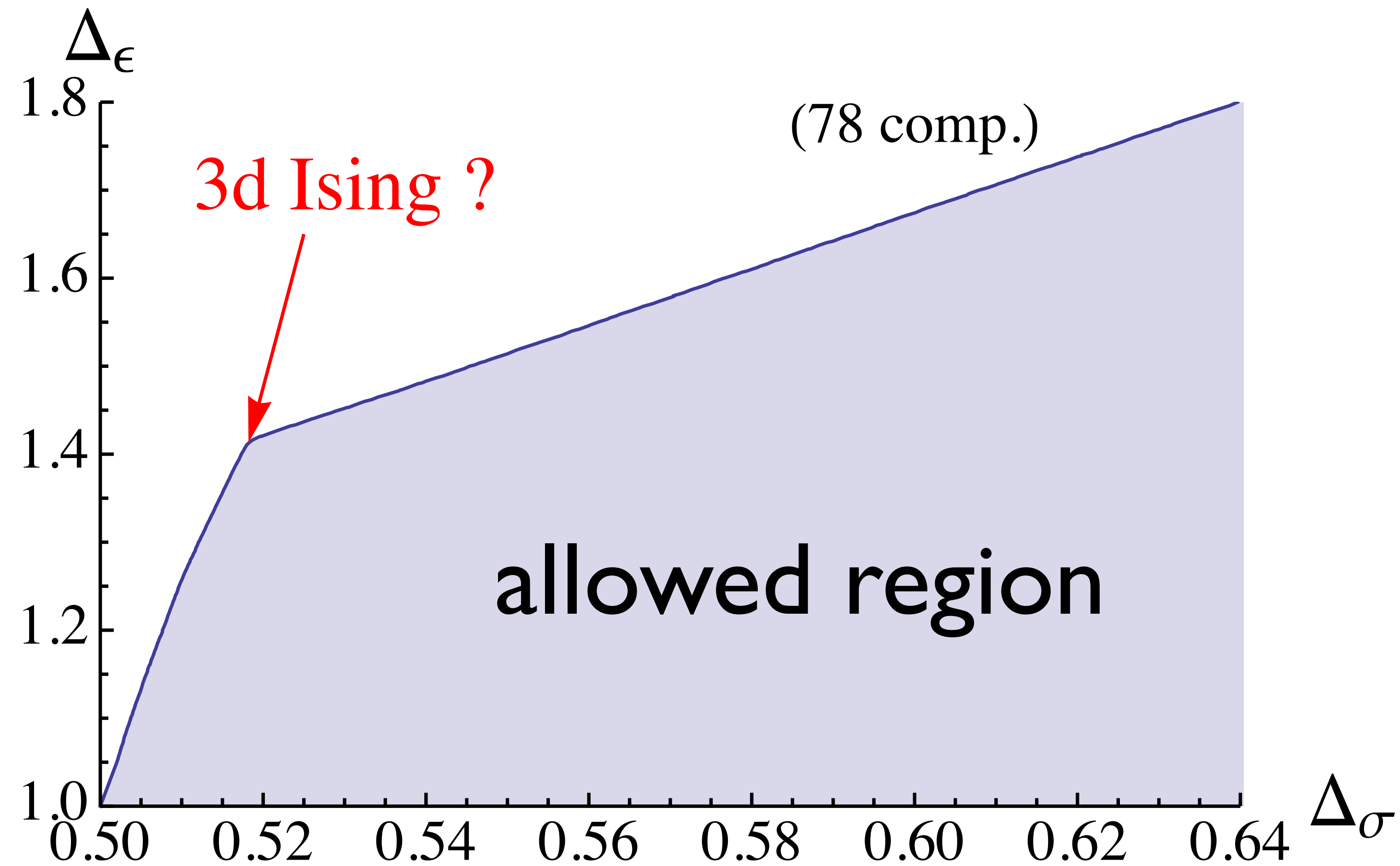
d=2 non-rational Virasoro bootstrap

Bootstrap on other geometries ($R^{d-1} \times S^1$)

Bootstrap analysis for 3d Ising

Two most important operators:
σ (Z_2 odd), ε (Z_2 even)

$\sigma \times \sigma = 1 + \varepsilon + \ldots$



(El-Showk, Paulos, Poland, Simmons-Duffin, S.R., Vichi, 2012)

Ingredients which go into this plot:
- conformal symmetry & unitarity
- bootstrap equation for $\langle\sigma\sigma\sigma\sigma\rangle$

Conjecture: 3d Ising spectrum maximizes scalar gap

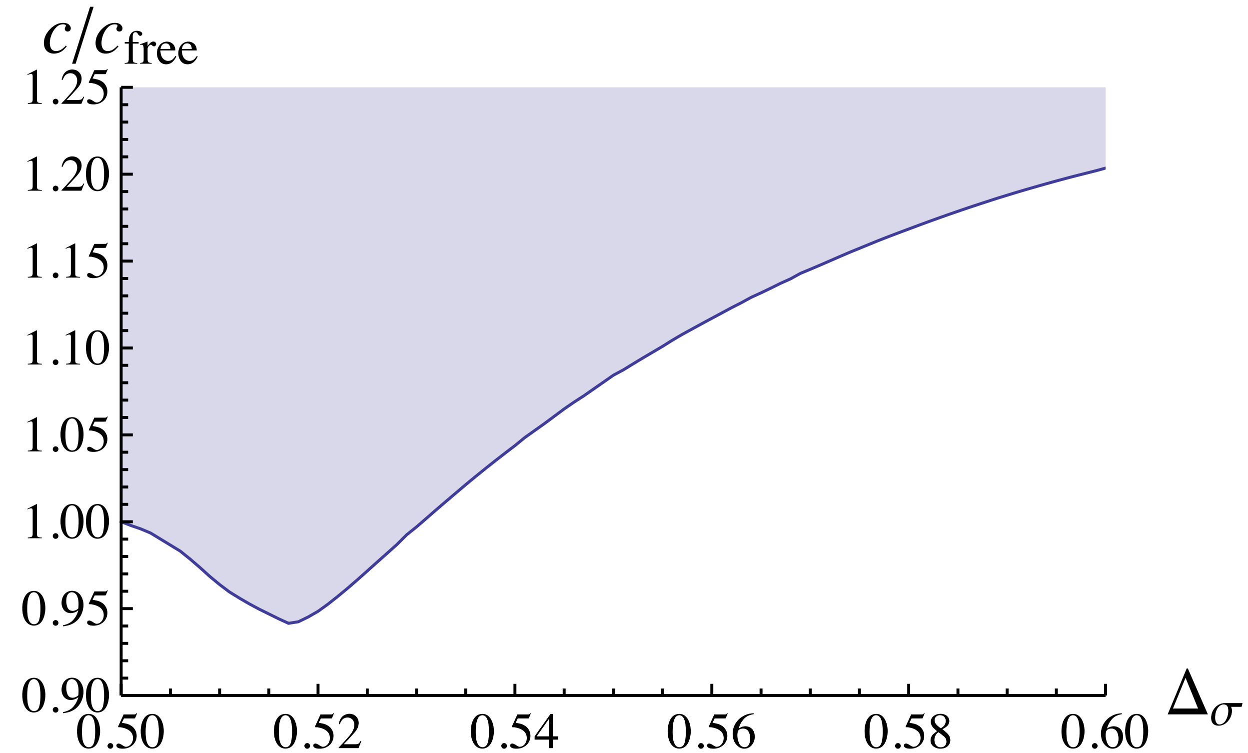
c-Minimization

Another interesting quantity is central charge *c*:

$$\langle T_{\mu\nu}(x) T_{\lambda\sigma}(y) \rangle \propto c$$

$\sigma \times \sigma = 1 + \varepsilon + T_{\mu\nu} + \ldots$

contribution goes as c^{-1}

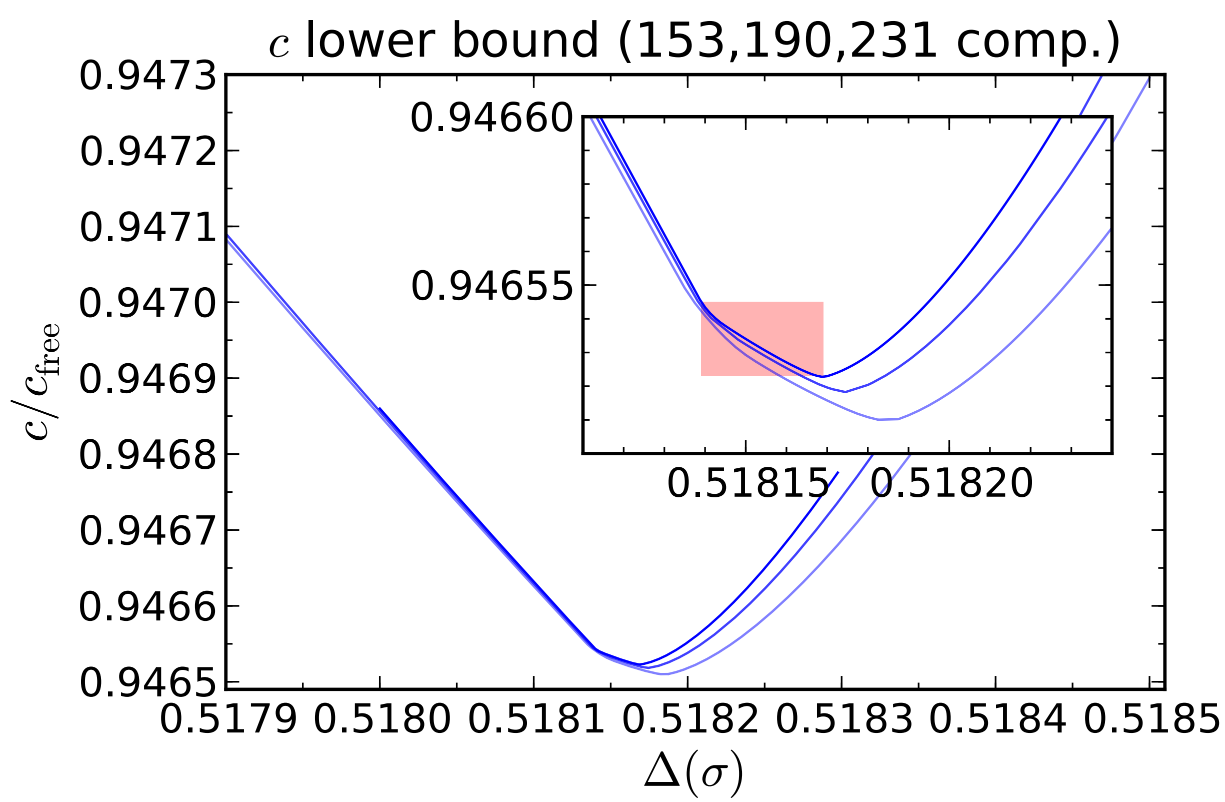


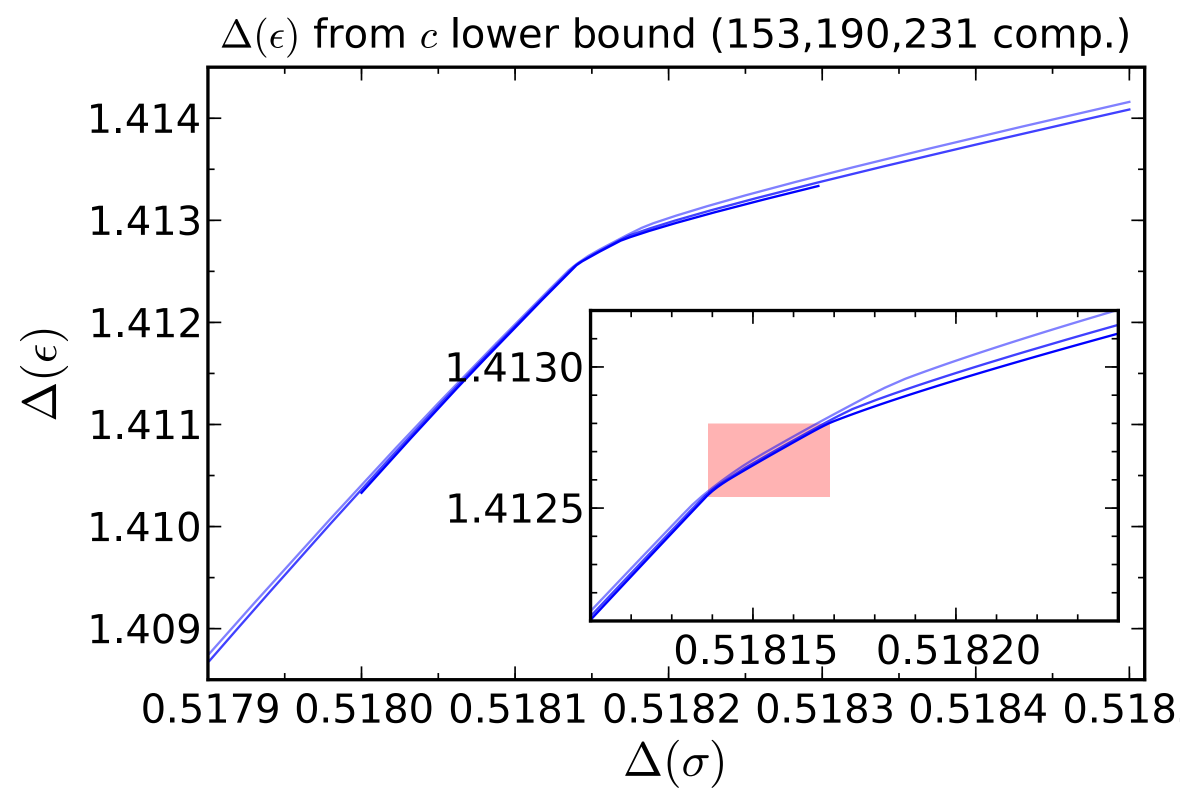
(El-Showk, Paulos, Poland, Simmons-Duffin, S.R., Vichi, 2012)

Conjecture: 3d Ising CFT has minimal central charge

Precision bootstrap

(El-Showk, Paulos, Poland, Simmons-Duffin, S.R., Vichi, 2014)





spin & $\mathbb{Z}_2$	name	Δ	OPE coefficient
$\ell = 0,\ \mathbb{Z}_2 = -$	σ	$0.518154(15)$	
$\ell = 0,\ \mathbb{Z}_2 = +$	ϵ ϵ'	$1.41267(13)$ $3.8303(18)$	$f^2_{\sigma\sigma\epsilon} = 1.10636(9)$ $f^2_{\sigma\sigma\epsilon'} = 0.002810(6)$
$\ell = 2,\ \mathbb{Z}_2 = +$	T T'	3 $5.500(15)$	$c/c_{\text{free}} = 0.946534(11)$ $f^2_{\sigma\sigma T'} = 2.97(2) \times 10^{-4}$

Results by traditional techniques

$\Delta_\sigma, \Delta_\epsilon, \Delta_{\epsilon'} \leftrightarrow \eta, \nu, \omega$

	ref	year	Method	ν	η	ω
Zinn-Justin	[22]	1998	ϵ-exp	0.63050(250)	0.03650(500)	0.814(18)
Zinn-Justin	[22]	1998	3D exp	0.63040(130)	0.03350(250)	0.799(11)
Vicari	[23]	2002	HT	0.63012(16)	0.03639(15)	0.825(50)
Blöte	[24]	2003	MC	0.63020(12)	0.03680(20)	0.821(5)
Hasenbusch	[25]	2010	MC	0.63002(10)	0.03627(10)	0.832(6)
		this work		0.62999(5)	0.03631(3)	0.8303(18)

Numerical resources:

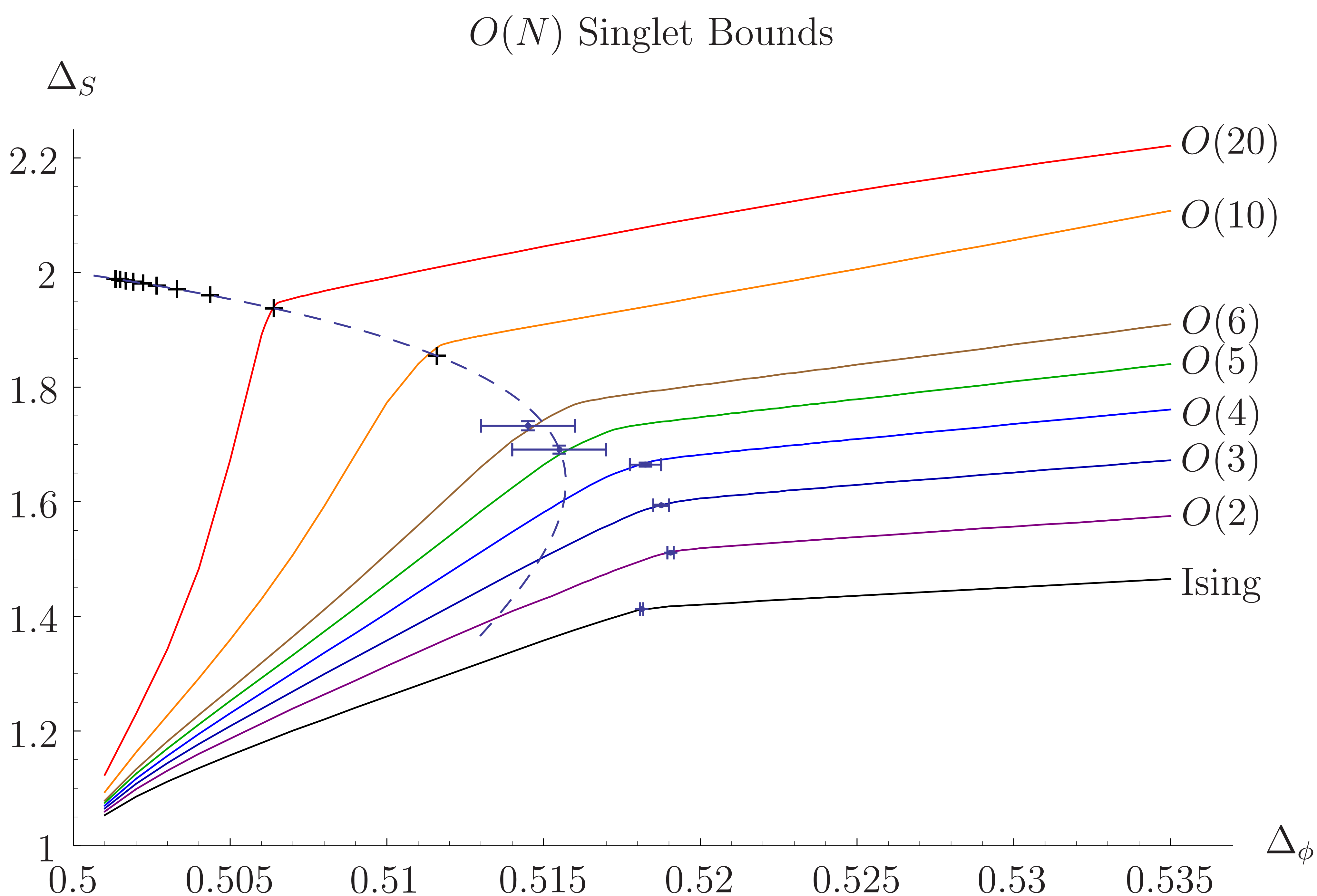
2-3 CPU years

vs 30 CPU years for Hasenbusch

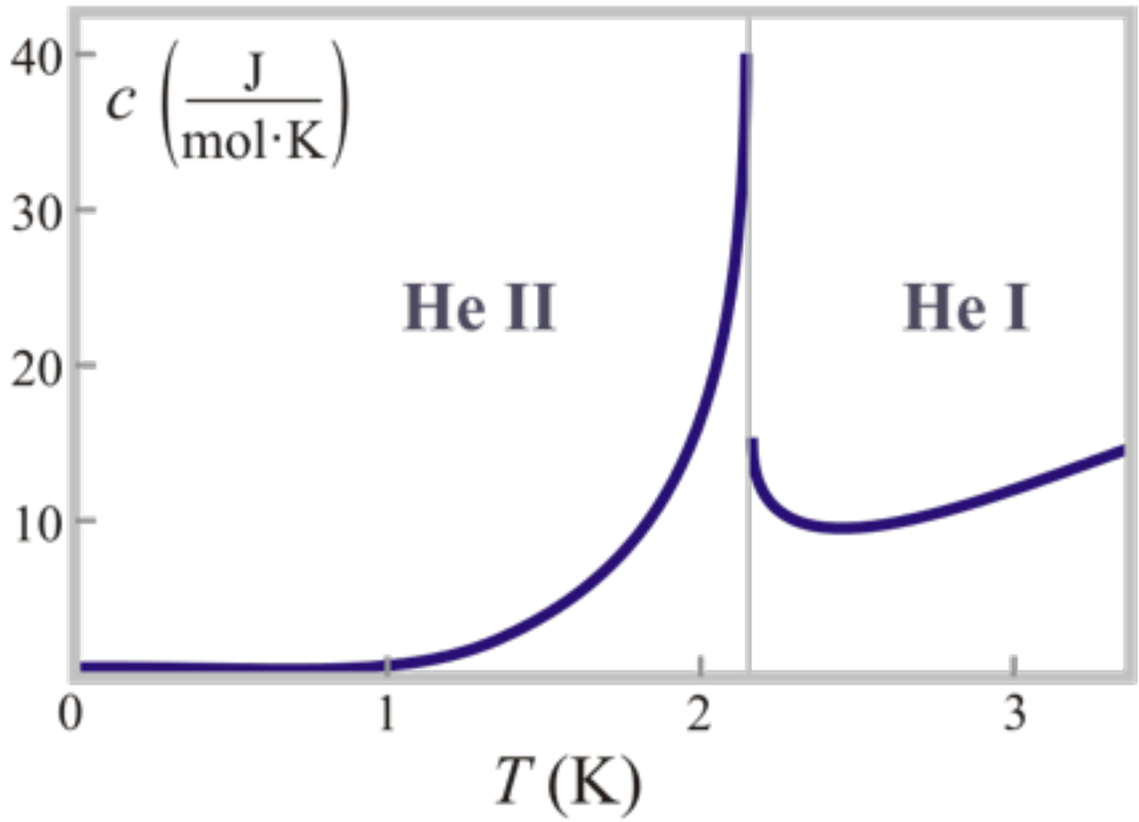
(1000 CPU years would be needed to reach our precision)

O(N) models

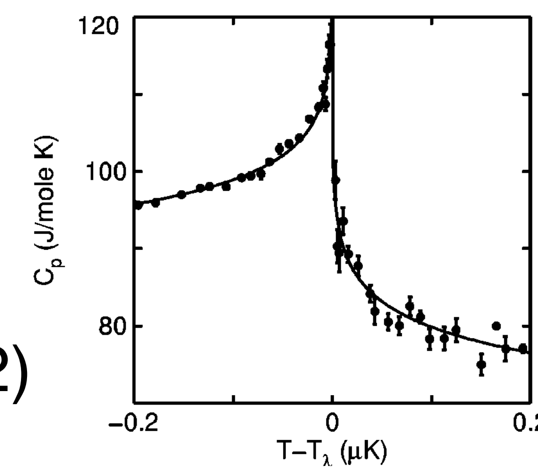
(Kos,Poland,Simmons-Duffin,2013)



O(2) controversy



λ -point of liquid Helium-4



Space shuttle measurement (1992): $\Delta_\varepsilon = 1.5094(2)$

8σ discrepancy

Best lattice prediction(2006): $\Delta_\varepsilon = 1.5112(2)$

Best RG prediction (Guida,Zinn-Justin 1998): $\Delta_\varepsilon = 1.5081(33)$ inconclusive

Bootstrap should be able to settle this

Conclusions

General lesson:
much to learn from conformal bootstrap,
which in $d \geq 3$ was unjustly neglected for 40 years

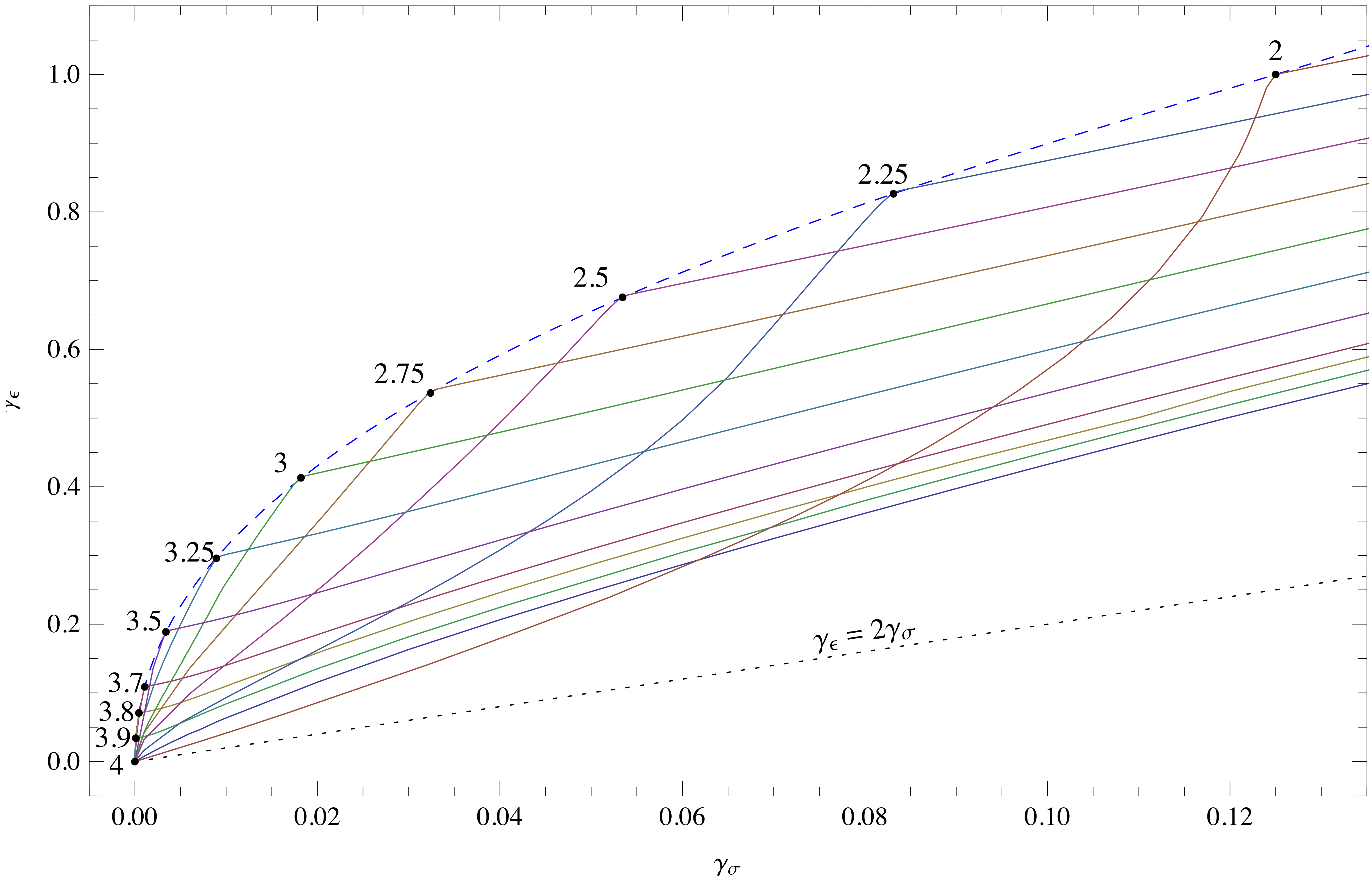
Particular lesson:
some models have extremal properties $\Rightarrow$ more tractable

3d Ising CFT is one of them. Does it mean it's exactly solvable?

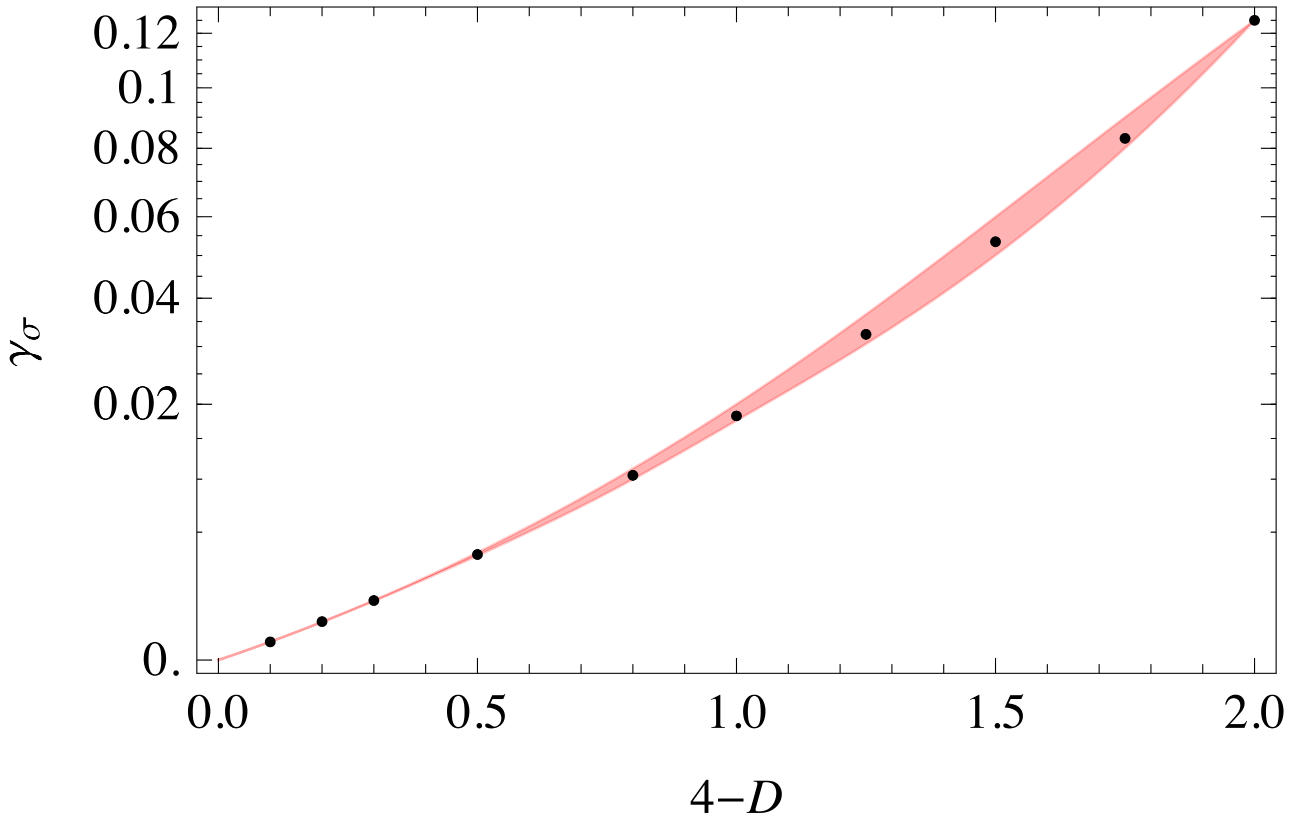
Big picture:
CFTs are important, but RG flows are even more important
They have to be solved as well

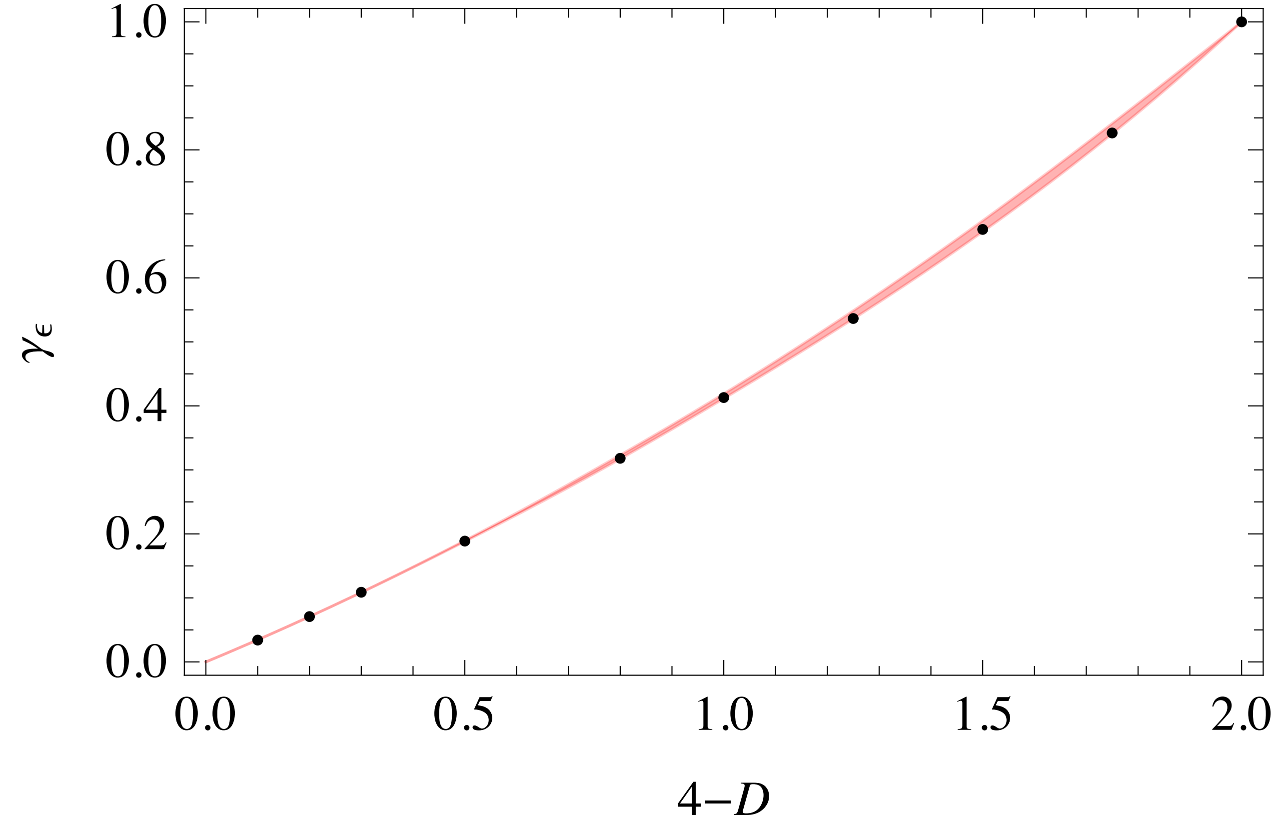
Backup

Fractional d

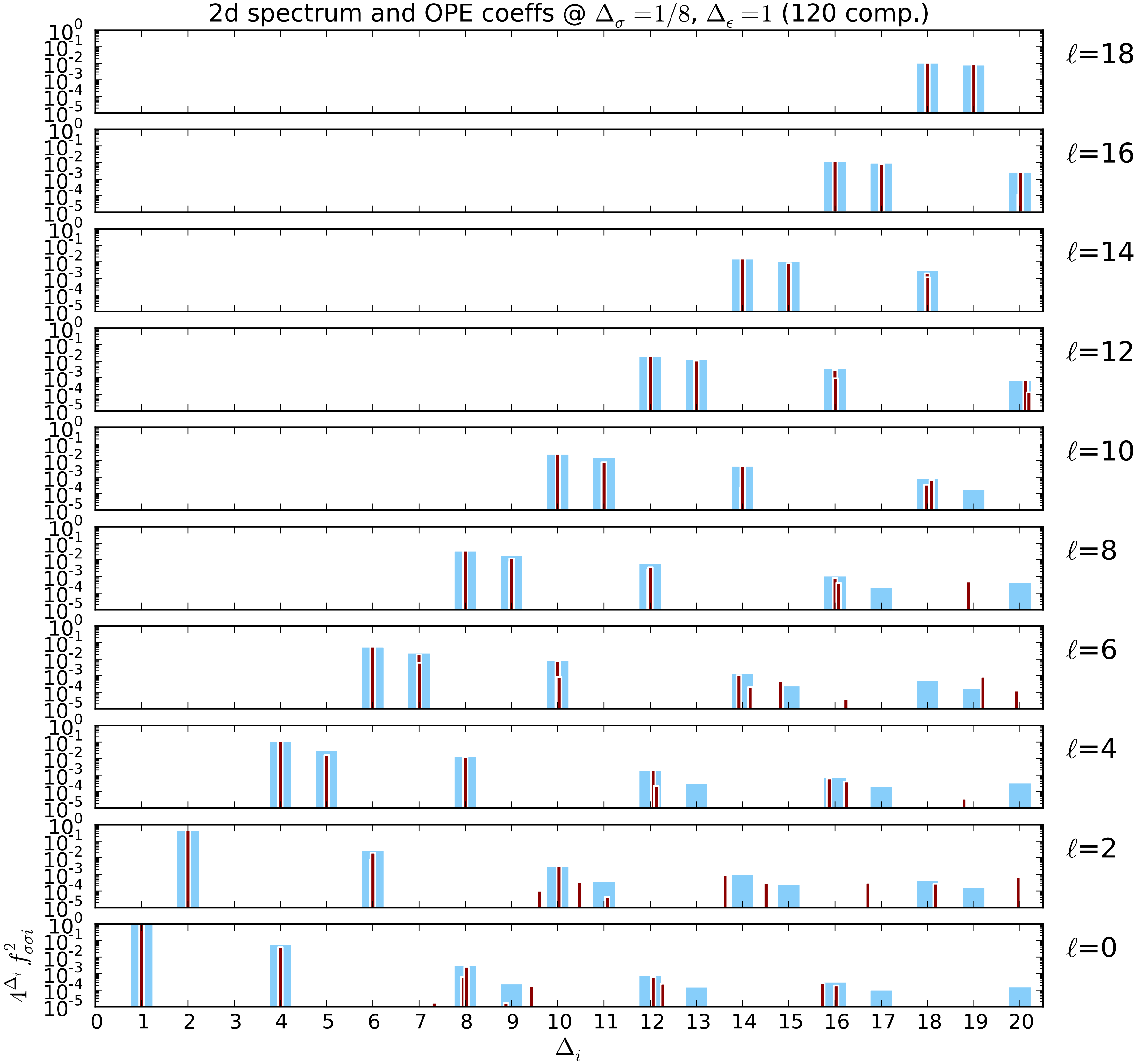


Comparing with 4-ε expansion





2d


2d spectrum and OPE coeffs @ $\Delta_\sigma = 1/8$, $\Delta_\epsilon = 1$ (120 comp.)
ℓ=18
ℓ=16
ℓ=14
ℓ=12
ℓ=10
ℓ=8
ℓ=6
ℓ=4
ℓ=2
ℓ=0
$4^{\Delta_i} f^2_{\sigma\sigma i}$
Δ_i
10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5}
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

2d