Emittances and Cooling
Lecture I: Revue of Cooling Methods

MICE Collaboration Meeting
02/25/14

R. B. Palmer (BNL)

- Introduction
- Schemes
  - Electron
  - Laser
  - Stochastic
  - Optical Stochastic
  - Coherent Electron
  - Synchrotron
  - Ionization
- Summary

Emittance Definitions

In the Muon world we usually use rms normalized emittances

$$\epsilon = \beta\gamma \frac{\text{rms Phase Space Area}}{\pi} \tag{1}$$

For transverse emittances the area is: θ_x vs x, or θ_y vs y. In longitudinal: it is dp/p vs z, or E vs. t.

If x and p_x are both Gaussian and uncorrelated then

$$\epsilon_{x,y} = (\gamma\beta_v)\sigma_{\theta_{x,y}}\sigma_{x,y} \qquad (m\ rad) \tag{2}$$

$$\epsilon_z == (\gamma\beta_v)\frac{\sigma_{p_z}}{p}\,\sigma_z \qquad (m\ rad) \tag{3}$$

When xy symmetric we often write $\epsilon_\perp$ for $\epsilon_x = \epsilon_y$, and $\epsilon_\parallel$ for ϵ_z

There are many other ϵ definitions

The above 'normalized' emittances are conserved by simple acceleration. 'Un-normalized' or 'geometric' emittances without the $\beta\gamma$ in $\epsilon \ = \ (\gamma\beta_v)\sigma_{\theta_{x,y}}\sigma_{x,y}$, fall with acceleration

$$\epsilon_{x,y}(geometric) \ = \ \sigma_{\theta_{x,y}}\sigma_{x,y}$$

95% emittances, usually un-normalized, are widely used for protons, using areas that contain approximately 95% of Gaussian distributions and have the transverse values:

$$\epsilon_{x,y}(95\%) \ = \ 6 \ \sigma_{\theta_{x,y}}\sigma_{x,y}$$

Longitudinal emittances are often given with the dimensions Energy $\times$ time

$$\epsilon_z(Energy, time) \ = \ \sigma_E \ \sigma_t$$

And which convention is being used, is not always clear

Cooling

For many applications it is desirable to reduce the emittance of a beam, and many ways have been discovered, used, or planned to be used (like our ionization cooling). An incomplete list:

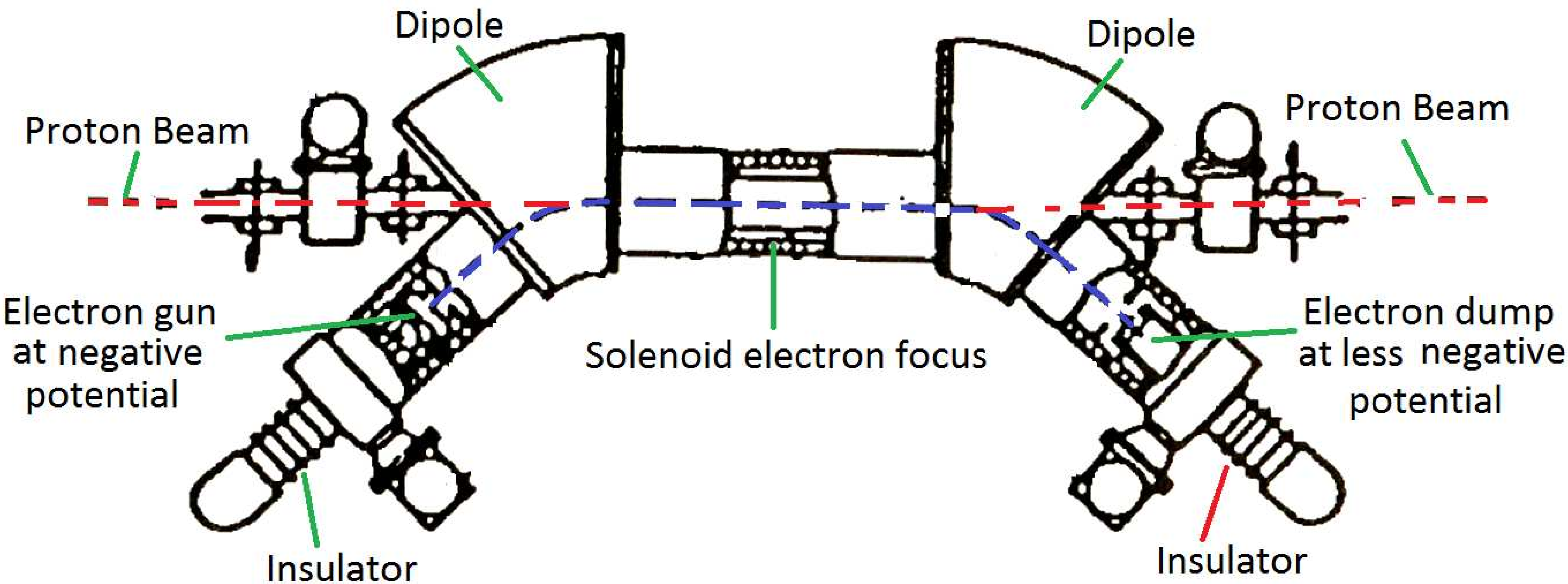
- One (electron cooling) that uses interaction with a colder beam
- One (Laser) that uses interactions with a cold laser
- Three (rf Stochastic, Optical Stochastic, and Coherent Electron) that use a Maxwell Demons
- And two (synchrotron radiation and ionization cooling) that rely on energy loss

In this lecture I will describe conceptually how they work. Tomorrow we will look at ionization cooling in more detail

For more details and references "try 'Handbook of Accelerator Physics and Engineering'; Chao, Mess, Tigner, Zimmermann.

Electron Cooling (G. I. Budker 1967)

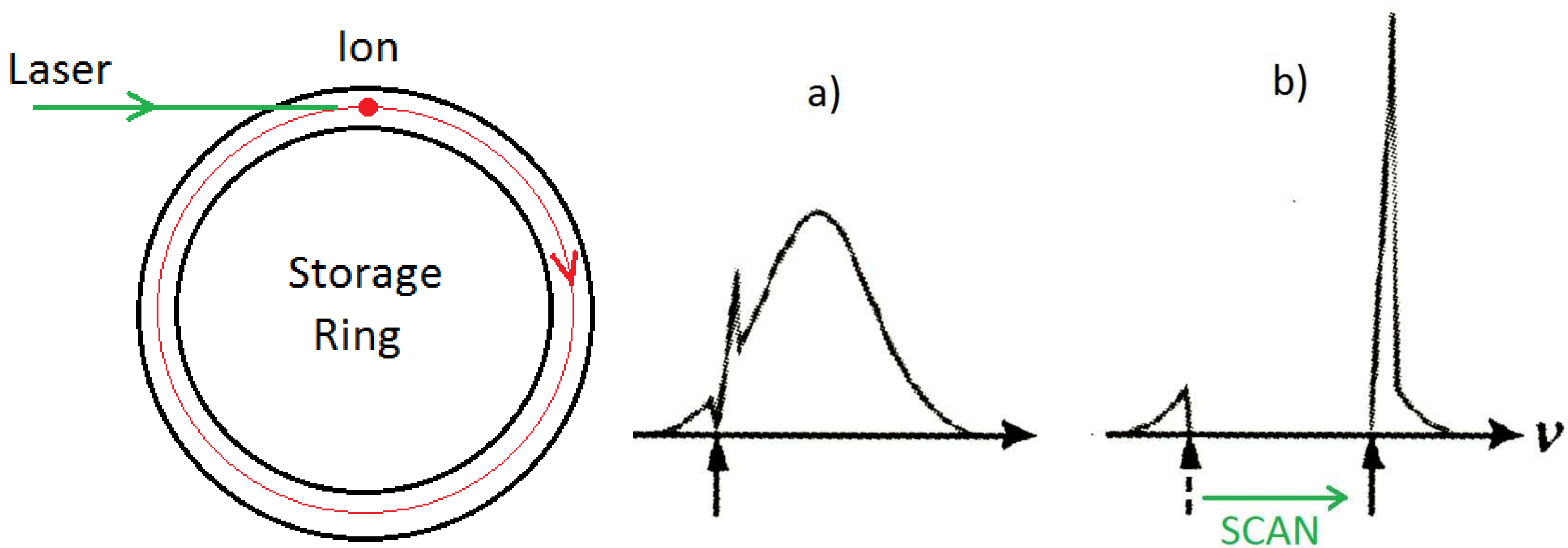
Take a bowl of hot water and add cold marbles: the water is cooled. Take a high emittance proton/ion beam and pass it down a transport along with a cold electron beam traveling at the same velocity, and the proton/anti-proton/ion beam will be cooled. The interactions between them is Coulomb scattering.



Laser Ion Cooling T. Haensch 1975

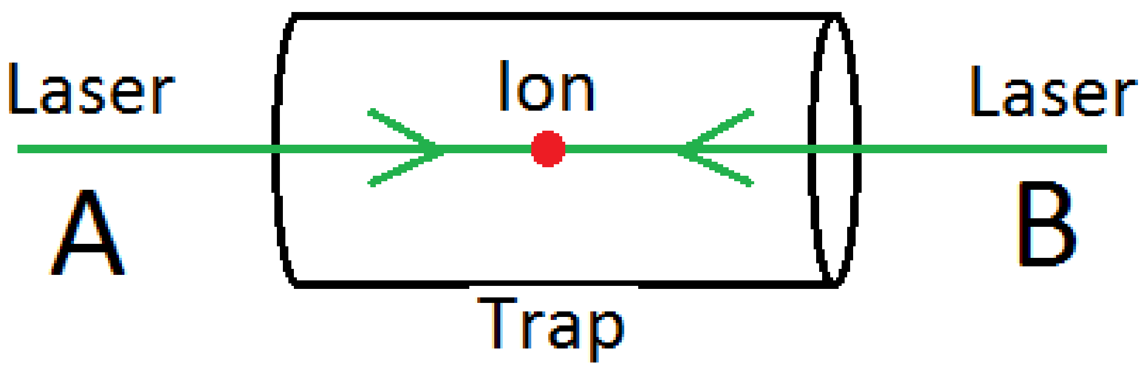
a) e.g. with one laser in a Ring

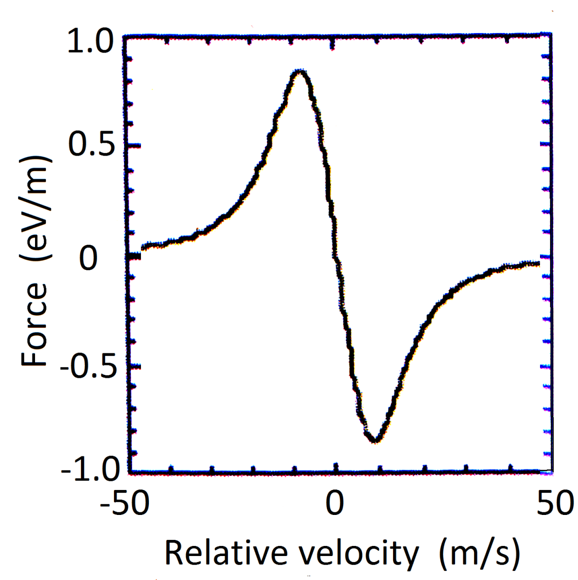
- At just the right, Doppler corrected (thus ion velocity dependent) laser frequency: a laser photon is absorbed, the ion raised to a higher unstable state, and the ion receives a forward kick.
- When the excitation spontaneously decays, photons are emitted isotropically, leaving, the forward kick.
- If the laser frequency is scanned, the velocity distribution of velocities can be 'bull-dozed' into a single narrower spectrum



a) e.g. with two lasers in a trap

- Two opposed lasers set just below the peak excitation frequency
- If the ion is stationary, forces ar balanced
- If moving towards laser A, its observed frequency from A is increased, frequency from B decreased
- force from A increases, that from B decreases
- Sum of forces are to right and velocity corrected
- And vice versa

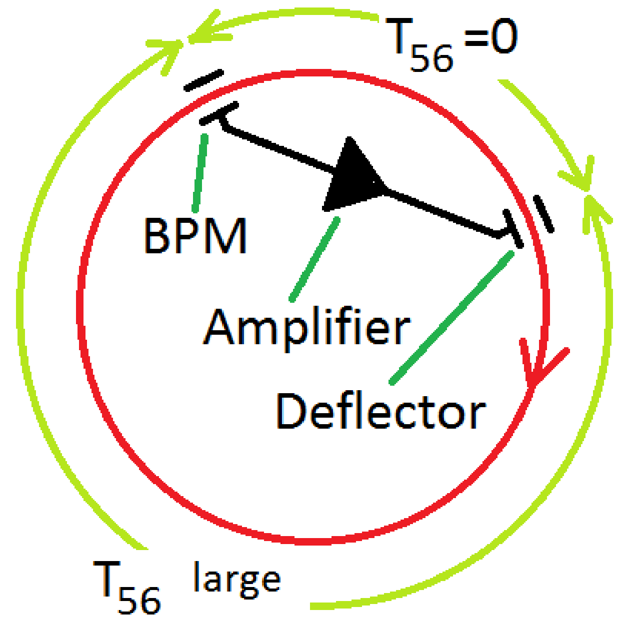




Sensitive to just a few m/sec   Cools to very low temperatures

Transverse rf Stochastic Cooling (S. Van de Meer 1972)

If, particle by particle, we could determine a transverse error, and then apply a deflection field to correct it, the beam will be instantly cooled.



Is this the unphysical "Maxwell Demon"? Yes/No, because a beam emittance is not a thermal temperature. It only shares some properties of one.

What is "Stochastic" about this ?

The band-width needed to determine the momentum of each particle in a useful beam, is way too high. The best we can do is measure an error of a selected set of N out of a total N_T, determined by a bandwidth W (s^{-1}). The greater W, the smaller the subset N.

And if we correct that subset's displacement, then we reduce the emittance of that subset by a small fraction $d\epsilon/\epsilon$.

If for each turn that subset is different (good mixing), then the emittance will be reduced again and again.

$$\frac{d\epsilon}{\epsilon}(\mathrm{per\ turn}) = \frac{1}{N} \qquad N = \frac{N_T\ \beta_v c}{W\ S} \qquad t(\mathrm{per\ turn}) = \frac{S}{\beta_v\ c}$$

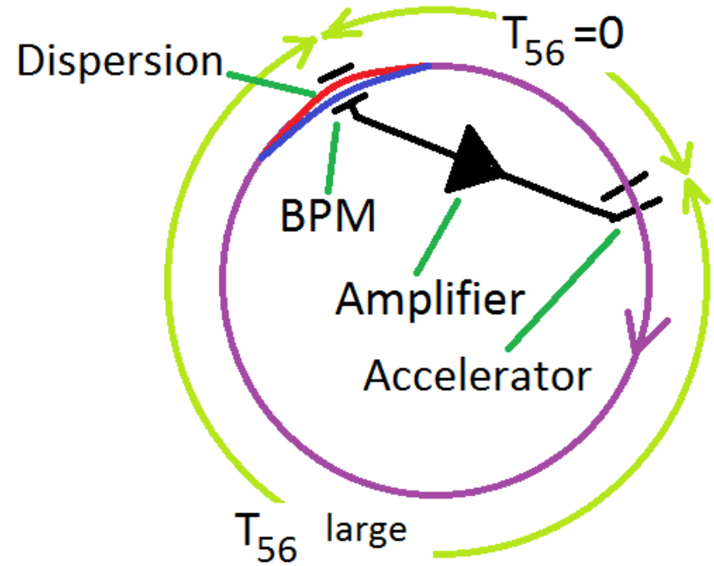
where S is the ring circumference; giving a cooling rate:

$$\frac{d\epsilon/\epsilon}{dt} = \frac{W}{N_T}$$

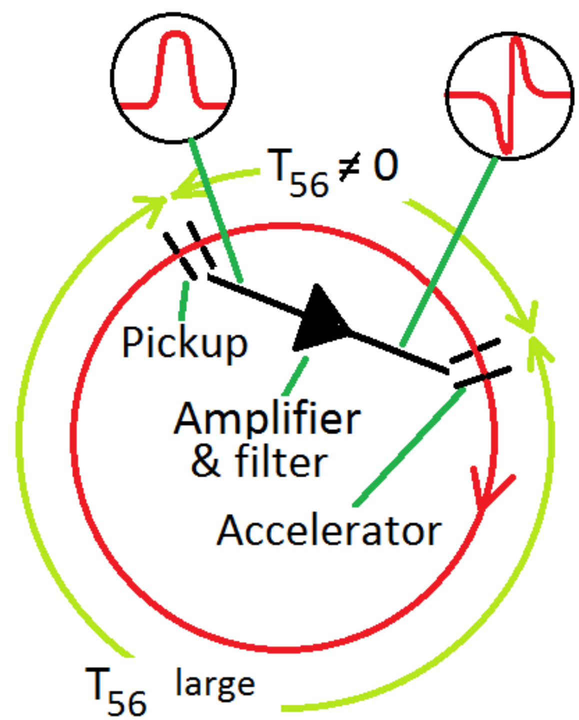
Longitudinal Cooling

Two methods:

The Palmer method uses a transverse pickup in a region of dispersion and an accelerator gap for energy correction.

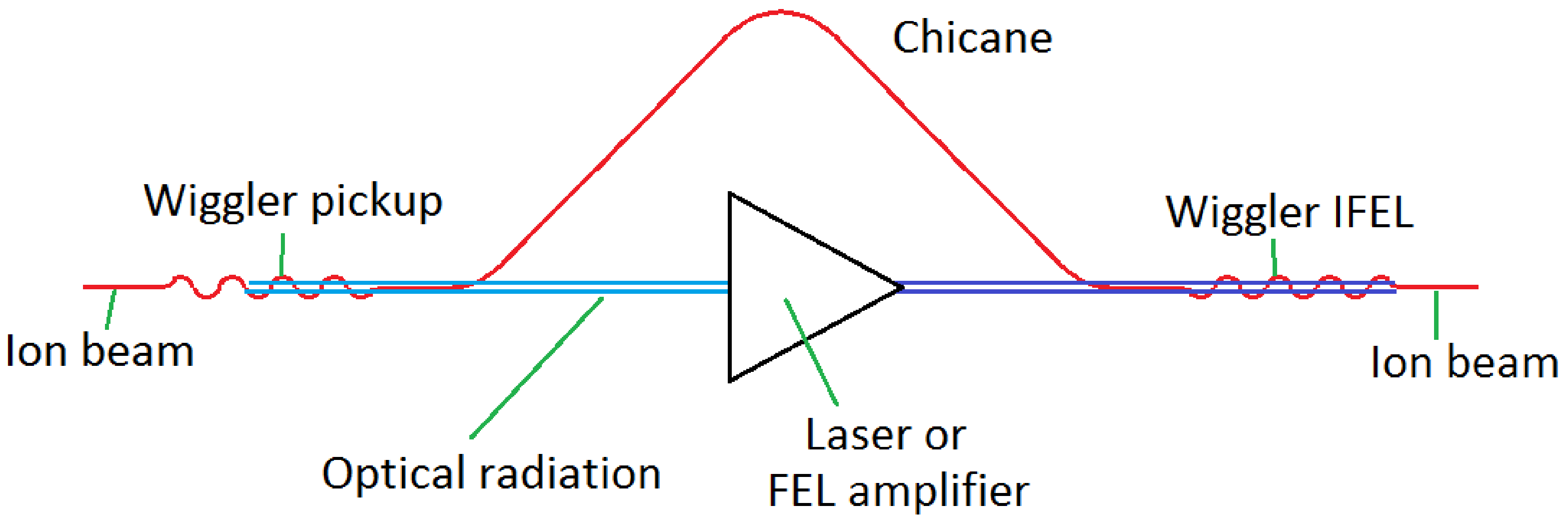


The Thorndahl method is much more elegant: a simple Schottky noise pickup's signal is differentiated and fed to the accelerator placed where the time of arrival depends on the particles energy.



Optical Stochastic Cooling
(M. S. Mikhailichenko, M. S. Zolotorev 1993

Conceptually, this is Thorndahl longitudinal cooling, but the pickup now is a magnetic wiggler, the signal is optical light, the amplifier is a laser, and the kicker is a Free Electron Laser.



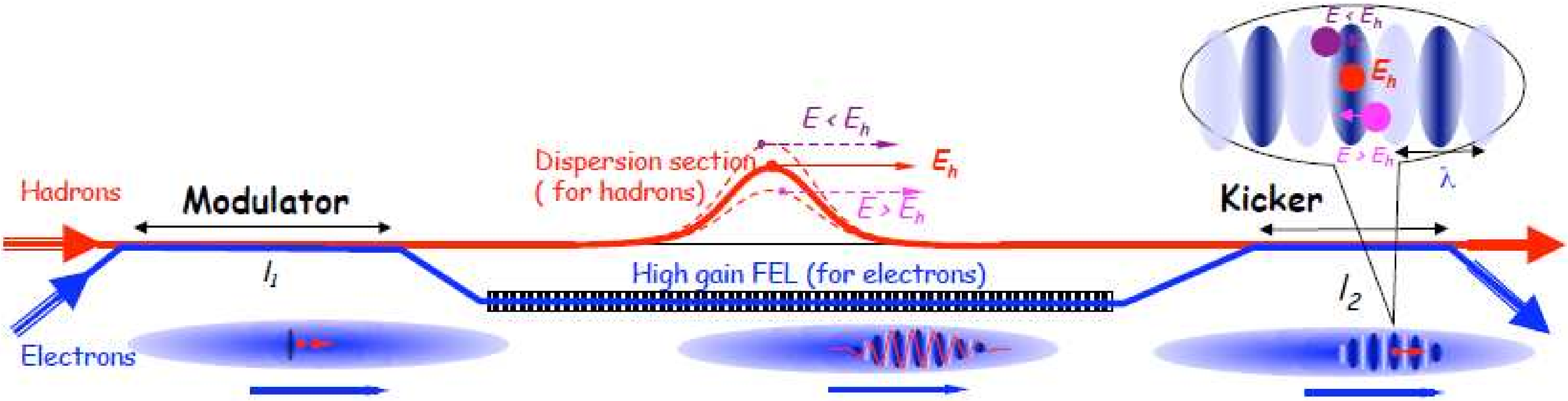
Transverse cooling is also possible with appropriate dispersions.

The bandwidth of a laser is many orders of magnitude higher than an rf amplifier, so N is smaller and the cooling faster, but it appears not fast enough for muons.

Not yet demonstrated, but planed experiment at Fermi Lab

Coherent Electron Cooling CEC (V. Litvinenko)

Coherent Electron Cooling, like Optical Stochastic Cooling, should have a huge bandwidth, but the signal now, instead of an electro-magnetic wave, is the temporal makeup of an electron beam.

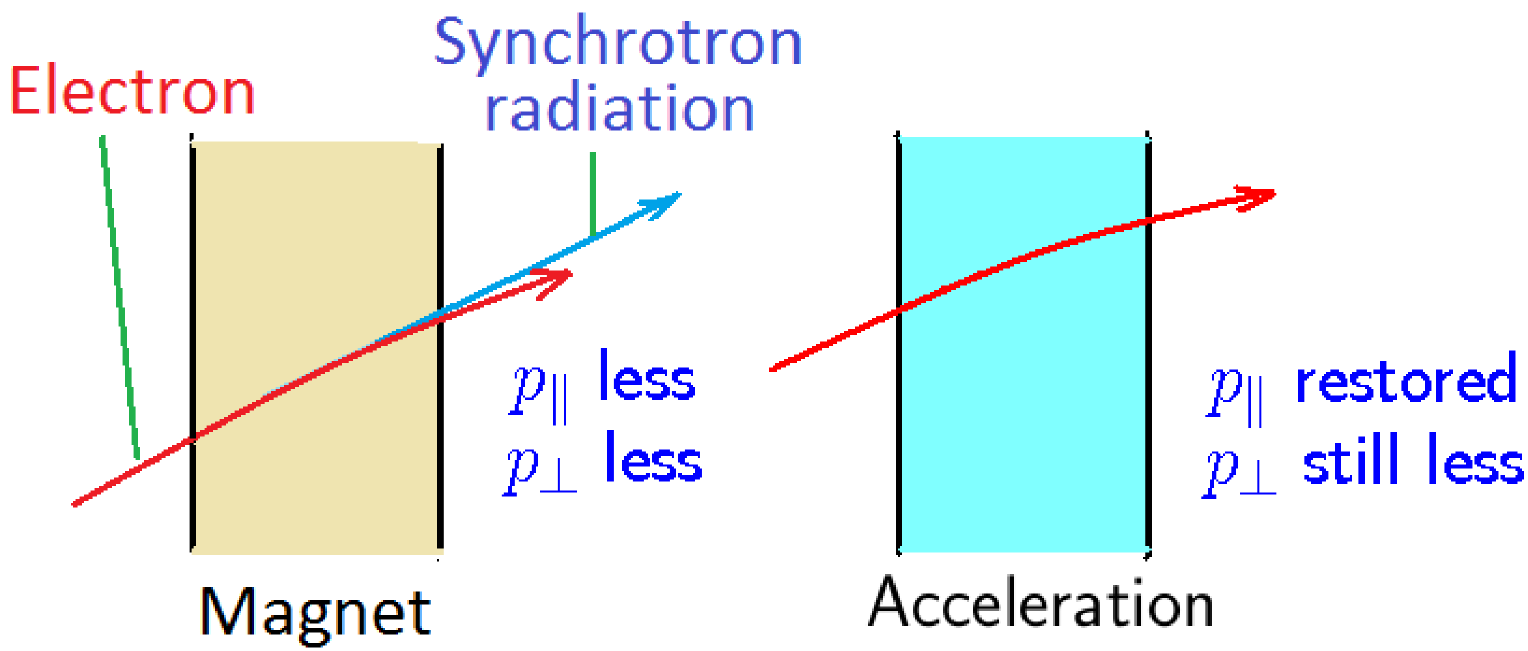


Both the pickup and the corrector are by electrostatic interactions between the electron beam and the ion beam being cooled. As in 'Electron Cooling' the velocities of ions and the electron beam must be the same. But these velocities do not have to be high, as in Optical Stochastic Cooling, since it does not require synchrotron radiation.

Not yet demonstrated. Experiment is planned at BNL's RHIC

Transverse Synchrotron Cooling (Damping)

A particle loses energy, and thus momentum, by synchrotron radiation in a magnetic field. If the particle has a transverse component, then that too is reduced. Subsequent rf acceleration restores the longitudinal component, but leaves the reduction in the transverse component.



The minimum emittance achieved is set by quantum fluctuations in the amount of radiation emitted

Transverse Partition Functions

From the definition

$$\epsilon \;=\; \beta\gamma\, \sigma_\theta\, \sigma_x \;=\; \frac{\sigma_{p_\perp}\sigma_x}{mc}$$

Since the emitting radiation does not change the beam size σ_x:

$$\frac{\Delta\epsilon}{\epsilon} \;=\; \frac{\Delta p_\perp}{p_\perp} \;=\; \frac{\Delta p}{p}$$

Defining (we will see why later)

$$J_x \;=\; \frac{\Delta\epsilon/\epsilon}{\Delta p/p}$$

We get

$$J_x \;=\; 1$$

Longitudinal Partition Functions

Longitudinal cooling arises naturally because the synchrotron energy loss is proportional to γ^2
From the definition

$$\epsilon_z \ = \ \beta\gamma \ \sigma_p/p \ \sigma_z \ = \ \frac{\sigma_E \sigma_t}{mc} \tag{4}$$

Since the radiation does not change a particles time t:

$$\frac{\Delta\epsilon_z}{\epsilon_z} \ = \ \frac{\Delta\sigma_E}{\Delta_E}$$

and if

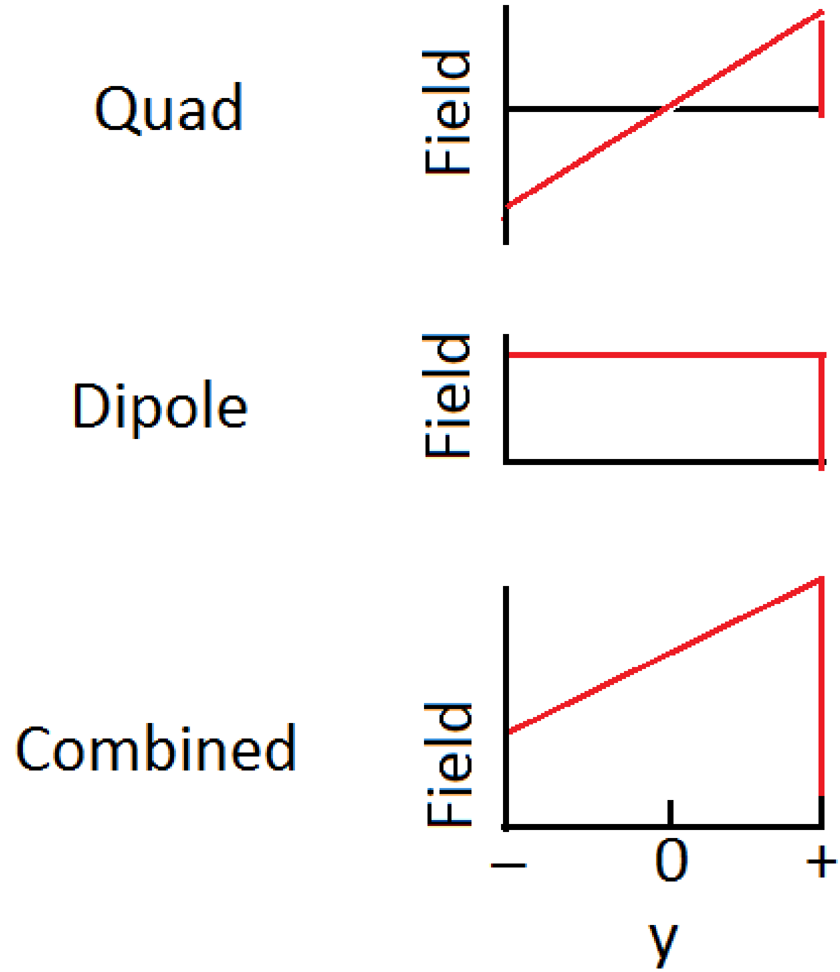
$$J_z \ = \ \frac{\Delta\epsilon_z/\epsilon_z}{\Delta E/E} \tag{5}$$

since the energy loss $\propto \ E^2$, we get

$$J_z \ = \ 2 \tag{6}$$

$$\text{and} \qquad J_6(\text{synchrotron}) \ = \ J_x \ + \ J_y \ + \ J_z \ = \ 4 \tag{7}$$

Can one increase longitudinal



- One can increase the longitudinal cooling using "combined function" magnets
- Plus dispersion so higher momentum particles have higher y
- The fields are higher on that side, causing higher radiation
- Reducing the energy spread further and thus increasing J_z

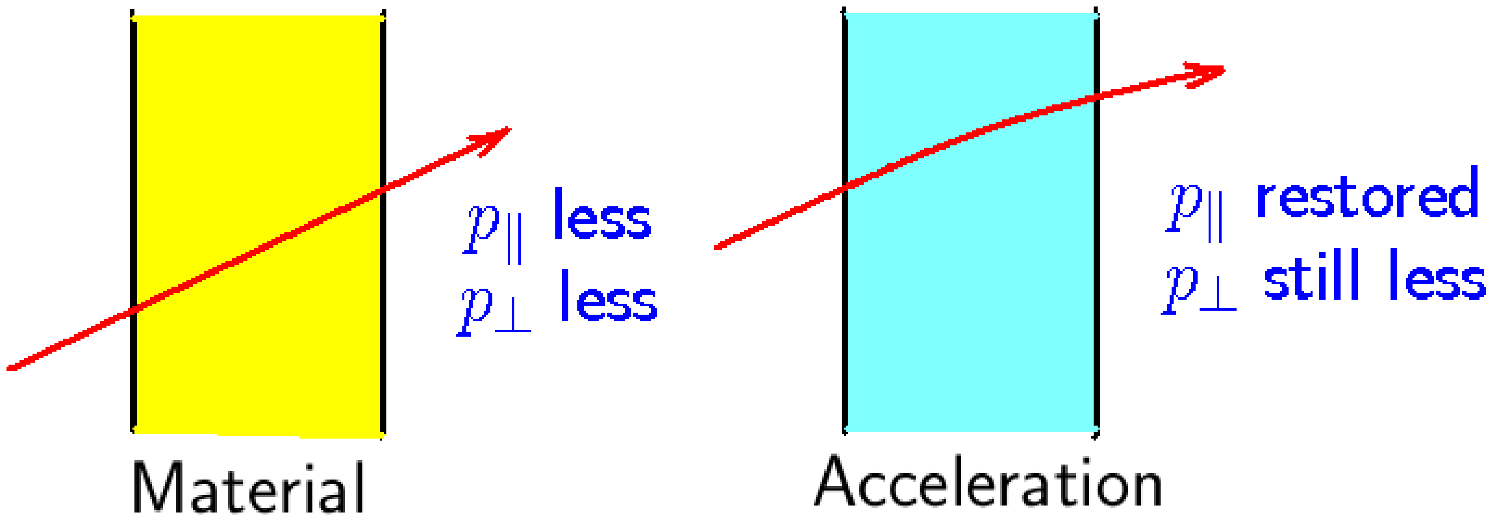
This works, but always increases J_x, or J_y so that

$$J_6 = J_x + J_y + J_z = 4 \tag{8}$$

is maintained

Transverse Ionization Cooling

As in Radiation Cooling, a particle loses energy, but instead of by synchrotron radiation, it is by ionization loss passing through material. The logic is the same: If the particle has a transverse component, then that is reduced. Subsequent rf acceleration restores the longitudinal component, but leaves the reduction in the transverse component.



The minimum emittance achieved is now set by Coulomb scattering in the material, and this we will address in more detail later.

Transverse Partition Functions

As for Radiation Cooling and the definition

$$\epsilon \;=\; \beta\gamma\ \sigma_\theta\ \sigma_x \;=\; \frac{\sigma_{p_\perp}\sigma_x}{mc}$$

And since the ionization does not change the beam size σ_x:

$$\frac{\Delta\epsilon}{\epsilon} \;=\; \frac{\Delta p_\perp}{p_\perp} \;=\; \frac{\Delta p}{p}$$

Defining

$$J_x \;=\; \frac{\Delta\epsilon/\epsilon}{\Delta p/p}$$

As for synchrotron cooling, we get

$$J_x \;=\; J_y \;=\; 1 \tag{9}$$

Longitudinal Ionization Cooling

Again:

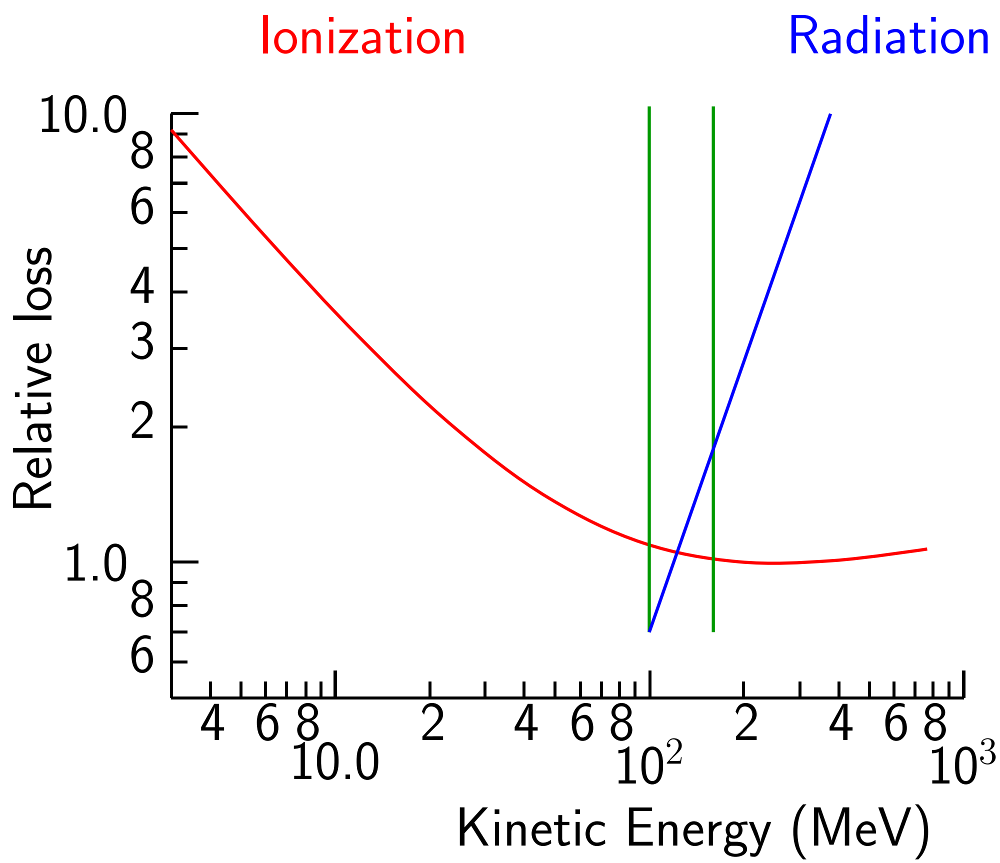
$$\epsilon_z = \beta\gamma \frac{\sigma_p}{p} \sigma_z = \frac{\sigma_E \sigma_t}{mc}$$

And ionization does not change time, so:

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_E}{\Delta_E}$$

and if $$J_z = \frac{\Delta\epsilon_z/\epsilon_z}{\Delta E/E}$$

$$J_z = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta E}{E}} = \frac{\left(\frac{d(d\gamma/ds)}{d\gamma/\gamma}\right)}{\left(\frac{d\gamma}{ds}\right)}$$

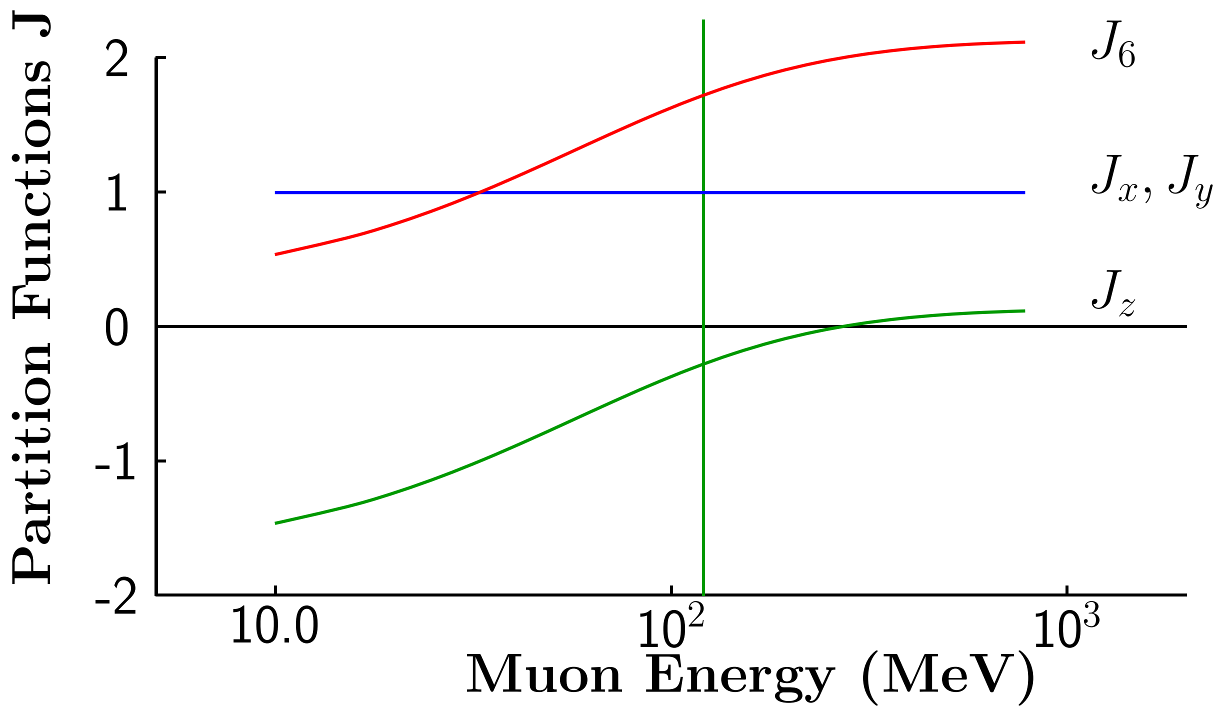


Unlike synchrotron radiation, ioization is more at low energies

Longitudinal Partition Function

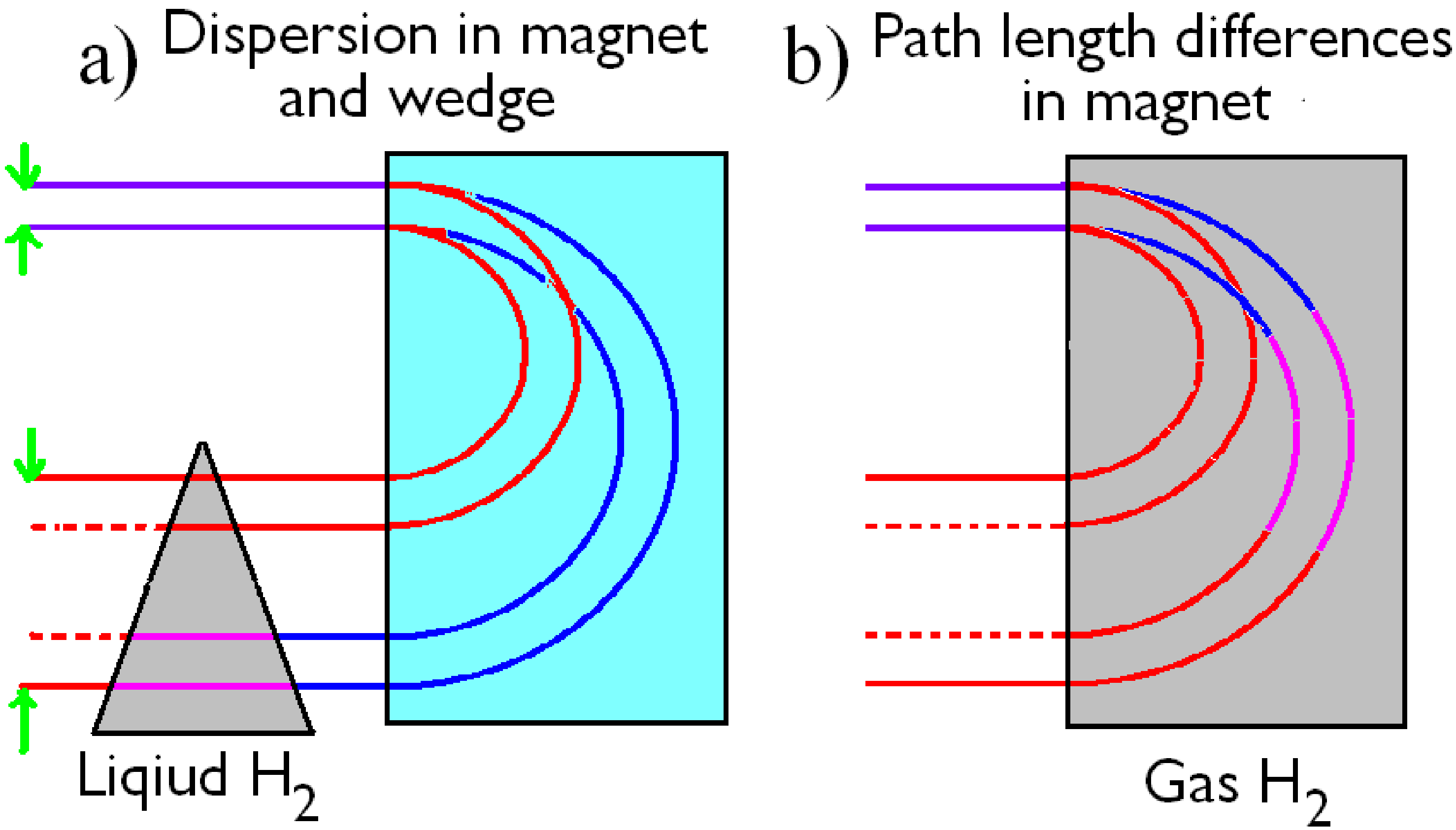
J_z is strongly negative at low energies (longitudinal heating), and barely positive at energies above 300 MeV/c. J_z is now energy dependent. In practice we cool at $\approx$ 130 MeV where is small but negative $J_z \approx -0.3$, i.e. heating.

However, the 6D cooling is still strong $J_6 \approx 1.7$.



Unlike synchrotron, emittance exchange needed even for stability

Emittance exchange



Higher momentum muons pass through more material than lower. Momentum spread and thus Longitudinal emittance is reduced. But the transverse beam size is increased.

For equal partition and J_6=1.7 we get $J_x = J_y = J_z \approx 0.6$

Summary of lecture I

- Many different cooling schemes
- All fascinating

- But muons decay requiring cooling to be very fast

- Only Ionization Cooling seems practical
- This is conceptually similar to Radiation Cooling
- But simple Ionization Cooling does not cool longitudinally
- Emittance Exchange is required

- Not yet demonstrated
- But MICE, even Stage IV, will do it
- MICE stage VI will do something else (next lecture)

Emittances and Cooling
Lecture II: Ionization Cooling

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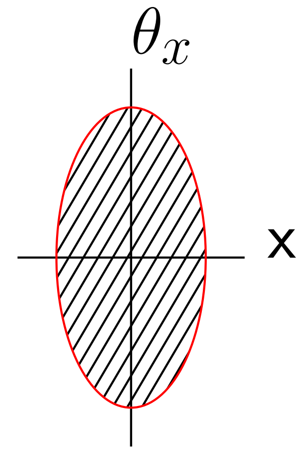
02/25/14

R. B. Palmer (BNL)

- Introduction
  - β functions
  - Solenoid focusing
  - Canonical Angular momentum
- Transverse Cooling Formulae
- Cooling in long solenoids
- Periodic focusing
- Super FOFO
- MICE

$\beta_{x,y}(Twiss)$ of Beam



For upright phase ellipse in θ_x vs x,

$$\beta_\perp \;=\; \left(\frac{\text{width}}{\text{height}} \quad \text{of phase ellipse}\right) \;=\; \frac{\sigma_x}{\sigma_\theta} \tag{10}$$

Then, using emittance definition:

$$\sigma_x \;=\sqrt{\epsilon_\perp\,\beta_\perp\,\frac{1}{\beta_v\gamma}} \qquad \text{true for any ellipse} \tag{11}$$

$$\sigma_\theta \;=\sqrt{\frac{\epsilon_\perp}{\beta_\perp}\,\frac{1}{\beta_v\gamma}} \qquad \text{only for upright ellipse} \tag{12}$$

When $\beta_x = \beta_y$ I will often refer to them as $\beta_\perp$

Units

I like to use strict MKS units, but when E, p, or m appear, add $1/e$, c/e, or c^2/e and put square brackets around them. The bracketed values are now - in electron Volts:

$[E/e]$

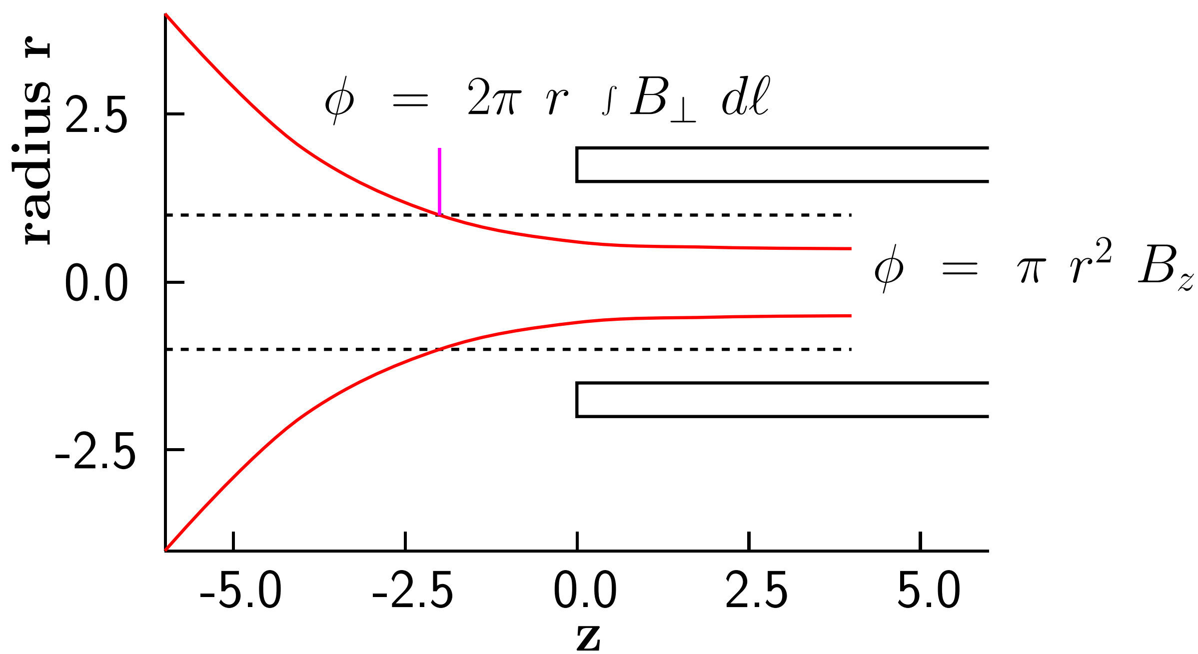
$[pc/e]$

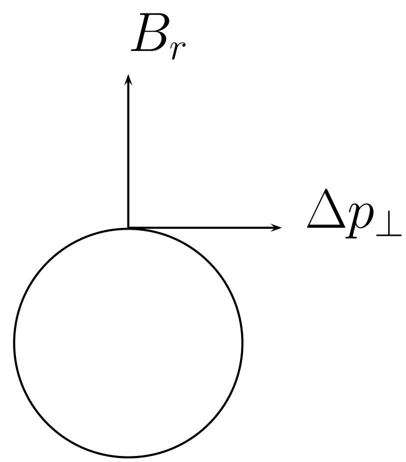
$[mc^2/e]$

For instance the curvature k of a 100 MeV beam in a 10 T magnetic field is

$$k \;=\; \frac{B_z\ c}{[pc/e]} \;=\; \frac{10\ \ 3\ 10^8}{10^8} \;=\; 30\ \ (\mathrm{m}^{-1})$$

Solenoid Focusing Entering a Solenoid





$$\Delta[pc/e]_\perp \;=\; \int B_r\ c\ dz \;=\; \frac{B_z\ r\ c}{2} \tag{13}$$

So for a case with zero initial transverse momentum,

$$[pc/e]_\perp \;=\; \frac{B_z\ r\ c}{2} \tag{14}$$

This azimuthal momentum, interacting with the axial field generated an inward focusing force. As r changes, the radial motion interacting with B_z maintains equation 14.

Canonical Angular momentum

If before entering a field, there is an initial 'Canonical' angular momentum $[pc/e]_{\perp\ \mathrm{can}}$, then after entering the solenoid field:

$$[pc/e]_\perp(1) \;=\; [pc/e]_{\perp\ \mathrm{can}} \;+\; \frac{B_z\ r\ c}{2}$$

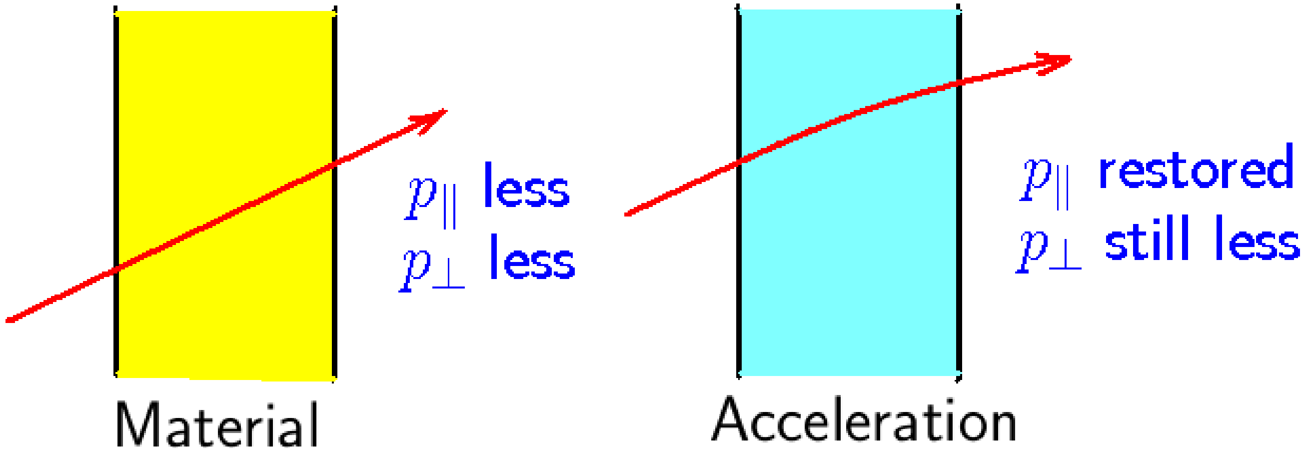
In the absence of material, when the particle comes out of the field, then there is a reverse angular kick and the angular momentum reverts to the initial 'Canonical' value.

$$[pc/e]_\perp(2) \;=\; \left([pc/e]_{\perp\ \mathrm{can}} \;+\; \frac{B_z\ r\ c}{2}\right) - \frac{B_z\ r\ c}{2} \;=\; [pc/e]_\perp(1)$$

When there is material inside the magnetic field, things get more interesting   More later

The σ_θs used to define $\beta_\perp$ in eq. 10 are defined in the Canonical frame

Back to: Transverse Ionization Cooling



Cooling rate vs. Energy

$$\text{eq. 2} \qquad \epsilon_{x,y} = \gamma\beta_v \, \sigma_{x,y} \, \sigma_{\theta_{x,y}}$$

If there is no Coulomb scattering, or other sources of emittance heating, then σ_θ and $\sigma_{x,y}$ are unchanged by energy loss, but p and thus $\beta\gamma$ are reduced. So the fractional cooling $d\epsilon\,/\epsilon$ is:

$$\frac{d\epsilon(\text{cooling})}{\epsilon} = \frac{dp}{p} = \frac{dE}{E}\,\frac{1}{\beta_v^2} \tag{15}$$

which, for a given energy change, favors cooling at low energy.

Heating Terms

$$\text{eq. 2} \qquad \epsilon_{x,y} = \gamma\beta_v\, \sigma_{x,y}\, \sigma_{\theta_{x,y}}$$

Between scatters the drifts conserves emittance (Liouiville).
When there is scattering, $\sigma_{x,y}$ is conserved, but σ_θ is increased.

$$d(\epsilon_{x,y})^2 = \gamma^2\beta_v^2\, \sigma_{x,y}^2 d(\sigma_\theta^2)$$

$$\text{eq.11} \qquad 2\epsilon\, d\epsilon = \gamma^2\beta_v^2 \left(\frac{\epsilon\beta_\perp}{\gamma\beta_v}\right)\, d(\sigma_\theta^2)$$

$$d\epsilon = \frac{\beta_\perp\gamma\beta_v}{2}\, d(\sigma_\theta^2)$$

$$\text{Rossi} \qquad d(\sigma_\theta^2) \approx \left(\frac{14.1\ 10^6}{[pc/e]\beta_v}\right)^2\, \frac{ds}{L_R}$$

$$d\epsilon(\text{heating}) = \frac{\beta_\perp}{\gamma\beta_v^3}\, dE\, \left(\left(\frac{14.1\ 10^6}{2[mc^2/e]_\mu}\right)^2 \frac{1}{L_R\, dE/ds}\right)$$

Minimum Emittance

Defining

$$C(mat,E) \;=\; \frac{1}{2}\left(\frac{14.1\ 10^6}{[mc^2/e]_\mu)}\right)^2 \frac{1}{L_R\ d\gamma/ds} \tag{16}$$

then

$$\frac{d\epsilon(\text{heating})}{\epsilon} \;=\; dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ C(mat,E)$$

Equating this with equation 15, for an equilibrium state

$$dE\ \frac{1}{\beta_v^2\ E} \;=\; dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ C(mat,E)$$

gives the equilibrium emittance without emittance exchange:

$$\epsilon_{x,y}(min) \;=\; \frac{\beta_\perp}{\beta_v}\ C(mat,E) \tag{17}$$

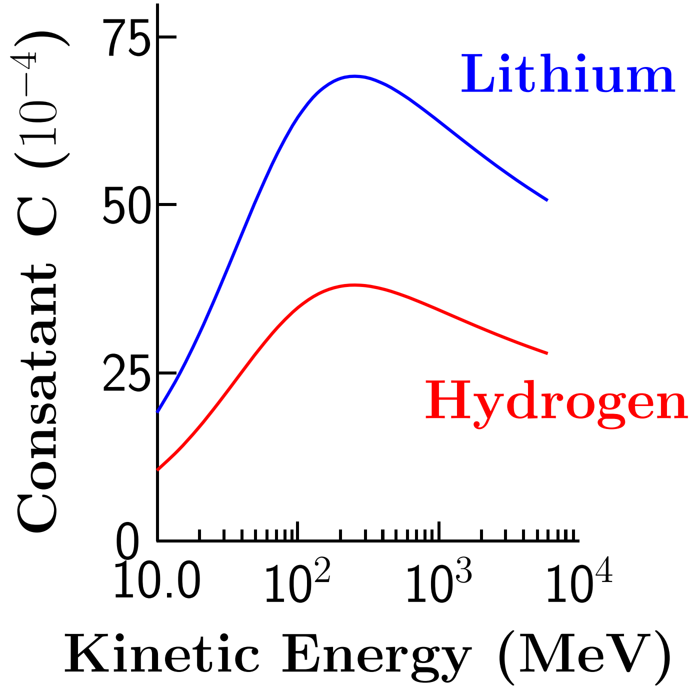
Or including possibility of emittance exchange:

$$\epsilon_{x,y}(min) \;=\; J_{x,y}\,\frac{\beta_\perp}{\beta_v}\ C(mat,E) \tag{18}$$

Choice of Materials

At energies such as to give minimum ionization loss, the constant C_o for various materials are approximately:

material	density kg/m^3	dE/dx MeV/m	L_R m	C_o 10^{-4}
Liquid H_2	71	28.7	8.65	38
Liquid He	125	24.2	7.55	51
LiH	820	159	0.971	61
Li	530	87.5	1.55	69
Be	1850	295	0.353	89
Al	2700	436	0.089	248



Liquid Hydrogen is the best material, even though it requires windows made of Aluminum or other material which somewhat degrade the performance.

Lower energies cool transverse emittance to lower emittances, but longitudinal emittances rise rapidly and 6D cooling is impossible.

Beam Divergence Angles

$$\text{eq.12} \qquad \sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp \; \beta_v \gamma}}$$

so, from equation 17: $\epsilon(min) = \beta_\perp \; C(mat, E)/\beta_v$

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp \; \beta_v \gamma} \; \frac{\beta_\perp \; C(mat, E)}{\epsilon(min) \; \beta_v}} = \sqrt{\left(\frac{\epsilon}{\epsilon(min)}\right) \; \frac{C(mat, E)}{\beta_v^2 \gamma}}$$

Independent of the emittance !

and for $\epsilon/\epsilon(min) = 2$, giving 50 % of maximum cooling rate (see below), and an aperture at 3 σ, the angular aperture at the absorber A_θ must be

$$A_\theta \; \geq \; 3\sqrt{2} \; \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}} \qquad (19)$$

For 130 MeV: H$_2$: $A \; \geq \; 0.25$ LiH: $A \; \geq \; 0.32$ Be: $A \; \geq \; 0.38$

Huge required acceptances $\rightarrow$ preference for H$_2$ at all emittances

Rate of Cooling

As one approaches the minimum emittance, the cooling rate will decrease:

$$\frac{d\epsilon_{x,y}}{\epsilon_{x,y}} = \left(1 - \frac{\epsilon_{\mathrm{min}}}{\epsilon}\right) J_{x,y} \frac{dp}{p} \qquad (20)$$

Using an $\epsilon >> \epsilon(min)$ is impractical because of the excessive required angular acceptance

Using $\epsilon(min) \rightarrow \epsilon$ implies slow cooling with resulting losses to decay

Thus efficient cooling requires a 'tapered' sequence of 'stages' with ever decreasing $\beta_{\perp}$s to keep $\epsilon/\epsilon(min) \approx 2$

Cooling with Long Solenoid Focusing

In a solenoid with axial field B_{sol}

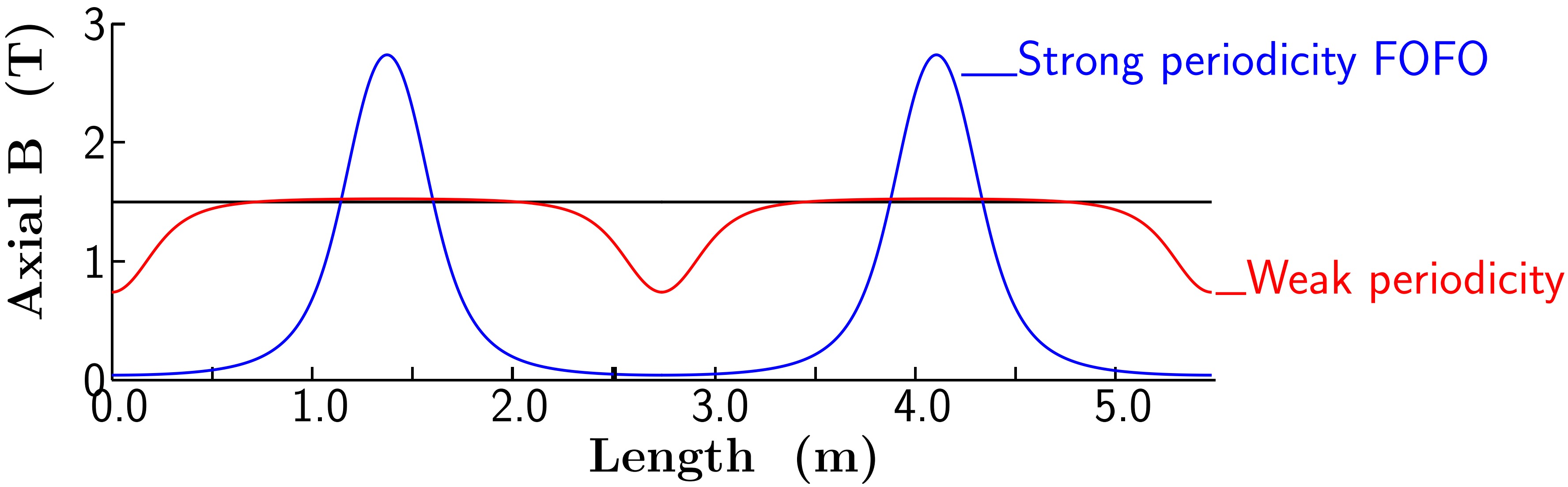
$$\beta_\perp \;=\; \frac{2\;[pc/e]}{c\;B_{sol}}$$

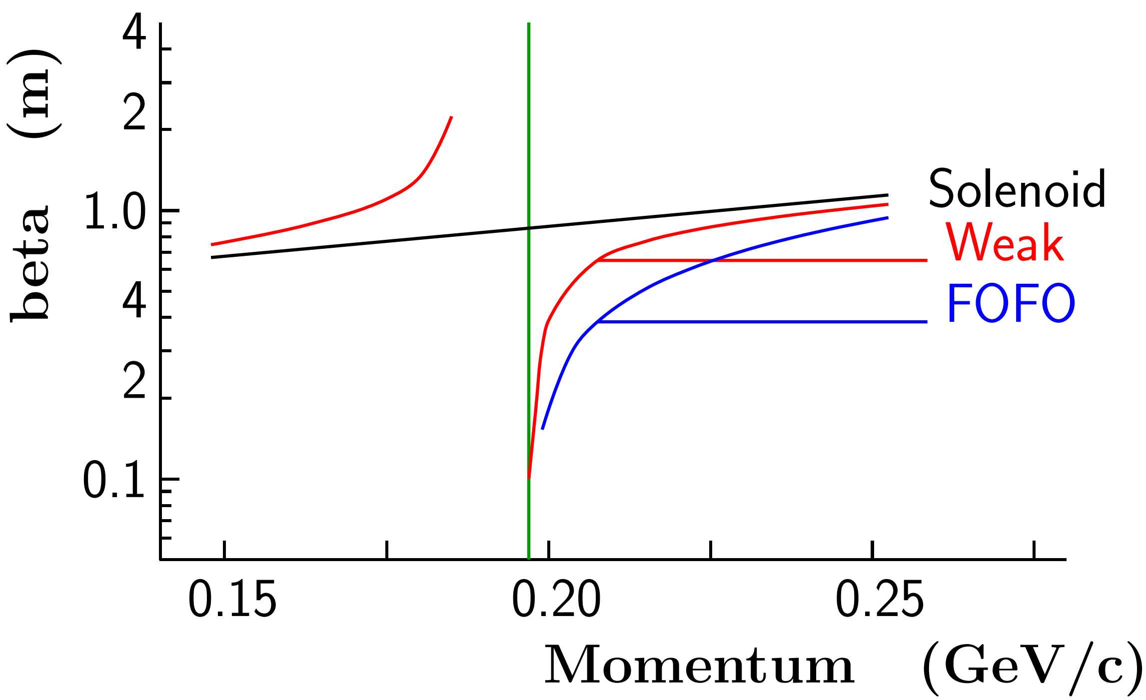
with no emittance exchange:

$$\epsilon_{x,y}(min) \;=\; C(mat,E)\;\frac{2\;\gamma\;[mc^2/e]_\mu}{B_{sol}\;c} \tag{21}$$

The minimum emittance depends on B_z. Can we do better without raising it further?

Decreasing $\beta_\perp$ in Solenoids by adding periodicity

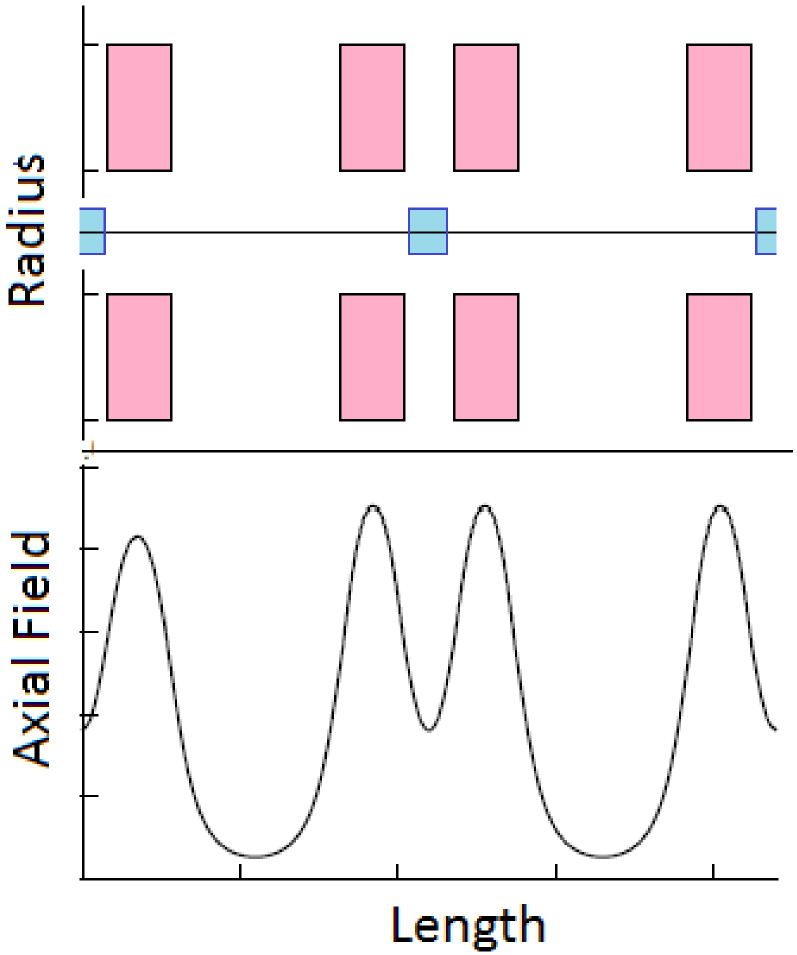


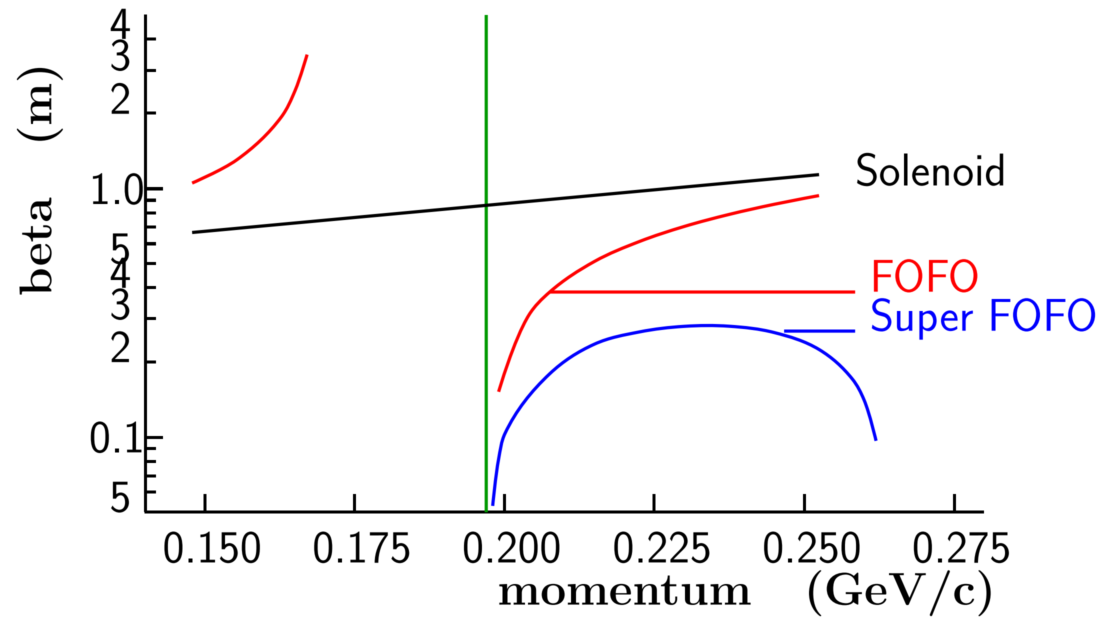


- Resonances introduced
- Betas reduced locally,
- But momentum acceptance small

Super FOFO (Sessler)

Double periodicity





- Beta lower over finite momentum range
- Beta lower by about 1/2 solenoid

Solenoid focusing is independent of sign

Lowest $\beta_\perp$ when "non-flip" as shown

But this has angular momentum, and other, problems

Angular Momentum with an Absorber

Assuming that initial Canonical angular momentum is zero, then in a focusing field B_z, the physical angular momentum will be:

$$\text{eq.14} \qquad [pc/e]_\phi = \frac{c\, B_z\, r}{2}$$

With material reducing all momenta by a factor K, there is cooling, and the physical angular momentum is also reduced:

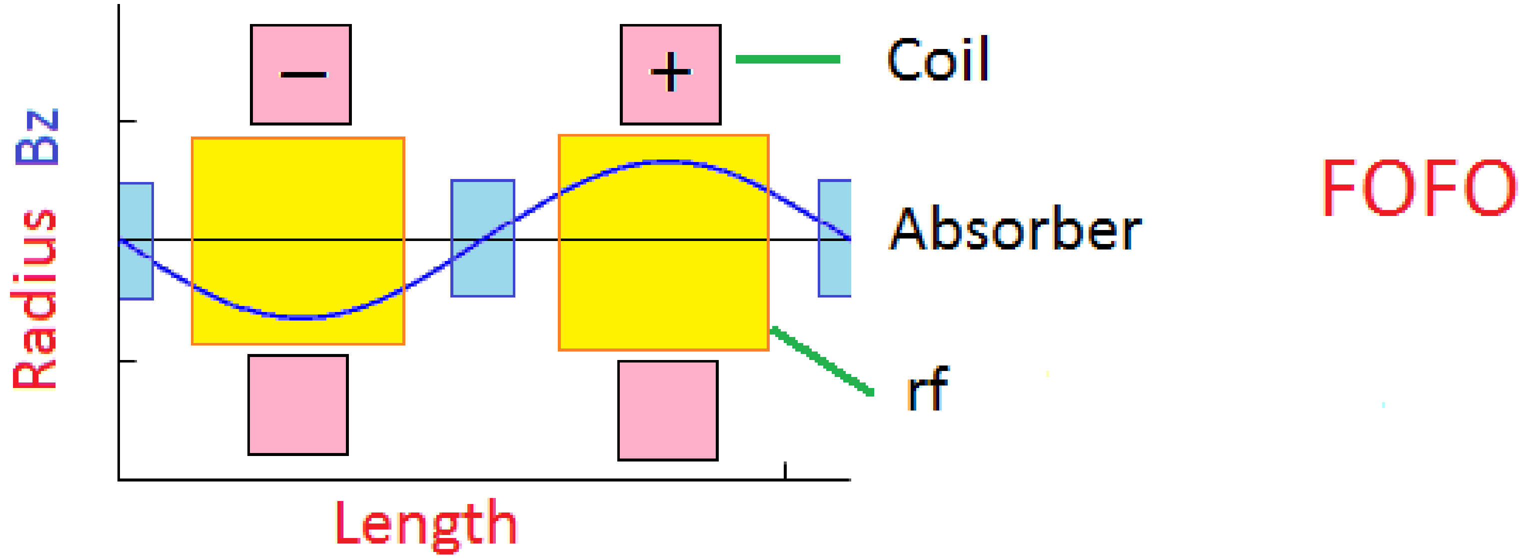
$$[pc/e]_\phi(\text{after absorber}) = \mathrm{K}\,\frac{\mathrm{c\ B_z\ r}}{2}$$

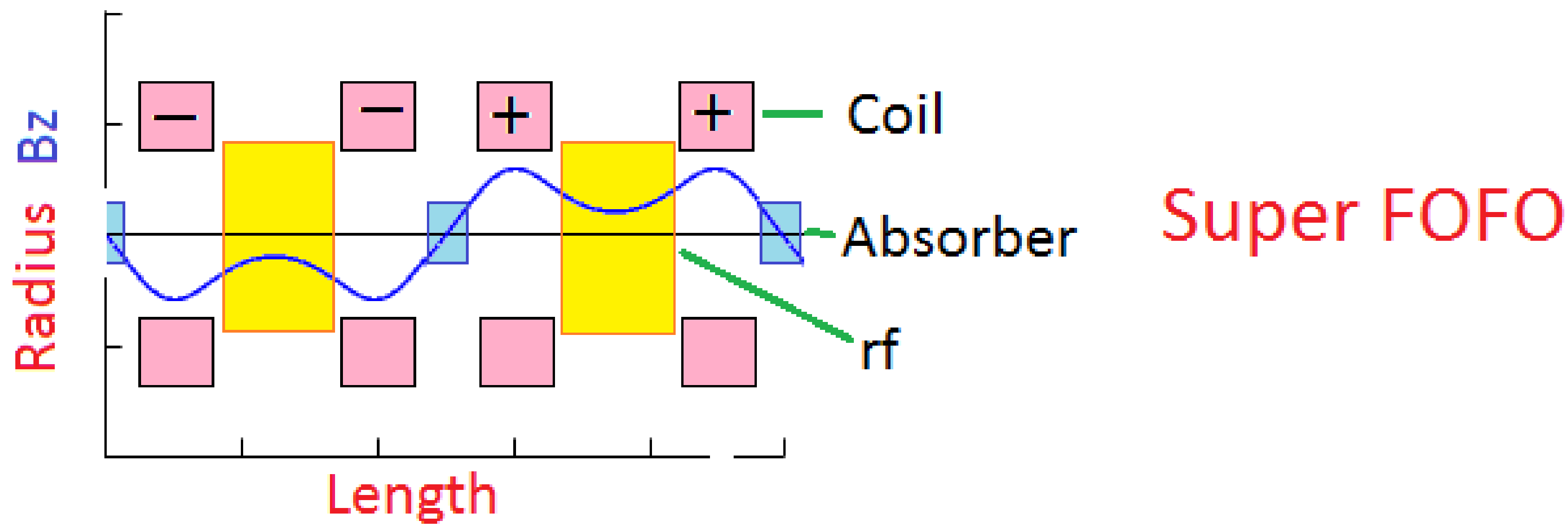
When a muon leaves the field, and its average angular momentum is also its Canonical value:

$$[pc/e]_\phi(\text{canonical}) = \mathrm{K}\,\frac{\mathrm{c\ B_z\ r}}{2} - \frac{\mathrm{c\ B_z\ r}}{2} = \frac{\mathrm{c\ B_z\ r}}{2}\,(\mathrm{K}-1) < 0$$

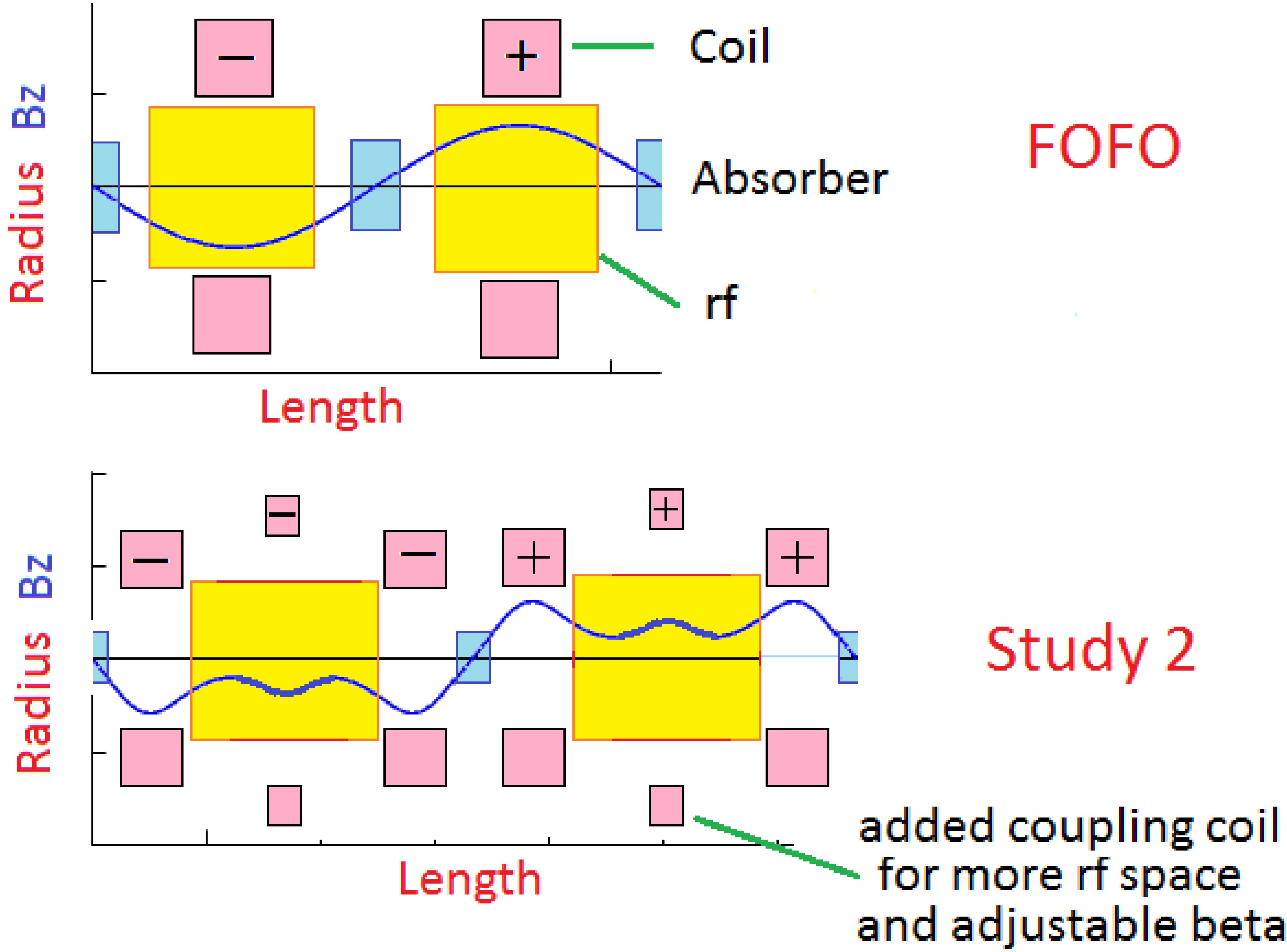
And it will continue to fall (or rise if B neg) with more cooling
One must reverse (flip) the field a finite number of times.

Lattices with many "flips"



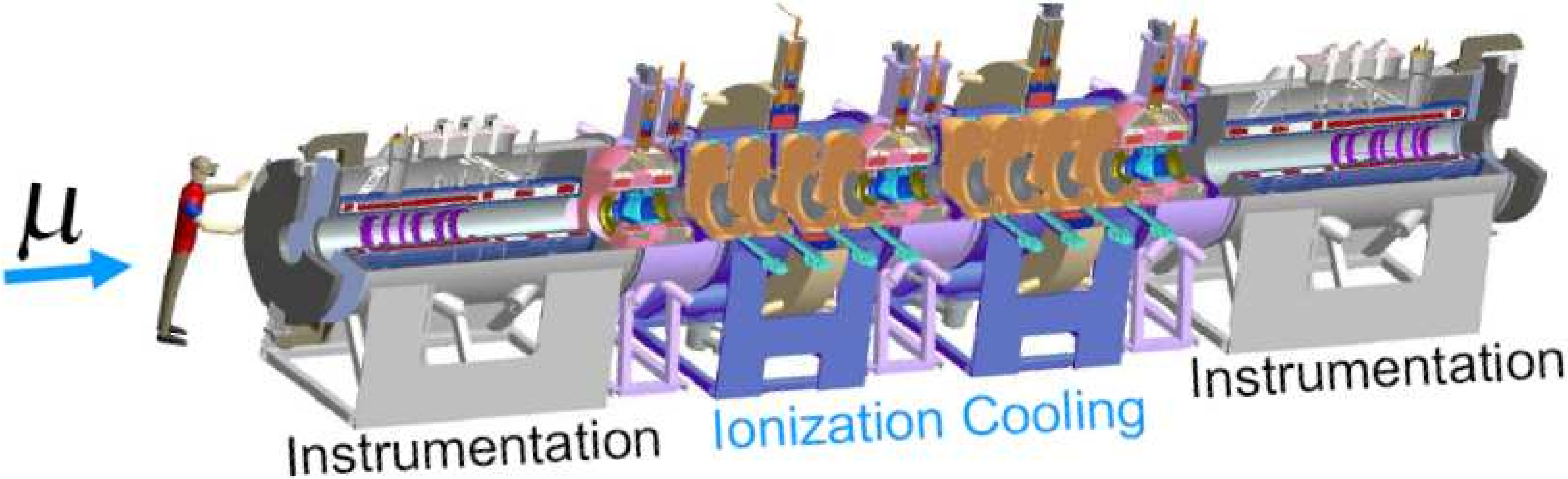


Lattices with many "flips"



SFOFO Lattice Engineering

Study 2 at Start of Cooling



- I think you have seen this before!
- It is the lattice to be tested in Muon Ionization Cooling Experiment (MICE) at RAL
- This is much more than a demonstration of Ionization Cooling
- It is a demonstration of a usable technology

Conclusion

- A particle entering a solenoid receives an azimuthal kick, but its 'Canonical' angular momentum does not change
- Focusing is generated by the interaction of this additional azimuthal momentum and the axial field
- Passing through an absorber cools transverse momenta, but Coulomb scattering heats them
- The minimum emittance is proportional to the beam $\beta_\perp$ at the absorber, and is least with a hydrogen absorber
- Beam $\beta_\perp$ can be less in a periodic lattice than in uniform B_z
- A bi-periodic lattice can have a wider momentum acceptance
- An alternating field lattice avoids accumulation of Canonical angular momenta
- MICE is demonstrating cooling in such a lattice