New Methods in Supersymmetric Theories and Emergent Gauge Symmetry

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- We are used to the idea that a scalar particle can be composite, for example the pion π has constituents which can be resolved if one does experiments at $\sim 1GeV$.
- We are even used to the idea that a massive spin 1 particle can be composite, e.g. the rho meson ρ_μ.
- The scale of compositeness is the scale when interactions become strong.

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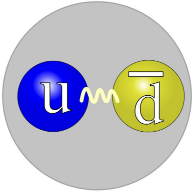
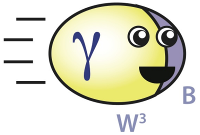


It is clearly an outstanding question to understand whether the W, Z bosons and the Higgs field are fundamental or composite.

But what about the photon?



Traditionally, one would argue that a massless gauge field A_μ cannot be composite.

- $A_\mu \simeq A_\mu + \partial_\mu \Lambda$. Necessary for unitarity. For example, if two fermions have a massless composite vector bound state, where would this gauge symmetry come from?
- Weinberg-Witten theorem: The gauge symmetry cannot be the manifestation of any conserved current in the system:

$$\langle VAC | j_\mu | A_\mu \rangle = 0 \quad \text{for all} \quad \partial^\mu j_\mu = 0$$

- A_μ couples to some conserved current (electron minus positron number). Where would this come from if, according to Witten-Witten, A_μ cannot "talk" to any current?

Where would the gauge symmetry come from if it did not exist in the fundamental theory?

Take a $U(1)$ Goldstone scalar in $2+1$-dimensional quantum field theory, $\pi \simeq \pi + f$. It can be transformed to a gauge potential

$$\partial_\mu \pi = \epsilon_{\mu\nu\rho} \partial^\nu A^\rho \ .$$

This transformation has the inherent ambiguity

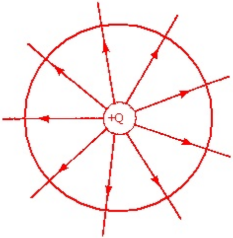
$$A_\mu \to A_\mu + \partial_\mu \Lambda \ .$$

The transformation from the fundamental degrees of freedom to the low energy degrees of freedom could have an inherent ambiguity.

Where does the current to which the gauge field couples come from?

By the Weinberg-Witten theorem, it cannot be one of the conserved currents in the theory.

But the current to which a gauge field couples is not a real current in the theory anyway: If we quantize the theory on $\mathcal{M} \times \mathbb{R}$ with compact $\mathcal{M}$, then the Hilbert space consists of gauge singlets (Gauss' law).



There is thus no theoretical obstruction for the compositeness of massless spin 1 particles. In nature, it could actually pertain to the photon and also the W, Z bosons (the Higgsing would come "later").

We will discuss the phenomenological possibilities at the end.

The simplest existence proof is provided by $d = 4$, $\mathcal{N} = 1$ theories [Seiberg].

The simplest possible case is that we start with $SU(4)$ gauge theory with 6 fundamental fields Q_i^A and 6 anti-fundamental fields $\tilde{Q}_A^i$. This theory has a negative beta function

$$\beta \sim -3 \times 3 + 4 = -5 \ .$$

It therefore develops strong coupling at some scale Λ_{QCD} and becomes intractable.

What happens in the infrared?

Seiberg has made a guess that the infrared theory is actually weakly coupled but in terms of different variables:

It is an $SU(2)$ gauge theory with 12 fundamentals and 36 neutral scalar fields.

The beta function is

$$\beta = -6 + 6 = 0$$

at one-loop, but it is positive at two loops. So the theory is free in the infrared.

The $SU(2)$ gauge fields have nothing to do with the original $SU(4)$ gauge theory. The $SU(2)$ gauge fields are new, emergent, weakly-coupled massless composite spin 1 particles.

We often refer to this guess of the low energy degrees of freedom as "duality." It is a duality in the sense that the original strongly-coupled $SU(4)$ description has a more useful, weakly-coupled, description as an $SU(2)$ gauge theory.

Seiberg's guess is essentially based on 't Hooft anomaly matching and comparing the vacua of the two theories.

Clearly, if the idea is correct, it may have far-reaching consequences both for particle physics and for condensed matter physics, so we would really like to test it thoroughly.

One of the main topics of the workshop is the recent development of methods which allow to make extremely detailed tests of this proposal.

Note that tests of this idea have to be non-perturbative because the duality is between a strongly coupled description and a weakly coupled description.

Suppose we have a theory with a conserved fermionic charge Q such that $Q^2 = H$ and $Q|boson\rangle = |fermion\rangle$ and vice versa.

Suppose $H|\Psi\rangle = E|\Psi\rangle$, $E \neq 0$. Then, $Q|\Psi\rangle = |\Psi'\rangle \neq 0$. Thus, $Tr(-1)^F = 0$ for all the states with $E \neq 0$.

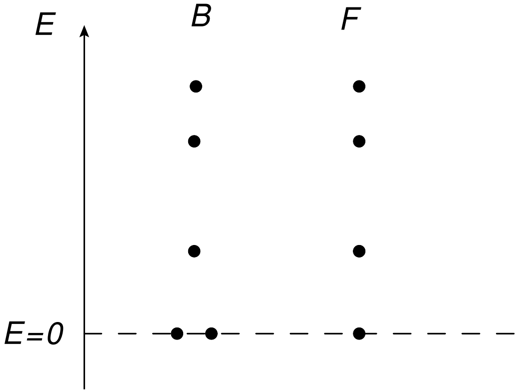
Define

$$I = Tr_{\mathcal{H}}(-1)^F$$

over the whole Hilbert space $\mathcal{H}$. The contributions only come from $H = 0$ states (vacua).

$$I = n_B - n_F$$

This is the Witten Index.



$I=2-1=1$

Since the index does no depend on the renormalization group scale, we would like to compare the Index I of our $SU(4)$ and $SU(2)$ gauge theories.

The trouble is that it diverges. This is due to the infinitely many SUSY vacua on both sides (flat directions).

We now know how to overcome this longstanding problem! The first steps were done by Römelsberger. We can study the theory on $\mathcal{M}_3 \times \mathbb{R}$ rather than on $\mathbb{R}^4$ and consider

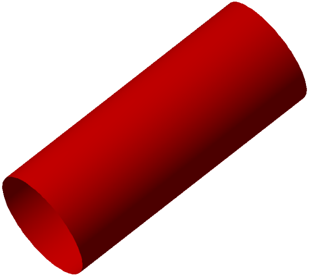
$$I(\mu_i) = Tr_{\mathcal{H}}((-1)^F e^{\mu_i q_i})$$

for various charges q_i that commute with the supercharge.

This is well defined for interesting theories such as our $SU(4)$ and $SU(2)$ theories.

In order to preserve sufficient supersymmetry, $\mathcal{M}_3$ must look locally as $S^1 \times \Sigma^{(2)}$ for some Riemann surface $\Sigma^{(2)}$ [Dumitrescu-Festuccia-Seiberg, Closset-Dumitrescu-Festuccia-ZK].

A particularly nice example is $\mathcal{M}_3 \sim S^3$ (topologically). This is what Römelsberger and subsequently many others studied.



The gauged charges do not correspond to chemical potentials because the corresponding charges vanish.

We can understand the effect of gauging as follows: We can imagine that we take $g_{YM} \to 0$ and remove the gauge fields. Then, the corresponding gauge chemical potentials r_i are allowed.

The effect of gauging with infinitesimal g_{YM} is to integrate over r_i. Hence,

$$I_{gauged}[\mu_i] \sim \int [dr_i] \, Tr_{\mathcal{H}_{ungauged}} \left((-1)^F e^{\mu_i q_i} e^{r_i Q_i} \right)$$

$[dr_i]$ is the Haar measure.

For $\mathcal{M}_3 = S^3$ the computation was done and one finds that if $SU(4)$ indeed flows to $SU(2)$ with massless emergent gauge fields, then the following identity should hold true (schematically):

$$(p,p)^2(q,q)^2 \int \prod_{i=1,..,4} [dr_i] \frac{\prod_{i,j\leq 4} \Gamma(\mu_i r_j, 1/(\tilde{\mu}_i r_j), p, q)}{\prod_{i,j\leq 4} \Gamma(r_i/r_j, r_j/r_i, p, q)}$$

$$= \left[\prod_{i,j\leq 2} \Gamma(\mu_i/\tilde{\mu}_j, p, q)\right] \int \prod_{i=1,2} [dr_i] \frac{\prod_{i,j\leq 2} \Gamma(\mu_i r_j, 1/(\tilde{\mu}_i r_j), p, q)}{\prod_{i,j\leq 2} \Gamma(r_i/r_j, r_j/r_i, p, q)}$$

$\Gamma(\cdot)$ is the elliptic hypergeometric gamma function. $(\cdot,\cdot)$ is the q-Pochhammer symbol.

It appears that mathematicians have independently proved such identities quite recently [Spiridonov, Rains, Rahman, van de Bult...]. In particular, the identity above can be confirmed!

The elliptic hypergeometric function Γ that appeared above is a "higher version" of the Jacobi theta function $\Theta_n(z;q)$. The latter are central in complex analysis in *two dimensions*.

It appears that the computations of

$$I(\mu_i) = Tr_{\mathcal{H}}((-1)^F e^{\mu_i q_i})$$

are closely linked with the theory of complex geometry in *four dimensions*:

- In order to preserve SUSY on some four-fold $\mathcal{M}_4$ the four-fold needs to be complex. [Klare-Tomasiello-Zaffaroni, Dumitrescu-Festuccia-Seiberg]
- If $\mathcal{M}_4 = \mathcal{M}_3 \times S^1$ then if locally $\mathcal{M}_3 = S^1 \times \Sigma^{(2)}$ we have complex structure and a holomorphic Killing vector, guaranteeing 2 supercharges.
- The Index is independent of the Hermitian metric on $\mathcal{M}_4$, and it depends only on the complex structure. [Closset-Dumitrescu-Festuccia-ZK]
- The Index I is thus a function of the complex structure parameters τ_i and of the moduli of holomorphic vector bundles. This is the geometric meaning of μ_i.
- The complex structure moduli space of $S^3 \times S^1$ is two-complex dimensional [Kodaira-Spencer], accounting for the parameters p, q that appeared above.

This is why choosing different metrics leads to essentially identical results and why in some cases the results are trivial.

The index only depends on non-metric data, i.e. the complex structure moduli.

This is somewhat reminiscent of twisting in $\mathcal{N} = 2$, where the theory becomes completely topological.

We see that one can also "twist" $\mathcal{N} = 1$ theories, but one gets a theory of the complex structure moduli space, rather than a topological theory.

A very partial summary of applications:

- Many checks of dualities, new dualities...
- Wilson loops expectation values (non-perturbative) [Pestun...]
- Exact computations of the metric in theory space and other previously inaccessible observables [Benini-Cremonesi, Doroud-Gomis-le Floch-Lee, Gerchkovit-Gomis-ZK....]
- Relations between field theories in different dimensions [Alday-Gaiotto-Tachikawa,...]
- Novel tests of AdS/CFT [see e.g. Martelli-Sparks, Cassani-Martelli,...]
- Monotonicity of Renormalization Group Flows in $d = 3$ [Jafferis-Klebanov-Pufu-Safdi]
- Many new relations to mathematics

Given that massless (or very light) spin 1 particles can be composite, we can ask whether this has any relevance to nature.

As we have seen in our $SU(4) \to SU(2)$ example, this is a strong/weak relation. It should be viewed as a generalization of Maxwell's Electric-Magnetic duality

$$E \to B \ , \qquad B \to -E \ ,$$

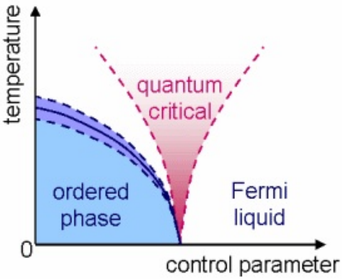
under which $e \to 1/e$.

It is thus a natural and rich idea, which would be a pity if nature does not realize...

Three-dimensional version of this idea may well be realizable in quantum phase transitions. For example [Maldacena-Zhiboedov, Aharony-GurAri-Yacoby...],

$$U(N_c)_{k+1/2} + \text{fund fermion} = U(|k|)_{N_c} + \text{fund critical boson.}$$

There are already claims of extremely similar quantum phase transitions with $k = 0$.



In QCD, there are the ρ_μ^i mesons with $i = 1, 2, 3$ and mass ~ 750MeV. One can describe them in the following way: imagine the unitary matrix of pions

$$U = e^{i\pi^a T^a} \ , \qquad U \to G_L U G_R \ , \qquad G_{L,R} \in SU(2)_{L,R}$$

and factorize it as

$$U = MN^\dagger$$

Now there is a gauge symmetry (i.e. redundancy) that appears [e.g. Georgi and refs]:

$$M \to Mh \ , \qquad N \to Nh \ , \qquad h \in SU(2)_{HLS}$$

We identify the gauge fields of this local symmetry as the rho mesons

$$\rho_\mu^i T^i \to h\rho_\mu^i T^i h^{-1} + ih^{-1}\partial_\mu h$$

Suppose we limit ourselves to 2-derivative Lagrangians that are invariant under the global $G_{L,R}$ symmetry and the local h symmetry. There are three couplings:

$$\mathcal{L} = -\frac{1}{g^2}F^2 + f_\pi^2(\partial_\mu \pi)^2 + af_\pi^2(\epsilon^{abc}\pi^a\partial_\mu\pi^b + \rho_\mu^c)^2 + \cdots \ ,$$

$F = \partial\rho + [\rho, \rho]$ the usual field strength. The mass of the rho meson fixes g and the decay constant of the pion fixes f_π. So there is one unknown parameter: a.

QCD phenomenology is best matched if one takes

$$a = 2 \ .$$

$a = 2$ reproduces the so-called "vector dominance."

If one could continuously deform QCD such that the rho mesons would become lighter and lighter, it is entirely conceivable that this $SU(2)_{LHS}$ would be our emergent massless gauge symmetry. Then, the two-derivative hypothesis would be rigorously justified for light rho mesons.

One could even justify $a = 2$ by appealing to a Weinberg-like sum rule.

In fact, in the SUSY duality we started from, there is an analog of a which is precisely equal to 2! [ZK]

To summarize, some crude aspects of QCD phenomenology are well described by treating the ρ mesons as if they are the light gauge bosons of some emergent "magnetic" $SU(2)_{HLS}$ gauge symmetry.

The challenge here is to make this precise: i.e. find a deformation of QCD for which the rho mesons become parametrically light.

Now we turn to some brief discussion of possible relevance to high-energy particle physics.

We would like to explain the fact that the photon is massless and the electroweak scale $M_{weak} \ll M_{Planck}$ by saying that they are composite states.

This is very different (=more ambitious) from garden variety Technicolor, where the W, Z, γ particles are introduced by hand and are not part of the actual dynamics.

What is the scale of compositeness? It should probably be the Landau pole scale. At least this is what happens in SUSY theories.

Without extra matter, the Landau pole is at too high energies to be interesting. Imagine that extra matter exists, say at $\sim 100\text{TeV}$, and drives the couplings to be strong at M_{GUT}. In fact, this actually happens in many models that people built.

In this case, it could be that **even the photon** is composite with constituents that we could resolve at $E \sim M_{GUT}$. This is not an idea that phenomenologists usually consider, but given that this is a beautiful theoretical idea and we know that this can happen in principle, perhaps one should contemplate the consequences.

Also it is conceivable that one can build low-energy (compared with M_{GUT}) models with a composite photon.

Conclusions

- Massless spin-1 particles can be composite.
- One has precise realizations with $\mathcal{N} = 1$ SUSY.
- One can test these ideas using very recent developments in $\mathcal{N} = 1$ SUSY dynamics. Intimate relations to modern mathematics.
- Remains to be seen if one can connect to nature in $2 + 1$ or $3 + 1$ dimensions. Many encouraging hints.

Thank You For Your Attention