General Instanton countings and 5d(6d) SCFTs

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Outline

- Motivation & Message
- ADHM Quantum Mechanics
- 5d SCFTs with enhanced global symmetry
- Conclusions

Motivation

- Much interests in computation of exact results in SYM
- Partition function on S^n, Superconformal index on $S^n \times S^1$, some other spaces
- Various 4d, 3d, 2d examples (Many people here)
- Evaluation can be done via localization
- Help to understand the nonperturbative aspects of SUSY field theories in various dimensions (Seiberg duality, mirror symmetry, AGT, understanding non Lagrangian SCFTs ...)
- It's known that nontrivial SCFTs exist in higher dimensions
- via String, M, F theory considerations
  ex)(0,2), (0,1) E_8, 5d SCFTs (Seiberg; Seiberg, Intriligator, Morrison;Bergman,Rodriguez-Gomez,Zafrir)
  various 6d (0,1)s (Apruzzi, Fazzi, Rosa, Tomasielle; Gaiotto, Tomasiello; Heckman, Morrison, Vafa; Del Zoto, Heckman, Morrison, Tomasiello, Vafa)
- Some of them are related to AdS^7, AdS^6

Motivation II

- Some of them obtained by considering the strong coupling limit of SYM in 5d
- One can obtain the partition function or the index of SCFTs in higher dimensions
- from the localization of 5d SYM on S^5, $S^4 \times S^1$
- though 5d SYM is not renormalizable

Motivation continued

- All of these are reduced to the evaluation of instanton partition of Nekrasov on R^4 or $S^1 \times R^4$ with various groups and with various matters
- e.g. It's known that N=1 5d $Sp(1)$ with $N_f = 5, 6, 7, 8$, one antisymmetric hyper
  exhibit E_6, E_7, E_8 global symmetry
- $Sp(1)$ with $N_f = 5, 6, 7, 8$ nonrenormalizable, no connections to 4d theory, how to define the instanton calculus?
- Related fact: 5d $U(1) N = 1*$ Nekrasov partition function necessarily involves small instanton singualrities
- Crucial to obtain the index of 1 M5-brane (0,2) theory

Motivation continued

- For the problem of the ambiguities in UV completion, we use the string theory as a guide
- Viewed 5d SYM as D4 systen, we consider D0-D4 SUSY Quantum mechanics as a proposal to compute the Nekrasov partition function (modulo removing string DOF irrelevant to field theory)

Message

- We solve the previously unsolved technical problem
- evaluation of the Nekrasov partition function is reduced to the evaluation of the residue
- Contour for this evaluation for general cases not known
- 5d N=1 vector multiplets worked out by Nekrasov
- 5d N=1 hypermultiplet of arbitrary representation ?
- We give the systematic derivation of the contour using ADHM QM
- related to evaluation of 2d elliptic genus using Jeffrey-Kirwan residue via dimensional reduction (Benini, Eager, Hori, Tachikawa I, II) + some additional subtleties
- Thereby establishing the basic tools to explore various 5d, 6d SCFTs
- Related work: Hori, Kim, Yi and Cordova, Shao

ADHM Quantum Mechanics

- Interested in 5d N=1 gauge theories (8 supercharges) and their instantons
- 8 supersymmetries $Q^A_\alpha, \bar{Q}^A_{\dot\alpha}$
- $\alpha = 1, 2, \dot\alpha = 1, 2$ index of rotation SO(4)=SU(2) X SU(2)
- $A = 1, 2$ doublet of $SU(2)_R$ symmetry
- topological $U(1)_I$ charge
  $k = \int TrF \wedge F \in Z$
- These are particles in 5d
- Instantons preserve 1/2 of SUSY $\bar{Q}^A_{\dot\alpha}$ (0,4) SUSY
- could be bound states of elementary partcles and instantonic particles
  $M = \frac{k}{g^2_{YM}} + Trv\Pi$
- v scalar vev of Coulomb branch, Π electric charge

ADHM Quantum Mechanics

- Nekrasov instanton partition function on $R^4 \times S^1$ = Witten index of BPS particles
  $Z_{Nek} = 1 + \sum_k Z_k q^k =$
  $Tr[(-1)^F q^k e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_+(2J_R^3+J_1+J_2)} e^{-\epsilon_-(J_1-J_2)} e^{-trv\Pi}]$
- $Q = -\bar{Q}^2_{\dot{2}}, Q^\dagger = \bar{Q}^1_{\dot{1}}, J_R^3 : SU(2)_R$ Cartan
- J_1, J_2 rotations of SO(4)
- computable from SUSY QM on the instanton modui space (with singularities)
- motivates D0-D4 QM

ADHM Quantum Mechanics as D0-D4 systems

- D4 Gauge group G= U(N), Sp(N), SO(N)
- D0 Gauge group $\hat{G} = U(k), O(k), Sp(k)$, k being instanton number
- D0-D4 bifundamentals of $G \times \hat{G}$ denoted as $q_{\dot{\alpha}}$
- D0-D0 adj, symmetric, antisymmetric of
  $\hat{G} = U(k), O(k), Sp(k)$
  $a_{\alpha\dot{\beta}} = a_m \sigma^m_{\alpha\dot{\beta}}$, $\sigma^m = (i\tau, i)$
  position muduli of instantons
- Supermultiplets
  $(q_{\dot{\alpha}}, \Psi^A), (a_{\alpha\dot{\beta}}, \lambda^A_\alpha)$

ADHM Quantum Mechanics as D0-D4 systems continued

- vector multiplets of $\hat{G}$
  A_t, $scalar\, \phi$, $\bar{\lambda}^A_{\dot{\alpha}}$
  3 auxiliary fields $D_{\dot{\alpha}\dot{\beta}} = D_{\dot{\beta}\dot{\alpha}}$

$$
\begin{aligned}
L \;=\; & \frac{1}{g_{QM}^2}\mathrm{tr}\left[\frac{1}{2}(D_t\varphi)^2+\frac{1}{2}(D_t a_m)^2+D_t q_{\dot{\alpha}} D_t \bar{q}^{\dot{\alpha}}+\frac{1}{2}[\varphi, a_m]^2-(\varphi\bar{q}^{\dot{\alpha}}-\bar{q}^{\dot{\alpha}}v)(q_{\dot{\alpha}}\varphi-vq_{\dot{\alpha}})\right.\\
&+\frac{1}{2}(D^I)^2-D^I\left((\tau^I)^{\dot{\alpha}}{}_{\dot{\beta}}\bar{q}^{\dot{\beta}}q_{\dot{\alpha}}+\frac{1}{2}(\tau^I)^{\dot{\alpha}}{}_{\dot{\beta}}[a^{\dot{\beta}\alpha}, a_{\alpha\dot{\beta}}]-\zeta^I\right)+\frac{i}{2}(\bar{\lambda}^{A\dot{\alpha}})^\dagger D_t\bar{\lambda}^{A\dot{\alpha}}+\frac{i}{2}(\lambda^A_\alpha)^\dagger D_t\lambda^A_\alpha\\
&+i(\psi^A)^\dagger D_t\psi^A+\sqrt{2}i\left((\bar{\lambda}^{A\dot{\alpha}})^\dagger\bar{q}^{\dot{\alpha}}\psi^A-(\psi^A)^\dagger q_{\dot{\alpha}}\bar{\lambda}^{A\dot{\alpha}}\right)+(\psi^A)^\dagger(\psi^A\varphi-v\psi^A)\\
&\left.+\frac{1}{2}(\bar{\lambda}^{A\dot{\alpha}})^\dagger[\varphi,\bar{\lambda}^{A\dot{\alpha}}]-\frac{1}{2}(\lambda^A_\alpha)^\dagger[\varphi,\lambda^A_\alpha]-i(\lambda^A_\alpha)^\dagger(\sigma^m)_{\alpha\dot{\beta}}[a_m,\bar{\lambda}^{A\dot{\beta}}]\right]\,. \qquad (2.5)
\end{aligned}
$$

- v vev of Coulomb branch of 5d, ζ^I $I = 1, 2, 3$ FI parameters

ADHM Quantum Mechanics as D0-D4 systems continued

- Additional matters give more QM degrees
- e.g., 5d N=1 theory with fundamentals give rise to Fermi multiplets
- 5d Chern-Simons term induces $L_{CS} = k(\phi + A_t)$
- $g_{QM} \to \infty$ Instantons in Higgs branch
  mass for ϕ given by $g_{QM}|q_{\dot\alpha}|$ large in the strong coupling
- Decomposition of (0,4) multiplets into (0,2) multiplets

$$\begin{aligned} \text{vector } (A_t, \varphi, \bar\lambda^A_{\dot\alpha}) &\to \text{vector } (A_t, \varphi, \bar\lambda^1_{\dot 1}, \bar\lambda^2_{\dot 2}) + \text{Fermi } (\bar\lambda^1_{\dot 2}, \bar\lambda^2_{\dot 1}) \\ \text{hyper } (\phi^{\dot\alpha}, \psi^A) &\to \text{chiral } (\phi^{\dot 1}, \psi^1) + \text{chiral } (\bar\phi_{\dot 2} = \bar\phi^{\dot 1}, \bar\psi_2) \ . \end{aligned}$$

- $J \equiv J_r + J_R$, J_r part of the SO(4) rotation, J_R $SU(2)_R$ symmetry play an important role, conjugate to ϵ_+

Witten Index of ADHM Quantum Mechanics

- Compute the index $\leftarrow$ (0,2) elliptic genus of Benini, Eager, Hori, Tachikawa,
- $Z_{inst} = \sum Z_k q^k$

$$Z^k_{\mathrm{QM}}(\epsilon_1, \epsilon_2, \alpha_i, z) = \mathrm{Tr}\left[(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_1(J_1+J_R)} e^{-\epsilon_2(J_2+J_R)} e^{-\alpha_i \Pi_i} e^{-zF}\right]$$

- J_1, J_2 Cartans of SO(4) J_R Cartan of $SU(2)_R$
  α_i chemical potential of electric charge Π_i, F flavor symmety

Witten Index of ADHM Quantum Mechanics continued

- zero mode integral A_t holonomy and scalar ϕ in the vector multiplet
- $\phi^I = \varphi^I + iA^I$ give rise to cylinder geometry $I = 1 \ldots rankG$

$$Z = \frac{1}{|W|}\oint e^{\kappa \mathrm{tr}(\phi)} Z_{\text{1-loop}} = \frac{1}{|W|}\oint e^{\kappa \mathrm{tr}(\phi)} Z_V \prod_{\Phi} Z_\Phi \prod_{\Psi} Z_\Psi$$

$$Z_V = \prod_{\alpha \in \mathrm{root}} 2\sinh\frac{\alpha(\phi)}{2} \prod_{I=1}^{r} \frac{d\phi_I}{2\pi i}$$

$$Z_\Phi = \prod_{\rho \in R_\Phi} \frac{1}{2\sinh\left(\frac{\rho(\phi)+J\epsilon_+ + Fz}{2}\right)}$$

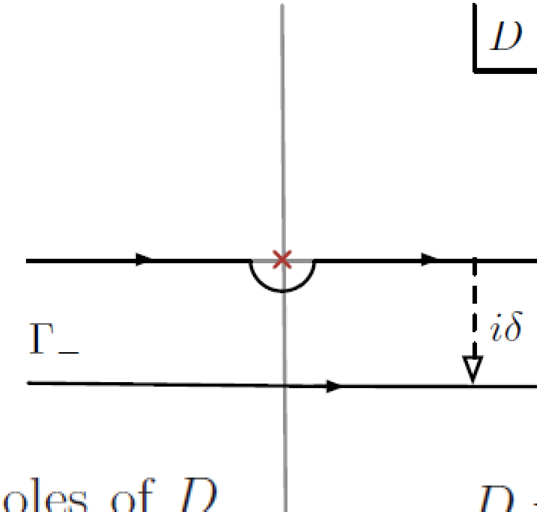
$$Z_\Psi = \prod_{\rho \in R_\Psi} 2\sinh\left(\frac{\rho(\phi)+J\epsilon_+ + Fz}{2}\right)$$

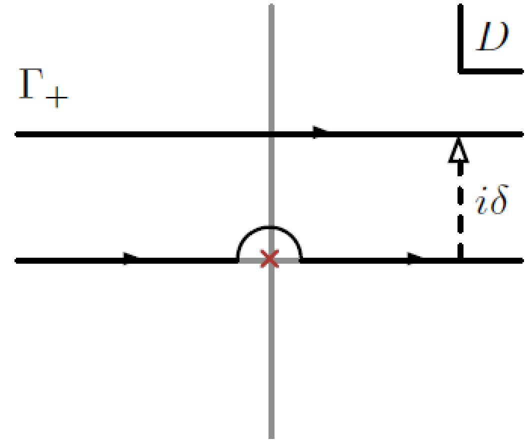
Witten Index of ADHM Quantum Mechanics continued

- Dangerous regions
  a.two noncompact regions of cylinder denoted by 0, ∞
  b. poles from chiral multiplets $\rho(\phi_*) + J\epsilon_+ + Fz = 0$
- poles from chiral multiplets; regularized by D terms and work out the pole regions carefully

$$Z_\Phi(\phi, \epsilon_+, z, D) = \prod_{\rho \in R_\Phi} \prod_{n=-\infty}^{\infty} \frac{-2\pi i n + \rho(\bar{\phi}) + J\bar{\epsilon}_+ + F\bar{z}}{|2\pi i n + \rho(\phi) + J\epsilon_+ + Fz|^2 + i\rho(D)}$$

$$Z = \int_{\mathbb{R}} dD \int_M d^2\phi f_{e,g}(\phi, \bar{\phi}, D) \exp\left(-\frac{D^2}{2e^2} - i\zeta D\right)$$





poles of D

$$D = \frac{i}{Q_i} |2\pi i n + Q_i \phi + J\epsilon_+ + Fz|^2$$

$$Z^+ = \sum_{Q_i>0} R_i + \frac{1}{2}(R_0 + R_\infty)\left[1 + \lim_{e,\varepsilon\to 0} \mathrm{erf}\left(\frac{e\zeta}{\sqrt{2}}\right)\right]$$

$$Z^- = -\sum_{Q_i<0} R_i - \frac{1}{2}(R_0 + R_\infty)\left[1 - \lim_{e,\varepsilon\to 0} \mathrm{erf}\left(\frac{e\zeta}{\sqrt{2}}\right)\right]$$

Witten indices in two limits $e\zeta = \pm\infty$

$$Z(e\zeta = \infty) = -\sum_{Q_i<0} R_i\,, \quad Z(e\zeta = -\infty) = \sum_{Q_i>0} R_i$$

$$Z(\zeta > 0) - Z(\zeta < 0) = R_0 + R_\infty$$

Multi-dimensional residues

- Given by Jeffrey-Kirwan residues
- Choose $\eta = -\zeta$ (FI) if needed

$$\text{JK-Res}(\text{Q}_*, \eta)\frac{d\phi_1 \wedge \cdots \wedge d\phi_r}{Q_{j_1}(\phi)\cdots Q_{j_r}(\phi)} = \begin{cases} |\det(Q_{j_1}, \cdots, Q_{j_r})|^{-1} & \text{if } \eta \in \text{Cone}(Q_{j_1}, \cdots, Q_{j_r}) \\ 0 & \text{otherwise} \end{cases}$$

$$Z = \frac{1}{|W|} \sum_{\phi_*} \text{JK-Res}(\mathbf{Q}(\phi_*), \eta)\ Z_{\text{1-loop}}(\phi, \epsilon_+, z)$$

Dependence on FI parameters; Wall Crossing?

- BPS states depend on the sign of FI parameters?
- (0,4) SUSY has tripet of FI parameters not expected to have such FI dependence
- BPS states breaks neither $SU(2)_r$ nor $SU(2)_R$ no dependence on FI
- expect to have $R_0 + R_\infty = 0$
- For most of the cases such conditions are satisfied but some of them are not
- at the infinities of cylinders Coulomb branch can be developed and D0 can escape
- not a field theory degrees of freedom
- $Z = Z_{QFT} Z_{string}$
- all FI dependence comes from Z_{string}

Heuristic Derivation of Contour

$$\frac{1}{2\sinh\left(\frac{Q_i(\phi)+J_i\epsilon_+ + F_i z}{2}\right)} \sim \frac{1}{e^{Q_i(\phi)} - e^{-J_i\epsilon_+}e^{-F_i z}}$$

- $exp(Q_i\phi) = exp(-J_i\epsilon_+)$
  For vectors $J_i > 0$ residue is inside the unit circle for positively charged fields (Nekrasov)
  For hypers $J_i < 0$ residue is outside the unit circle for positively charged fields (Observed by Hollands, Keller, Song)
- These prescriptions agree with the the above derivations from ADHM QM
- For 5d $N = 1^* U(N)$ theory, the contribution from hypers are vanishing and only vectors contribute

Example

- 5d $N = 1^* U(N) theory$
- Contribution from vector multiplets are classified by Young diagrams

$$\frac{1}{k!}\oint\left[\prod_{I=1}^{k}\frac{d\phi_I}{2\pi i}\right] Z_{\rm vec}(\phi,\alpha,\epsilon_{1,2}) Z_{\rm adj}(\phi,\alpha,\epsilon_{1,2},m)$$

For $G = U(N)$ and $\hat{G} = U(k)$

$$Z_{\rm vec} = \frac{\prod_{I\neq J} 2\sinh\left(\frac{\phi_{IJ}}{2}\right)\cdot\prod_{I,J=1}^{k} 2\sinh\left(\frac{\phi_{IJ}+2\epsilon_+}{2}\right)}{\prod_{I=1}^{k}\prod_{i=1}^{N} 2\sinh\left(\frac{\phi_I-\alpha_i+\epsilon_+}{2}\right)\cdot 2\sinh\left(\frac{\alpha_i-\phi_I+\epsilon_+}{2}\right)\prod_{I,J=1}^{k} 2\sinh\left(\frac{\phi_{IJ}+\epsilon_1}{2}\right)\cdot 2\sinh\left(\frac{\phi_{IJ}+\epsilon_2}{2}\right)}$$

$$Z_{\rm adj} = \frac{\prod_{I=1}^{k}\prod_{i=1}^{N} 2\sinh\left(\frac{\phi_I-\alpha_i+m}{2}\right)\cdot 2\sinh\left(\frac{\alpha_i-\phi_I+m}{2}\right)\prod_{I,J=1}^{k} 2\sinh\left(\frac{\phi_{IJ}\pm m-\epsilon_-}{2}\right)}{\prod_{I,J=1}^{k} 2\sinh\left(\frac{\phi_{IJ}\pm m-\epsilon_+}{2}\right)}$$

$$Y_i = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \\ \cline{1-1}\end{array} \qquad \phi_1 - \alpha_i + \epsilon_+ = 0\ , \quad \phi_{21} + \epsilon_1 = 0\ , \quad \phi_{31} + \epsilon_2 = 0$$

$$Y_i = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array}$$

$$\phi_1 - \alpha_i + \epsilon_+ = 0\ , \quad \phi_{21} + \epsilon_1 = 0\ , \quad \phi_{31} + \epsilon_2 = 0\ , \quad \phi_{43} + \epsilon_1 = 0\ , \quad \phi_{42} + \epsilon_2 = 0$$

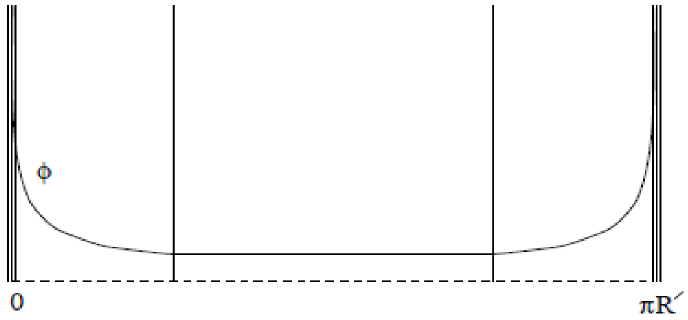
$$Z_k = \sum_{\sum_i |Y_i| = k} \prod_{i,j=1}^{N} \prod_{s \in Y_i} \frac{\sinh \frac{E_{ij}+m-\epsilon_+}{2} \sinh \frac{E_{ij}-m-\epsilon_+}{2}}{\sinh \frac{E_{ij}}{2} \sinh \frac{E_{ij}-2\epsilon_+}{2}}$$

$$E_{ij} = \alpha_i - \alpha_j - \epsilon_1 h_i(s) + \epsilon_2 (v_j(s) + 1)$$

- $h_i(s)$ distance from the box to the right edge
  $v_j(s)$ distance from the box to the bottom edge

Sp(N) theory

- 5d $N = 1 Sp(N)$ with N_f flavors and one antisymmetric hyper
- This arise from D0-D4-D8-O8 of Type I'
- The strong coupling limit corresponds to heterotic $E_8 \times E_8$ with Wilson lines
- $N_f = 5, 6, 7$ have E_6, E_7, E_8 global symmetry in the strong coupling limit
- $N_f = 8$ corresponds to 6d (0,1) theory with E_8 global symmetry (M5-M9 system)



Sp(N) theory continued

- Important point; antisymmetric hyper
- Sp(1) antisymmetric hyper perturbatively trivial
- but nontrivial nonperurbatively, ADHM QM is different
- Should exclude D0-D8 contribution
- This has applications to the superconformal index for Sp(N) defined on $S^1 \times S^4$

$$I(t,u,m_i,q) = \mathrm{Tr}\left[(-1)^F e^{-\beta\{Q,S\}} t^{2(J_r+J_R)} u^{2J_l} e^{-F\cdot m} q^k\right]$$

- J_r, J_l rotations of SO(4) on S^4, J_R Cartans of $SU(2)_R$ of F(4) superconformal symmetry
  k instanton numer F global symmetry

$$\{Q,S\} = E - 2J_r - 3J_R \geq 0$$

$$I(t,u,m_i,q) = \int [da] Z_{\text{pert}}(ia,t,u,m_i) Z_{\text{inst}}(ia,t,u,m_i,q) Z_{\text{inst}}(-ia,t,u,-m_i,q^{-1})$$

$$Z_{\text{pert}} = \mathrm{PE}\left[f_{\text{vec}}(t,u,e^{ia}) + f_{\text{fund}}(t,u,e^{ia},e^{m_l}) + f_{\text{anti}}(t,u,e^{ia},e^{m})\right]$$

$$\begin{aligned} f_{\text{vec}} &= -\frac{t(u+u^{-1})}{(1-tu)(1-t/u)}\left[\sum_{i<j}^{N} e^{\pm ia_i \pm ia_j} + \sum_{i=1}^{N} e^{\pm 2ia_i} + N\right] \\ f_{\text{fund}} &= \frac{t}{(1-tu)(1-t/u)} \sum_{i=1}^{N}\sum_{l=1}^{N_f} e^{\pm ia_i \pm m_l} \\ f_{\text{anti}} &= \frac{t(e^m + e^{-m})}{(1-tu)(1-t/u)}\left[\sum_{i<j}^{N} e^{\pm ia_i \pm ia_j} + N\right] . \end{aligned}$$

- Instanton part is localized at the north and south pole of S^4 and reduced to the Nekrasov instanton partition function

$$
\begin{aligned}
I = 1 &+ \chi^{E_7}_{133} t^2 + \chi_2(u)\left[1+\chi^{E_7}_{133}\right] t^3 + \left[1+\chi^{E_7}_{7371} + \chi_3(u)\left(1+\chi^{E_7}_{133}\right)\right] t^4 \\
&+ \left[\chi_2(u)\left(1+\chi^{E_7}_{133}+\chi^{E_7}_{7371}+\chi^{E_7}_{8645}\right) + \chi_4(u)\left(1+\chi^{E_7}_{133}\right)\right] t^5 \\
&+ \left[2\chi^{E_7}_{133}+\chi^{E_7}_{8645}+\chi^{E_7}_{238602}+\chi_3(u)\left(2+2\chi^{E_7}_{133}+\chi^{E_7}_{1539}+2\chi^{E_7}_{7371}+\chi^{E_7}_{8645}\right)\right. \\
&\qquad \left. +\chi_5(u)\left(1+\chi^{E_7}_{133}\right)\right] t^6 + \mathcal{O}\left(t^7\right) \ ,
\end{aligned}
$$

$$
\begin{aligned}
I = 1 &+ \chi^{E_8}_{248} t^2 + \chi_2(u)\left[1+\chi^{E_8}_{248}\right] t^3 + \left[1+\chi^{E_8}_{27000} + \chi_3(u)\left(1+\chi^{E_8}_{248}\right)\right] t^4 \\
&+ \left[\chi_2(u)\left(1+\chi^{E_8}_{248}+\chi^{E_8}_{27000}+\chi^{E_8}_{30380}\right) + \chi_4(u)\left(1+\chi^{E_8}_{248}\right)\right] t^5 \\
&+ \left[2\chi^{E_8}_{248}+\chi^{E_8}_{30380}+\chi^{E_8}_{1763125}+\chi_3(u)\left(2+2\chi^{E_8}_{133}+\chi^{E_8}_{3875}+2\chi^{E_8}_{27000}+\chi^{E_8}_{30380}\right)\right. \\
&\qquad \left. +\chi_5(u)\left(1+\chi^{E_8}_{248}\right)\right] t^6 + \mathcal{O}\left(t^7\right) \ ,
\end{aligned}
$$

- For $Sp(2)$ expected enhanced global symmetry observed

Features of the superconformal index

- It exhibits the global symmetry E_7, E_8
- Instanton plays the crucial role
- $E_7 \to SO(12) \times U(1)_I$
  $E_8 \to SO(14) \times U(1)_I$ where $U(1)_I$ denotes the instanton charge
- The character of $E_5 = SO(10) \to SO(8) \times U(1)_I$

$$\mathbf{45} \to \mathbf{1}_0 + \mathbf{28}_0 + (\mathbf{8}_s)_1 + (\mathbf{8}_s)_{-1}$$

$$\chi^{E_5}_{\mathbf{45}} = \chi^{SO(8)}_{\mathbf{1}} + \chi^{SO(8)}_{\mathbf{28}} + q\chi^{SO(8)}_{\mathbf{8}_s} + q^{-1}\chi^{SO(8)}_{\mathbf{8}_s}$$

Comparison of the previous work

- $\frac{Z(\textit{with antisym})}{Z(\textit{without antisym})} = Z(\textit{superparticle index } D0 - D8 - O8)\ N_f \leq 5$
- Explains the previous work by Kim, Kim, Lee on 5d SCFTs

Conclusions

- We derive the Nekrasov instanton partition in a systematic way using ADHM QM
- This holds for classical gauge groups with arbitrary matter representations
- Instanton partition function for ABCDEFG?
- With basic tools available, one can explore conformal zoo in 5d, 6d SCFTs
- Obvious examples: (0,1) E_8, Orbifolds of (0,2), (0,1) E_8, 5d SCFTs but much more
  (Some of them are done, others to be done)
- Relation to local singularites in F theory and M theory on Calabi-Yau