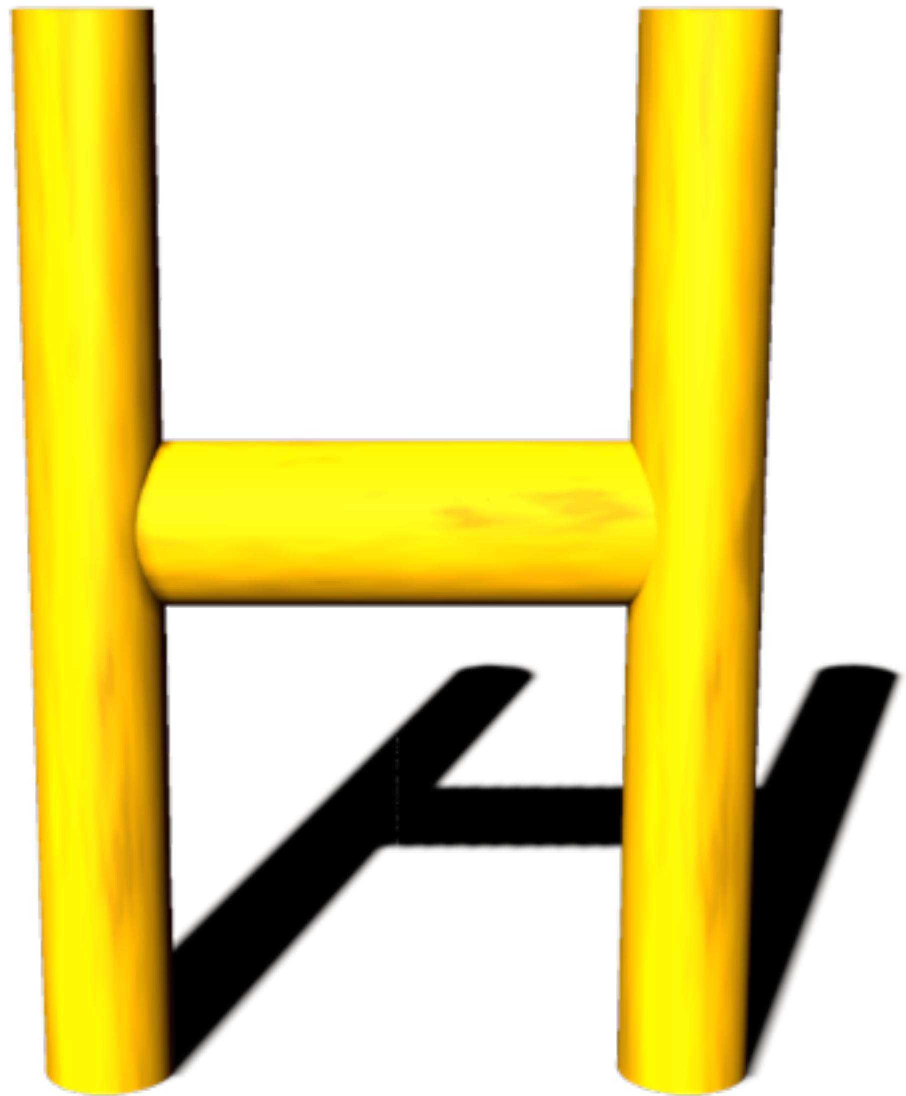
The Higgs in the Golden Channel



CERN TH BSM Forum
13 January, 2014

James "Jamie" Gainer
University of Florida

Outline

- Matrix Element Method (MEM)
  - Lightning Review of Relevant Statistics
  - Matrix Element Method

- Measuring Higgs Properties with the MEM

  in the "Golden" $H \to Z Z^* \to 4 \ell$ channel

How to Tell Where You Are

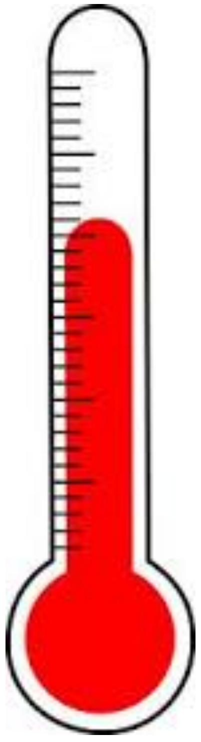
- Florida?

- Geneva?

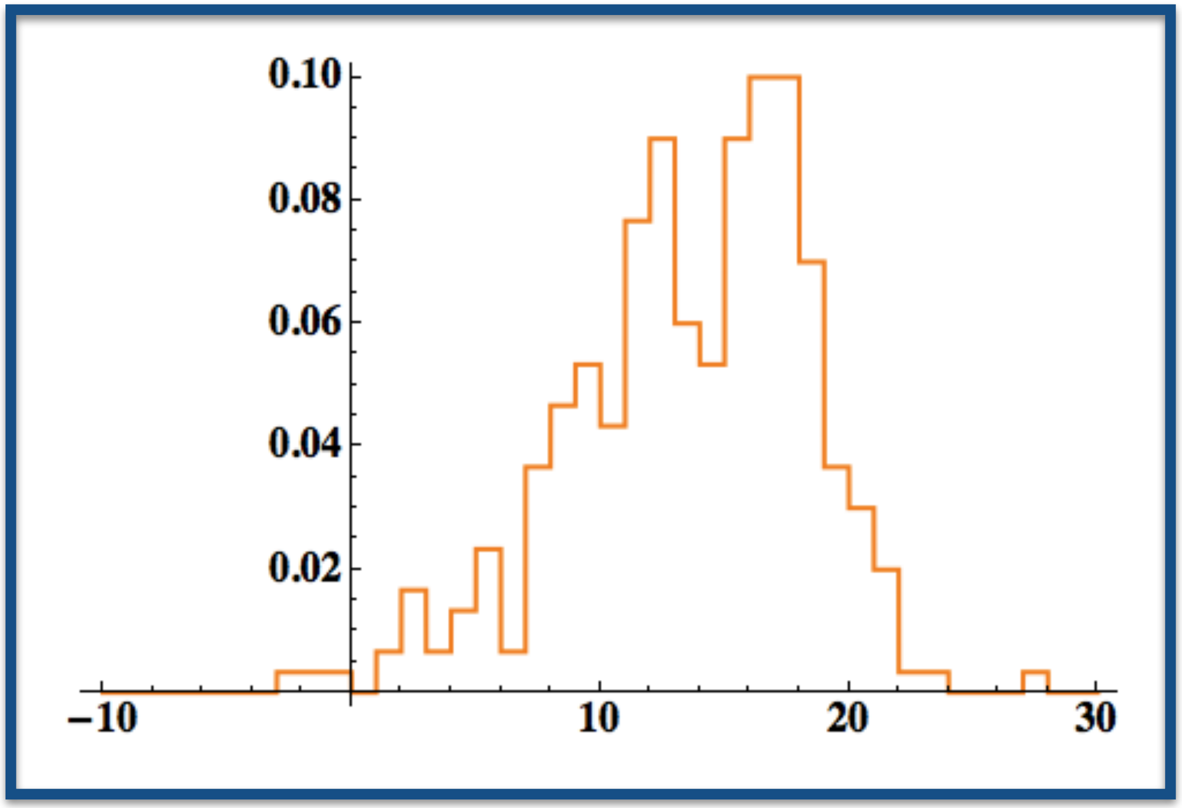
How to Tell Where You Are

- For the moment we'll ignore such sensitive variables as the presence or absence of alligators, etc.

- Instead try to make the determination only by daily measurements with an (integer) thermometer

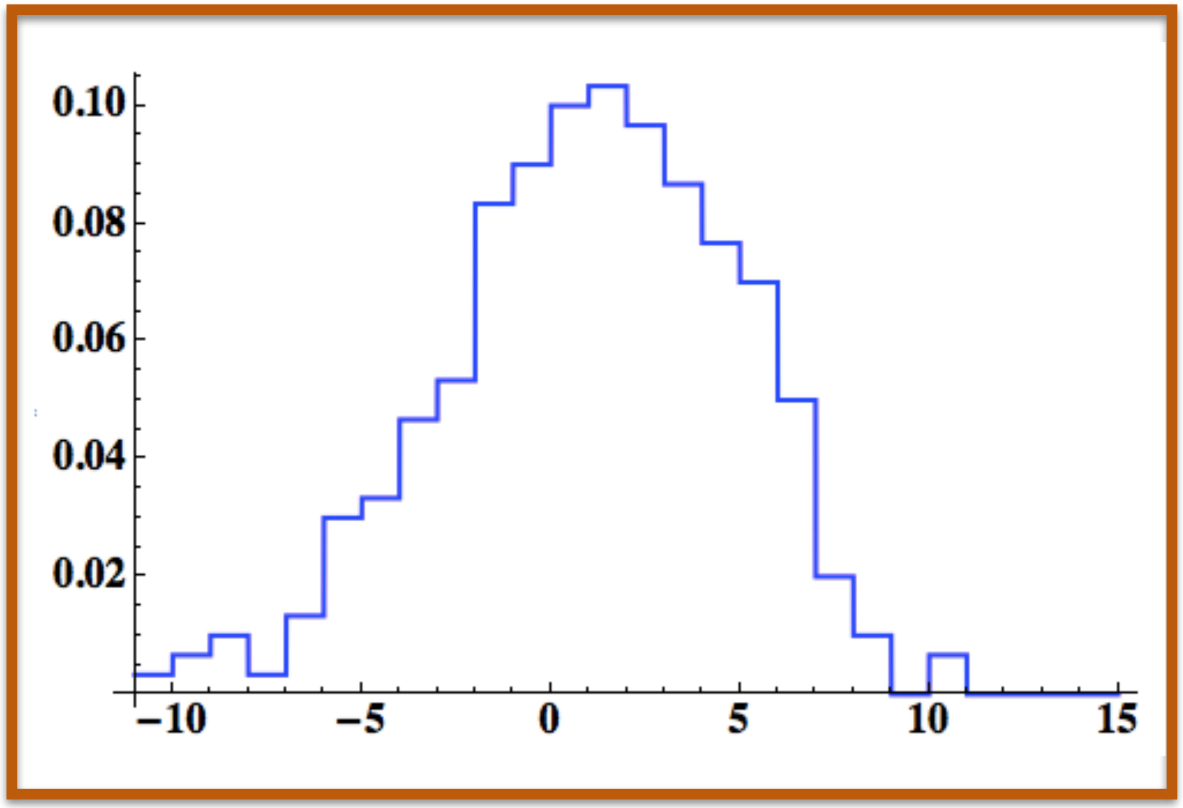


Temperature



P(T| Florida)

Temperatures in Celsius



P(T| CERN)

Temperatures in Celsius

Data for January high temperatures.
Data is made up, but based on a true story.

Likelihood Ratios and All That

After a temperature measurement T_1

Posterior Odds

Prior Odds

Likelihood Ratio

$$\frac{p(\text{CERN} \mid T_1)}{p(\text{Florida} \mid T_1)} = \frac{p(\text{CERN})}{p(\text{Florida})} \times \frac{p(T_1 \mid \text{CERN})}{p(T_1 \mid \text{Florida})}$$

What we want to know

What we think before the measurement

Ratio of probabilities of observing T_1 at CERN and at Florida

From Bayes Theorem

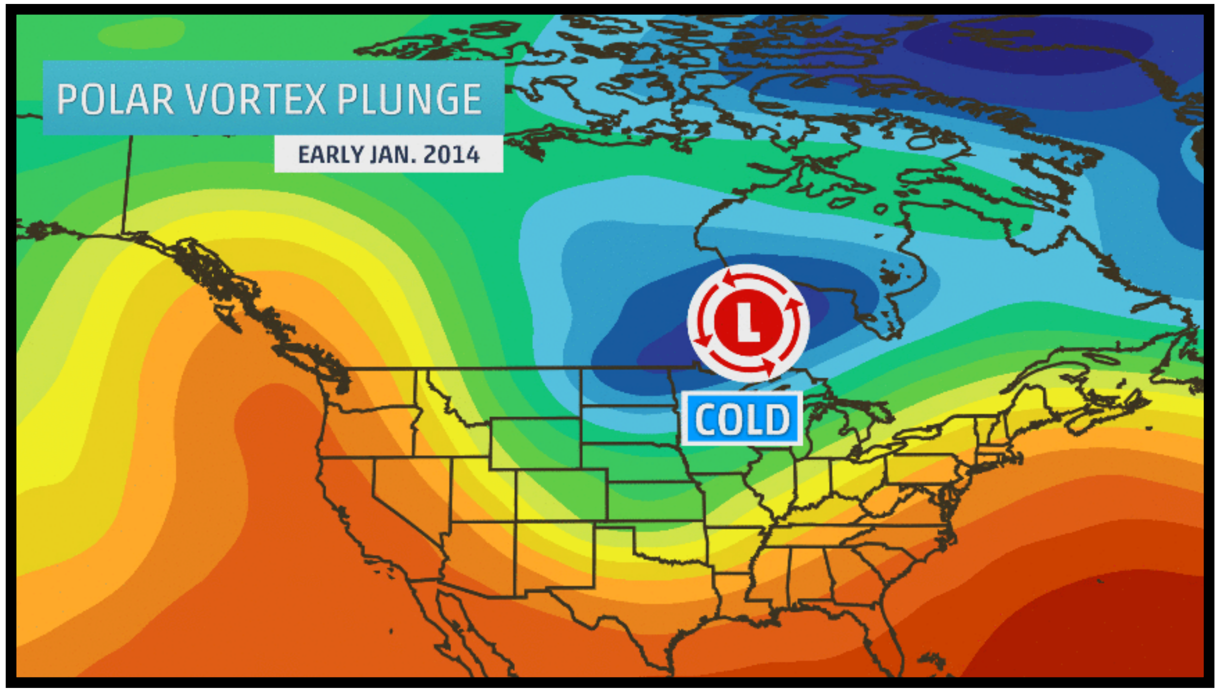
Likelihood Ratios and All That

- So if we have prior odds = 1:1 "flat priors"
  likelihood ratio quantifies how likely it is that one hypothesis fits the data as compared with the other

- Extension to many measurements is straightforward as if measurements are "independent" (uncorrelated)
  and "identical" (involve the same pdf), then, e.g.

$$p(T_1, T_2, T_3 | \text{ CERN}) = p(T_1 | \text{ CERN})\ p(T_2 | \text{ CERN})\ p(T_3 | \text{ CERN})$$

- Amusingly, our temperature measurements are in general neither **independent** (cold snaps, heat waves, …) nor **identical** (if we wait a few months for the second measurement…)



Neyman-Pearson Lemma

- We can use the likelihood ratio to get an idea of whether we're at CERN or in Florida
- But we might be wrong.
  We could either falsely decide we're in Florida, when we're really at CERN, or falsely decide we're at CERN when we're really in Florida.
- More generally, if one hypothesis is "null" and the other is "alternative", then

  α = probability of false positive (choose "alternative" when "null" is true)

  β = probability of false negative (choose "null" when "alternative" is true)

  $1 - \beta$ = "power", so "most powerful" = "minimum false negative rate"

Neyman-Pearson Lemma

Data may involve many events and many variables per event

$$\text{Let } \Lambda(\bar{\mathbf{x}}) = \frac{p(\bar{\mathbf{x}} \mid \text{null})}{p(\bar{\mathbf{x}} \mid \text{alternative})}$$

Then the test for excluding the null hypothesis

$\Lambda(\bar{\mathbf{x}}) < \eta$ "likelihood ratio test"

minimizes the false negative rate, β, (or maximizes power) for a given false positive rate, $\alpha(\eta)$

Neyman-Pearson Lemma

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- Likelihood ratio is optimal test statistic

test statistic = function of the data that yields a real number for hypothesis testing

Neyman-Pearson Lemma

- Note that our optimal log likelihood test requires that we use the likelihood ratio as a function of all of our data, **x**.
- If we, e.g., integrate over one variable to obtain the likelihood as a function of a smaller number of variables, then we no longer have an optimal test statistic
- This makes intuitive sense: ignoring data is throwing away information
- We should use all the variables at our disposal.
- When distinguishing between CERN and Florida, we shouldn't integrate out the alligators.

Why Does This Matter?

- If we want to distinguish two hypotheses at the LHC, we can use the same methods
- Instead of using probabilities like P(T | CERN), use, e.g.,

  P ($\bar{\mathbf{x}}$ | SM Higgs) and P ($\bar{\mathbf{x}}$ | Psuedoscalar)

  $\bar{\mathbf{x}}$: measured value of some set of kinematic variables (e.g. momenta of various particles) in an event or events

  "SM Higgs": hypothesis that 125 GeV particle discovered at the LHC is a Standard Model Higgs boson

  "Psuedoscalar": hypothesis that 125 GeV particle discovered at the LHC is a Standard Model Higgs boson

So how do we obtain P ($\bar{\mathbf{x}}$ | SM Higgs)?

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Note that the relative likelihood of obtaining $\bar{x}_1$ or $\bar{x}_2$, assuming the SM Higgs hypothesis is given by

$$P(\bar{x}_1 \mid \text{SM Higgs}) \,/\, P(\bar{x}_2 \mid \text{SM Higgs}) =$$

$$\left.\frac{d\sigma}{d\bar{x}}\right|_{\bar{x}_1} \Big/ \left.\frac{d\sigma}{d\bar{x}}\right|_{\bar{x}_2}$$

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But $\frac{d\sigma}{d\bar{x}}$ is not a probability, as

$$\int \frac{d\sigma}{d\bar{x}} = \sigma$$

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But $\frac{d\sigma}{d\bar{x}}$ is not a probability, as

$$\int \frac{d\sigma}{d\bar{x}} = \sigma$$

So we must normalize by the total cross section (after acceptances, etc.):

$$P(\bar{x}_1 \mid \text{SM Higgs}) = \left.\frac{d\sigma}{d\bar{x}}\right|_{\bar{x}_1} \Big/ \sigma$$

Probability of observing observed momenta as a function of model parameters, α

Sum over initial states

pdfs

$$\mathcal{P}(\mathbf{p}_i^{\text{vis}}|\alpha) = \frac{1}{\sigma(\alpha)} \sum_{k,l} \int dx_1 dx_2 \frac{f_k(x_1) f_l(x_2)}{2 s x_1 x_2} \times \left[\prod_{j \in \text{inv.}} \int \frac{d^3 p_j}{(2\pi)^3 2E_j} \right] |\mathcal{M}_{kl}(p_i^{\text{vis}}, p_j; \alpha)|^2$$

Normalization

Squared Matrix Element

Integrate over invisible particle momenta

Take into account finite detector resolution using "transfer functions": more integrals

Probability of observing observed momenta as a function of model parameters, α

Sum over initial states

pdfs

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Normalization

Squared Matrix Element

Integrate over invisible particle momenta

Take into account finite detector resolution using “transfer functions”: more integrals

This gives us P ($\bar{\mathbf{x}}$ | SM Higgs) for one event

Events are independent and “identical” so likelihood for all events is product of P ($\bar{\mathbf{x}}_i$ | SM Higgs)

The Matrix Element Method

- We have seen that the likelihood ratio is the optimal way to distinguish between hypotheses
- We have also seen that we can, at least in principle, calculate this likelihood ratio from the differential cross section
- The use of the likelihood ratio, calculated in the manner described, is the "Matrix Element Method" (MEM)
- In addition to the use in Higgs physics, which I will discuss, the MEM has seen significant use, e. g., in top studies (mass measurement, single top, etc.) and B physics

The Matrix Element Method

- Its great advantage (besides optimality) is transparency
- Not a black box; based on differential cross section which (obviously) is connected to the underlying physics
- Great disadvantage is that it is computationally intensive due to integrations over particle momenta
- This is not as important in leptonic final states, so 4ℓ avoids this problem
- In general, other MVAs such as BDTs or NNs achieve similar sensitivity with less computation (though also less transparency)

The Matrix Element Method

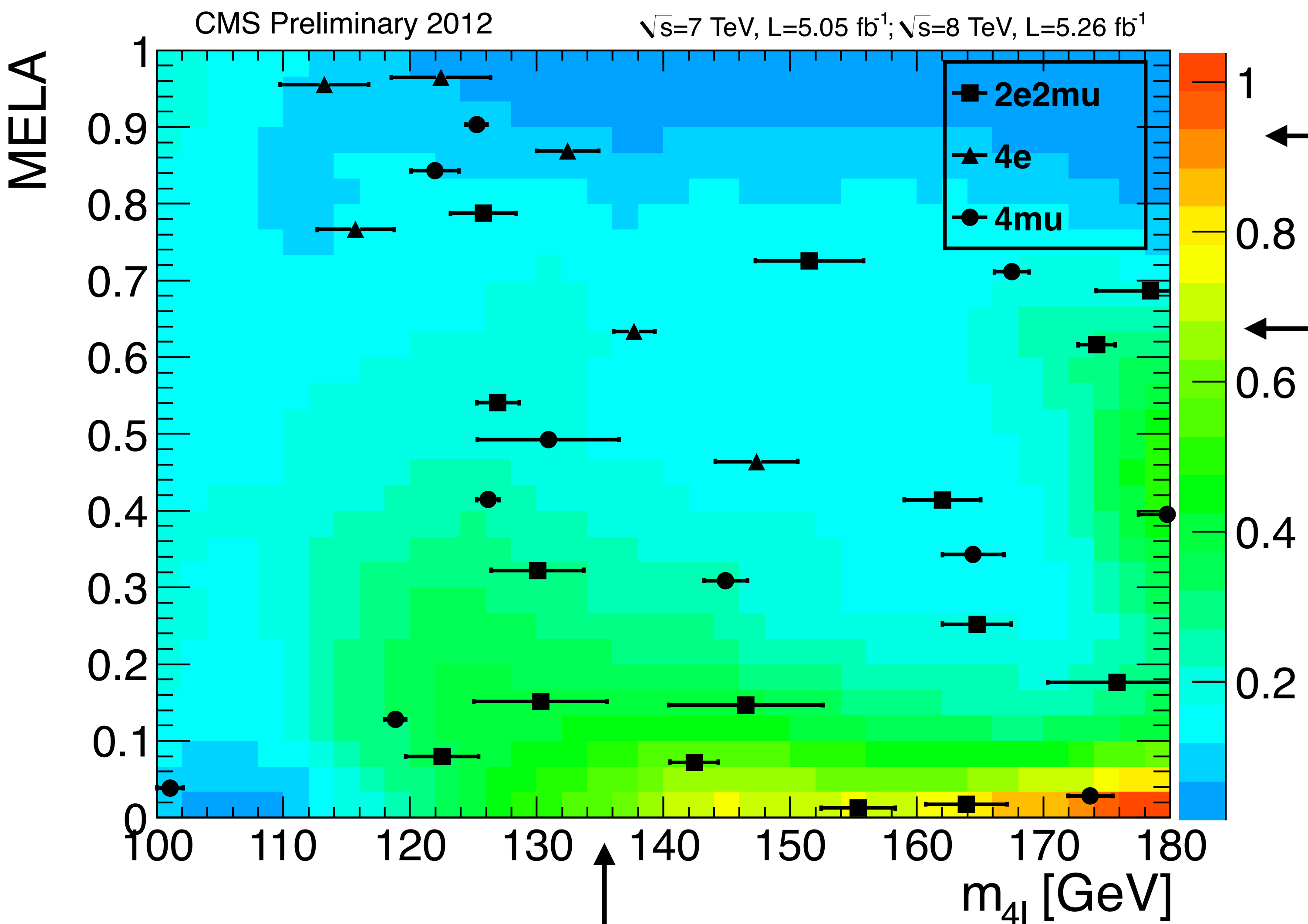
- Often MEM probabilities, P($\bar{x}$ | hypothesis) are used to construct an event-by-event kinematic variable/ discriminant

- Lose Neyman-Pearson optimality

- Much easier to include the effects of reducible backgrounds, etc.

- Examples MELA, MEKD, etc.

The Matrix Element Method



Data at discovery, CMS-PAS-HIG-12-016

Background likelihood above 2 M_Z (not pictured :)
at Higgs discovery time from
JG, Kumar, Low, and Vega-Morales (2011)

- Gao, Gritsan, Guo, Melnikov, Schulze, Tran (2010)
- Bolognesi, Gao, Gritsan, Melnikov, Schulze, Tran, Whitbeck (2012)
- CMS 1212.6639, 1312.5353, etc.

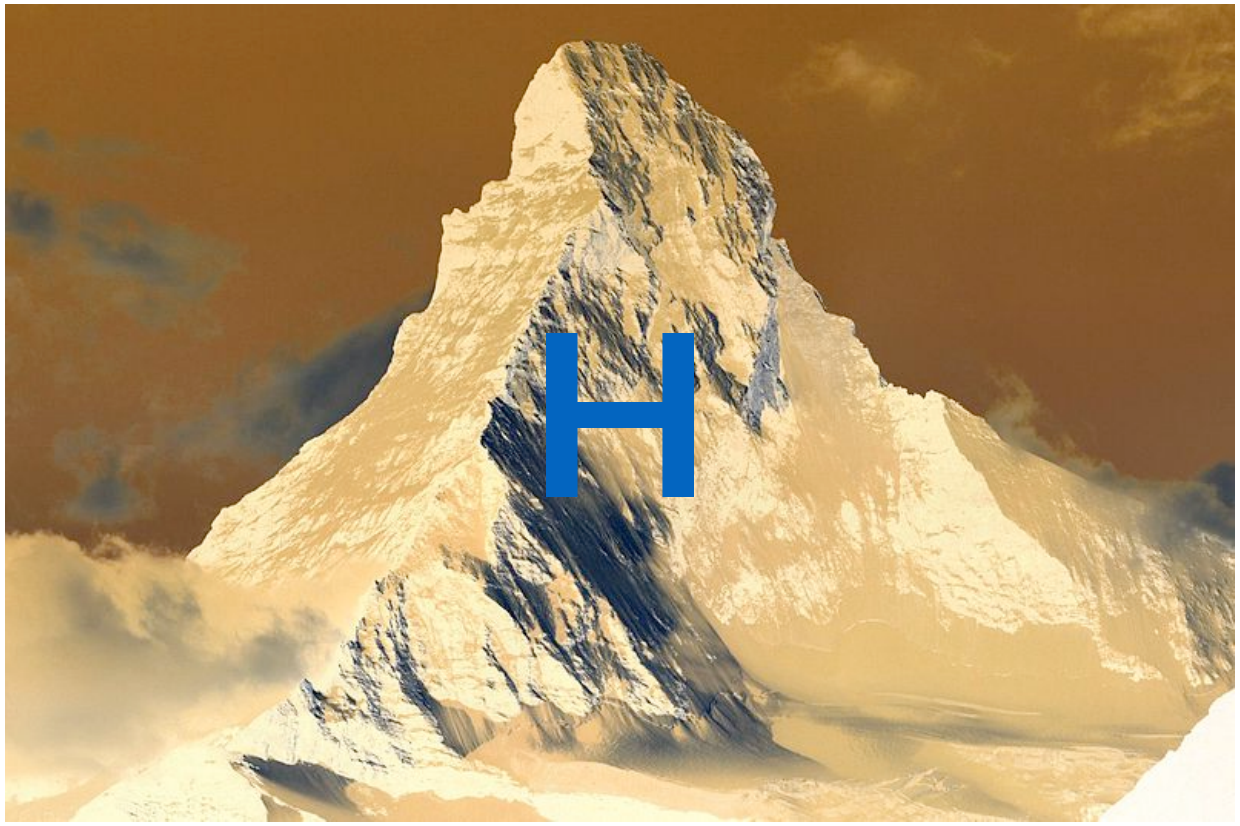
Phys.Rev. D87 (2013) 055006
Avery, Bourilkov, Chen, Cheng, Drozdetskiy, JG, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, and Snowball.
(1210.0896)

MELA: $P_s/(P_s + P_b)$

MEKD: ln (P_s/P_b)

MEM for Higgs Properties Measurement

or



Goal: Measure XZZ Couplings

Mainly Following

- Phys.Rev. D87 (2013) 055006
Avery, Bourilkov, Chen, Cheng, Drozdetskiy, JG, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, and Snowball.
(1210.0896)

- PRL 111 (2013) 041801
JG, Lykken, Matchev, Mrenna, and Park.
(1304.4936)

- arXiv:**1310.1397** (accepted by PRD)
Chen, Cheng, Gainer, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, and Snowball

- In progress (arXiv: **1401.SOON)**
JG, Lykken, Matchev, Mrenna, and Park

(I'm going to refer to the ≈125 GeV scalar as "X", since we are trying to determine whether it really is an "H")

but also

- Gao, Gritsan, Guo, Melnikov, Schulze, Tran (2010)
- De Rujula, Lykken, Pierini, Rogan, Spiropulu (2010)
- Bolognesi, Gao, Gritsan, Melnikov, Schulze, Tran, Whitbeck (2012)
- Artoisenet, de Aquino, Demartin, Frixione, Maltoni, Mandal, Mathews, Mawatari, Ravindran, Seth, Torrielli, Zaro, 2013
- …
- Many, more, see e.g. the bibliography of the journal version of 1310.1397

I'm not going to say anything about other channels.

Experimental Status

- Pure pseudoscalar couplings definitively ruled out
- Pure CP-even non-(LO)-SM coupling strongly constrained
- CMS(1312.5353) used MELA framework
(with matrix elements from JHUGEN— same refs as MELA),
MEKD for verification
- $f_{a3} < 0.51$ at 95% confidence level $$f_{a3} = \frac{|a_3|^2\sigma_3}{|a_1|^2\sigma_1 + |a_2|^2\sigma_2 + |a_3|^2\sigma_3}$$ with a_2=0
- ATLAS (ATLAS-CONF-2013-013) used MELA and BDT
- See also projections in ATLAS-PHYS-PUB-2013-013

Experimental Status

J^P model	J^P production	Expected ($\mu = 1$)	Obs. 0^+	Obs. J^P	CL_s
0^-	any	2.4σ (2.7σ)	-0.9σ	$+3.6\sigma$	0.09%
0^+_h	any	1.7σ (1.9σ)	-0.0σ	$+1.8\sigma$	7.1%
1^-	$q\bar{q} \to X$	2.6σ (2.7σ)	-1.4σ	$+4.8\sigma$	0.001%
1^-	any	2.6σ (2.6σ)	-1.7σ	$+4.9\sigma$	0.001%
1^+	$q\bar{q} \to X$	2.1σ (2.3σ)	-1.5σ	$+4.1\sigma$	0.03%
1^+	any	2.0σ (2.1σ)	-1.9σ	$+4.5\sigma$	0.01%
2^+_m	$gg \to X$	1.7σ (1.8σ)	-0.8σ	$+2.6\sigma$	1.9%
2^+_m	$q\bar{q} \to X$	1.6σ (1.7σ)	-1.6σ	$+3.6\sigma$	0.03%
2^+_m	any	1.5σ (1.5σ)	-1.3σ	$+3.0\sigma$	1.4%
2^+_b	$gg \to X$	1.6σ (1.8σ)	-1.2σ	$+3.1\sigma$	0.9%
2^+_h	$gg \to X$	3.7σ (4.0σ)	$+1.8\sigma$	$+1.9\sigma$	3.1%
2^-_h	$gg \to X$	4.0σ (4.5σ)	$+1.0\sigma$	$+3.0\sigma$	1.7%

CMS, 1312.5353

		BDT analysis				J^P-MELA analysis			
		tested J^P for an assumed 0^+		tested 0^+ for an assumed J^P	CL_S	tested J^P for an assumed 0^+		tested 0^+ for an assumed J^P	CL_S
		expected	observed	observed*		expected	observed	observed*	
0^-	p_0	0.0037	0.015	0.31	0.022	0.0011	0.0022	0.40	0.004
1^+	p_0	0.0016	0.001	0.55	0.002	0.0031	0.0028	0.51	0.006
1^-	p_0	0.0038	0.051	0.15	0.060	0.0010	0.027	0.11	0.031
2^+_m	p_0	0.092	0.079	0.53	0.168	0.064	0.11	0.38	0.182
2^-	p_0	0.0053	0.25	0.034	0.258	0.0032	0.11	0.08	0.116

ATLAS-CONF-2013-013

Theory

Theory

(Spin 0)

- To use the MEM, to distinguish between signal hypotheses, we need to specify these hypotheses precisely

- To specify XZZ coupling use either

    - EFT with all terms up to a specified mass dimension

    - Amplitude with all structures up to a specified mass dimension

In both cases there is freedom in how couplings are parameterized.

1401.SOON

EFT

$$\mathcal{L} \supset \sum_{i=1}^{5} \kappa_i \mathcal{O}_i = -\kappa_1 \frac{m_Z^2}{v} X Z_\mu Z^\mu - \frac{\kappa_2}{2v} X F_{\mu\nu} F^{\mu\nu} - \frac{\kappa_3}{2v} X F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\kappa_4 m_Z^2}{M_X^2 v} \Box X Z_\mu Z^\mu + \frac{2\kappa_5}{v} X Z_\mu \Box Z^\mu$$

- **Most general EFT with all terms with mass dimension ≤ 5 (+1 if we assume Higgs mechanism)**
- **I'll say more about all of these operators later.**

1401.SOON

Amplitude

$$\mathcal{A} = -\frac{2i}{v}\epsilon_1^{*\mu}\epsilon_2^{*\nu}\left(a_1 g_{\mu\nu} + a_2 p_{1\nu}p_{2\mu} + a_3\epsilon_{\mu\nu\rho\sigma}\, p_1^\rho p_2^\sigma\right)$$

$$a_1(p_1, p_2) = a_{10} + \frac{1}{\Lambda^2}\left(a_{11}(p_1 \cdot p_2) + a_{13}(p_1^2 + p_2^2)\right)$$

cf. analogous expressions in Bolognesi et al. (2012) and Gao et al. (2010)

- Most general Lorentz invariant, Bose symmetric coupling of scalar, X, to Z bosons with momenta p_1 and p_2, containing only terms with two or fewer powers of momenta

1401.SOON

Translation

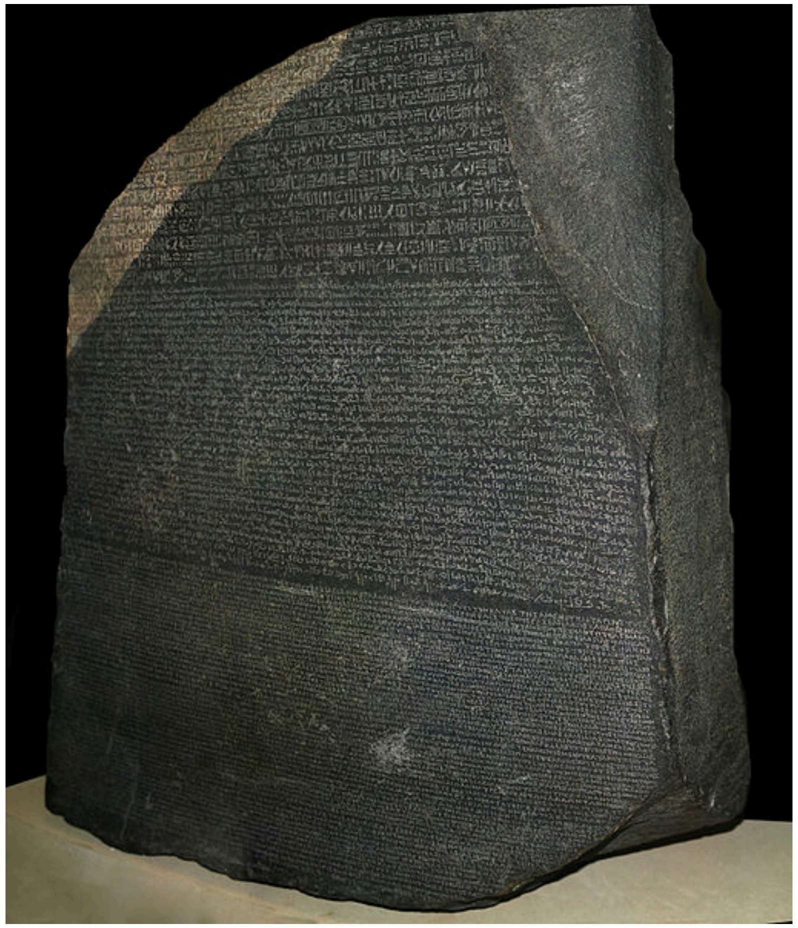
$$i\,\epsilon_1^*\cdot\epsilon_2^* \quad\Longleftrightarrow\quad -\frac{1}{2}XZ_\mu Z^\mu$$

$$i\,(p_1\cdot p_2)(\epsilon_1^*\cdot\epsilon_2^*) \quad\Longleftrightarrow\quad \frac{1}{2}X\partial_\mu Z_\nu\partial^\mu Z^\nu$$

$$i\,(p_1\cdot\epsilon_2^*)(p_2\cdot\epsilon_1^*) \quad\Longleftrightarrow\quad \frac{1}{2}X\partial_\mu Z_\nu\partial^\nu Z^\mu$$

$$i\,\epsilon_{\mu\nu\rho\sigma}\epsilon_1^{*,\mu}\epsilon_2^{*,\nu}p_1^\rho p_2^\sigma \quad\Longleftrightarrow\quad -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\mu Z^\nu\partial^\rho Z^\sigma$$

$$i\,(p_1^2+p_2^2)(\epsilon_1^*\cdot\epsilon_2^*) \quad\Longleftrightarrow\quad XZ_\mu\Box Z^\mu$$



$$a_1 \equiv \kappa_1 m_z^2 + (2(m_Z^2/m_X^2)\kappa_4 - \kappa_2)p_1\cdot p_2 + ((m_Z^2/m_X^2)\kappa_4+\kappa_5)(p_1^2+p_2^2),$$

$$a_2 \equiv \kappa_2,$$

$$a_3 \equiv \kappa_3.$$

1401.SOON

Keep it Real?

- If the couplings described by the Lagrangian are tree-level
  or
  if the couplings described by the Lagrangian result from loops involving heavy particles
  then
  the couplings must be real.

- However, loop effects involving light ($\lesssim$ 63 GeV) particles give complex contributions to the amplitude (optical theorem)

1310.1397

Keep it Real?

- Generically we expect loop contributions to be small:
  - Large loop contribution (for loop-induced coupling to generate observed HZZ coupling assuming SM-ish production rates)* must come from non-SM state
  - Large loop contribution from that state necessarily implies a large partial width for Higgs decay that non-SM state and hence a large total Higgs width**
  - non-SM state must also couple to Z, strongly constrained if mass $\lesssim M_Z/2$
- Longer discussion in 1310.1397

*Subdominant loop corrections are fine and in fact expected.

**Large loop-induced couplings are easier to reconcile with an SM-like Higgs total width if there is a large multiplicity of new states.

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How General?

$$\mathcal{L} \supset \sum_{i=1}^{5} \kappa_i \mathcal{O}_i = -\kappa_1 \frac{m_Z^2}{v} X Z_\mu Z^\mu - \frac{\kappa_2}{2v} X F_{\mu\nu} F^{\mu\nu} - \frac{\kappa_3}{2v} X F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\kappa_4 m_Z^2}{M_X^2 v} \Box X Z_\mu Z^\mu + \frac{2\kappa_5}{v} X Z_\mu \Box Z^\mu$$

- Can constrain all couplings
- Makes sense to narrow focus somewhat in early running based on sensitivity, likelihood of deviations, etc.

1401.SOON

Simplify Lagrangian

Keeping only the lowest dimensional operators with each of three symmetry properties:

1. CP-even, (naively) gauge invariance violating
2. CP-even, gauge invariant
3. CP-odd, gauge invariant

$$\mathcal{L} = -X\left[\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu + \frac{\kappa_2}{2v} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa_3}{2v} F_{\mu\nu} \tilde{F}^{\mu\nu}\right]$$

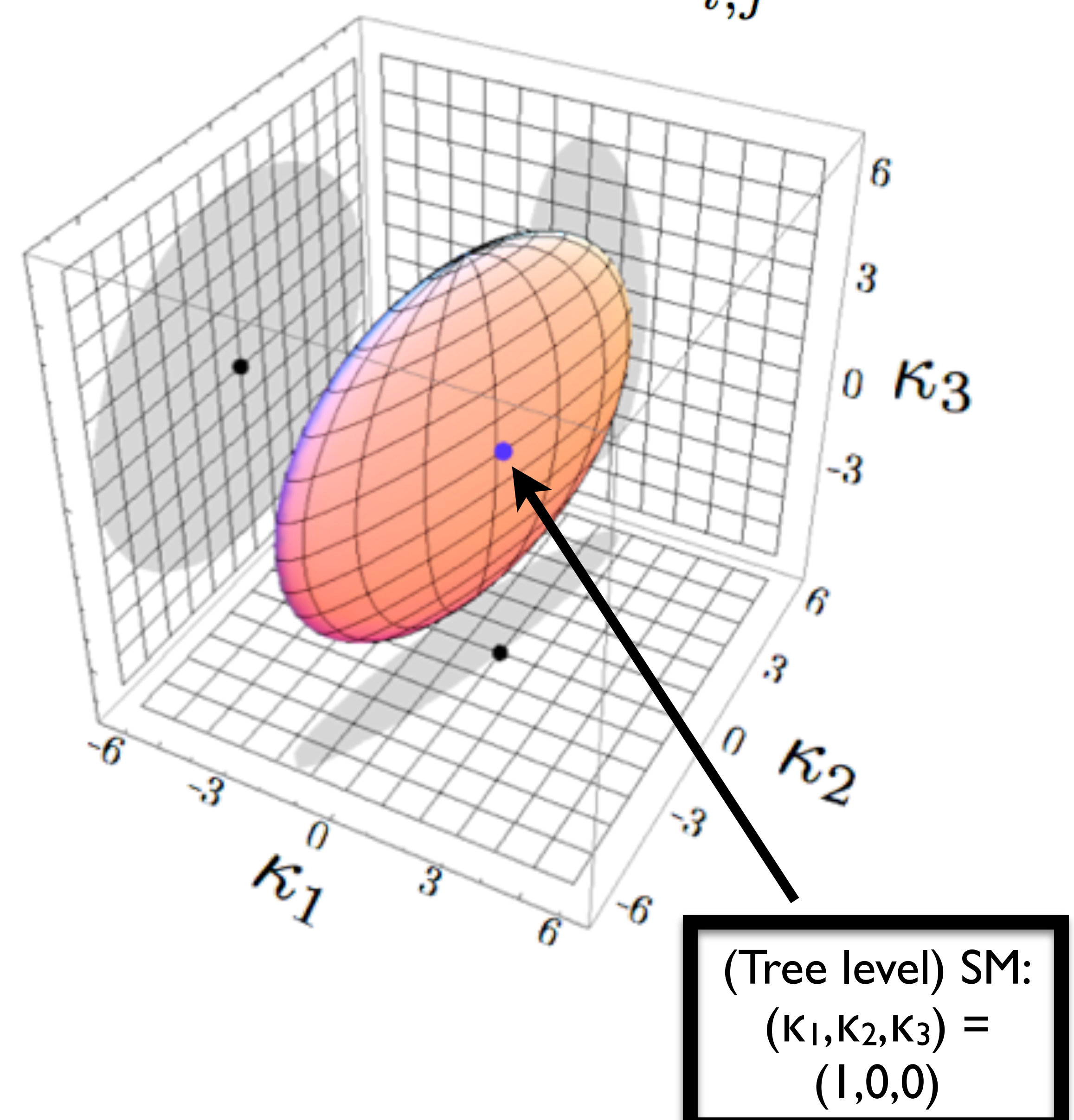
These operators contribute to the three Lorentz structures in the amplitude, a_1, a_2, and a_3.

1304.4936

Apply Rate Constraint

- In terms of these three real couplings, we can write the partial width for a point in parameter space as
- Measured rate implies correlations among couplings
- Defines an ellipsoidal "pancake" in κ space
- But shape stays the same
- Separates rate— leaves us with two parameters that are independent of rate
- Helpful when maximizing likelihoods when using the MEM, as likelihoods involve normalized probabilities

$$\Gamma(X \to ZZ) = \Gamma_{SM} \sum_{i,j} \gamma_{ij} \kappa_i \kappa_j$$



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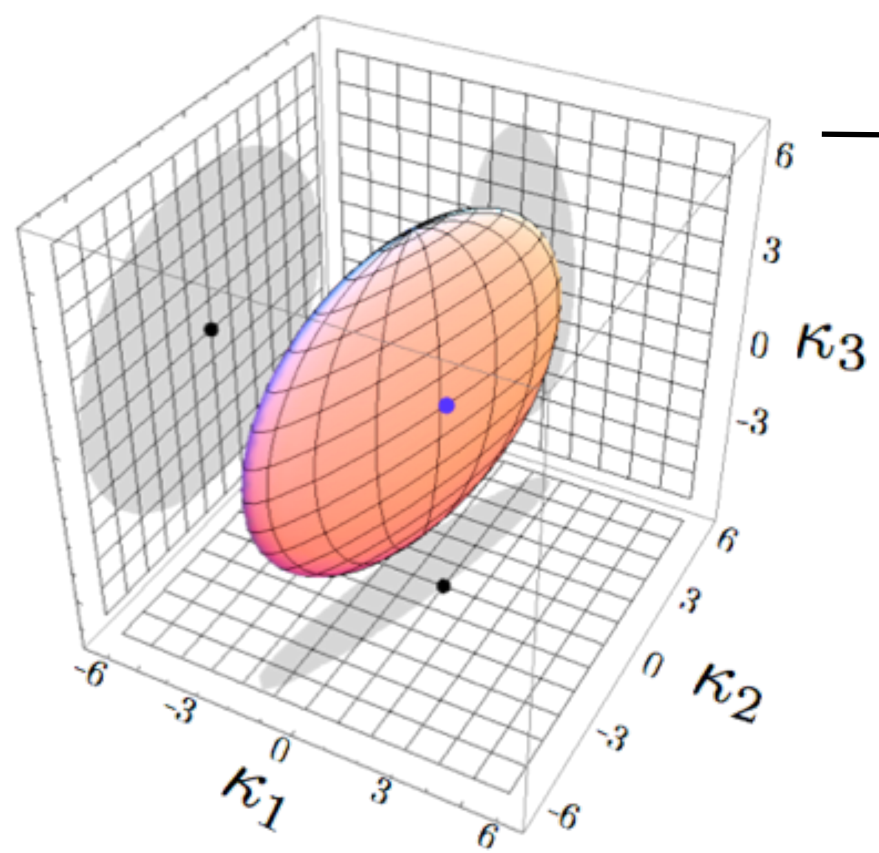
Parametrize the Pancake

- One can describe points on the equal rate "pancake" using, e.g., spherical coordinates
- Alternatively one can change variables to deform the pancake into an "equal rate sphere"
- This involves a linear transformation:

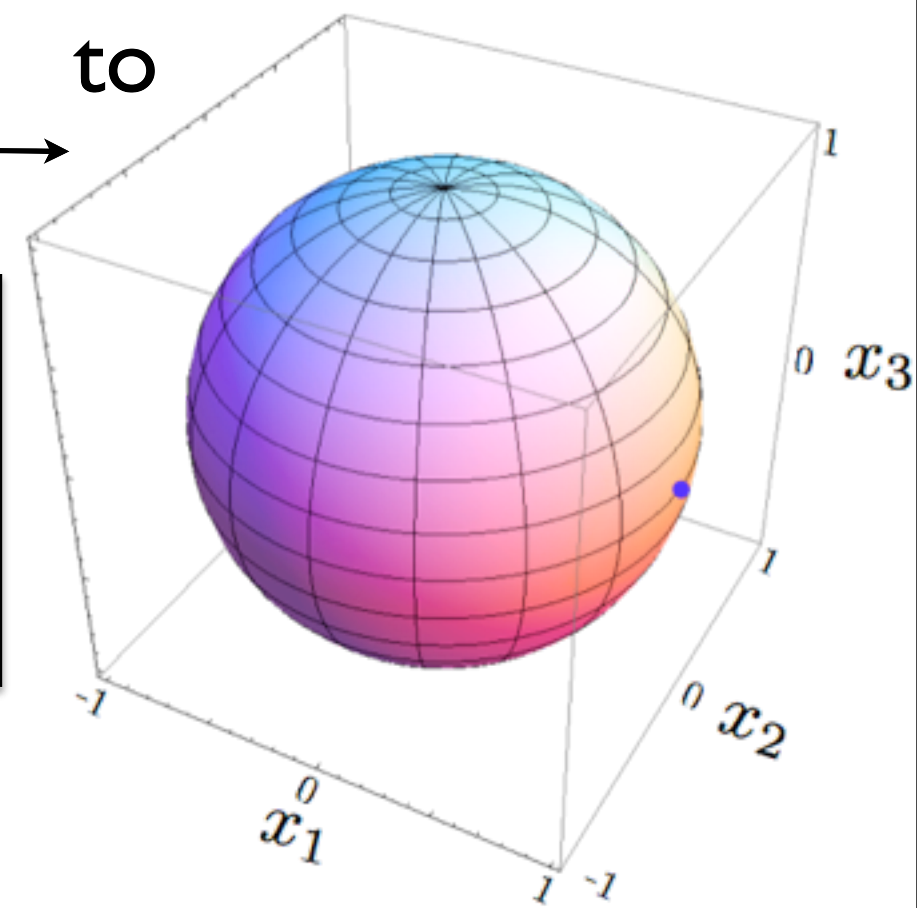


We go from to

using

$$
\begin{aligned}
x_1 &= \kappa_1 - 0.25\ \kappa_2 \\
x_2 &= 0.17\ \kappa_2 \\
x_3 &= 0.19\ \kappa_3
\end{aligned}
$$

DF, before cuts



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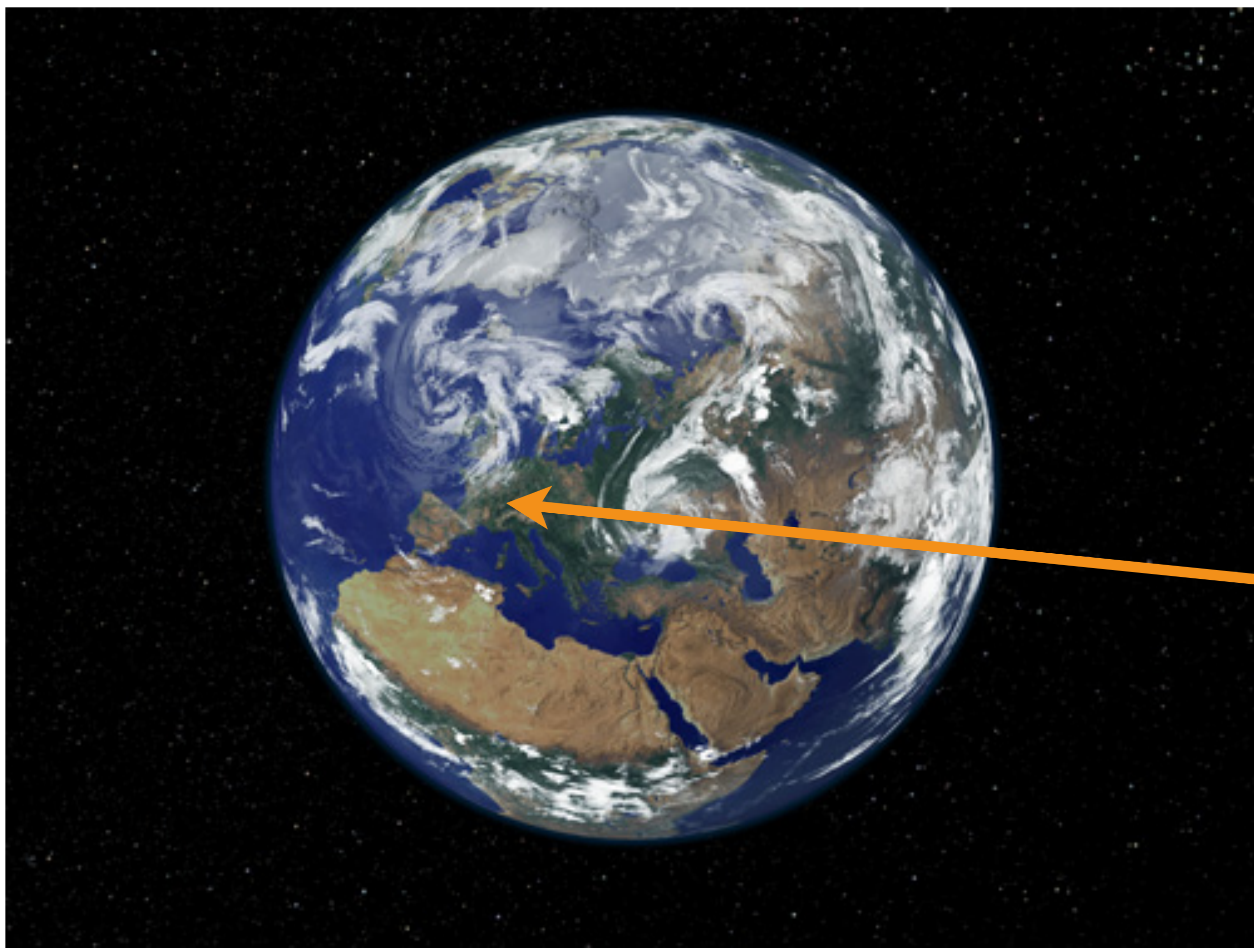
Geolocating the Higgs



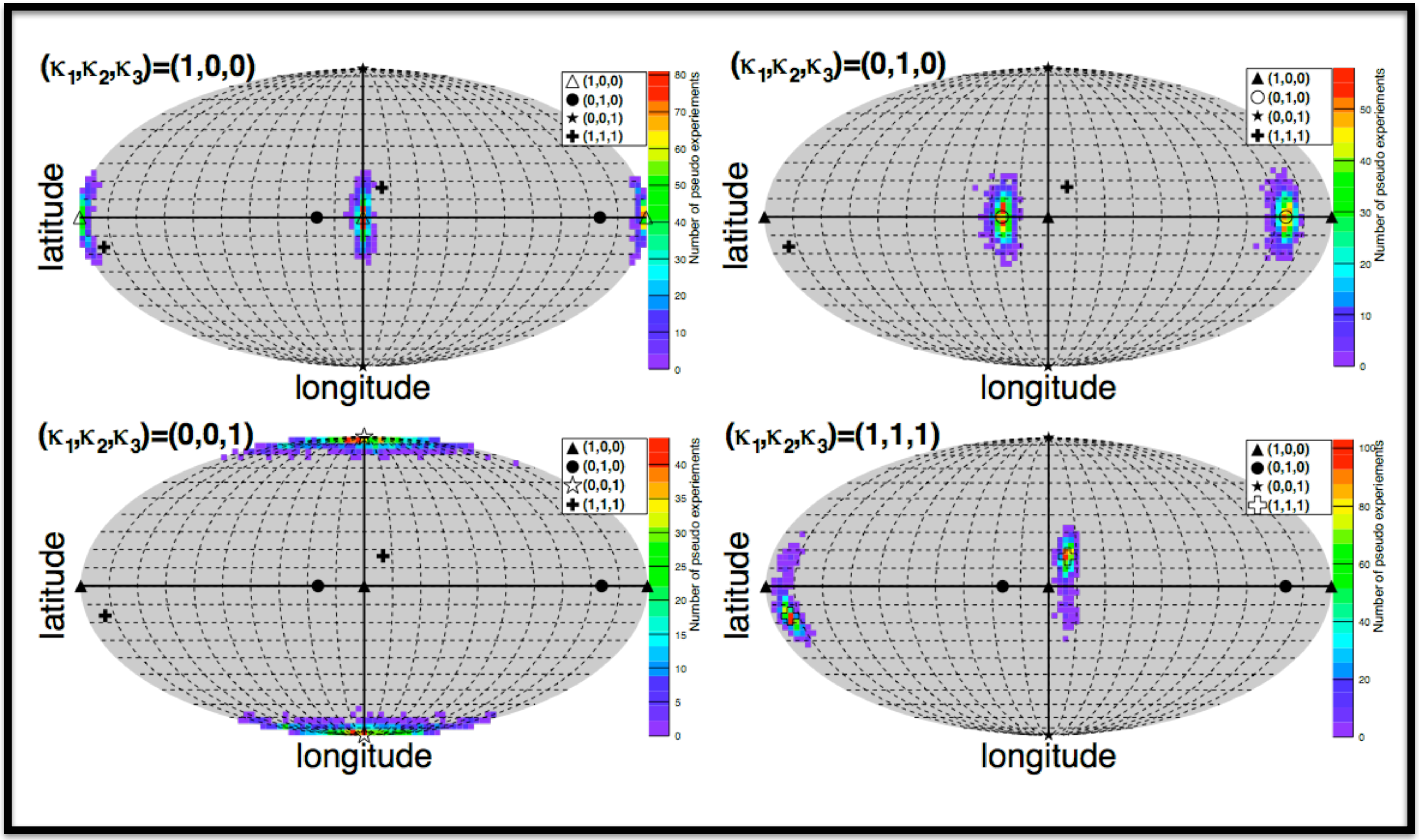
Any given value of
$(\kappa_1, \kappa_2, \kappa_3)$,
corresponding to a given rate,
maps to a point on the sphere

"Measuring parameters of Higgs-ZZ couplings = "Geolocating the Higgs"

1304.4936

Geolocation Example

- We illustrate the use of the sphere for displaying results with a toy analysis
- We generate 1000 pseudoexperiments
  - 300 DF signal events for each of 4 benchmark points (~300 fb-1 at 14 TeV): three pure states and one completely mixed state
  - Impose cuts (pT, |η|, MZ1, MZ2)
  - Find the point on the sphere that maximizes the likelihood for each pseudoexperiment and plot



Note: a point and its antipode are effectively equivalent

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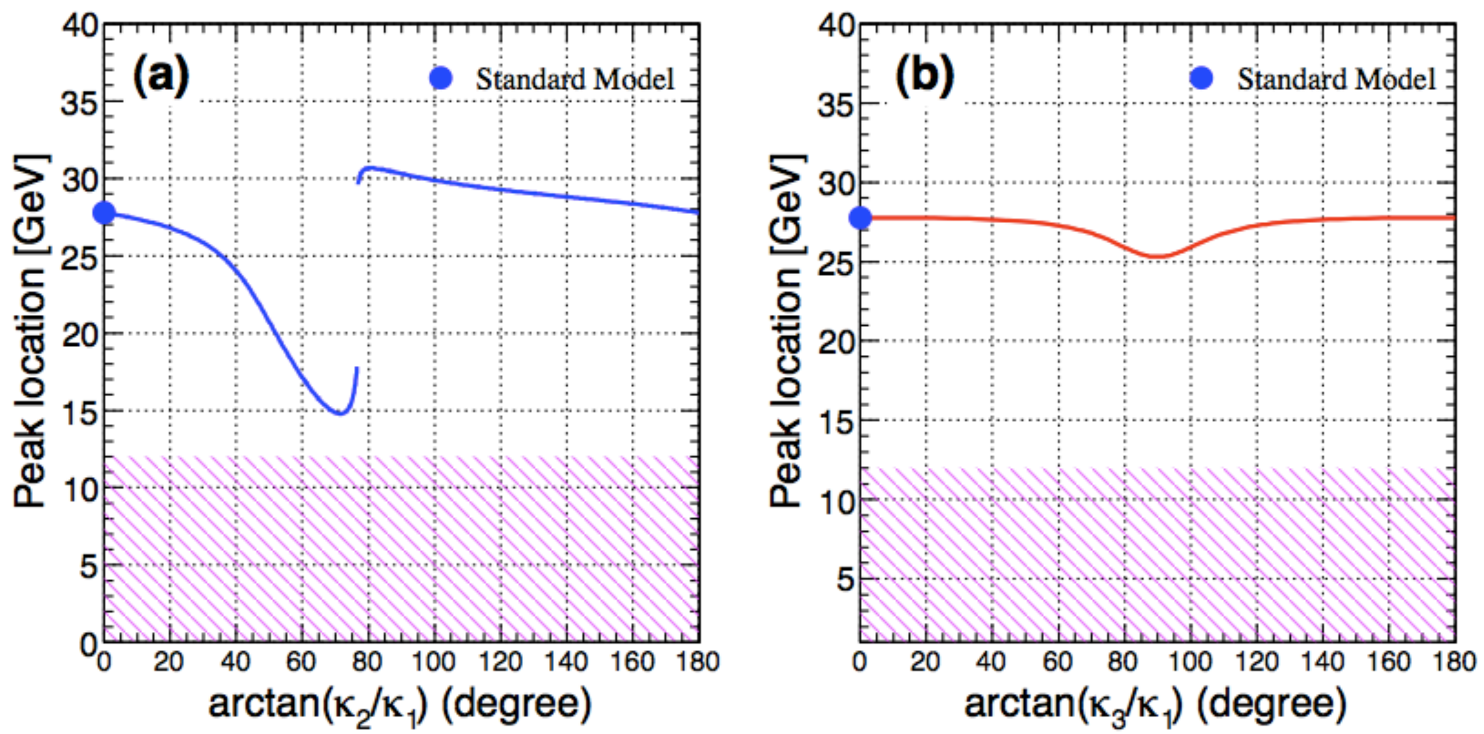
Importance of Interference

One goal of the geolocating framework is to have measurements with multiple non-zero couplings, due to the potential importance of interference between operators.



Peak of M_{Z2} distribution displays “first order phase transition” from κ_1-κ_2 interference , no such feature when considering κ_1 and κ_3

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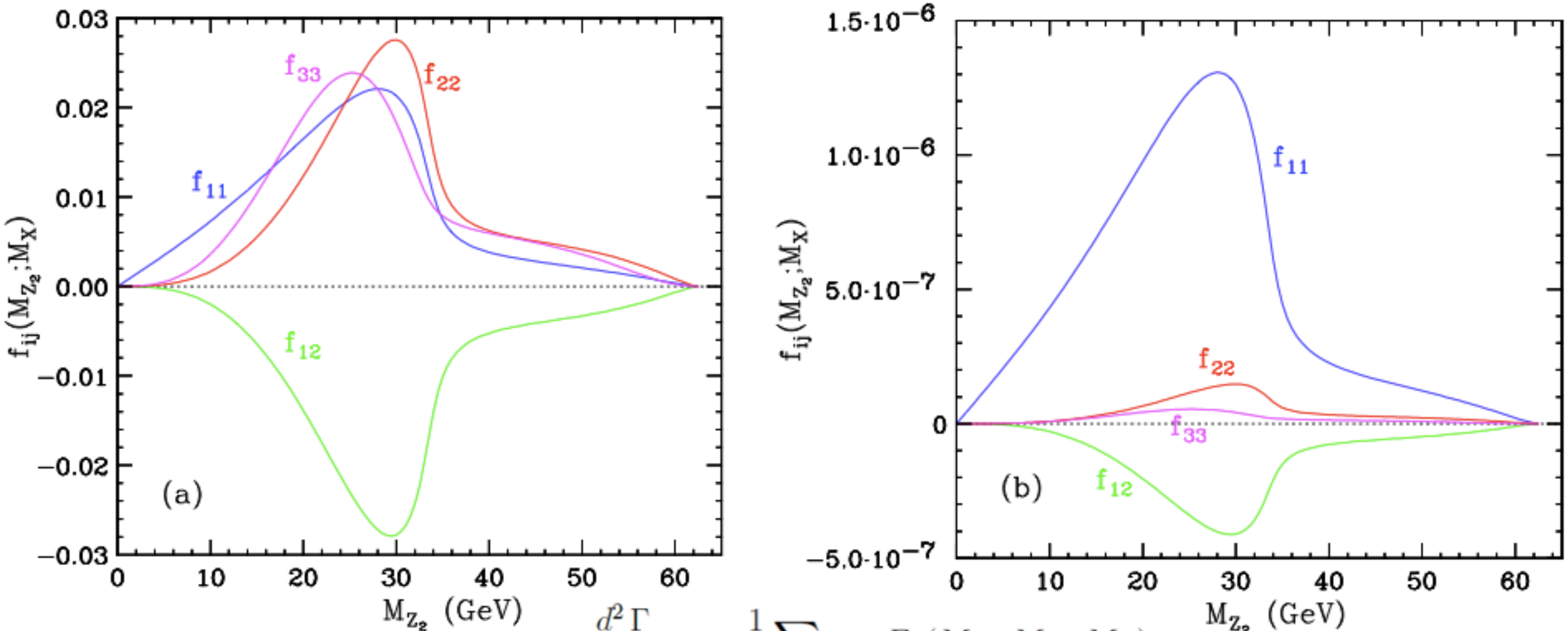
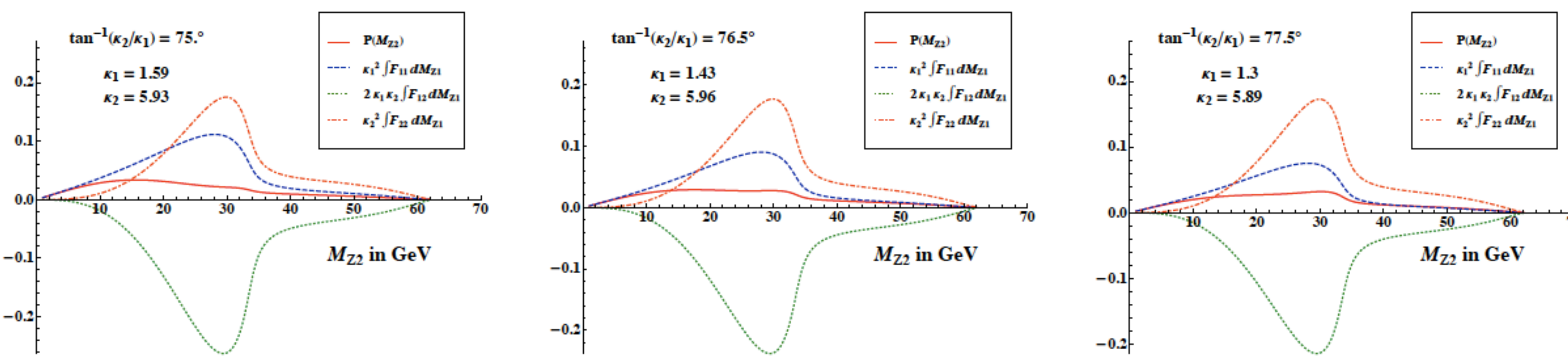


$$\frac{d^2\Gamma}{dM_{Z_1}dM_{Z_2}} = \frac{1}{v}\sum_{i,j}\kappa_i\kappa_j F_{ij}(M_{Z_1}, M_{Z_2}; M_X)$$

obtain f_{ij} from integrating F_{ij} over M_{Z1}



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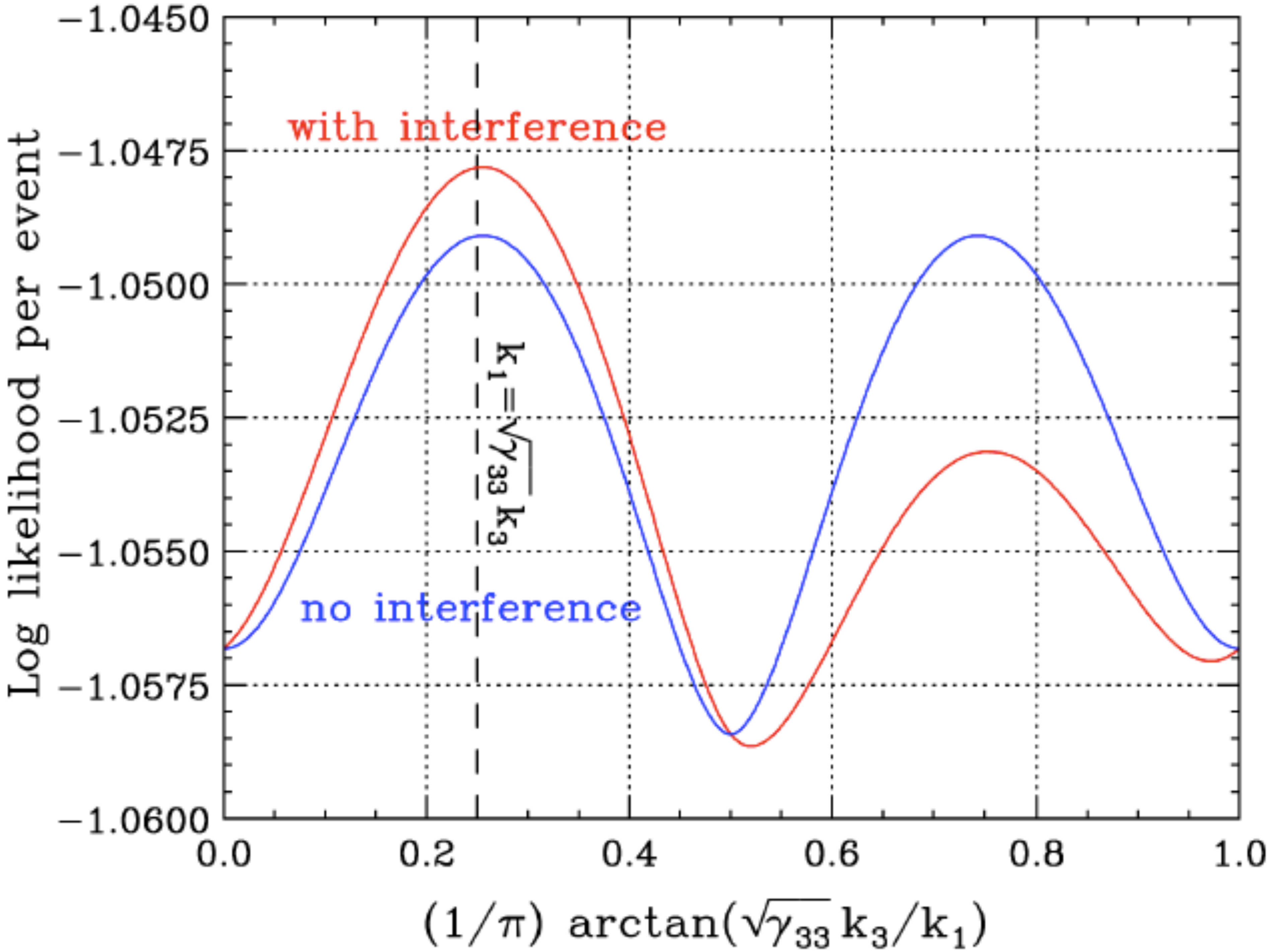
Importance of Interference



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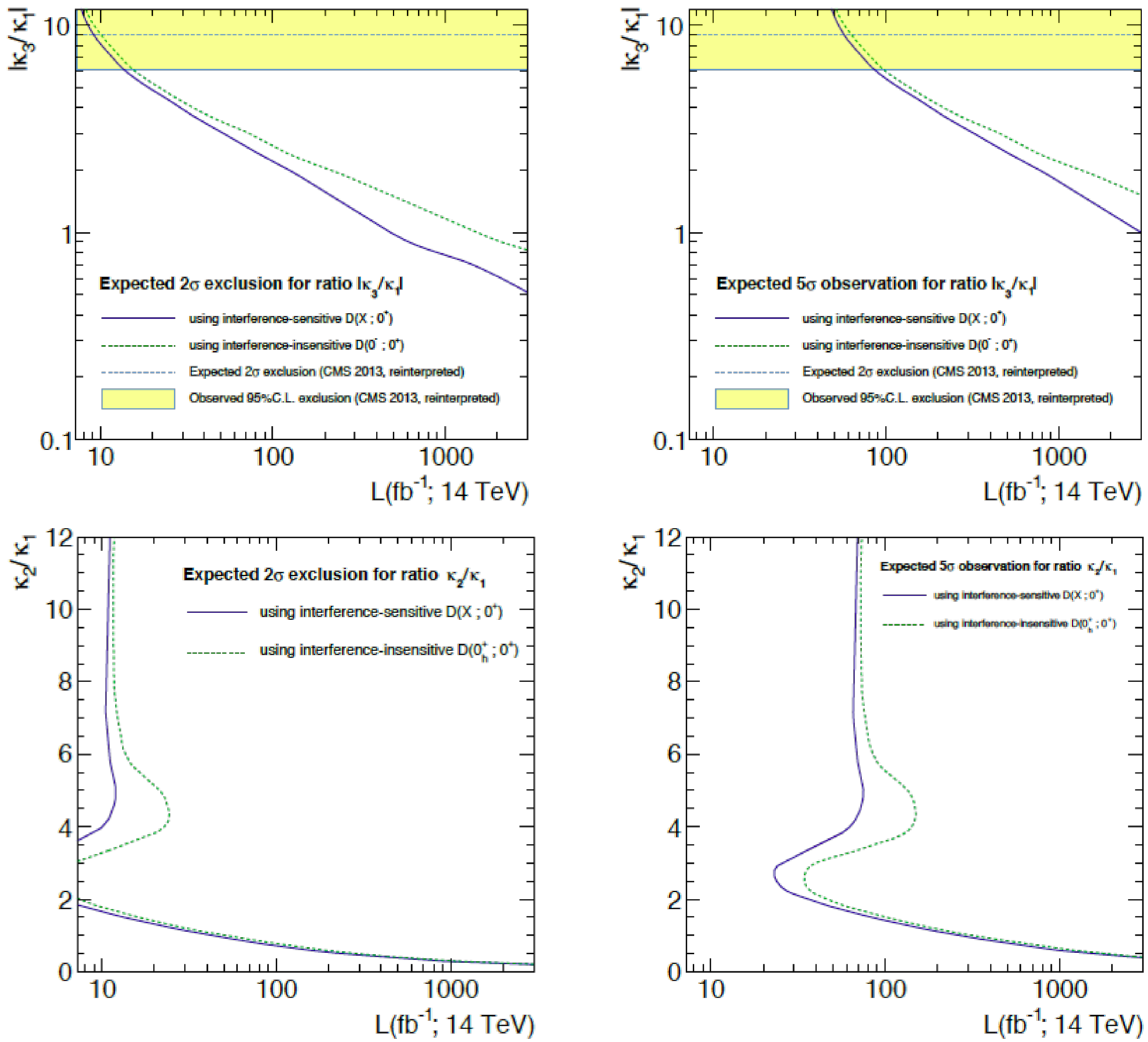
Projections



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The Other Operators

- Remember, we started with the EFT Lagrangian

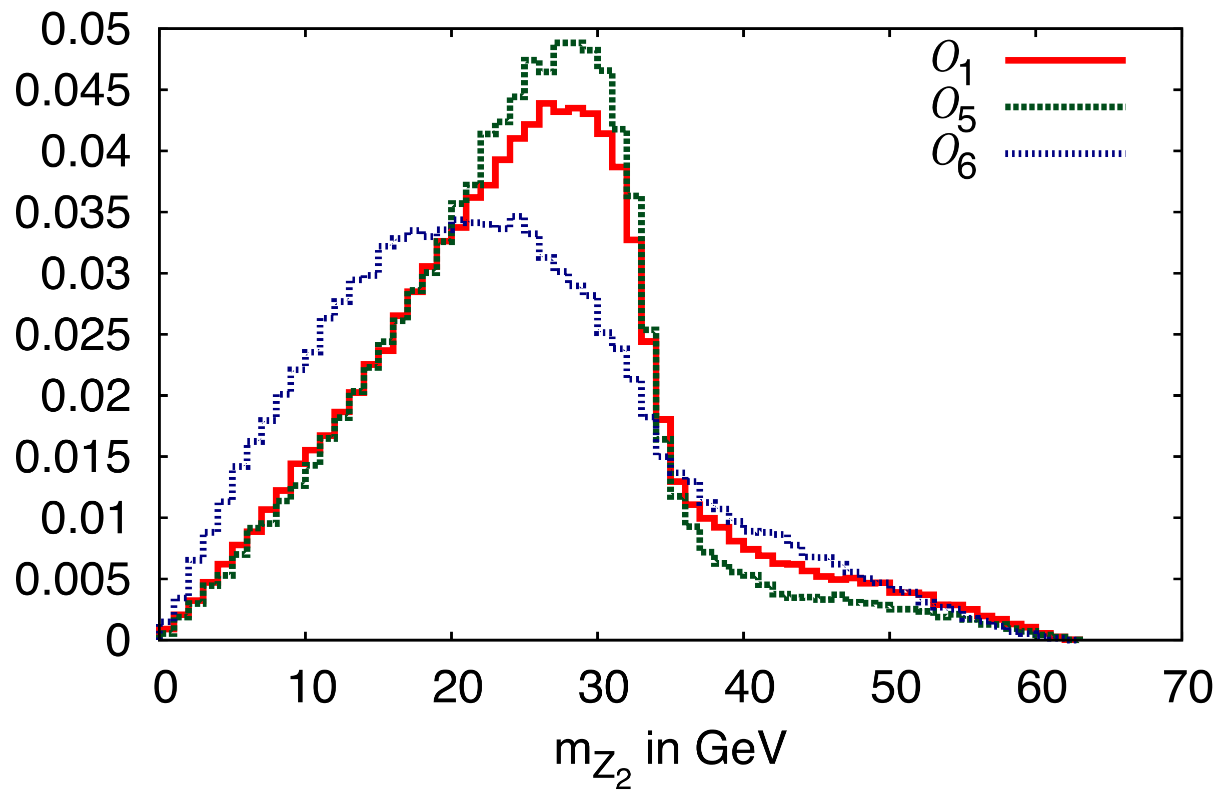
$$\mathcal{L} \supset \sum_{i=1}^{5} \kappa_i \mathcal{O}_i = -\kappa_1 \frac{m_Z^2}{v} X Z_\mu Z^\mu - \frac{\kappa_2}{2v} X F_{\mu\nu} F^{\mu\nu} - \frac{\kappa_3}{2v} X F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\kappa_4 m_Z^2}{M_X^2 v} \Box X Z_\mu Z^\mu + \frac{2\kappa_5}{v} X Z_\mu \Box Z^\mu$$

but only considered the first three operators.

- κ_5 operator can also be written as

$$Z_\nu \partial_\mu Z^{\mu\nu}$$

- Its main effect is a subtle modification of the M_{Z2} distribution



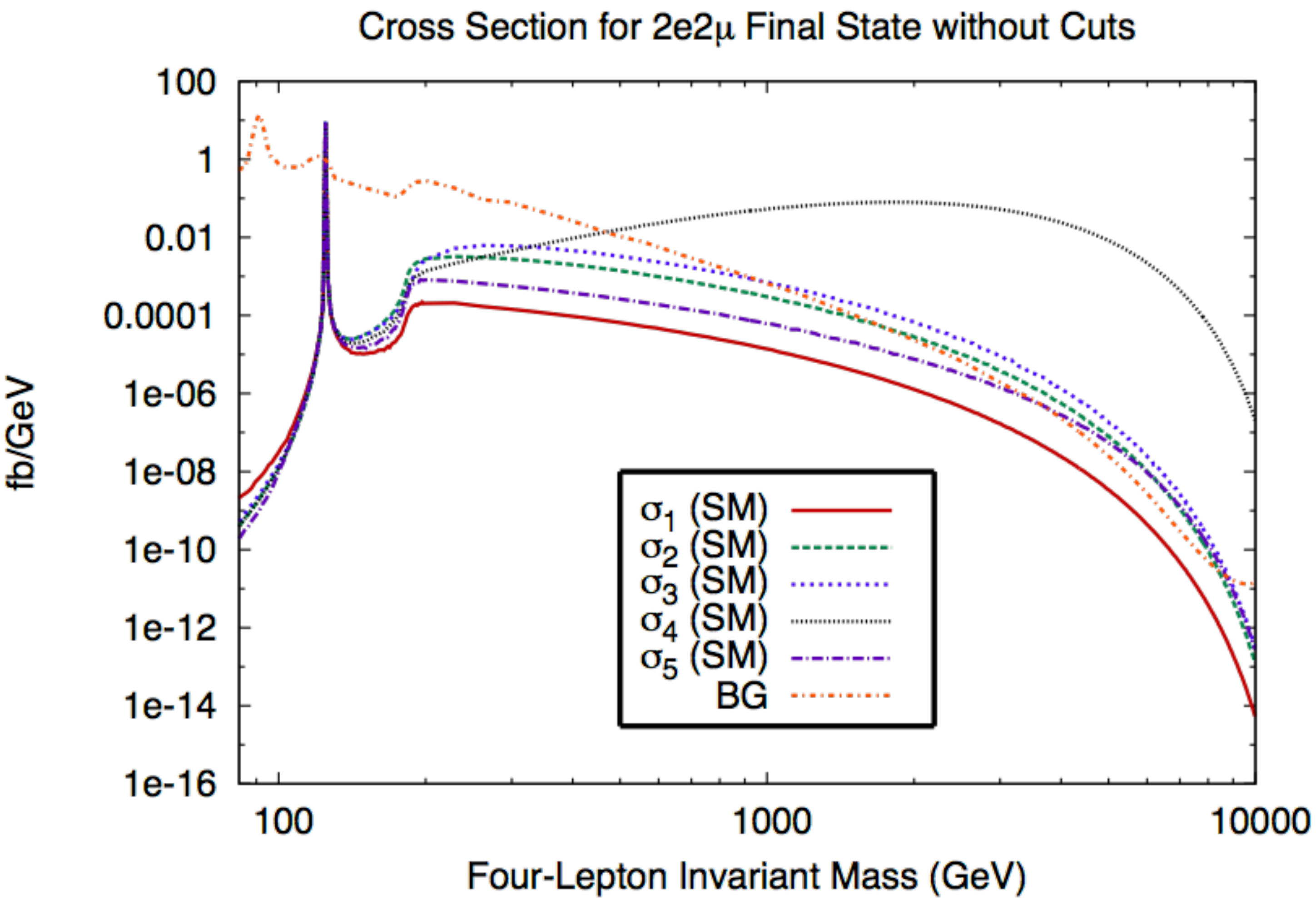
1401.SOON

The Other Operators

- The κ_4 operator $\Box X Z_\mu Z^\mu$

  is indistinguishable from the κ_1 (SM) operator on the Higgs peak

- Much larger cross section for off-shell Higgs production



- **Strong unitarity bounds on this operator**

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The Other Operators

	$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$	$\mathcal{O}_4$	$\mathcal{O}_5$	$\mathcal{O}_6$
$2\langle\Delta \log \mathcal{L}\rangle_{SM}$	0	-0.747	-1.017	0	-0.178	-0.503
Events for 3σ Limit	———	12.0	8.85	———	50.5	17.9

- To quantify the sensitivity of various operators, we can calculate the difference in log likelihood, with respect to the SM, per signal event.
- Since in the high statistics limit, $2\ \Delta \log \mathscr{L} \sim \Delta\ \chi^2$, we can estimate the number of events needed to get a 3σ limit, (assuming true hypothesis is SM) by dividing 3^2 by twice the average per-event difference in log likelihood
- Here we show values for various pure operator couplings, normalized to give the SM rate on peak.
- These values are on the small side because we are assuming a perfect detector and turning off backgrounds.
- Still, they suggest that the easiest pure state to exclude is the pure pseudo scalar, followed by the k_2 coupling, followed by the k_5 coupling. (k_4 can not be distinguished from the SM using only on-peak events.)

1401.SOON

Conclusions

- The Matrix Element Method played an important role in Higgs discovery in the 4ℓ final state,
- and is playing an important role in Higgs properties measurements in the 4ℓ final state and beyond.
- In order to use the MEM for Higgs properties measurements in the 4ℓ final state we consider XZZ couplings in generality
- A motivated Ansatz yields a 2-parameter description of XZZ couplings "Geolocating the Higgs"
- Considering XZZ couplings in greater generality gives two additional operators

- One of which can be constrained using the MEM, via its impact on the MZ2 distribution

- The other of which can be constrained via off-shell behavior

- I look forward to continuing Higgs measurements when the LHC resumes operation next year!

Bonus

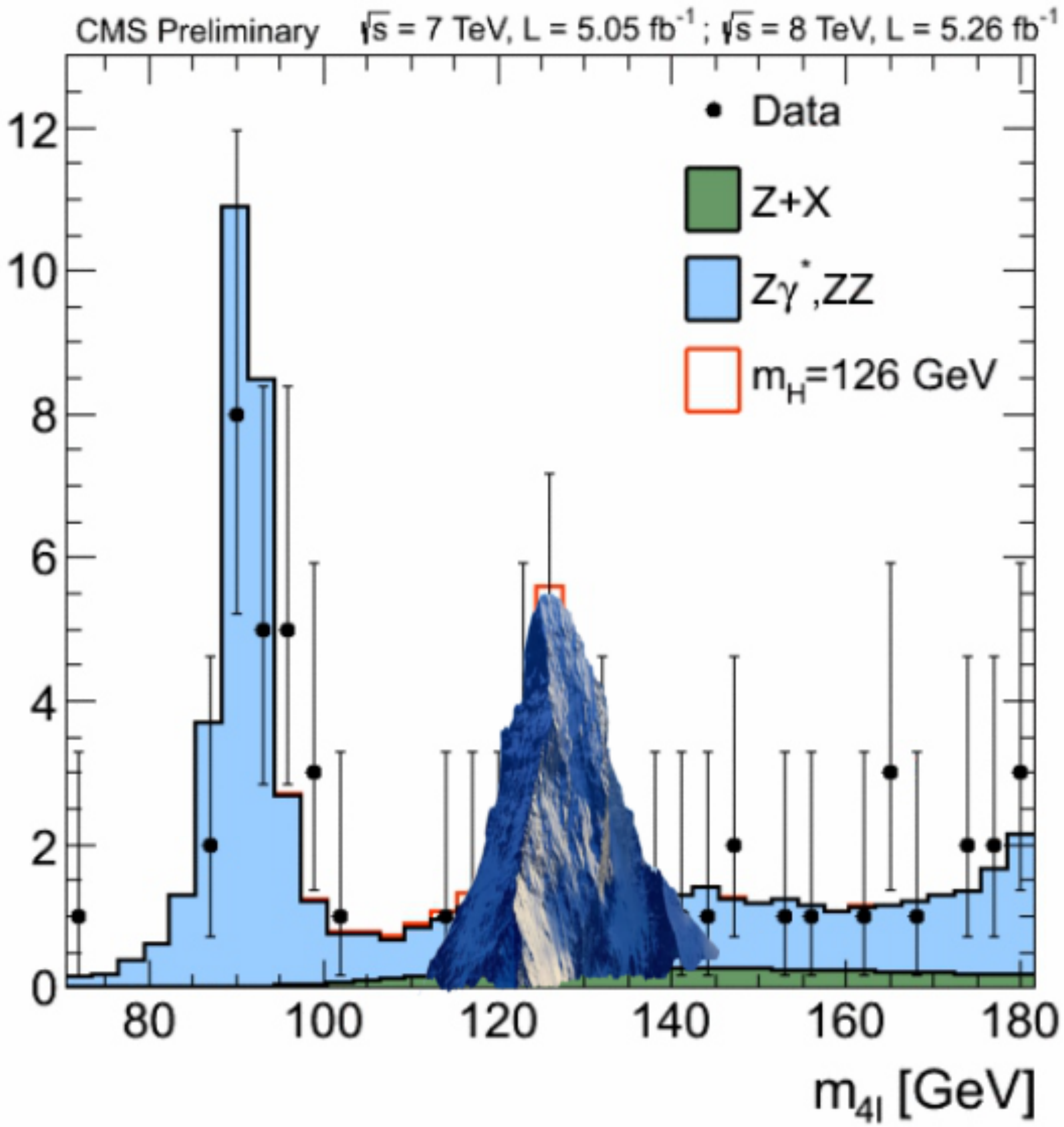
The Matrix Element Method

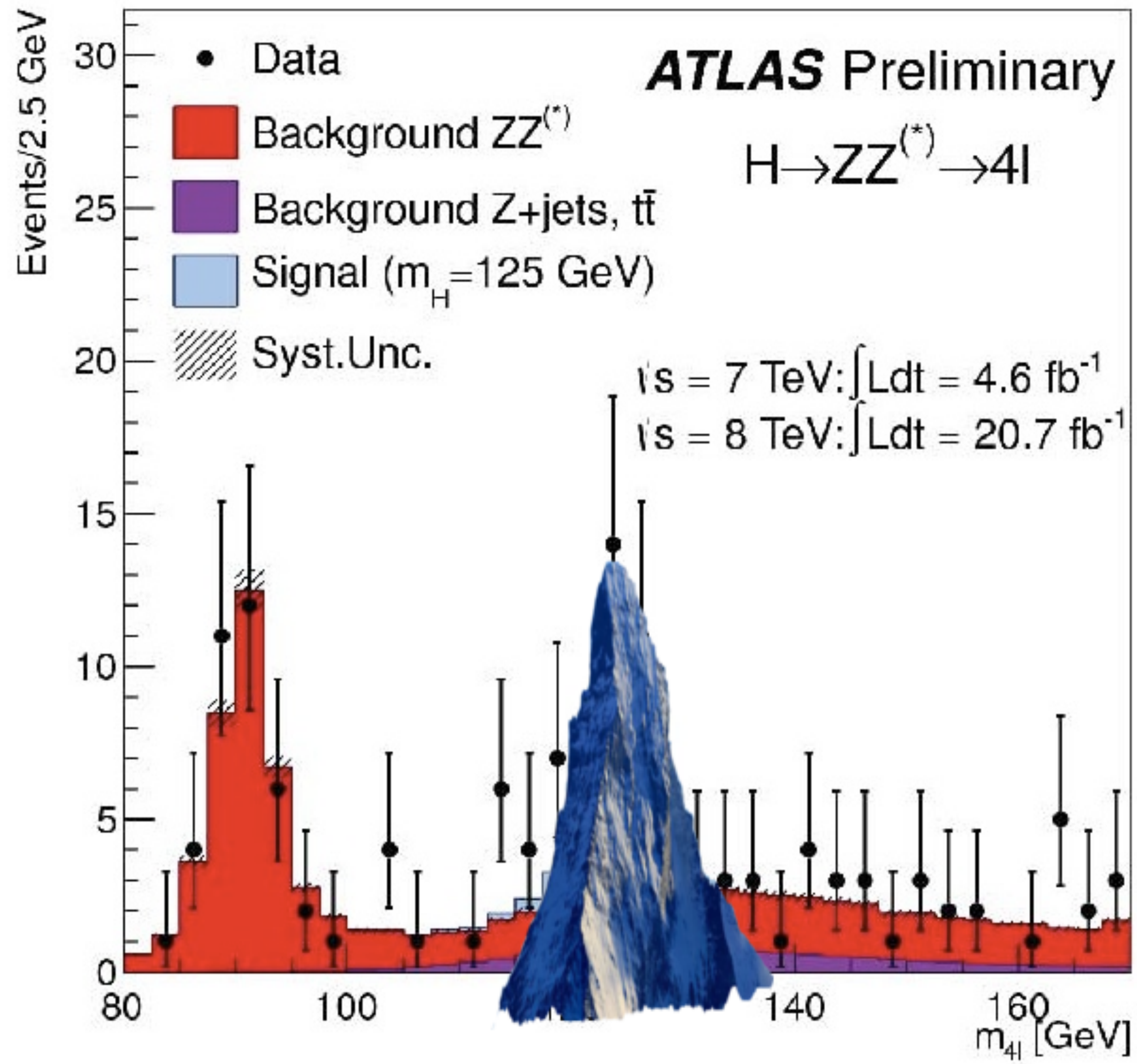
- Of course the true likelihood for some particle physics process is more than the leading order partonic differential cross section

- Work has been done to include the effects of extra radiation/ incorporate NLO cross sections
  Alwall, Freitas, Mattelaer (2010)
  Campbell, Giele, Williams (2012) x 2
  Campbell, Ellis, Giele, Williams (2013)
  …

- Work has been done to include parton shower/ substructure effects
  Soper, Spannowsky (2011) (2012), …

MEM for Higgs Discovery in H→ZZ →4ℓ





- Can the MEM help distinguish Higgs signal from background for 4ℓ final states?

- If so, how? What is the physical reason for the sensitivity?

Physics of Signal-Background Separation

- **Initial state radiation.**
  SM signal is mostly gg, irreducible background is from $q\bar{q}$.
  cf., e.g., Campbell, Giele, Williams (2012)

- **Rapidity distributions.**
  Also sensitive to different initial states, though demanding four central leptons forces $|\eta|$ for the event to be relatively small, reducing the ability to separate signal and background.

- **Z polarizations.**
  Z polarizations different for different signal, background processes. See next slide.

- **Propagators.**
  Invariant mass distribution of off-shell "Z" different depending on likelihood for that resonance to be γ^* versus Z^*.

ZZ Polarizations

$$\Delta\lambda = \pm 2 : \quad \mathcal{A}^{\Delta\sigma}_{\pm\mp} = -\sqrt{2}(1+\beta_1\beta_2) \ ,$$

$$\Delta\lambda = \pm 1 : \quad \mathcal{A}^{\Delta\sigma}_{\pm 0} = \frac{1}{\gamma_2(1+x)}\left[(\Delta\sigma\Delta\lambda)\left(1+\frac{\beta_1^2+\beta_2^2}{2}\right) - 2\cos\Theta \right.$$
$$\left. -(\Delta\sigma\Delta\lambda)(\beta_2^2-\beta_1^2)x - 2x\cos\Theta - (\Delta\sigma\Delta\lambda)\left(1-\frac{\beta_1^2+\beta_2^2}{2}\right)x^2\right]$$

$$: \quad \mathcal{A}^{\Delta\sigma}_{0\pm} = \frac{1}{\gamma_1(1-x)}\left[(\Delta\sigma\Delta\lambda)\left(1+\frac{\beta_1^2+\beta_2^2}{2}\right) - 2\cos\Theta \right.$$
$$\left. -(\Delta\sigma\Delta\lambda)(\beta_2^2-\beta_1^2)x + 2x\cos\Theta - (\Delta\sigma\Delta\lambda)\left(1-\frac{\beta_1^2+\beta_2^2}{2}\right)x^2\right]$$

$$\Delta\lambda = 0 : \quad \mathcal{A}^{\Delta\sigma}_{\pm\pm} = -(1-\beta_1\beta_2)\cos\Theta - \lambda_1\Delta\sigma(1+\beta_1\beta_2)x \ ,$$

$$\Delta\lambda = 0 : \quad \mathcal{A}^{\Delta\sigma}_{00} = 2\gamma_1\gamma_2\cos\Theta\left[((1-x)\beta_1+(1+x)\beta_2)\sqrt{\frac{\beta_1\beta_2}{1-x^2}} - (1+\beta_1^2\beta_2^2)\right]$$

JG, Kumar, Low, Vega-Morales, 2011

TABLE 8
Coefficients for the helicity amplitudes for the processes
$e^+e^- \to ZZ$ and $e^+e^- \to Z\gamma$

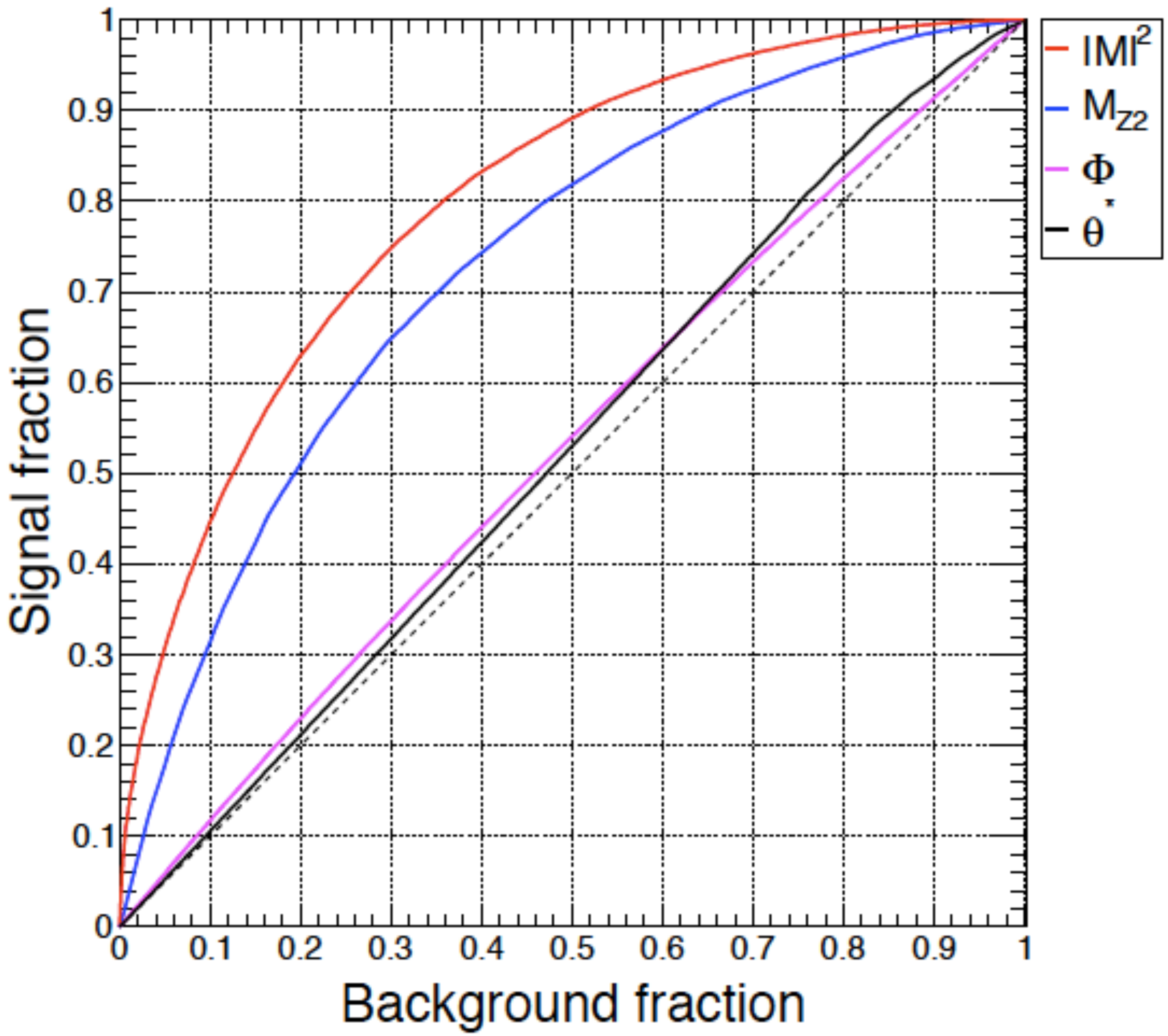
$\Delta\lambda$	$(\lambda_1\lambda_2)$	$\mathscr{A}_{\lambda_1\lambda_2}$	$\mathscr{B}_{\lambda_1\lambda_2}$
± 2	$(\pm\mp)$	$-\sqrt{2}(1+\beta^2)$	$\sqrt{2}$
± 1	(± 0)	$\gamma^{-1}[\Delta\sigma\cdot\Delta\lambda(1+\beta^2)-2\cos\Theta]$	
± 1	$(0\pm)$	$\gamma^{-1}[\Delta\sigma\cdot\Delta\lambda(1+\beta^2)-2\cos\Theta]$	$2r(\cos\Theta+\Delta\sigma\cdot\lambda_2)$
0	$(\pm\pm)$	$-\gamma^{-2}\cos\Theta$	$r^2(\cos\Theta+\Delta\sigma\cdot\lambda_2)$
0	(00)	$-2\gamma^{-2}\cos\Theta$	

Hagiwara, Peccei, Zeppenfeld, Hikasa, 1987

- At high energies the dominant polarizations of the Zs are +- for background
- Amplitudes non-vanishing for all choices for Z_1 and Z_2 polarizations
- For spin-zero resonances, only ZZ polarization possibilities are ++, - -, and 00
- So Z polarizations → lepton angular distributions, Z invariant mass distributions allows for signal and background separation for heavy Higgses ($M_H \gtrsim 2\ M_Z$), cf., e.g., JG, Kumar, Low, Vega-Morales, 2011

Propagator Drives Sensitivity for 125 GeV Higgs

- Full Matrix Element best variable for signal-background discrimination
- Invariant mass of lighter "Z", M_{Z2}, also very sensitive
- Φ (angle between decay planes) and θ^* (angle between Z_1 direction and collision axis in X rest frame) are less sensitive.
- Sensitivity to M_{Z2} due to very different signal and background distributions:
    - Background distribution peaks at M_{Z2} due to γ^* propagator
    - Signal distribution does not.

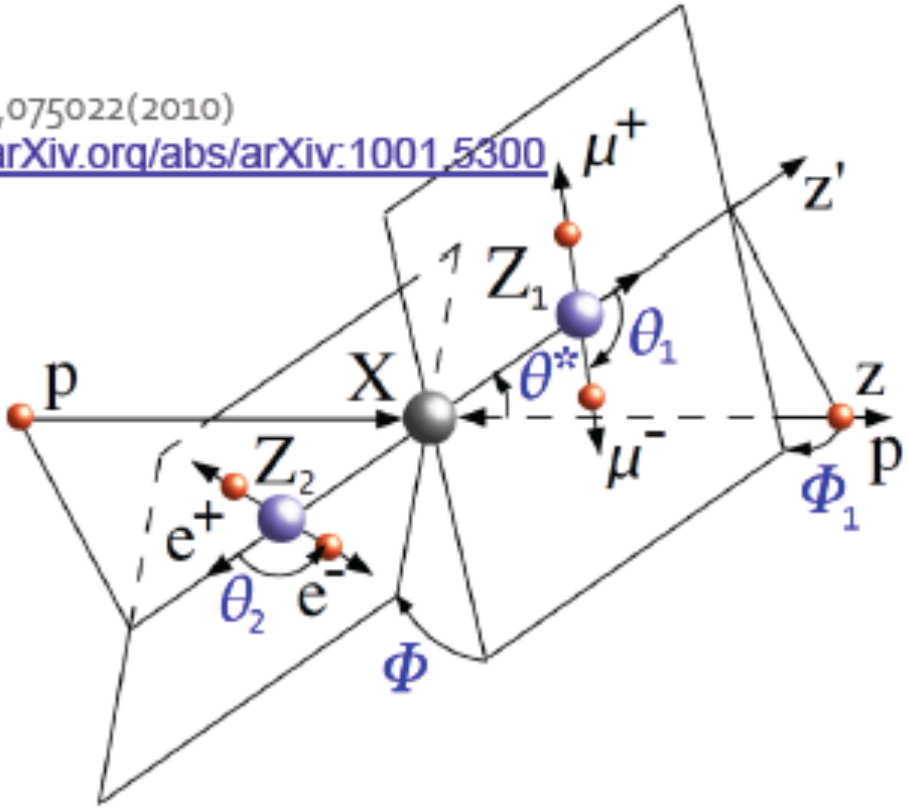


from Phys.Rev. D87 (2013) 055006 (Avery, Bourilkov, Chen, Cheng, Drozdetskiy, JG, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, and Snowball.
In this paper we presented **MEKD**, a publicly available tool for MEM analyses in this channel

1210.0896

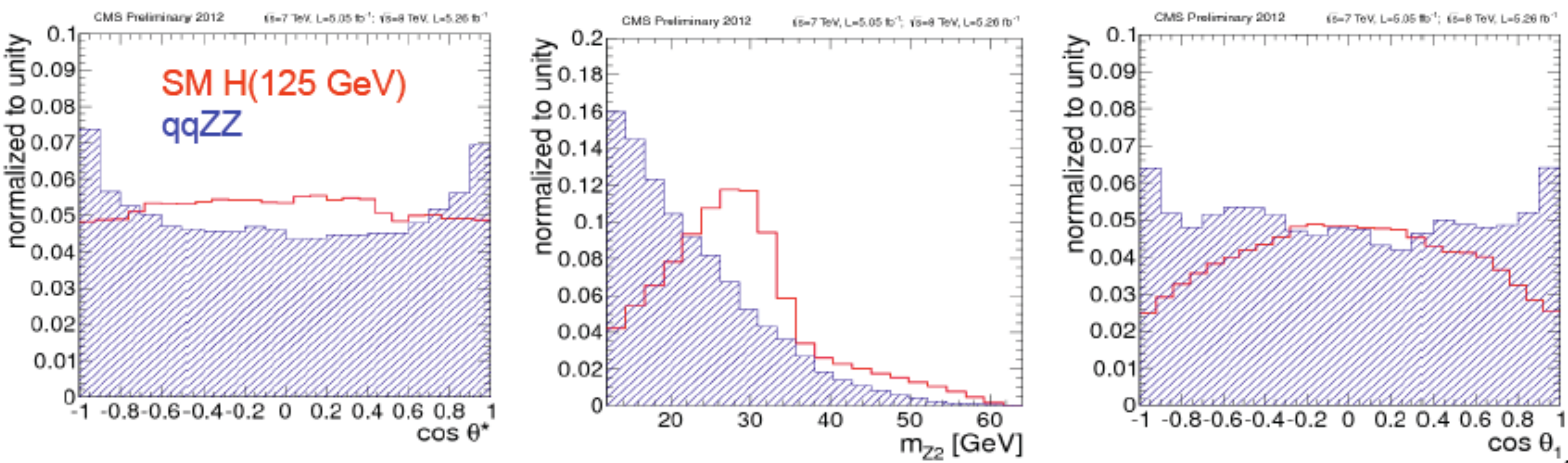
MELA

PRD81,075022(2010)
http://arXiv.org/abs/arXiv:1001.5300



Matrix Element Likelihood Analysis:
uses kinematic inputs for
signal to background discrimination
$\{m_1, m_2, \theta_1, \theta_2, \theta^*, \Phi, \Phi_1\}$

$$\mathrm{MELA} = \left[1 + \frac{\mathcal{P}_{\mathrm{bkg}}(m_1, m_2, \theta_1, \theta_2, \Phi, \theta^*, \Phi_1 | m_{4\ell})}{\mathcal{P}_{\mathrm{sig}}(m_1, m_2, \theta_1, \theta_2, \Phi, \theta^*, \Phi_1 | m_{4\ell})}\right]^{-1}$$



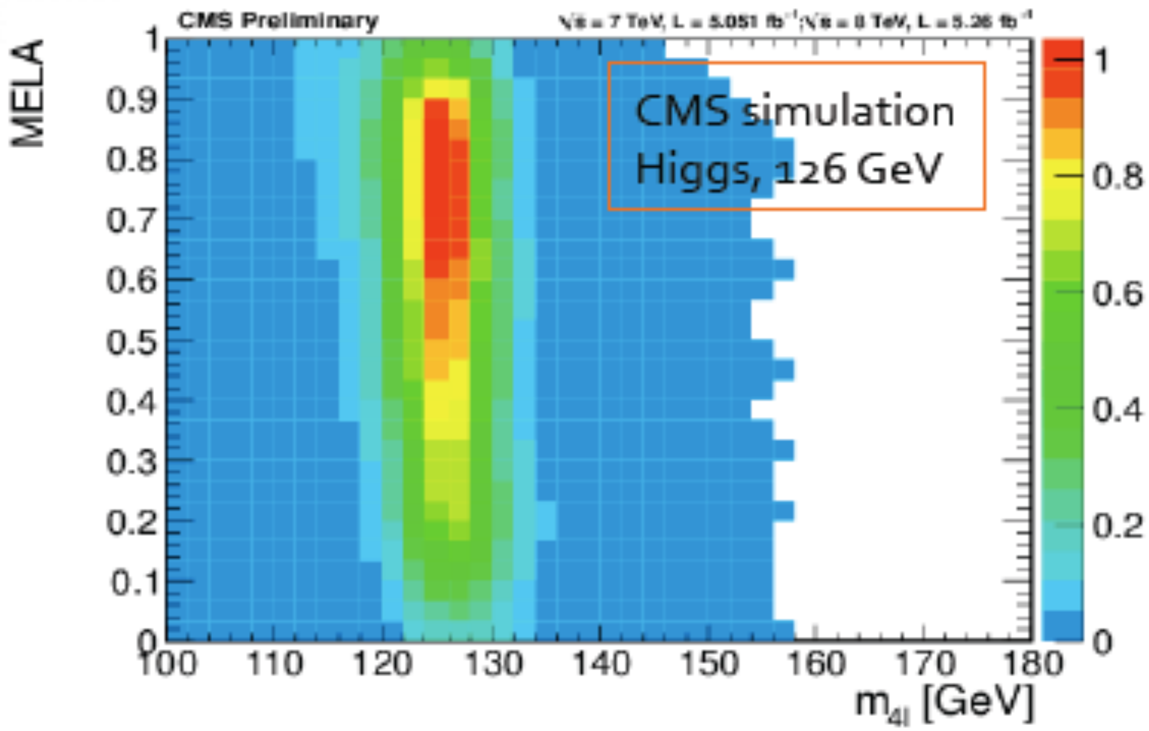
(Slide from CMS Higgs Discovery Talk)

MELA

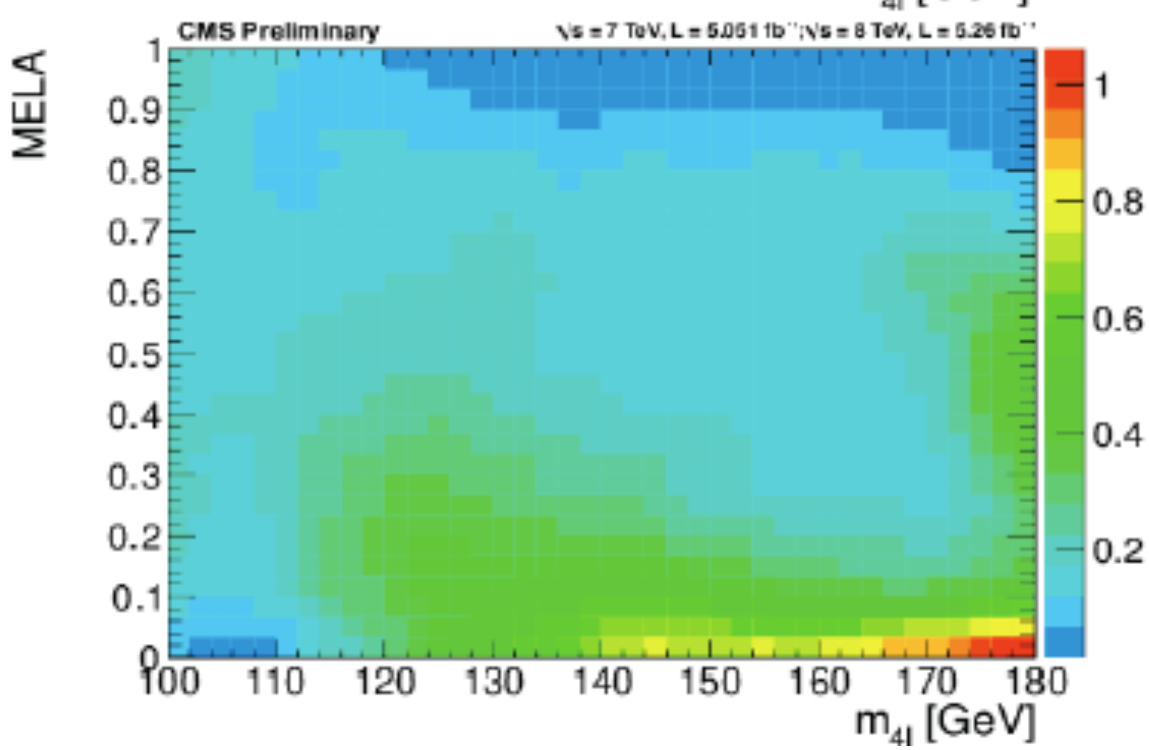
July 4th 2012 The Status of the Higgs Search J. Incandela for the CMS COLLABORATION

2D analysis using {m_{4l}, MELA}

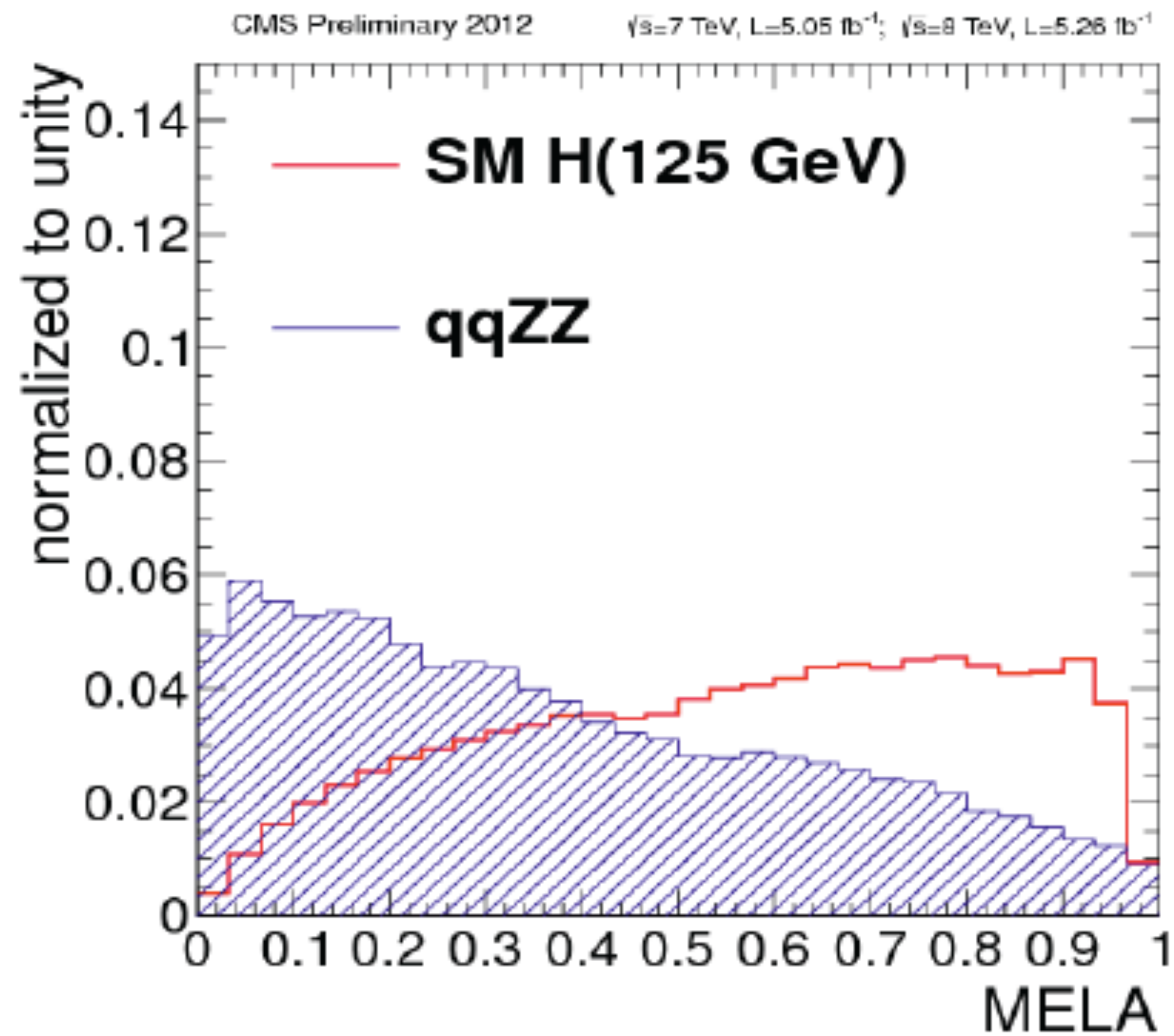
signal



background



MELA offers powerful discrimination of background



technique applicable for signal hypothesis testing

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(Slide from CMS Higgs Discovery Talk)