Space Charge Collaboration Meeting 2014 / CERN / 21th of May 2014

Numerical Model and Scaling Law of PIC Noise

Frederik Kesting

University of Frankfurt / GSI Darmstadt

1. 2.5D Space Charge Model
2. Noise in 2D PIC Calculations
3. Tracking of Noisy Beams

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Tracking with Space Charge Kicks

Basic idea of maps:

$$\begin{pmatrix} x_i \\ x_i' \\ y_i \\ y_i' \end{pmatrix} \rightarrow \begin{pmatrix} x_f \\ x_f' \\ y_f \\ y_f' \end{pmatrix} = \mathcal{M}(s_i, s_f) \begin{pmatrix} x_i \\ x_i' \\ y_i \\ y_i' \end{pmatrix}$$

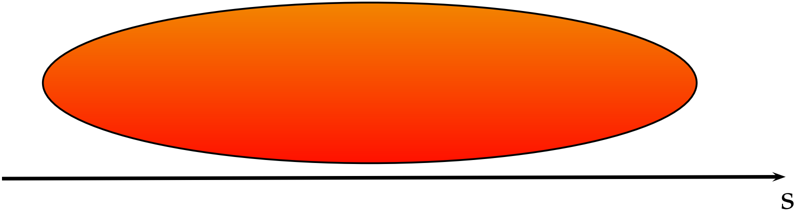
Space charge can be modeled by non-linear kicks:

$$\begin{pmatrix} x_i \\ x_i' \\ y_i \\ y_i' \end{pmatrix} \rightarrow \begin{pmatrix} x_f \\ x_f' \\ y_f \\ y_f' \end{pmatrix} = \begin{pmatrix} x_i \\ x_i' + \Delta s \frac{qE_{SC,x}(x,y,z)}{v_0 p_0} \\ y_i \\ y_i' + \Delta s \frac{qE_{SC,y}(x,y,z)}{v_0 p_0} \end{pmatrix}$$

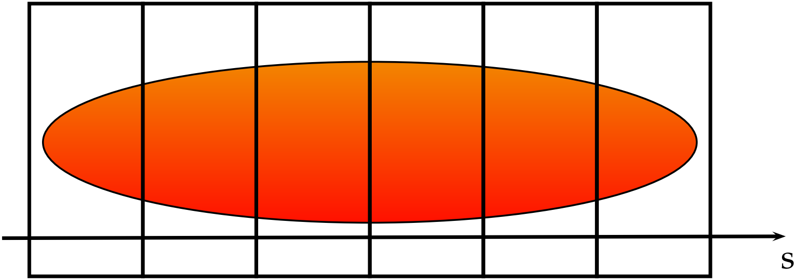
$\Rightarrow$ $E_x(x, y, z)$ and $E_y(x, y, z)$ have to be calculated for every particle.

for example 10^{13} protons at the SIS100/FAIR

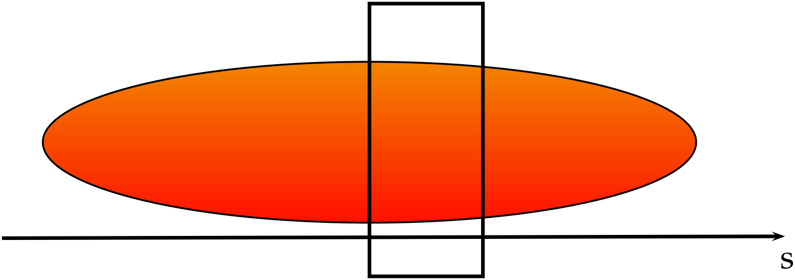
2.5D Space Charge Model



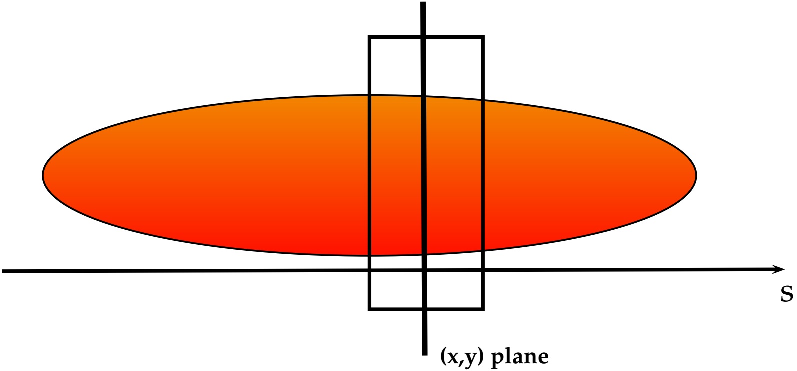
2.5D Space Charge Model



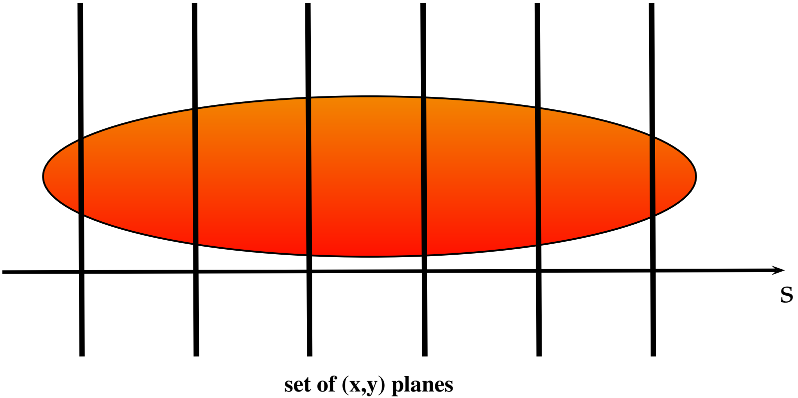
2.5D Space Charge Model



2.5D Space Charge Model



2.5D Space Charge Model



⇒ Solution on planes of $E_x(x, y)$ and $E_y(x, y)$ via 2D Poisson solver.

⇒ Interpolation inbetween planes to get $E_x(x, y, z)$ and $E_y(x, y, z)$.

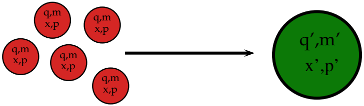
2D Poisson Solver

Our 2-dimensional Problem:

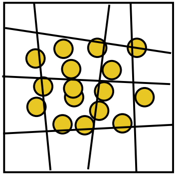
$$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0}$$

Find proper approximations:

$\Rightarrow$ use M macro-particles



$\Rightarrow$ and N grid points

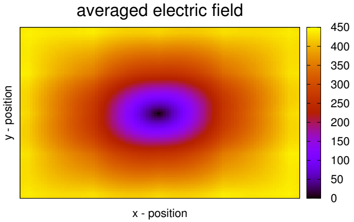


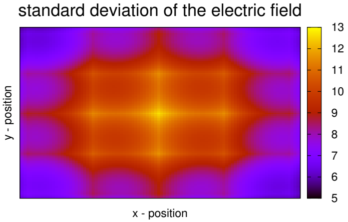
Choose a fixed number of grid points N and macro-particles M.

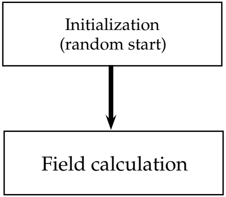
$\Rightarrow$ Can we reasonably decrease the error?

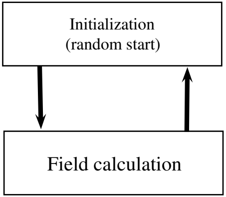
2D Poisson Solver

Sample output for a Gaussian beam profile

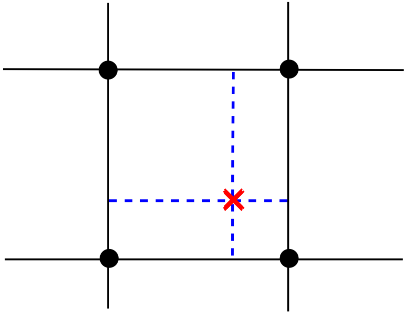


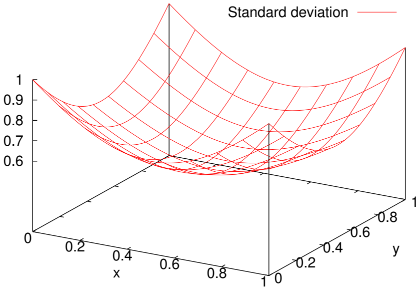






What happens in-between those grid points?





$$f(x,y) = 1/4\left\{(1-x)(1-y)g_{00} + x(1-y)g_{10} + (1-x)yg_{01} + xyg_{11}\right\}$$

$\Rightarrow$ Error calculation

$\Rightarrow$ Periodic extension for the whole grid

2D Poisson Solver - Mathematical noise description

Space charge field for a Gaussian beam profile:

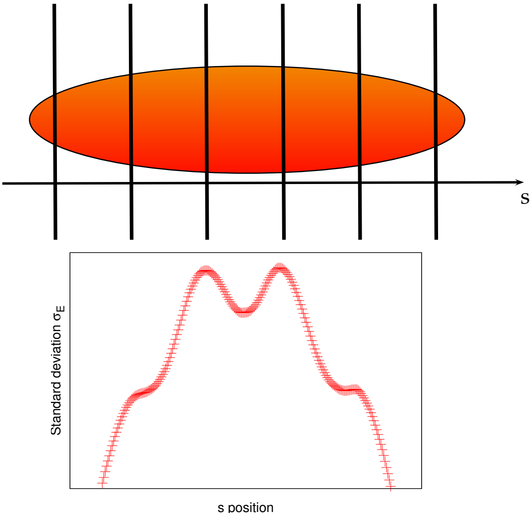
$$E(r) = \frac{E_0}{r}\left(1 - e^{-\frac{r^2}{2\sigma_r^2}}\right) \tag{1}$$

with a standard deviation of:

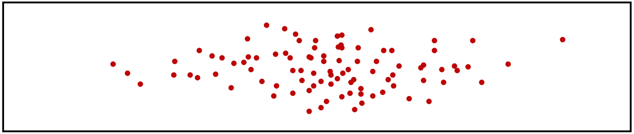
$$\delta E_{SC}(r) = \sigma_0 \; \xi(x, y)\sqrt{\frac{\sqrt{N_G}}{N_M}} e^{-\frac{r^2}{4\sigma_r}} \tag{2}$$

N_G ... Number of grid points
N_M ... Number of macro - particles
$\xi(x, y)$ ... Fourier extension of the grid

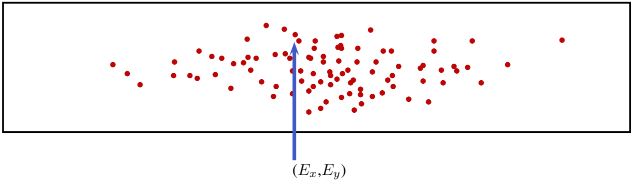
2D Poisson Solver - Longitudinal interpolation



Tracking of noisy beams



Tracking of noisy beams

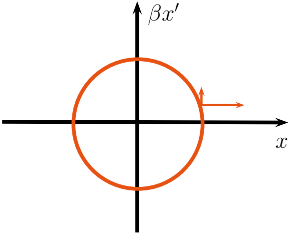


Spectral analysis of the electric field (E_x, E_y)

- Flat spectrum $\Rightarrow$ Random walk modelling possible!
- Peaks in spectrum $\Rightarrow$ Extended random walk modelling?

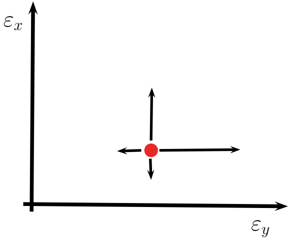
Random Walk Modelling

Phase Space Description



$x \to x \pm dx \qquad x' \to x' \pm dx'$

Emittance Description



$\varepsilon_x \to (1+\Delta)\,\varepsilon_x$

Gives (in the regular case) an understanding of

⇒ the effect of noise on a single particle

⇒ the effect of macroparticles and grid points

⇒ the effect of the optics of a machine

Summary

⇒ Mathematical modeling of PIC noise in 2D

$$\delta E_{SC}(r) = \sigma_0 \; \xi(x,y)\sqrt{\frac{\sqrt{N_G}}{N_M}e^{\frac{-r^2}{4\sigma_r}}}$$

⇒ Extension for 2.5D bunched beams
⇒ Random walk modelling for regular cases

Outlook:

⇒ Studing the irregular cases in more detail
⇒ Benchmarking of codes

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