GRID INDUCED NOISE in 3D PIC SIMULATION

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Ingo Hofmann

GSI/TUD Darmstadt

- Theory overview
- 3D bunches in periodic solenoid
- Temperature anisotropy
- 3D in FODO
- Conclusions

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O. Boine-Frankenheim (collaborator)
J. Struckmeier (discussions)

Theory overview

following work by Struckmeier (1994,1996,2000)

$$\frac{d}{ds}\ln\,\epsilon_x(s)\epsilon_y(s)\epsilon_z(s) = \frac{k_f}{3}\,I_A = \frac{k_f}{3}\left(\frac{(1-r_{xy})^2}{r_{xy}} + \frac{(1-r_{xz})^2}{r_{xz}} + \frac{(1-r_{yz})^2}{r_{yz}}\right) \geq 0$$

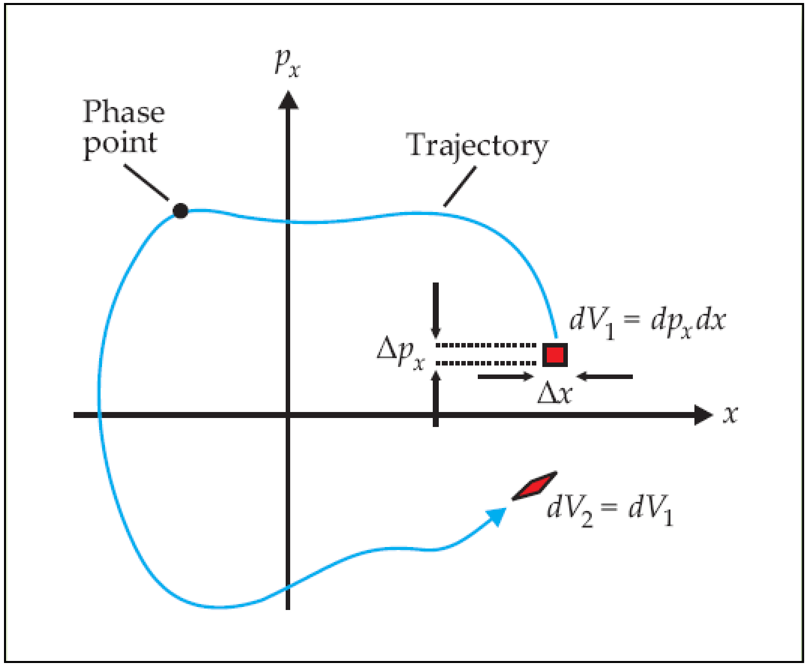
$$r_{xy}(s) \equiv \frac{T_y(s)}{T_x(s)},\ r_{xz}(s) \equiv \frac{T_z(s)}{T_x(s)},\ r_{yz}(s) \equiv \frac{T_z(s)}{T_y(s)}$$

$$\frac{T_y(s)}{T_x(s)} \equiv \frac{\epsilon_y^2/\left\langle y^2\right\rangle}{\epsilon_x^2/\left\langle x^2\right\rangle} = \frac{\epsilon_y k_y}{\epsilon_x k_x}$$

$$\frac{1}{k}\frac{dS}{ds} = \frac{d}{ds}\ln\,\epsilon_x(s)\epsilon_y(s)\epsilon_z(s) = \frac{k_f}{3}\,I_A \geq 0$$

Entropy
Liouville - infinite resolution – coarse graining

Liouville

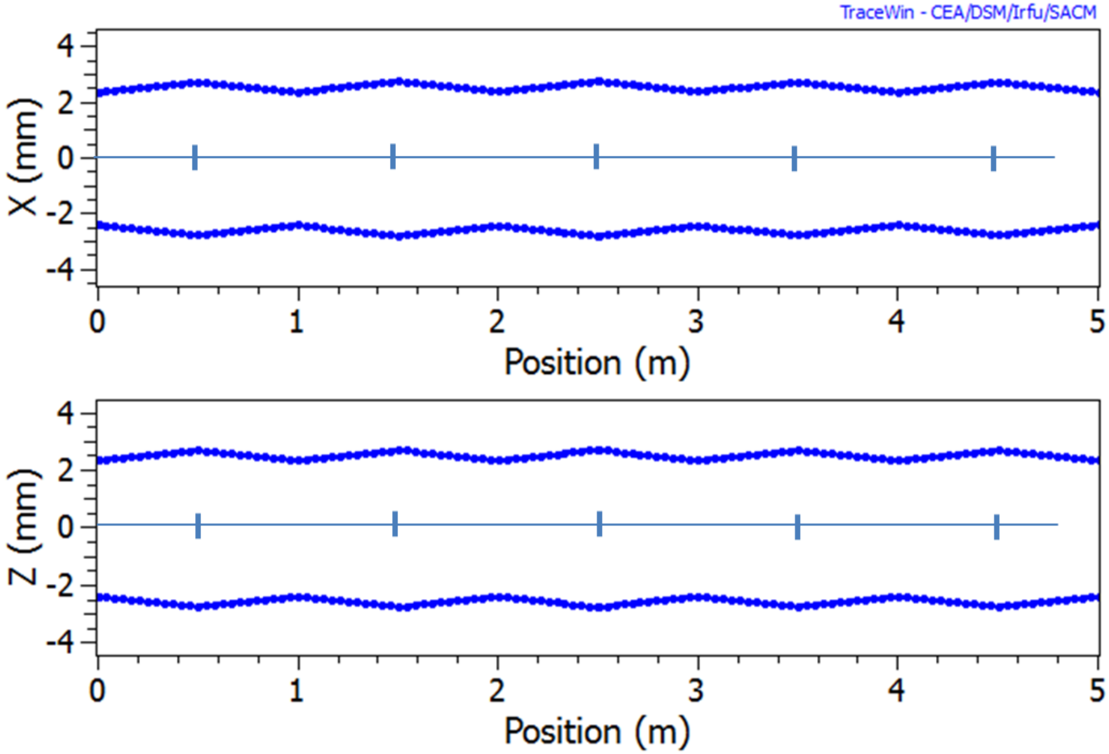


- Liouville: exact area in 2d, 4d, 6d phase space invariant for Hamiltonian flow
- for „infinite resolution“ self-consistent space charge potential (as in PIC) exact areas or volumes invariant → **no growth of „infinite resolution“ entropy**
- for exactly resolved motion of directly interacting particles (collisions) in 6d:
  - „infinite resolution“ entropy may grow
  - in time-independent external potential total energy invariant → $\varepsilon_{6d,rms}$ basically constant
- for exactly resolved collisional motion in 6Nd no growth of „infinite resolution“ entropy (Hamiltonian flow in 6Nd)
- Gibbs entropy in 6Nd needs finite resolution or „coarse graining“ to allow entropy growth

TRACEWIN code (CEA Saclay)

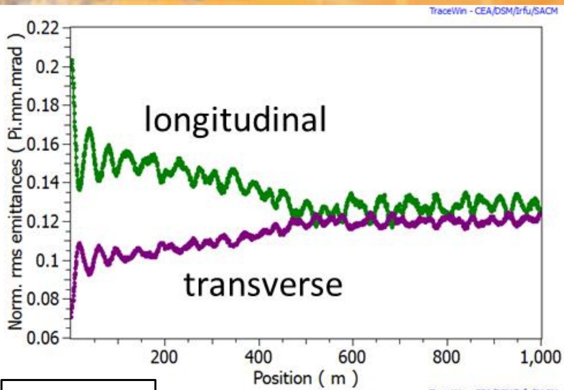
- 3d PIC code primarily for linac applications („commercial“)
- rz and xyz (PICNIC) Poisson solvers and grids
- analytical continuation of fields in halo region
- hard edge elements or field maps
- variable number of space charge kicks per element (drift and field maps)
- plots: rms emittance and their products etc.

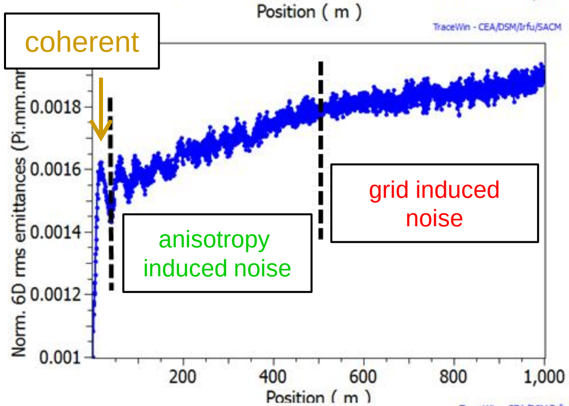
Periodic solenoid lattice

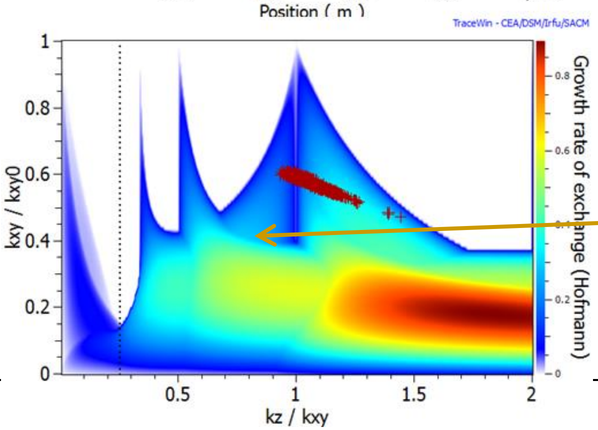


- thin lenses & rf kicks
- → option of identical envelopes and „temperatures“ in x,y,z
- 1000 cells as reference to determine emittance & entropy growth

Different regimes in anisotropic case







- $k_{0x,y,z}$=60° and $\varepsilon_z / \varepsilon_{xy}$=3 → anisotropy generated initially $T_{x,y} << T_z$
- space charge: $k_{x,y,z}$=32°/32°/38°
- N=1000: enhances effects over 1000 cells

3 distinct regimes:

- coherent exchange < 20 cells
  - resonant equipartition driven by space charge octupole
  - → $\varepsilon_{6d,rms}$ not suitable as entropy since also decreasing and ~ independent of N
- anisotropy (+ grid) induced **noise** → cell 500 (equi-partition)
- **purely grid induced noise** → continuing

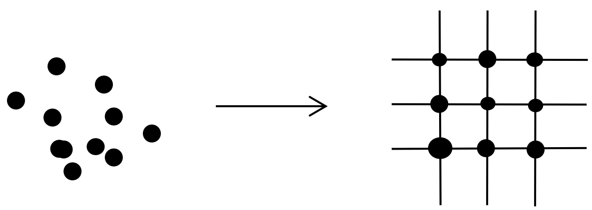
stop-band for resonant coherent exchange of „temperatures“ (→ equipartition) ($2k_z - 2k_{xy}$ ~ 0)

Poisson solver grid & noise

Purely grid induced noise:

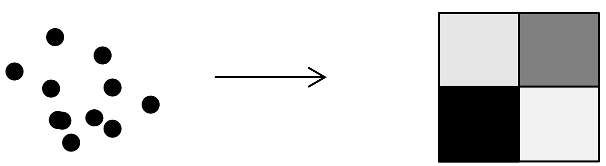
- not included in original „collisional“ approach of Struckmeier
- assume several sources:
  - **non-Liouvillean effect by fluctuating charges on grid**
  - focussing modulation „fast“
  - coherent flow vs. incoherent „temperature“

PIC: replaces infinite resolution charge density by “grid distribution”



Phase space remains infinitely resolved

Gibbs entropy: replaces fine-grained (infinite resolution) 6N-d phase space by “Coarse Grained"



$$S_{CG}(\rho(X)) = -k \int \bar{\rho}(X)[\ln \bar{\rho}(X)] d\Gamma$$

$\bar{\rho}$ is averaged over a cell

$$\Rightarrow \frac{dS_{CG}}{dt} \geq 0 \quad \text{in isolated system}$$

$$= 0 \quad \text{if infinitely resolved}$$

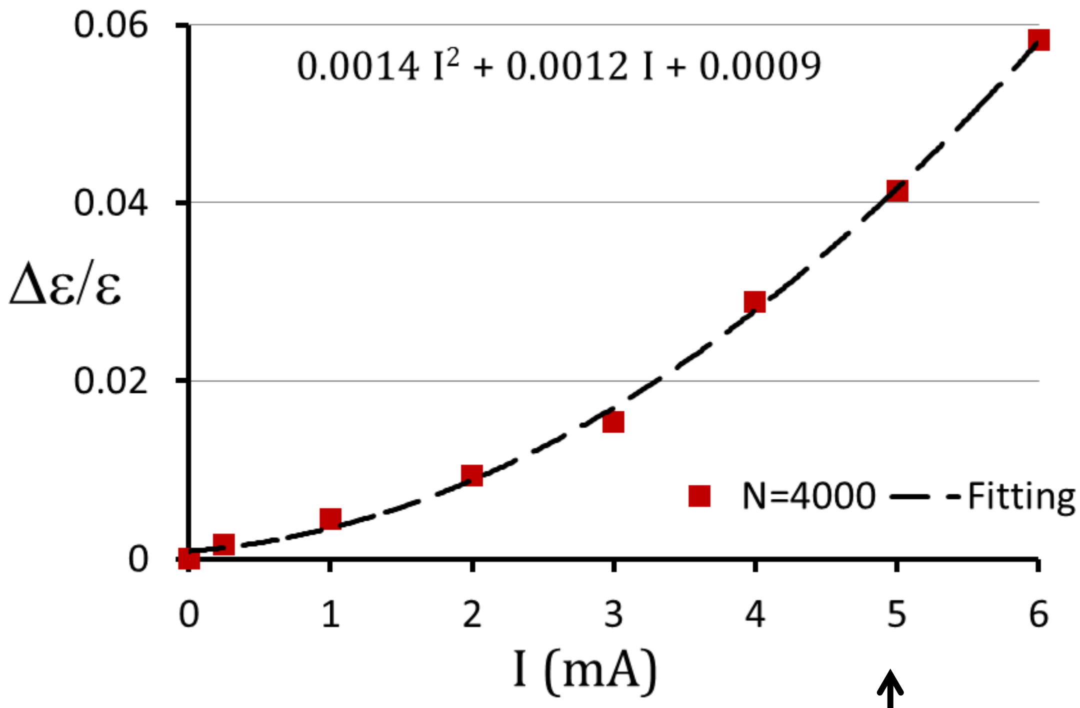
Averaging over cells:

- ✓ **<u>loss of Hamiltonian character</u> of phase space flow**
- ✓ **present and future states not connected by Hamiltonian mechanics**
- ✓ **→ <u>growth of entropy</u>**

Space charge dependency

approximately ~ I^2

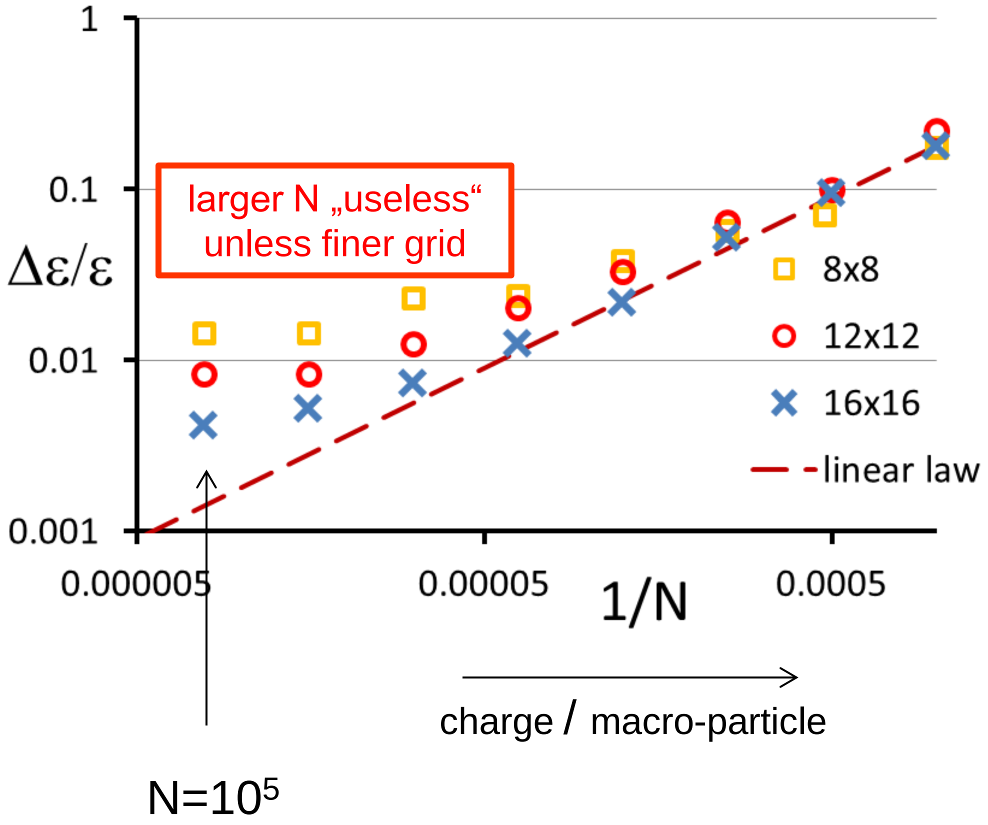
I_A=0 (isotropic)



$k_{x,y,z}$=60°→33°

N – dependence

grid limitation for large N!



I_A=0 (isotropic)

$$\Delta\epsilon_{6d}/\epsilon_{6d} \propto N^{-1}$$

if grid resolution is increased accordingly !

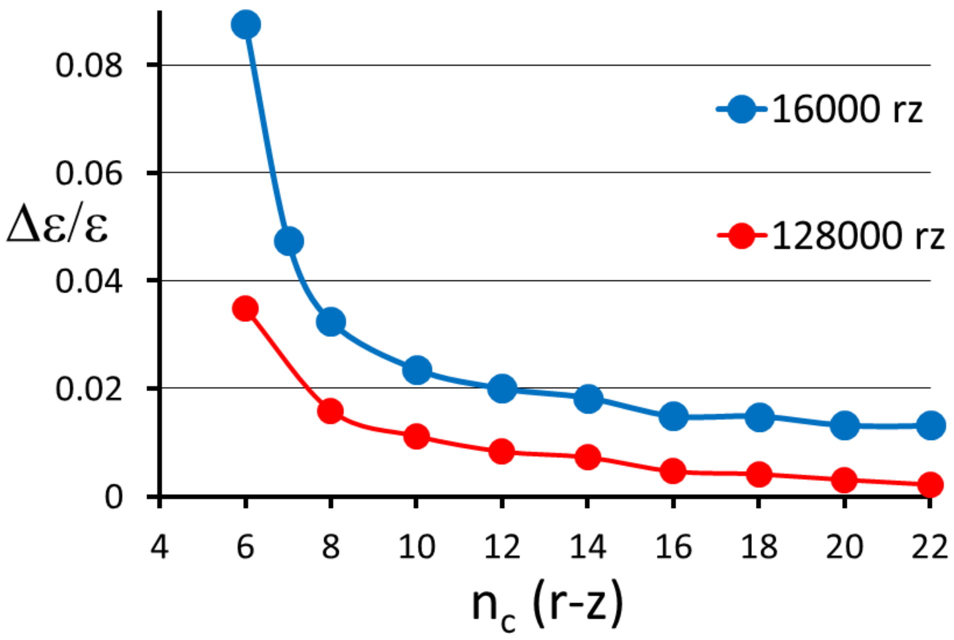
n_c=8:

- 8 cells in r from 0...3σ
- 8 cells in z from -3σ...+3σ
- analytical beyond

„Grid dominated“ noise

grid dominated

I_A=0 (isotropic)

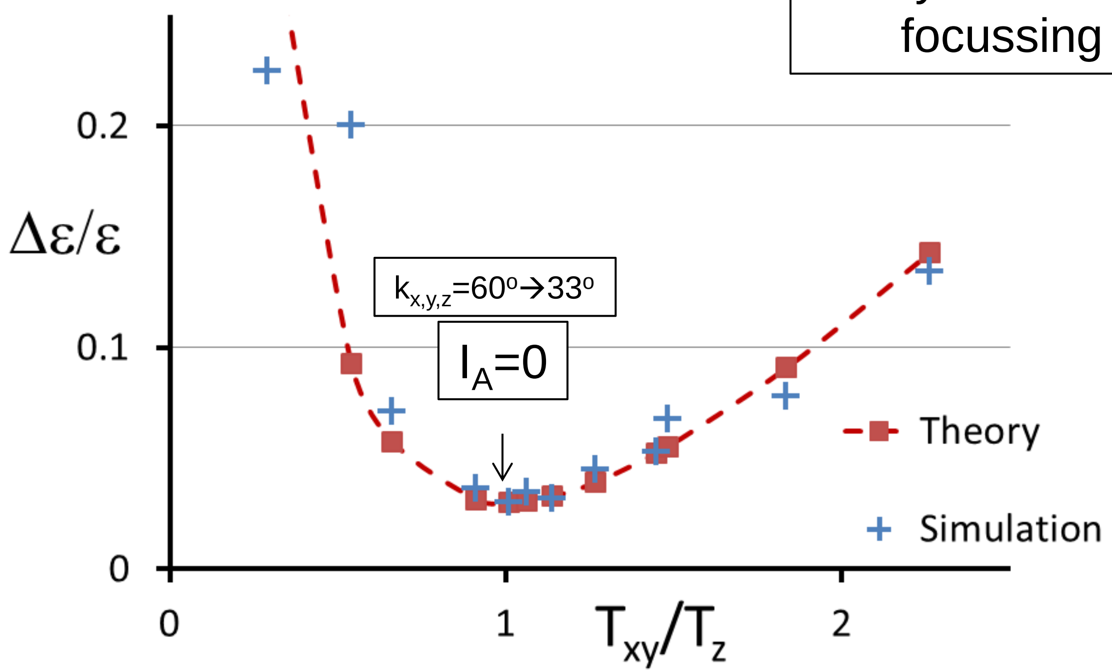


statistics (N) limited

Anisotropy and grid effect

→ **fits to theory with I_{GN} and k_f^* fitted to data**

symmetrical external focussing $k_{0x,y,z}$=60°



$$\frac{d}{ds}\ln\,\epsilon_x(s)\epsilon_y(s)\epsilon_z(s)=\frac{k_f}{3}\,I_A=$$
$$\frac{k_f}{3}\left(\frac{(1-r_{xy})^2}{r_{xy}}+\frac{(1-r_{xz})^2}{r_{xz}}+\frac{(1-r_{yz})^2}{r_{yz}}\right)\geq 0$$

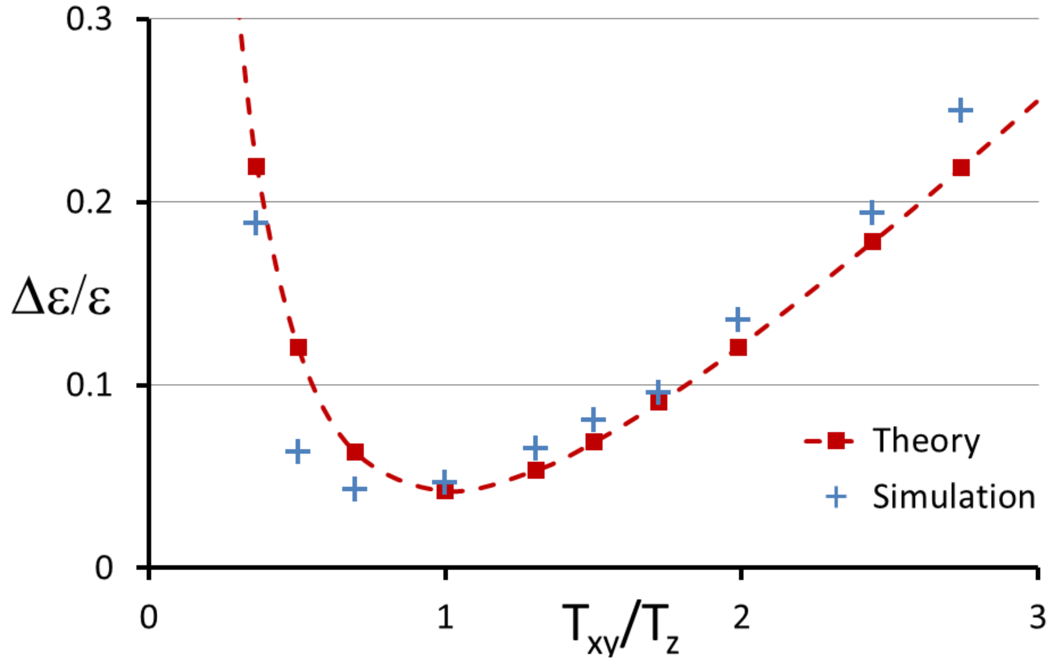
→

$$\frac{d}{ds}\ln\,\epsilon_x(s)\epsilon_y(s)\epsilon_z(s)=\frac{k_f^*}{3}\,(I_A+I_{GN})\,.$$

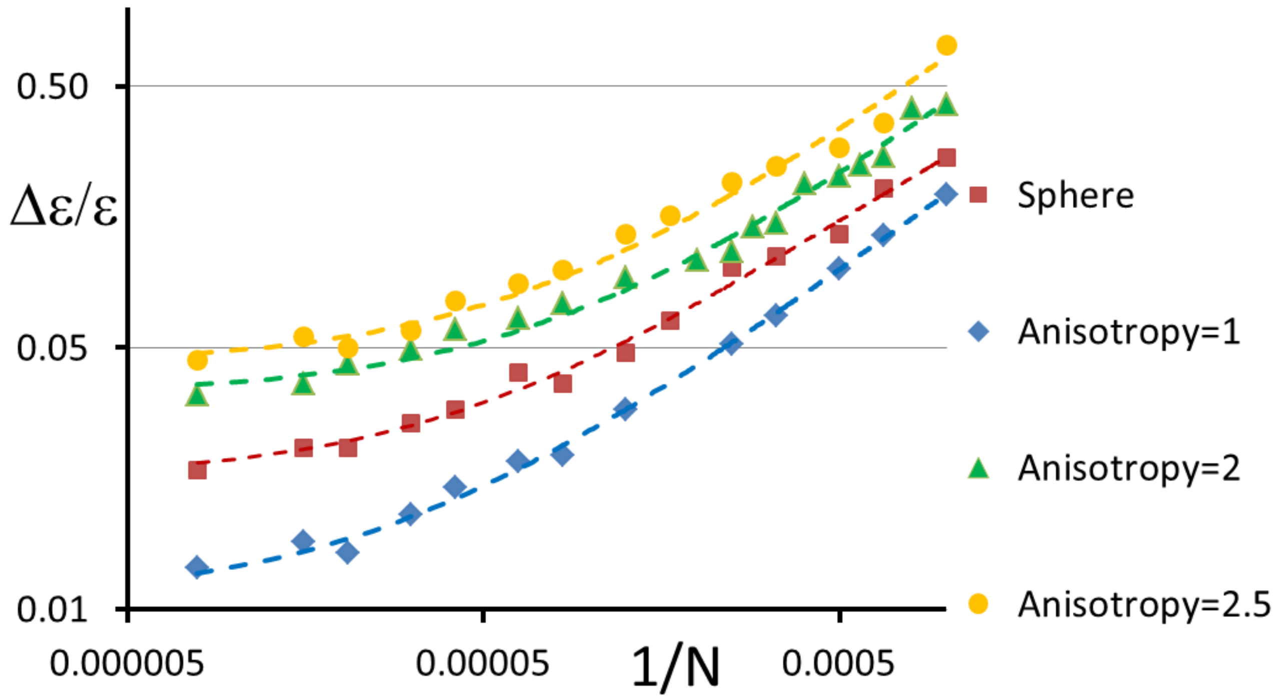
$$\frac{1}{k}\Delta S=\frac{\Delta\epsilon_{6d}}{\epsilon_{6d}}=\Delta s\frac{k_f^*}{3}\,(I_A+I_{GN})$$

Unsymmetrical external focussing
$k_{0x,y,z}$=60º / 60º / 47º

Basis for more unsymmetric focussing (synchrotron motion)

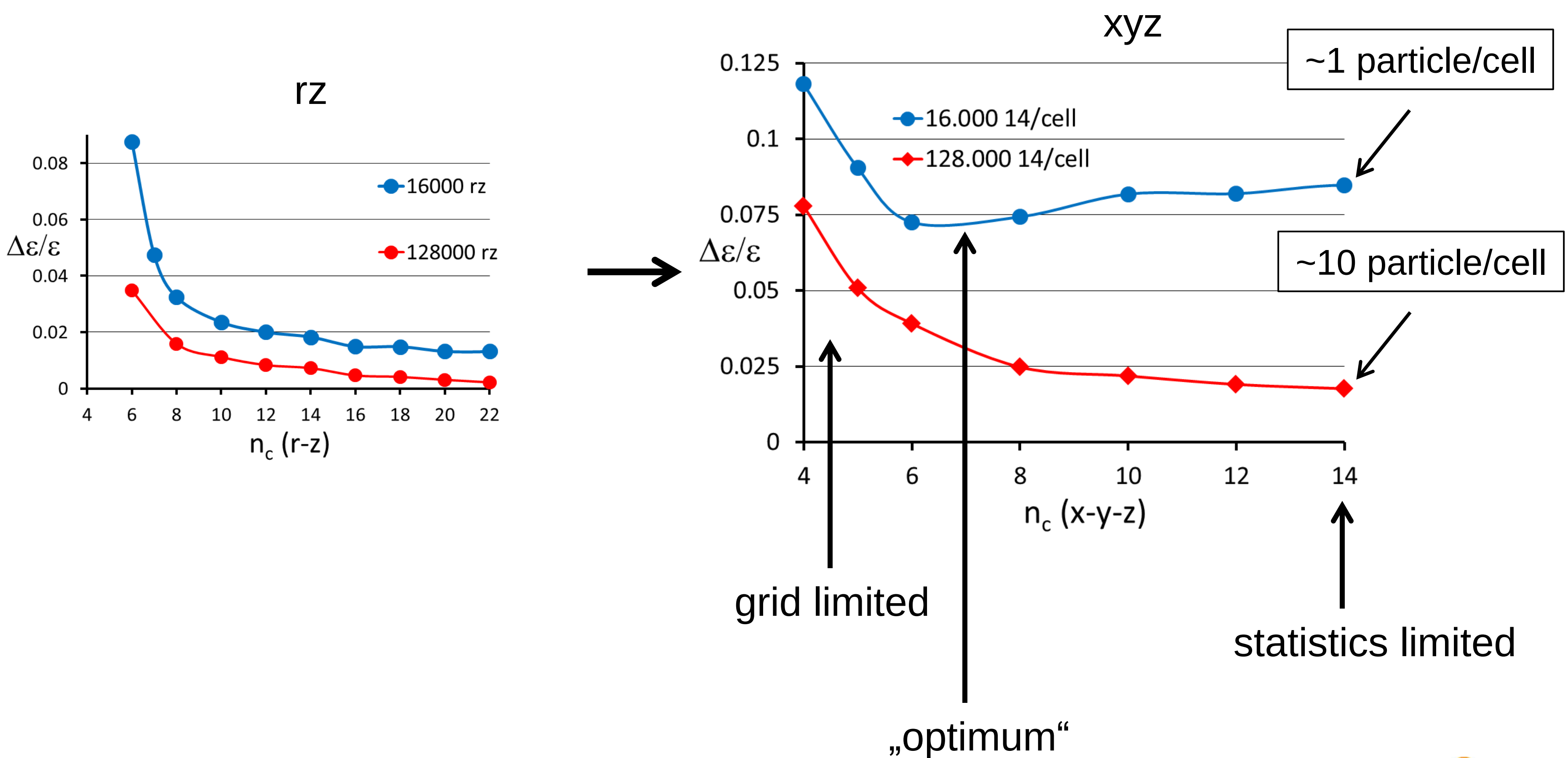


Anisotropy and N
→ grid limitation universal feature



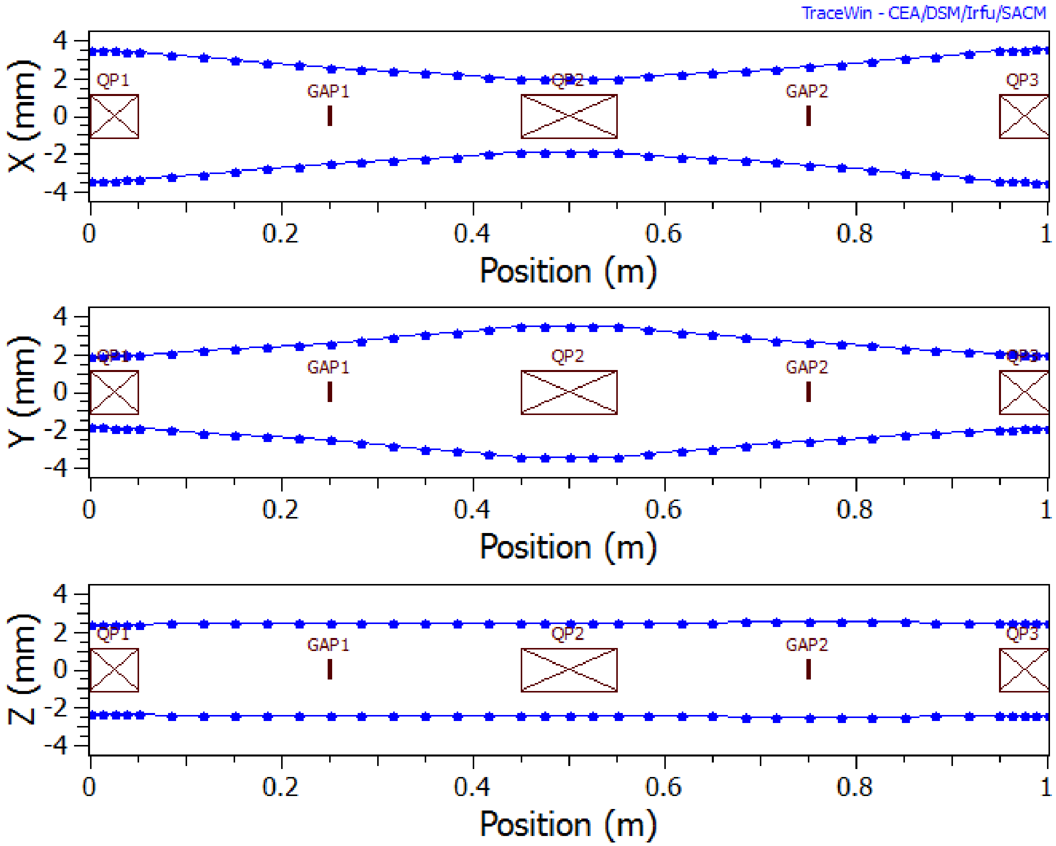
rz → xyz Poisson solver

same periodic solenoid lattice → enhanced noise



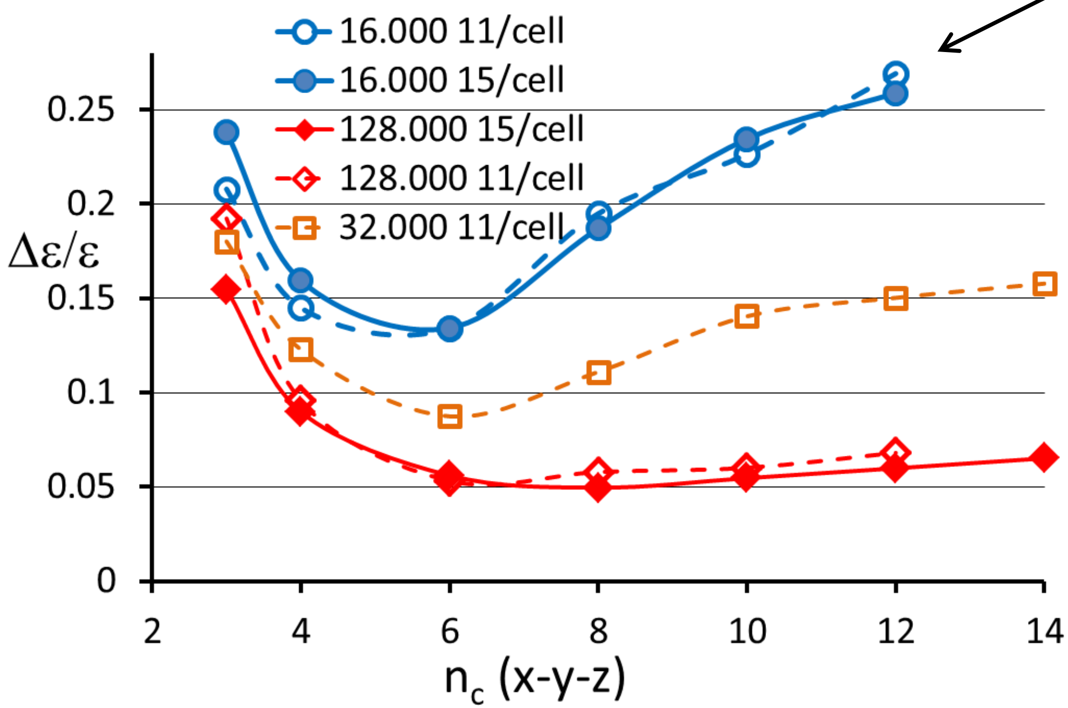
FODO lattice + xyz Poisson solver

same emittances, current, focussing strenghts, cell lengths



Statistics limitation more pronounced

Dependence on # space charge kicks / cell is negligible

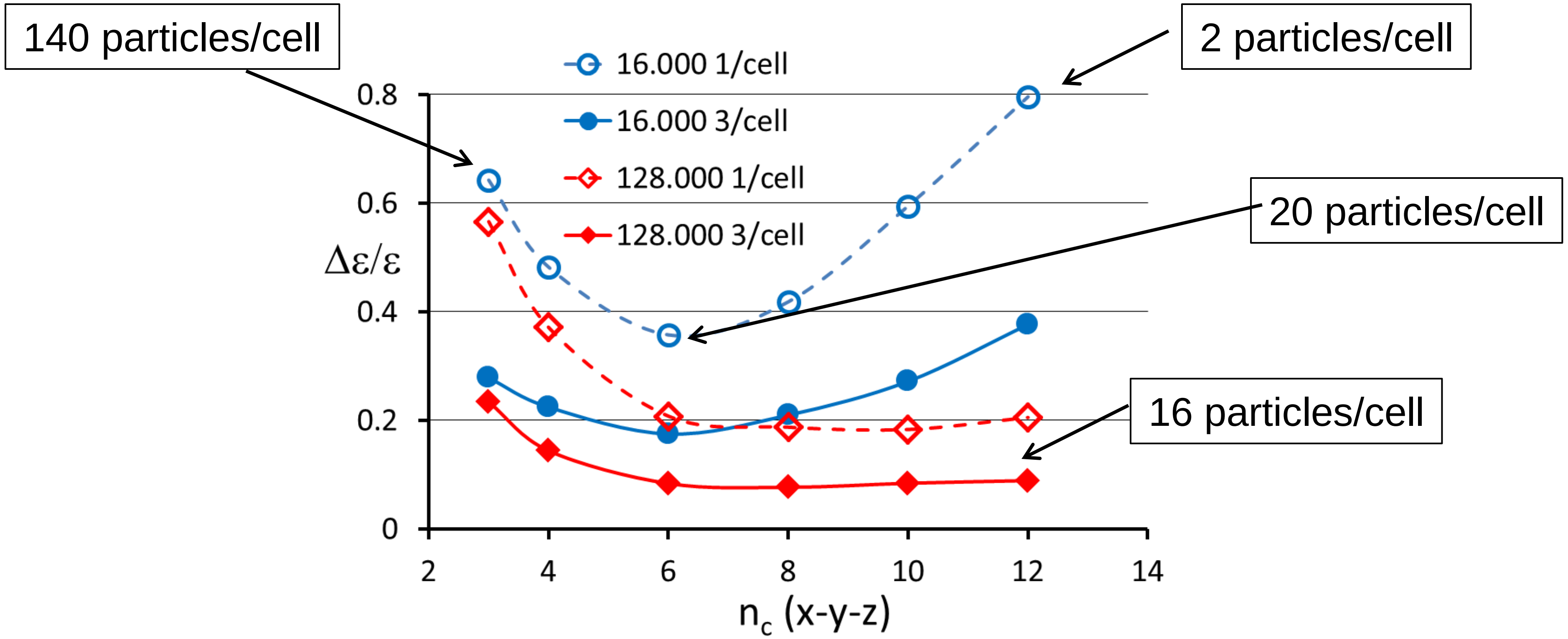


2 particles/cell

„flat" in n_c only for $> 10^5$ particles

Low # of space charge kicks

speeded-up for linac design: DRIFT TUBE LINAC CELL OPTION
1 or 3 kicks per DTL cell ($k_0=60^0$)



avoid 1 kick/cell and low N: relevant for statistical error studies
with 1000's of different linacs (error sets)

Conclusions & outlook

- 6d rms emittance evidenced as practical measure for noise + entropy
- „grid induced“ noise differs from „collisional“
  - collisional requires temperature anisotropy to drive it
  - grid induced noise component not
  - $I_A(r_{xy},r_{xz},r_{yz}) + I_{GN}$
- Limitations by enhanced non-Liouvillean effects:
  - grid resolution limited (small n_c)
  - statistics limited (small N)
- Other limitations to be further explored:
  - Time step not sufficiently small compared with typical transit time across grid cell
  - “Coherent flow” in periodic focusing – against temperature-like incoherent flow