

B Physics

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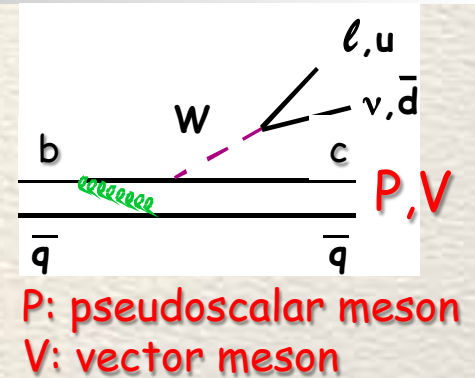


Introduction

- The b quark is the heaviest quark that produces bound states, like the Υ resonances ($b\bar{b}$), B mesons ($\bar{b}q$) and B baryons (bqq' , bbq , bbb), $q,q'=u,d,s,c$
→ we focus here on B meson physics
- b quarks are produced in pairs
 - at e^+e^- colliders → $\Upsilon(4S)$: CUSB, CLEO, BABAR, Belle, Belle II
→ 90 GeV (LEP, SLC): ALEPH, DELPHI, L3, OPAL, SLD
 - at hadron colliders → Tevatron: CDF, D0
→ LHCb: LHCb, ATLAS, CMS
- At $\Upsilon(4S)$, b quark hadronizes into B^+ and B^0_d only with no additional pions
 - cross section is 1 nb → need high luminosity get hundreds of 10^6 B mesons
 - excellent laboratory to study B mesons → well-defined initial conditions
- At LEP, Tevatron and LHC, b quark hadronizes into B^+ , B^0_d , B^0_s , and b baryons in fractions of $(0.402 \pm 0.007):(0.402 \pm 0.007):(0.105 \pm 0.006):(0.092 \pm 0.015)$
 - cross section increases with \sqrt{s} → at 14 TeV $\sigma \approx 0.5$ b
 - initial conditions are not well defined, use p_T (conserved)

General Remarks

- The reason why isospin is a good symmetry in QCD is that $m_d - m_u \ll \Lambda_{\text{QCD}} \cong 200 \text{ MeV} \cong 1/r$, where $r=1 \text{ fm}$
- For heavy quarks it was noticed that there is another symmetry of QCD, $m_Q \gg \Lambda_{\text{QCD}}$ (exact symmetry for $m_Q \rightarrow \infty$)
- This symmetry arises because once a quark becomes sufficiently heavy, its mass becomes irrelevant to the non perturbative dynamics of the light degrees-of-freedom (DOF) of QCD \rightarrow framework to calculate $b \rightarrow c$ transitions, also the spin of the heavy quark decouples from that of the light DOF
- In the heavy quark limit, an effective theory (HQET) exists that allows exact calculations of the 6 form factors in $B \rightarrow PW$ and $B \rightarrow VW$ in terms of a universal function \rightarrow since m_b is finite, corrections need to be included
- The heavy quark symmetry also justifies the calculation of hadronic decay properties in inclusive decays in terms of the quark decay properties plus an expansion in powers of $1/m_b$ (heavy quark expansion)
 - \rightarrow for many quantities the $1/m_b$ term vanishes



General Remarks II

- As an example, let's look at B inclusive semileptonic decay rate

- The decay is at short distance → calculable

- Hadronization occurs at long distance
→ is non perturbative

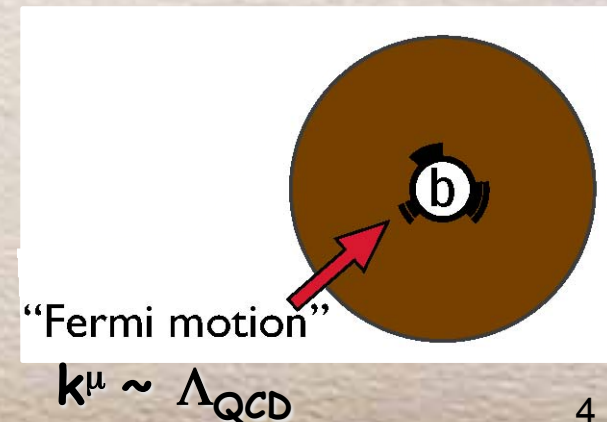
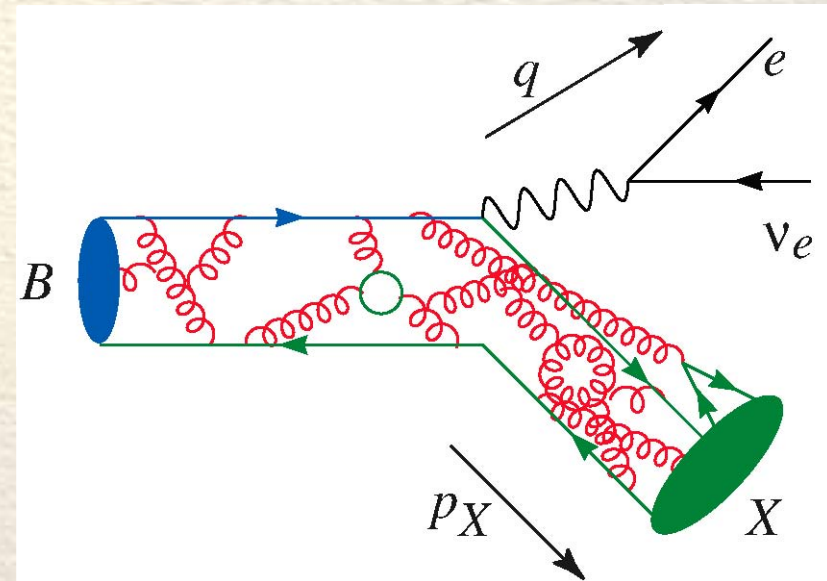
- At leading order, short distance and long distance are cleanly separated and the probability to hadronize is 100%

- Thus, the semileptonic decay rate can be approximated by

$$\frac{d\Gamma}{d(\text{LIPS})} \sim \frac{d\Gamma}{d(\text{LIPS})} (\text{parton model}) + \sum_n z_n \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^n \quad (1)$$

- In this way, decay rate is calculated at ~1% accuracy

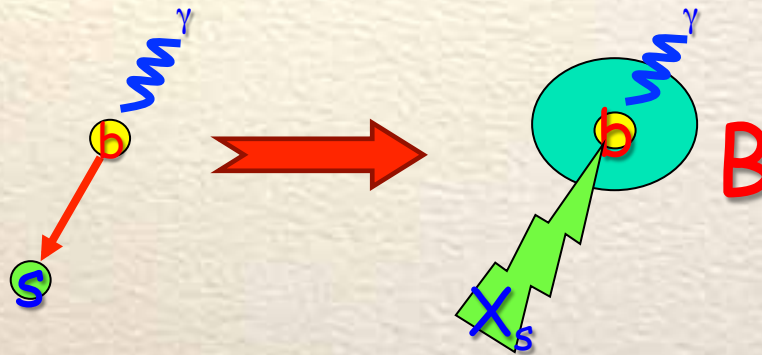
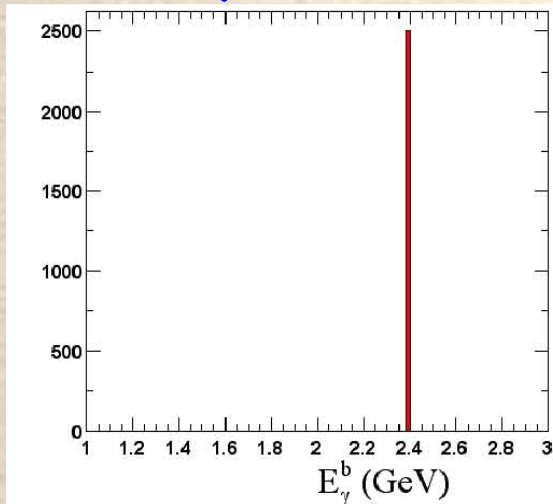
- Most of the time details of b-quark wave function are not relevant, only averages matter (k^2)



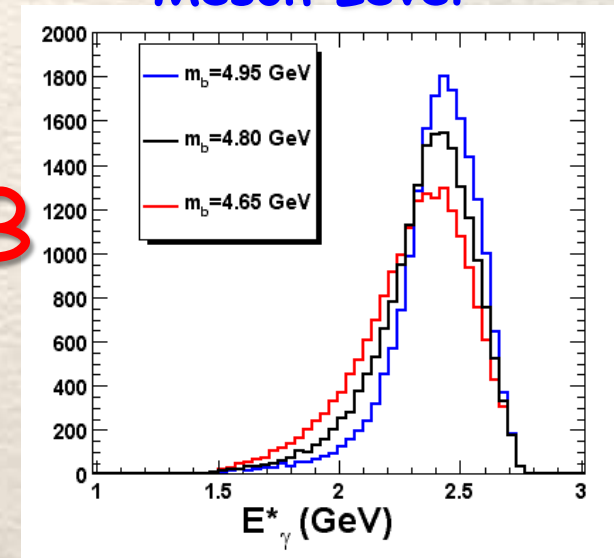
General Remarks III

- Lets illustrate the effect of the Fermi motion in the decay $B \rightarrow X\gamma$
- The b-quark is confined inside the B meson \rightarrow the Fermi motion affects the shape of the photon energy spectrum (m_b)

Quark Level



Meson Level



- Since the b-quark motion is universal arising from hadronic b-quark interactions inside the B meson, we expect it to determine the dynamics of decays like $B \rightarrow X\gamma$ and $B \rightarrow X\ell\nu$ in the same way



So for many effects, we need to deal with the motion of the b quark inside the B hadron

General Remarks IV

- Why is B physics so interesting?
- B decays have relatively simple decay topologies and a secondary vertex
 - so powerful criteria can be defined to discriminate signal from backgrounds
 - particularly at the $\Upsilon(4S)$ with well-defined initial conditions
 - dedicated experiments can fully exploit the physics
- For most observables predictions in the Standard Model (SM) are rather precise → deviations would indicate new physics contributions
- With high statistics samples, measurements are rather precise to test the SM and check for new physics contributions
- There are many decays and observables that can be measured → we get a complete picture which is important for deciding if an effect is real or is a fluctuation

Outline

- Study of B_c^+ Meson
- Rare Decays
 - $B \rightarrow X\gamma$
 - $B \rightarrow K\ell^+\ell^-$, $B \rightarrow K^*\ell^+\ell^-$, $B_s \rightarrow \phi\mu^+\mu^-$, and $B \rightarrow X_s\ell^+\ell^-$
 - $B \rightarrow \tau\nu$
 - $B \rightarrow \mu^+\mu^-$
- $B \rightarrow D^{(*)}\tau\nu$
- $B^0\bar{B}^0$ mixing
- CP violation
 - Direct CP violation
 - Measurement of β
 - Measurement of β_s
 - Measurement of α
 - Measurement of γ
 - Unitarity Triangle

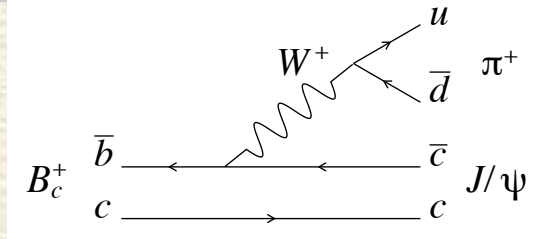
Summary and outlook



B_c^+ Meson

Study of the B_c Meson

- The B_c (discovered by CDF in 1998) is only the B meson that consists of 2 decaying quarks thus shortening its lifetime wrt to that of other B mesons

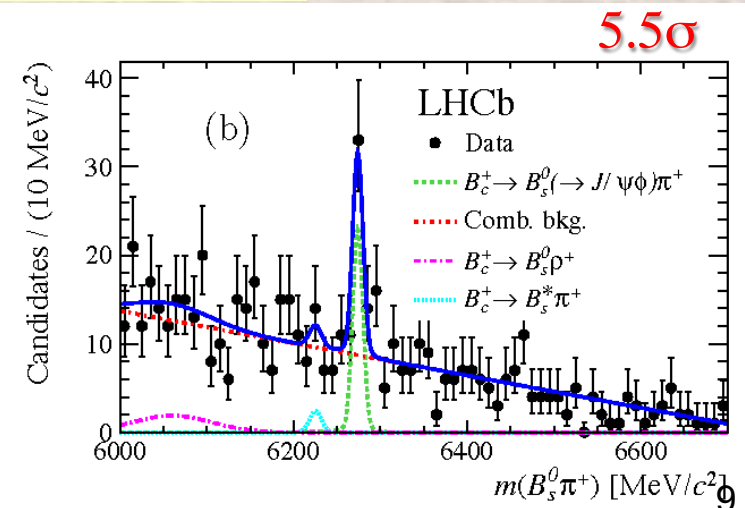
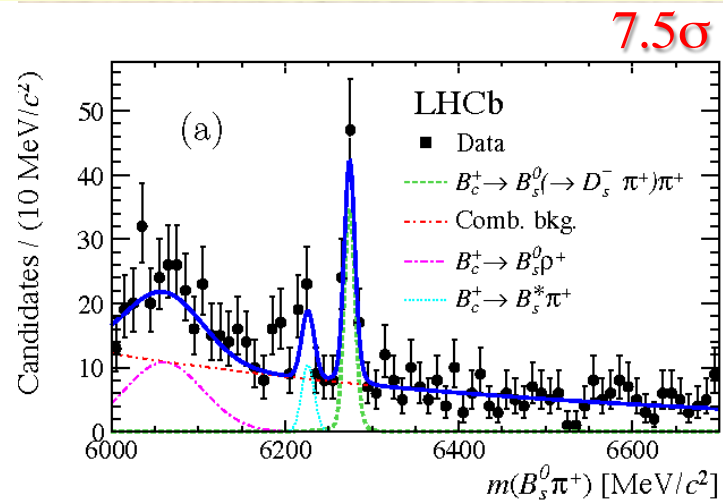
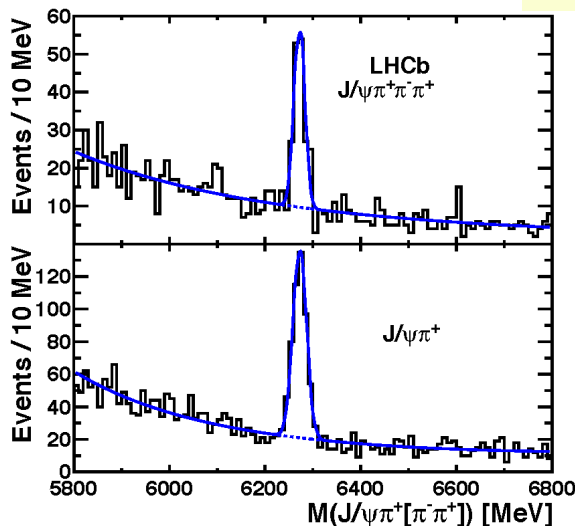
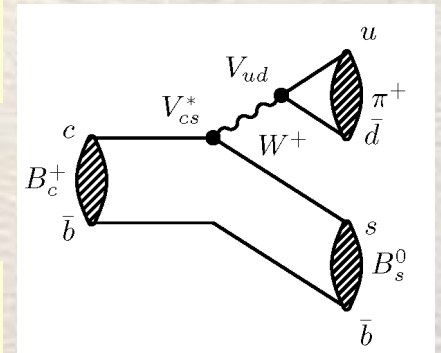


- LHCb observes 414 ± 14 $J/\psi\pi^+$ events, and 145 ± 14 $J/\psi\pi^+\pi^+\pi^-$ events (0.8 fb^{-1}) yielding a branching fraction ratio:

$$\frac{\mathcal{B}(B_c^+ \rightarrow J/\psi\pi^+)}{\mathcal{B}(B_c^+ \rightarrow J/\psi\pi^+\pi^+\pi^-)} = 2.41 \pm 0.30 \pm 0.33$$

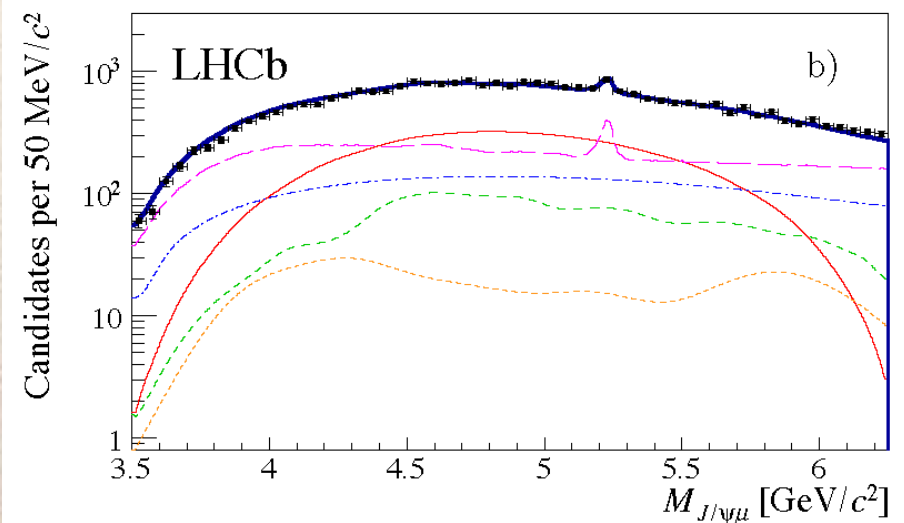
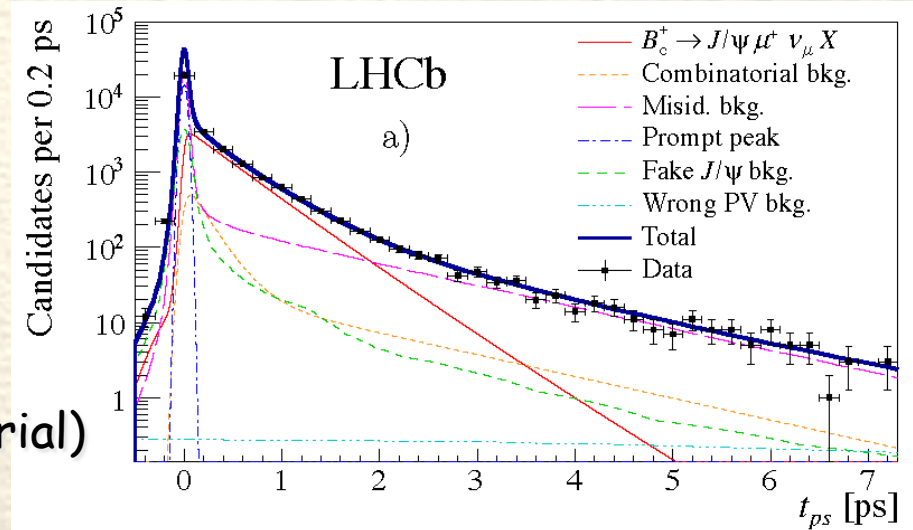
- LHCb observed for the first time the decay to $B_s\pi^+$ (3 fb^{-1}) measuring the ratio of B_c to B_s cross sections in the $B_s\pi^+$ mode:

$$\frac{\sigma(B_c^+) \times \mathcal{B}(B_c^+ \rightarrow B_s^0\pi^+)}{\sigma(B_s^0)} = (2.37 \pm 0.31_{\text{stat}} \pm 0.11_{\text{sys}} \pm 0.17^{+0.17}_{-0.13\tau_{B_c^+}}) \times 10^{-3}$$



Study of the B_c Meson

- LHCb has also measured the B_c lifetime using a sample 2 fb^{-1} of $J/\psi X_{\mu\nu}$ events
- Backgrounds:
 - $J/\psi +$ hadron misidentified as μ
 - false $J/\psi +$ correct μ
 - $J/\psi + \mu$ from primary vertex (prompt)
 - $J/\psi + \mu$ from different vertices (combinatorial)
 - $J/\psi + \mu$ from same vertex ($B \rightarrow 3\mu$)
- The lifetime is extracted from a 2D unbinned maximum likelihood fit to the lifetime and the $J/\psi\mu$ mass
- Signal t distribution is exponential, $J/\psi\mu$ mass is modeled from simulation
- Observe 8995 ± 103 signal events

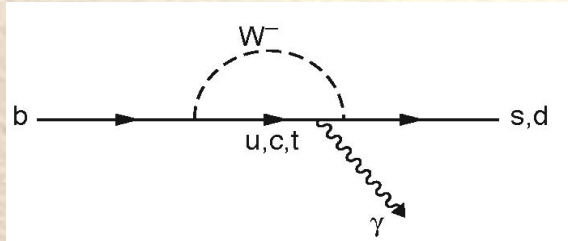


Measure B_c^+ lifetime: $\tau_{B_c^+} = 0.5087 \pm 0.0077 \text{ ps}$ \rightarrow compared to $\tau_{B_s^0} = 1.516 \pm 0.011 \text{ ps}$

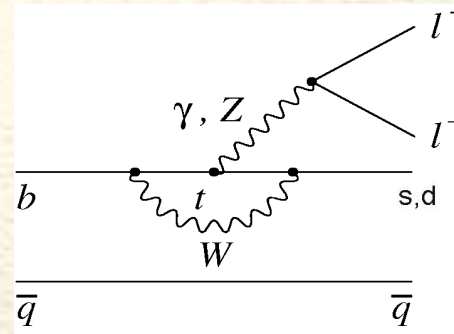
Rare Decays

Rare Decays

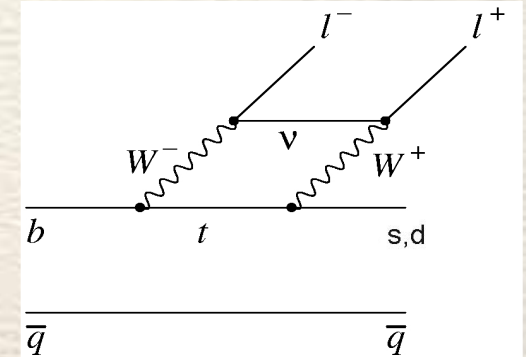
- $B \rightarrow X\gamma$ & $B \rightarrow X\ell^+\ell^-$ are flavor-changing neutral current (FCNC) processes, forbidden in SM at tree level



C_7^{eff} (EM penguin)



C_9^{eff} & C_{10}^{eff} (V & A parts of weak penguin and box)



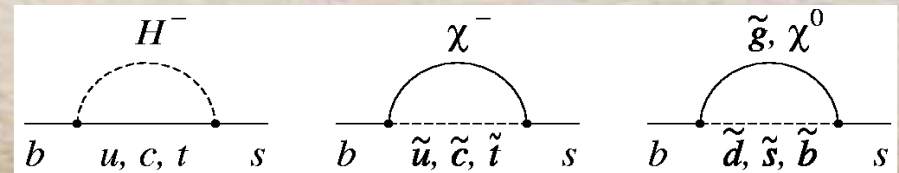
- Effective Hamiltonian factorizes short-distance from long-distance effects [$\mathcal{O}(\alpha_s)$]

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_i V_{tb}^* V_{ts,d} C_i(\mu) \mathcal{O}_i \quad (2)$$

→ 4 effective Wilson coefficient: C_7^{eff} , C_8^{eff} , C_9^{eff} , C_{10}^{eff}

- New physics adds new loops with new particles → modifies SM values of Wilson coefficients and may introduce new terms, e.g. C_S and C_P

- Probe here new physics at a scale of a few TeV



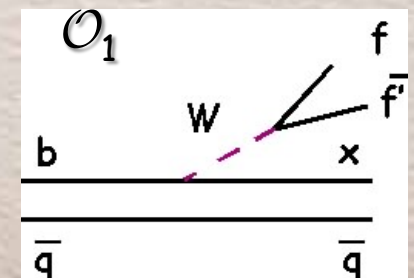
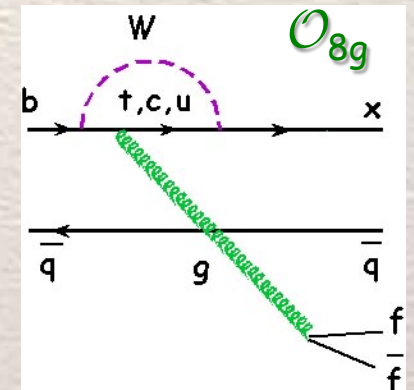
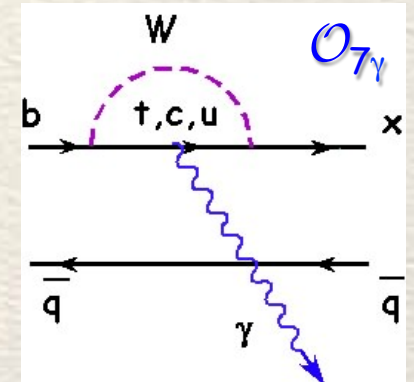
B → X_sγ Study

- The $\bar{B} \rightarrow X_s \gamma$ transition is dominated by the magnetic dipole operator $O_{7\gamma}$ with $C_7^{\text{eff}}(\text{SM}) \approx -0.33$
 → may be enhanced by new physics contributions
- In addition, O_1 and O_{8G} contribute via mixing
- It is customary to use the spectator in which $\bar{B} \rightarrow X_s \gamma$ is approximated by the quark decay $b \rightarrow s \gamma$ that is then related to the inclusive semileptonic decay

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma) \equiv \frac{\Gamma(\bar{B} \rightarrow X_s \gamma)}{\Gamma(\bar{B} \rightarrow X_c e \bar{\nu}_e)} \approx \frac{\Gamma(b \rightarrow s \gamma)}{\Gamma(b \rightarrow c e \bar{\nu}_e)} \mathcal{B}(\bar{B} \rightarrow X_c e \bar{\nu}_e) \quad (3)$$

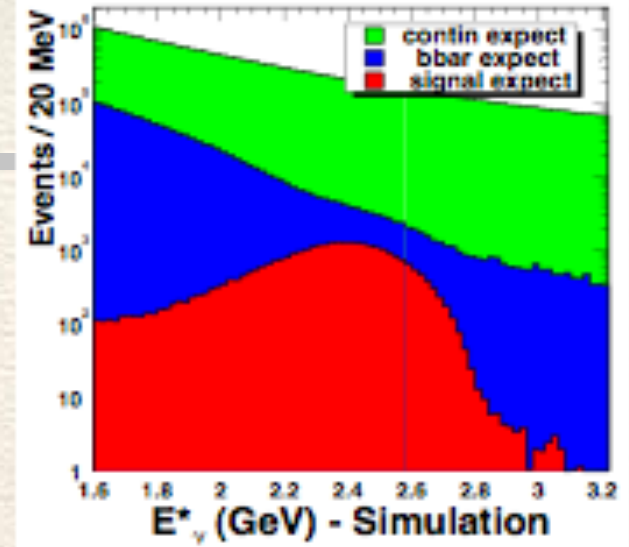
- Here short-distance QCD effects are included
- Normalization to the semileptonic rate removes $(m_b)^5$ factor & reduces uncertainties in CKM parameters
- In NNLO, the SM prediction yields

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4} \quad E_\gamma > 1.6 \text{ GeV}$$



$\bar{B} \rightarrow X_s \gamma$ Results

- Experimental challenge is to remove γ s from π^0 and η decays
- Thus, use 2 approaches
 - fully inclusive method with B tag on opposite side and stringent π^0 and η vetoes
 - sum of exclusive final states, 1 K^\pm (K^0_S) plus 4π ($\leq 1 \pi^0$) \rightarrow 38 final states



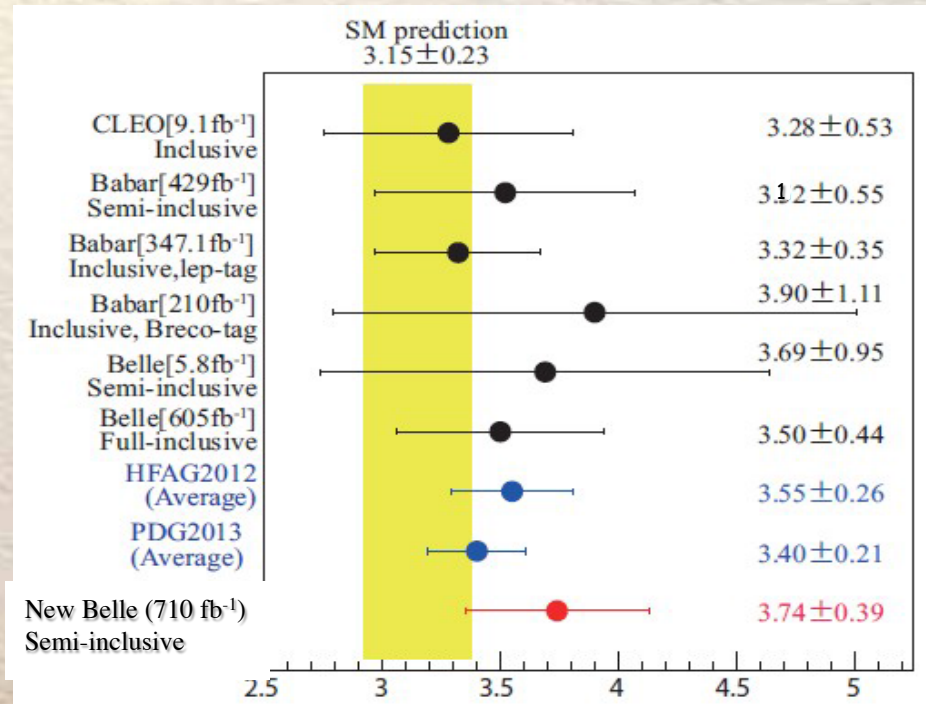
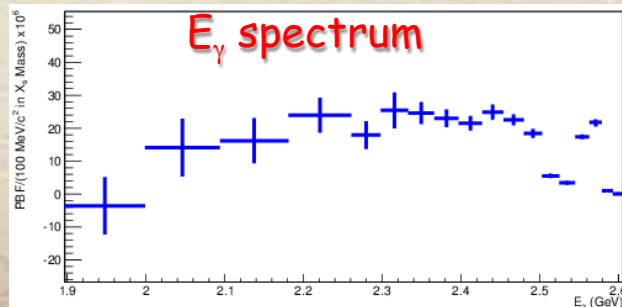
- Measured branching fraction (WA)

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

stat+sys shape b \rightarrow d γ
function

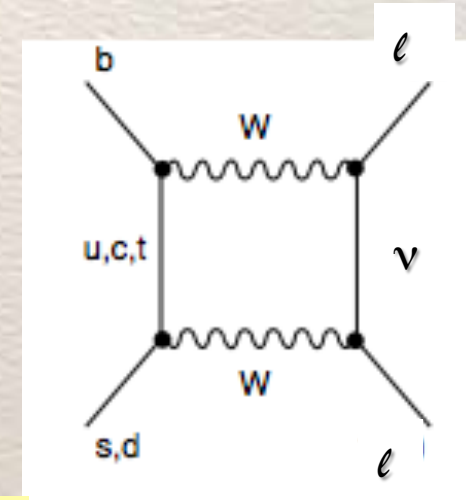
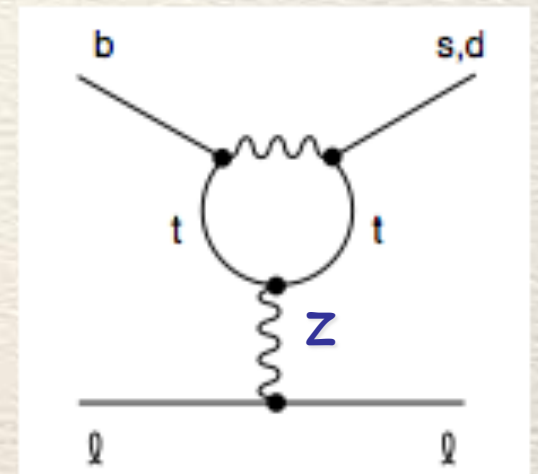
agrees well with the SM prediction

- We will see constraints in m_H -tan β plane



$B \rightarrow K \ell^+ \ell^-$ and $B \rightarrow K^* \ell^+ \ell^-$ Expectations

- In addition to the magnetic dipole operator $\mathcal{O}_{7\gamma}$, the weak penguin and box diagrams contribute, where the linear combination of vector currents form \mathcal{O}_9 and that of the axial-vector currents form \mathcal{O}_{10}
- So we encounter 2 new Wilson coefficients $C_9^{\text{eff}}(q^2)$ and C_{10}^{eff}
 $C_9^{\text{eff}}(q^2) - Y(q^2) = 4.211$, $C_{10}^{\text{eff}} = -4.103$,
 $Y(q^2)$ increases faster than exponential for $q^2 < 5.76$
for $q^2 > 5.76$ decreases \sim exponentially
- The decay rate is again normalized to the semileptonic rate
- Predictions for $1 < q^2 < 6 \text{ GeV}^2$ and for $q^2 > 14.2 \text{ GeV}^2$ have smallest uncertainties
- SM prediction (for $1 < q^2 < 6 \text{ GeV}^2$)



$$\mathcal{B}(B \rightarrow K^* \ell^+ \ell^-) = \left(2.60^{+1.82}_{-1.34} \right) \times 10^{-7}$$

$$\mathcal{B}(B^0 \rightarrow K_s^0 \ell^+ \ell^-) = \left(1.59^{+0.59}_{-0.35} \right) \times 10^{-7}$$

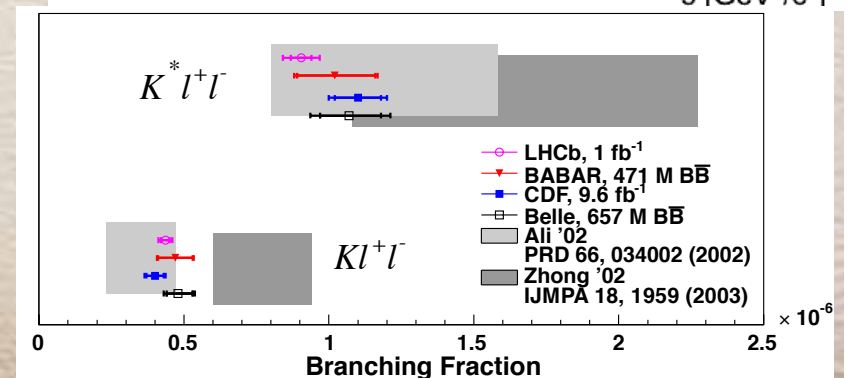
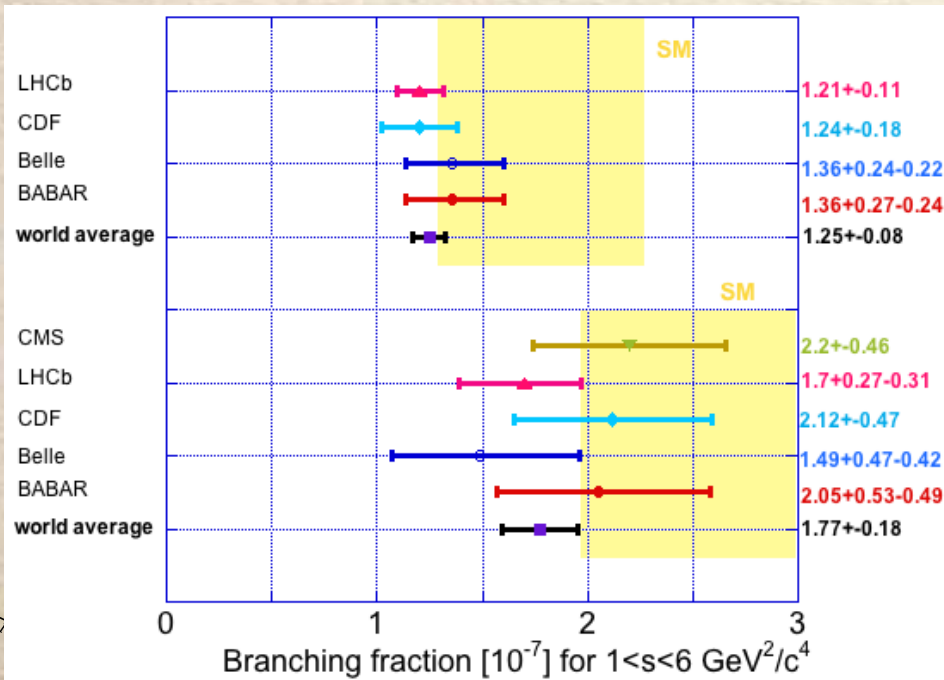
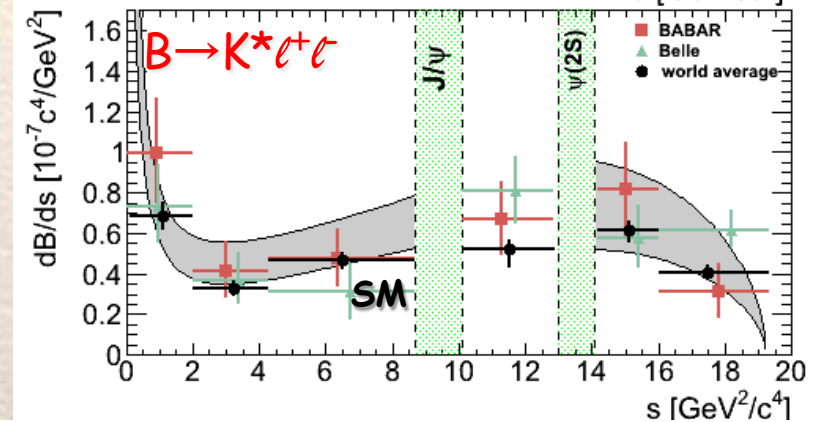
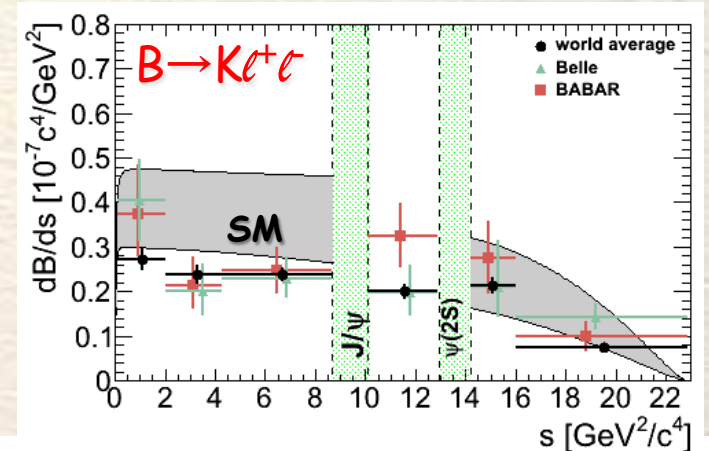
$$\mathcal{B}(B^+ \rightarrow K^+ \ell^+ \ell^-) = \left(1.75^{+0.64}_{-0.38} \right) \times 10^{-7}$$

$B \rightarrow K \ell^+ \ell^-$ and $B \rightarrow K^* \ell^+ \ell^-$ Branching Fractions

- BABAR, Belle, CDF, LHCb and CMS measured differential branching fractions of $B \rightarrow K \ell^+ \ell^-$ & $B \rightarrow K^* \ell^+ \ell^-$ decays
WA is dominated by LHCb
- Branching fractions for $1 < s < 6 \text{ GeV}^2$

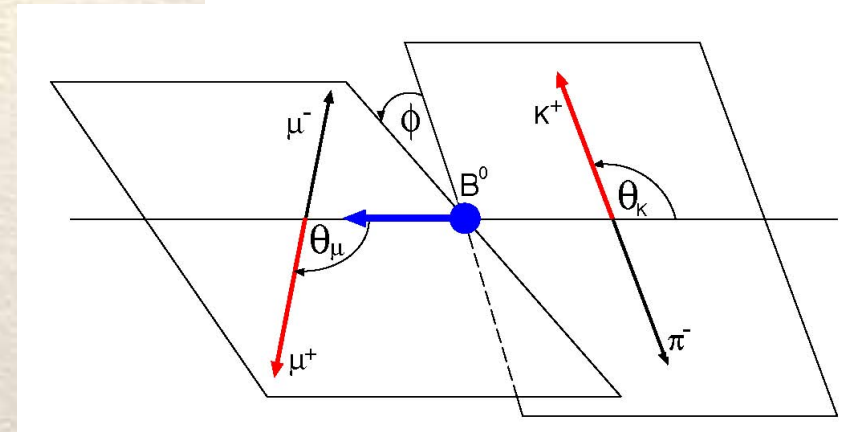
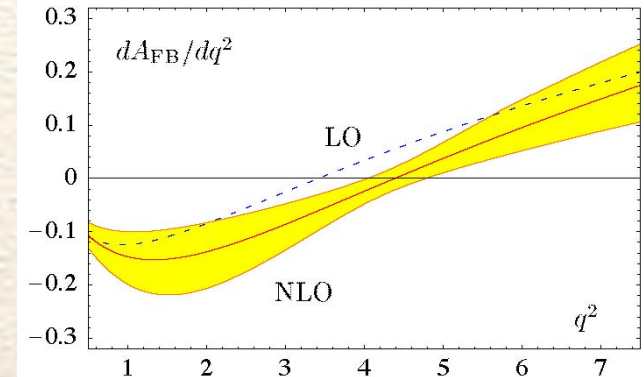
$$\mathcal{B}(B \rightarrow K \ell^+ \ell^-) = (1.25 \pm 0.08) \times 10^{-7}$$

$$\mathcal{B}(B \rightarrow K^* \ell^+ \ell^-) = (1.77 \pm 0.18) \times 10^{-7}$$



$B \rightarrow K \ell^+ \ell^-$ and $B \rightarrow K^* \ell^+ \ell^-$ Angular Analysis

- From angular distributions we can measure the forward-backward asymmetry \mathcal{A}_{FB} (ℓ^+ is in same -opposite hemisphere of the B meson) and the K^* longitudinal polarization \mathcal{F}_L
- 3 angles determine the decay rate:
 - θ_μ : angle between ℓ^+ & B momenta in the $\ell^+ \ell^-$ CM frame
 - θ_K : angle between K & B momenta in K^* rest frame
 - ϕ : angle between 2 decay planes
- We extract parameters from the 1-dimensional angular distributions



$$W(\cos \theta_K) = \frac{3}{2} \mathcal{F}_L \cos^2 \theta_K + \frac{3}{4} (1 - \mathcal{F}_L) \sin^2 \theta_K \quad (4)$$

$$W(\cos \theta_\ell) = \frac{3}{4} \mathcal{F}_L \sin^2 \theta_\ell + \frac{3}{8} (1 - \mathcal{F}_L) (1 + \cos^2 \theta_\ell) + \mathcal{A}_{FB} \cos \theta_\ell \quad (5)$$

B → K* ℓ+ ℓ- Forward-backward Asymmetry

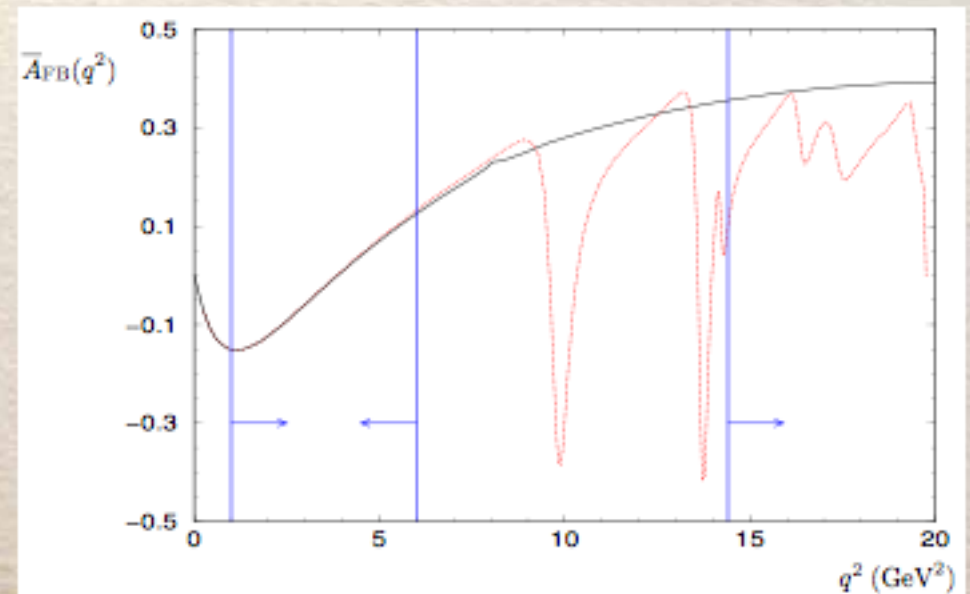
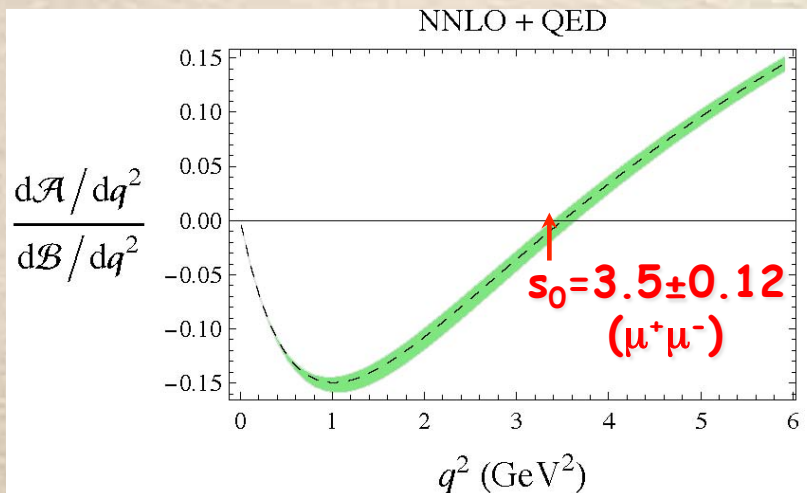
- In $\mathcal{O}(1)$ the lepton forward-backward asymmetry is

$$\frac{dA_{FB}(q^2)}{ds} \propto - \left\{ \text{Re} \left[C_9^{\text{eff}}(q^2) C_{10} \right] V(q^2) A_1(q^2) + \frac{m_b m_B}{q^2} \text{Re} \left[C_7^{\text{eff}} C_{10} \right] \left[V(q^2) T_2(q^2) \left(1 - \frac{m_{K^*}}{m_B} \right) + A_1(q^2) T_1(q^2) \left(1 + \frac{m_{K^*}}{m_B} \right) \right] \right\} \quad (6)$$

- A_{FB} has a zero crossing that provides a powerful signature for searching for new physics effects

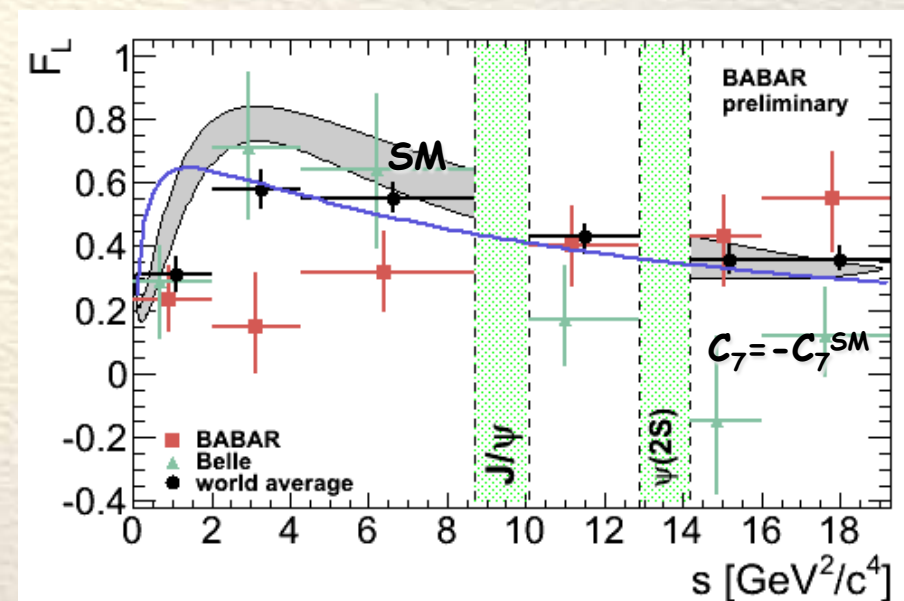
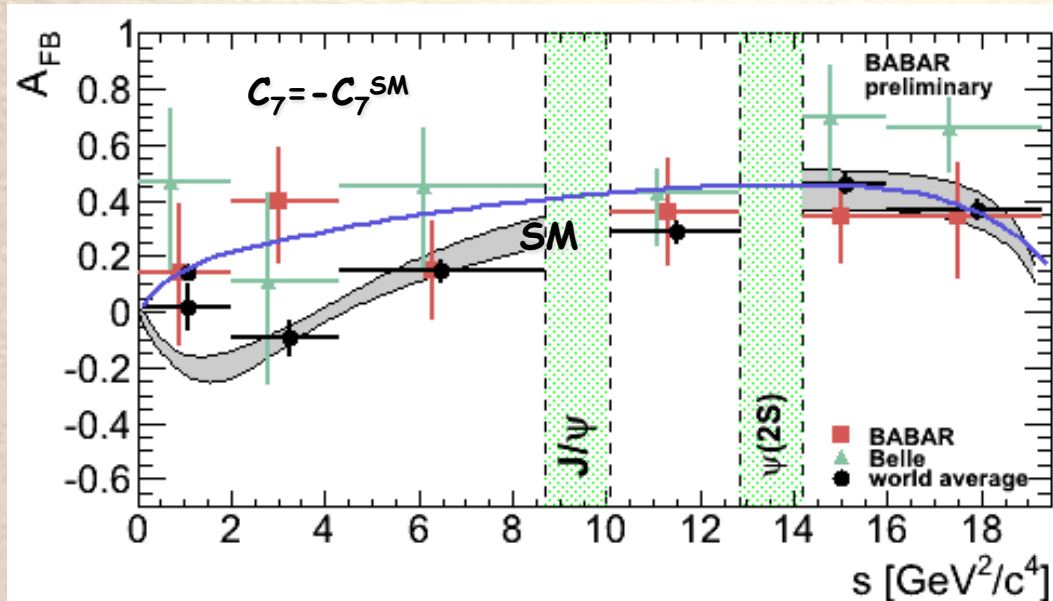
$V(q^2), A_1(q^2), T_1(q^2), T_2(q^2)$ are form factors that increase with q^2 in a monopole or dipole form and are positive

- In $\mathcal{O}(\alpha_s)$ the scale dependence is small



$B \rightarrow K^* \ell^+ \ell^-$ Results for A_{FB} and \mathcal{F}_L

- BABAR, Belle, CDF, LHCb, CMS & ATLAS measured A_{FB} & \mathcal{F}_L in 6 bins of q^2



- In the low q^2 region, the A_{FB} and \mathcal{F}_L averages over all measurements yield

$$A_{FB}^{WA}(K^* \ell \ell) = -0.074^{+0.047}_{-0.048} \quad \text{and} \quad \mathcal{F}_L = 0.523^{+0.047}_{-0.044}$$

WA is dominated by LHCb

- This is in good agreement with the SM prediction

$$A_{FB}^{SM} = -0.0494^{+0.0281}_{-0.0252} (K^{*0} \ell^+ \ell^-) \quad \text{and} \quad \mathcal{F}_L^{SM} = 0.735^{+0.06}_{-0.07} (K^{*0} \ell^+ \ell^-)$$



B → K* ℓ+ ℓ- Angular Observables

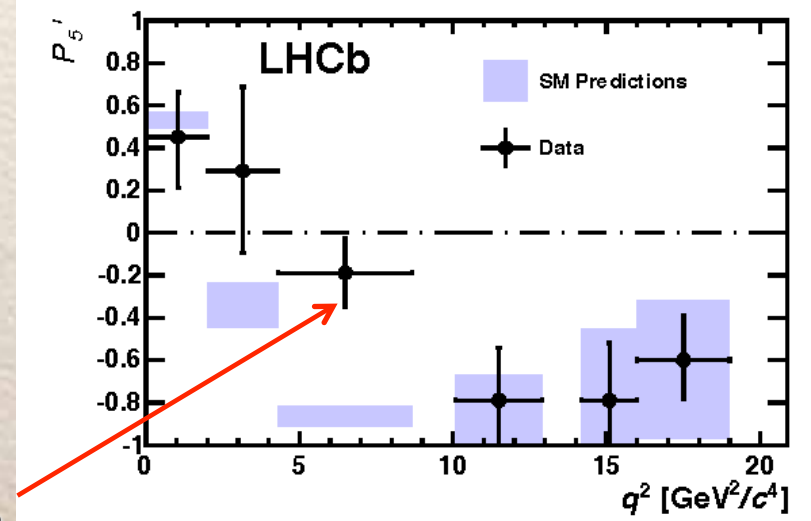
- The full angular distribution for B → K* ℓ+ ℓ- is given by

$$\frac{1}{d\Gamma/dq^2} \frac{d^4\Gamma}{d\cos\theta_\ell d\cos\theta_K d\phi dq^2} = \frac{9}{32\pi} \left[\frac{3}{4} (1 - F_L) \sin^2\theta_K + F_L \cos^2\theta_K + \frac{1}{4} (1 - F_L) \sin^2\theta_K \sin 2\theta_\ell \right. \\ \left. - F_L \cos^2\theta_K \cos 2\theta_\ell + S_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi \right. \\ \left. + S_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + S_5 \sin 2\theta_K \sin \theta_\ell \cos \phi \right. \\ \left. + S_6 \sin^2\theta_K \cos \theta_\ell + S_7 \sin 2\theta_K \sin \theta_\ell \sin \phi \right. \\ \left. + S_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + S_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi \right] \quad (7)$$

- Define new observables in 6 bins of q^2

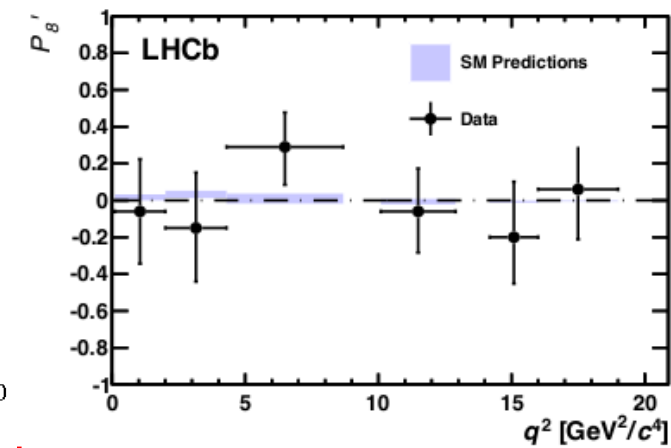
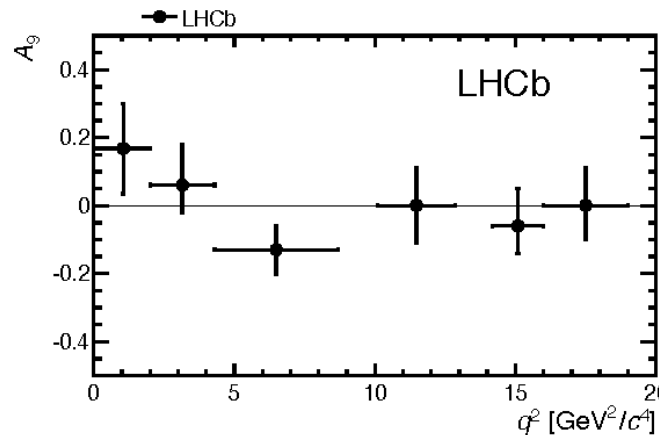
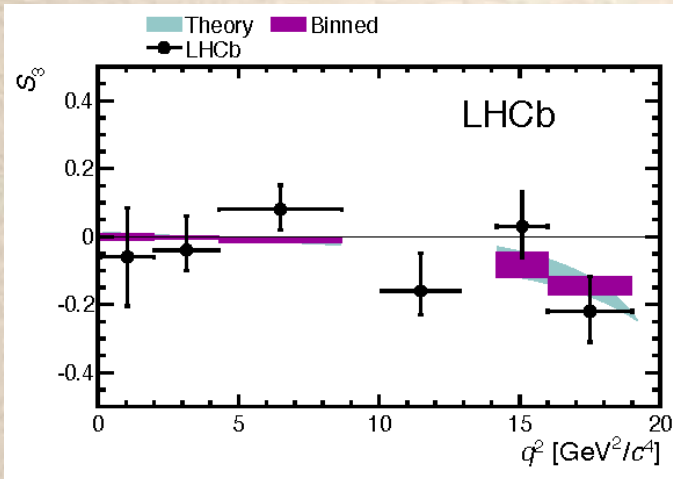
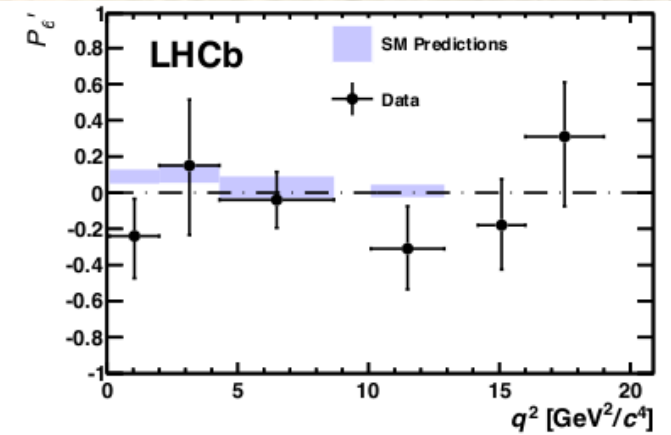
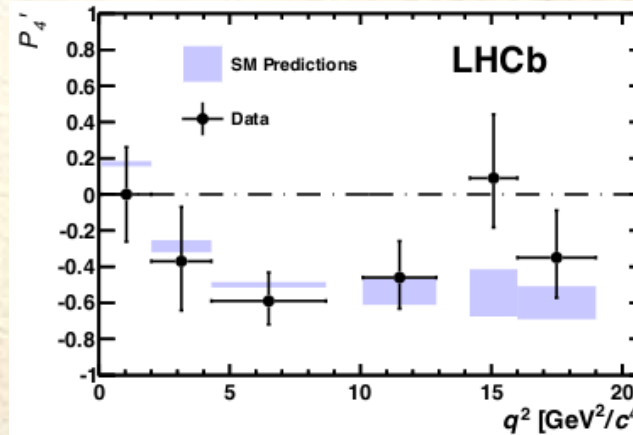
$$P'_{4,5,6,8} = \frac{S_{4,5,7,8}}{\sqrt{F_L(1-F_L)}}$$

- At low q^2 , observables are free from form factor uncertainties
- LHCb explored individual S_i/P'_i distributions
- P'_5 shows a deviation from the SM in bin 3



Measurement of $B \rightarrow K^* e^+ e^-$ Angular Observables

- Other distributions agree with the SM
- LHCb estimates new physics scale in $b \rightarrow s e^+ e^-$ at $\Lambda_{\text{NP}} > 15 \text{ TeV}$

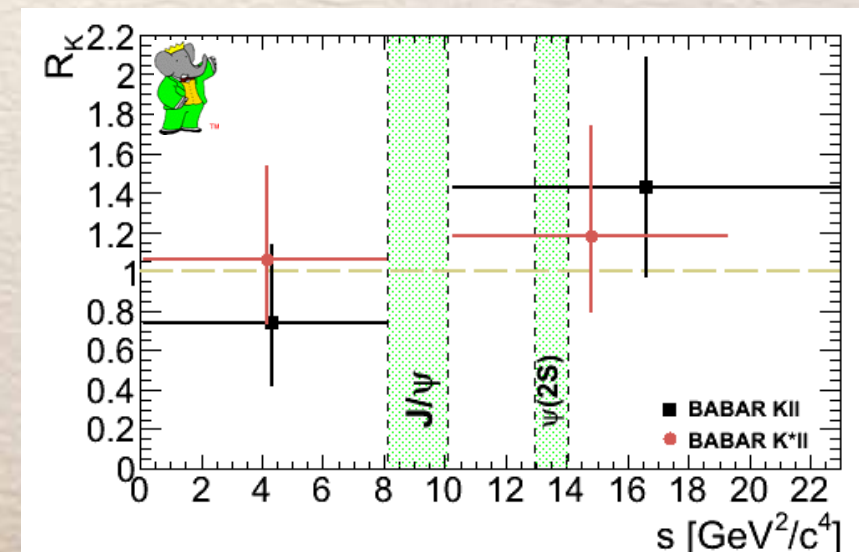
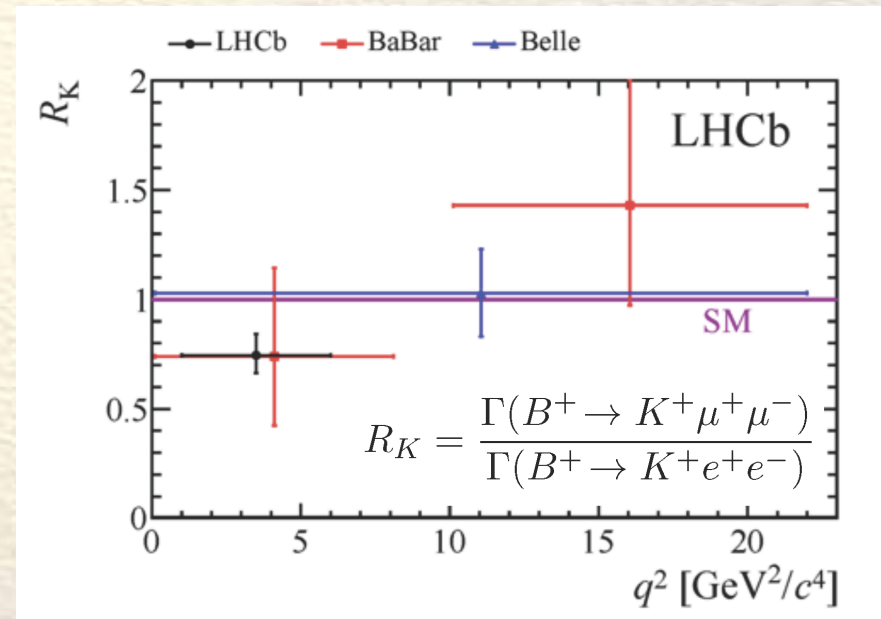


$B \rightarrow K e^+ e^-$ and $B \rightarrow K^* e^+ e^-$ Lepton Flavor Ratio

- Since $B \rightarrow K^{(*)} e^+ e^-$ and $B \rightarrow K^{(*)} \mu^+ \mu^-$ may receive different contributions from new physics, it is interesting to look at the lepton flavor ratios

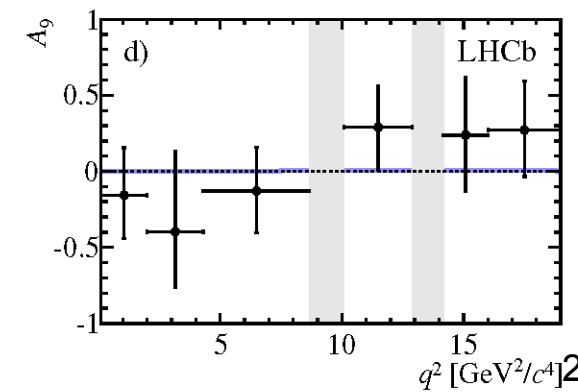
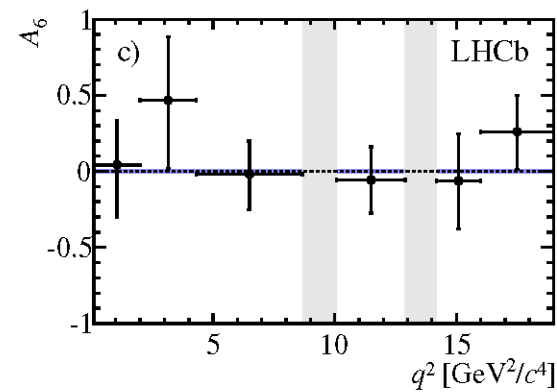
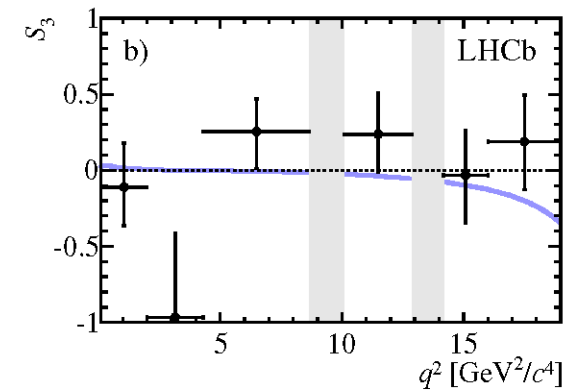
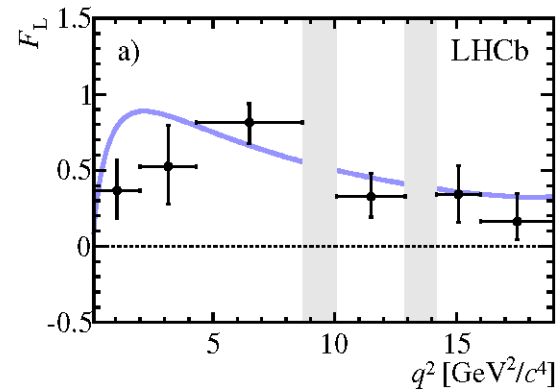
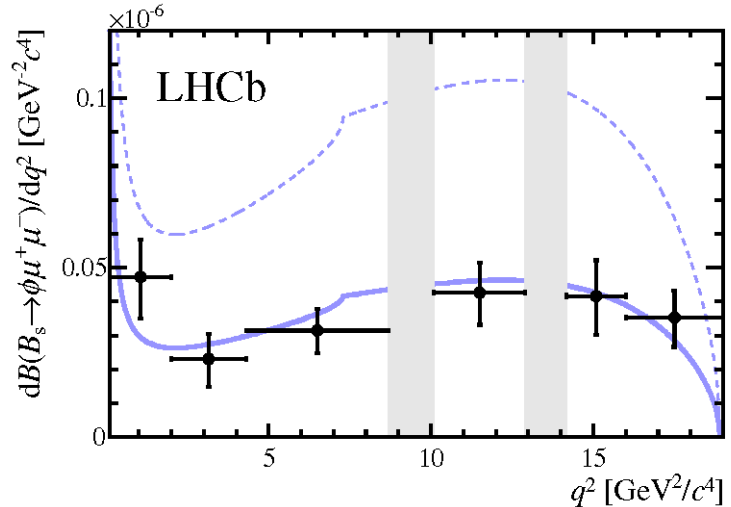
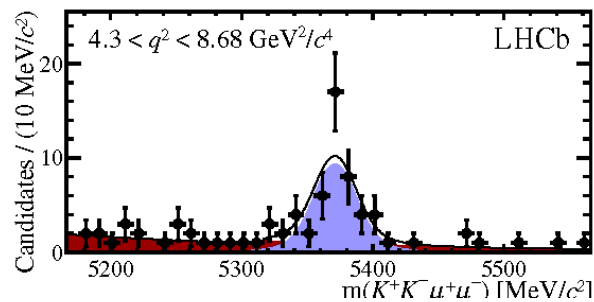
$$\mathcal{R}_{K^{(*)}} \equiv \frac{\mathcal{B}(B \rightarrow K^{(*)} \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow K^{(*)} e^+ e^-)} \quad q^2 \geq (2m_\mu)^2$$

- LHCb has measured R_K below the J/ψ resonance
- BABAR measured both R_K and R_{K^*} below and above the J/ψ resonance
- BABAR $R_{K^{(*)}}$ results are consistent and agree with the SM prediction
- The LHCb R_K result is consistent with the SM within 2σ



$B_s \rightarrow \phi \mu^+ \mu^-$ Study

- Replacing the spectator d quark with and s quark yields $B_s \rightarrow \phi \mu^+ \mu^-$
- LHCb has observed $B_s \rightarrow \phi \mu^+ \mu^-$ measuring differential branching fractions and angular observables in 6 bins of q^2
- All results agree with the SM prediction
- The total branching fraction is $\mathcal{B}(B_s \rightarrow \phi \mu^+ \mu^-) = (7.07^{+0.64}_{-0.56} \pm 0.17 \pm 0.71) \times 10^{-7}$



B → X_S e⁺e⁻

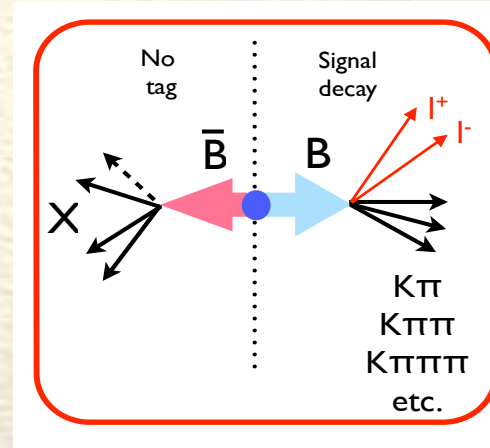
- Reconstruct 20 exclusive final states with K (K_S⁰ → π⁺π⁻), ≤2π[±] & ≤1π⁰(→γγ) for branching fraction and 14 self-tagging modes (red) for CP asymmetry measurements

- The 20 exclusive modes represents 70% of the inclusive rate with m_{X_S} < 1.8 GeV accounting for K_L⁰ modes, K_S⁰ → π⁰π⁰ and π⁰ Dalitz decays

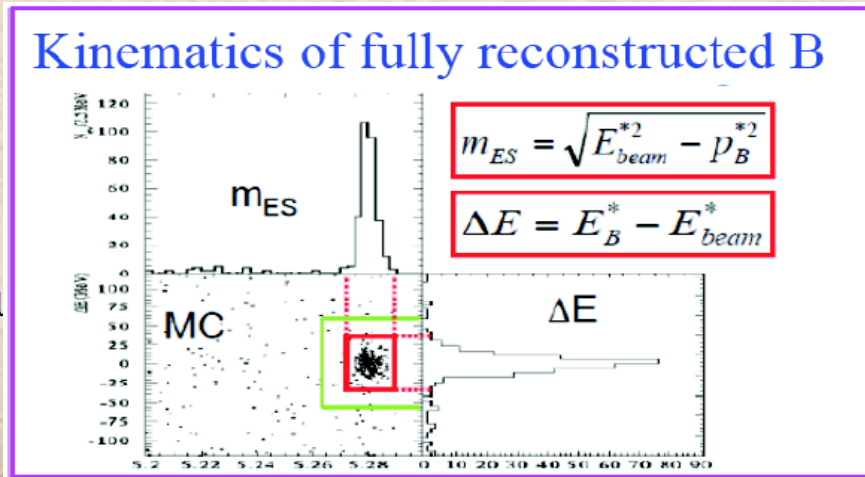
- Extrapolate for the missing modes and those with m_{X_S} > 1.8 GeV using JETSET fragmentation and theory predictions

- Kinematic constraints
 - m_{ES} > 5.225 GeV
 - 0.1 < ΔE < 0.05 for X_S ee
 - 0.05 < ΔE < 0.05 for X_S μμ

Use no tag for B



- $B^0 \rightarrow K_S^0 \mu^+ \mu^-$
- $B^+ \rightarrow K^+ \mu^+ \mu^-$
- $B^0 \rightarrow K_S^0 e^+ e^-$
- $B^+ \rightarrow K^+ e^+ e^-$
- $B^0 \rightarrow K^{*0} (K_S^0 \pi^0) \mu^+ \mu^-$
- $B^+ \rightarrow K^{*+} (K^+ \pi^0) \mu^+ \mu^-$
- $B^+ \rightarrow K^{*+} (K_S^0 \pi^+) \mu^+ \mu^-$
- $B^0 \rightarrow K^{*0} (K^+ \pi^-) \mu^+ \mu^-$
- $B^0 \rightarrow K^{*0} (K_S^0 \pi^0) e^+ e^-$
- $B^+ \rightarrow K^{*+} (K^+ \pi^0) e^+ e^-$
- $B^+ \rightarrow K^{*+} (K_S^0 \pi^+) e^+ e^-$
- $B^0 \rightarrow K^{*0} (K^+ \pi^-) e^+ e^-$
- $B^0 \rightarrow K_S^0 \pi^+ \pi^0 \mu^+ \mu^-$
- $B^+ \rightarrow K^+ \pi^- \pi^0 \mu^+ \mu^-$
- $B^+ \rightarrow K_S^0 \pi^+ \pi^- \mu^+ \mu^-$
- $B^0 \rightarrow K^+ \pi^+ \pi^- \mu^+ \mu^-$
- $B^0 \rightarrow K_S^0 \pi^+ \pi^0 e^+ e^-$
- $B^+ \rightarrow K^+ \pi^+ \pi^0 e^+ e^-$
- $B^+ \rightarrow K_S^0 \pi^+ \pi^- e^+ e^-$
- $B^0 \rightarrow K^+ \pi^+ \pi^- e^+ e^-$



$B \rightarrow X_s e^+ e^-$

- For $1 < q^2 < 6 \text{ GeV}^2$ BABAR measures

$$\mathcal{B}(B \rightarrow X_s \mu^+ \mu^-) = (0.66^{+0.82+0.30}_{-0.76-0.24} \pm 0.07) \times 10^{-6}$$

$$\mathcal{B}(B \rightarrow X_s e^+ e^-) = (1.93^{+0.47+0.21}_{-0.45-0.16} \pm 0.18) \times 10^{-6}$$

$$\mathcal{B}(B \rightarrow X_s l^+ l^-) = (1.60^{+0.41+0.17}_{-0.39-0.13} \pm 0.18) \times 10^{-6}$$

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- Agrees well with the SM prediction

$$\mathcal{B}(B \rightarrow X_s \mu^+ \mu^-) = (1.59 \pm 0.11) \times 10^{-6}$$

$$\mathcal{B}(B \rightarrow X_s e^+ e^-) = (1.64 \pm 0.11) \times 10^{-6}$$

- For $q^2 > 14.2 \text{ GeV}^2$ BABAR measures

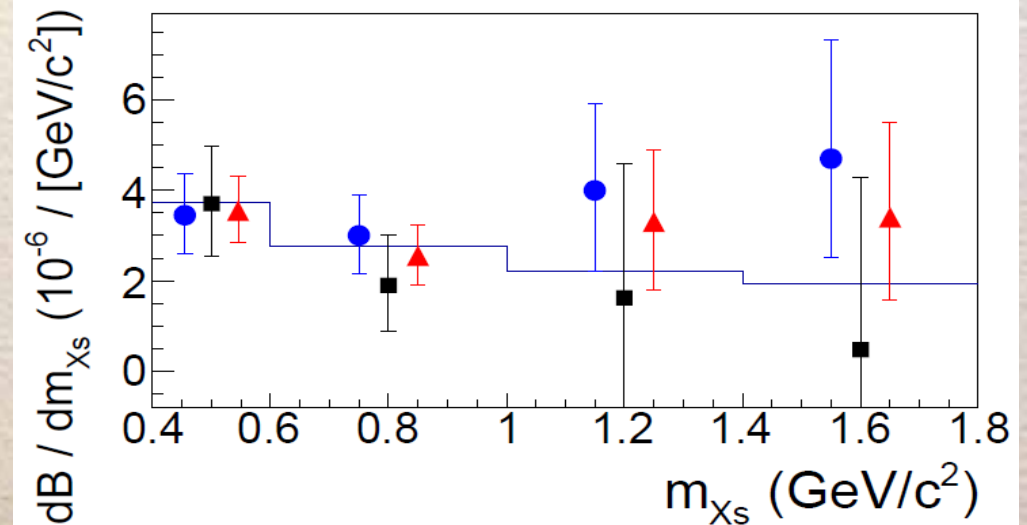
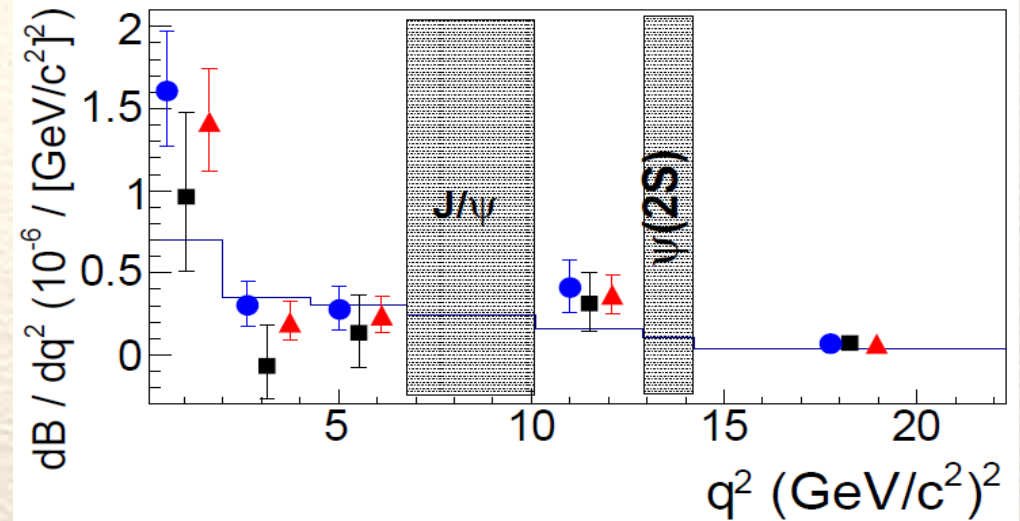
$$\mathcal{B}(B \rightarrow X_s \mu^+ \mu^-) = (0.60^{+0.31+0.05}_{-0.29-0.04} \pm 0.00) \times 10^{-6}$$

$$\mathcal{B}(B \rightarrow X_s e^+ e^-) = (0.56^{+0.19+0.03}_{-0.18-0.03} \pm 0.00) \times 10^{-6}$$

$$\mathcal{B}(B \rightarrow X_s l^+ l^-) = (0.57^{+0.16+0.03}_{-0.15-0.02} \pm 0.00) \times 10^{-6}$$

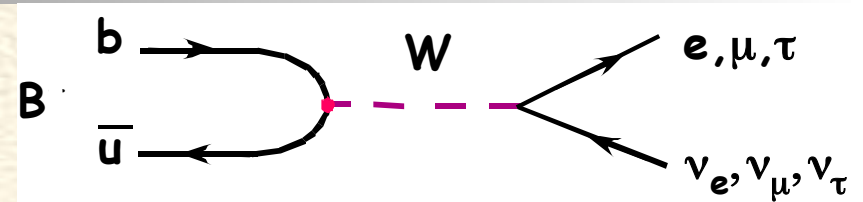
- Consistent with SM prediction at $\sim 1\sigma$

$$\mathcal{B}(B \rightarrow X_s \mu^+ \mu^-) = (0.25^{+0.07}_{-0.06}) \times 10^{-6}$$



Leptonic Decays

- Pure leptonic decays are mediated by W annihilation



- The relevant ME here is

$$\langle 0 | \mathbf{J}_\mu^{V-A} | \mathbf{B} \rangle$$

(8)

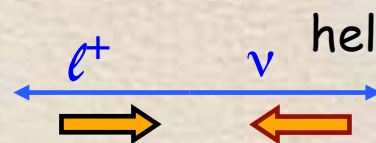
- The vector current vanishes while the axial vector currents yields

$$\langle 0 | \bar{u} \gamma_\mu \gamma_5 b | \mathbf{B} \rangle = i f_B p_\mu^B \quad (9)$$

f_B : B decay constant
 p_B : B momentum

- With this relation of the hadronic ME to the decay constant we can express the branching fraction by

$$\mathcal{B}(B^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} |V_{ub}|^2 f_B^2 m_B m_\ell^2 \left[1 - \frac{m_\ell^2}{m_B^2} \right]^2 \quad (10)$$



helicity suppression

$L=0$, since B is pseudoscalar

phase space

- In the SM, the branching fractions are predicted to be

$$\mathcal{B}_{SM} = 1.0 \times 10^{-11} \quad \text{for } \ell=e$$

$$\mathcal{B}_{SM} = 3.5 \times 10^{-7} \quad \text{for } \ell=\mu$$

$$\mathcal{B}_{SM} = 1.1 \times 10^{-4} \quad \text{for } \ell=\tau$$



Measurement of $B^+ \rightarrow \tau^+ \nu$

- New Physics may enhance the branching fraction by a factor of

$$r_H = \left(1 - \frac{m_B^2}{m_H^2} \frac{\tan^2 \beta}{1 + 0.01 \times \tan \beta} \right)^2 \quad (11)$$

m_H : charged Higgs mass
 $\tan \beta$: ratio of vacuum expectation values
 for the 2 higgs doublets

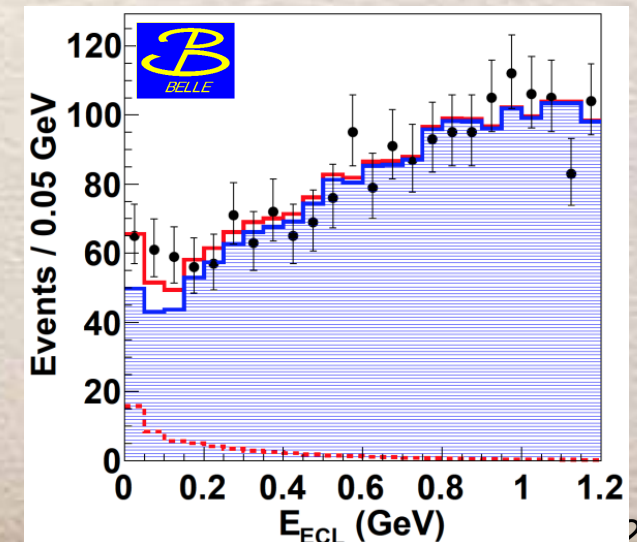
- BABAR and Belle have observed $B^+ \rightarrow \tau^+ \nu$ and measured its branching fraction
- With reconstructing the other B meson fully in an exclusive final state, no additional particles should appear in the decay besides the τ decay products
- So BABAR and Belle examine the extra energy measured in the calorimeter
- BABAR observes 89 ± 44 signal events
- Belle sees 62^{+23}_{-22} events
- The branching fractions measurements yield



$$\mathcal{B}(B \rightarrow \tau \nu) = (1.76 \pm 0.49) \times 10^{-4}$$

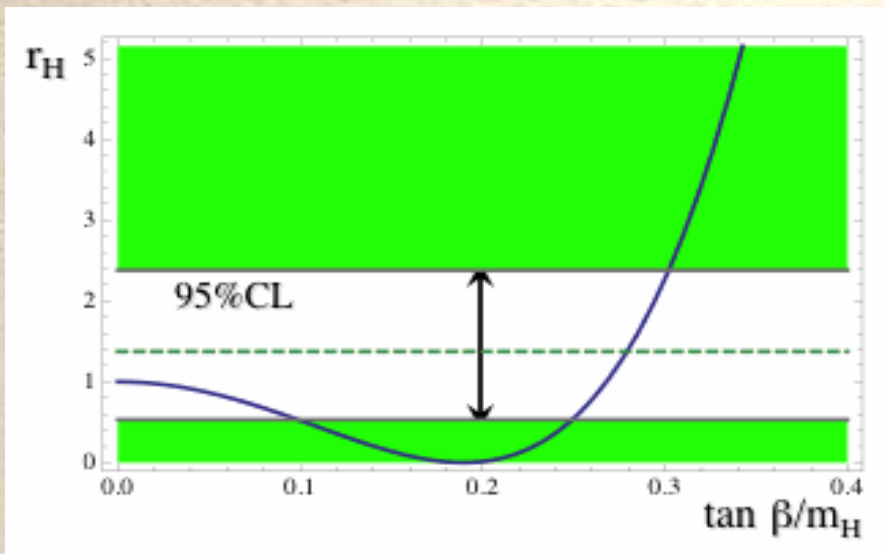


$$\mathcal{B}(B \rightarrow \tau \nu) = (0.72^{+0.27+0.11}_{-0.25-0.11}) \times 10^{-4}$$

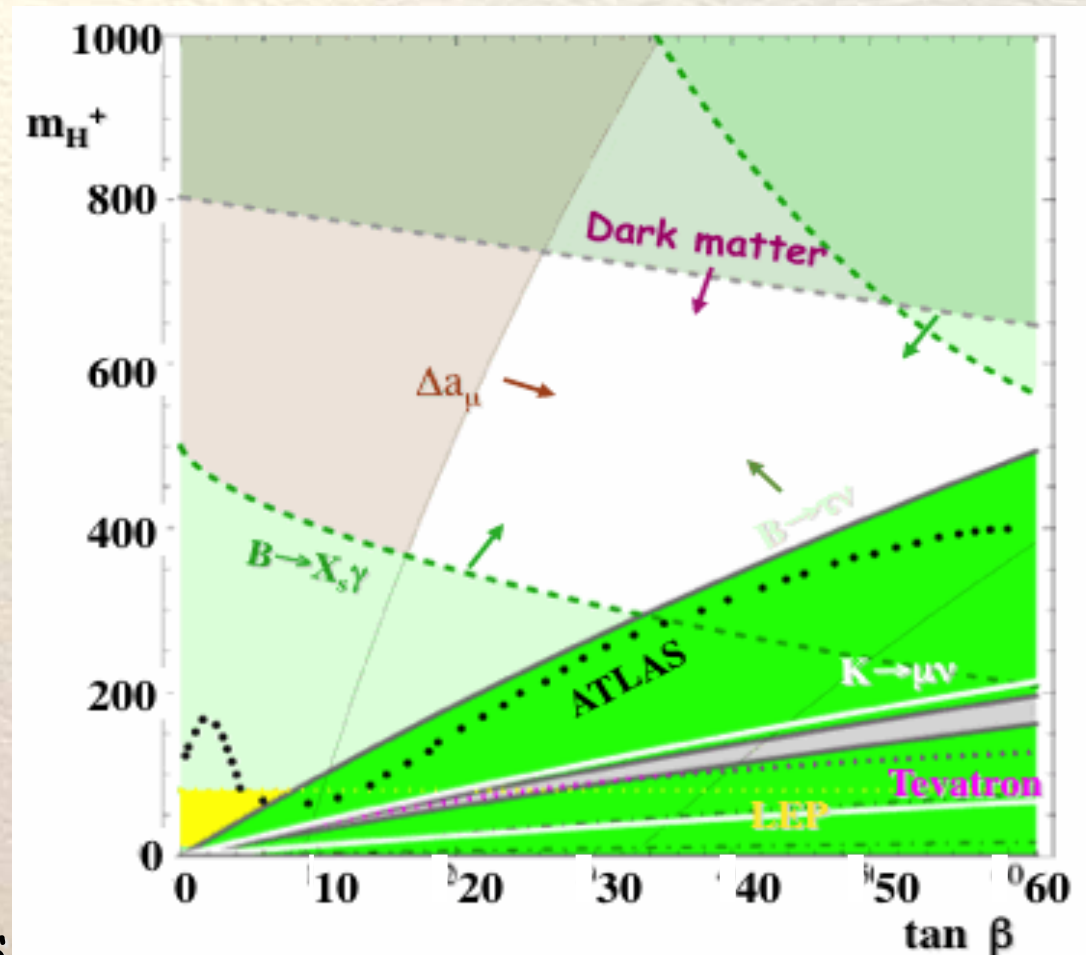


Implications of $B^+ \rightarrow \tau^+ \nu$ Measurements

- $B^+ \rightarrow \tau^+ \nu$ is a useful probe to search for new physics contributions
- Extra contributions may arise from a charged Higgs boson increasing $\mathcal{B}(B^+ \rightarrow \tau^+ \nu)_{SM}$ by r_H



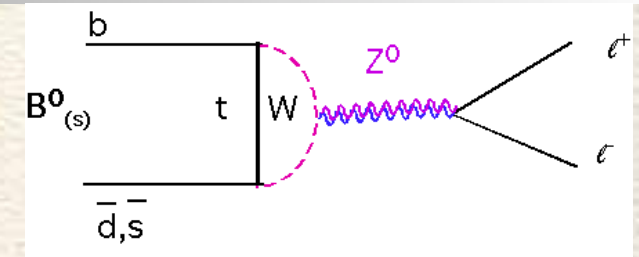
- Using the present WA of $\mathcal{B}(B^+ \rightarrow \tau^+ \nu) = (1.15 \pm 0.23) \times 10^{-4}$, we can set a limit on the H^\pm mass vs $\tan \beta$



$\mathcal{B}(B^+ \rightarrow \tau^+ \nu)$ plus other measurements impose stringent constraints on the H^\pm - $\tan \beta$ plane

Study of $B \rightarrow \mu^+ \mu^-$ Decays

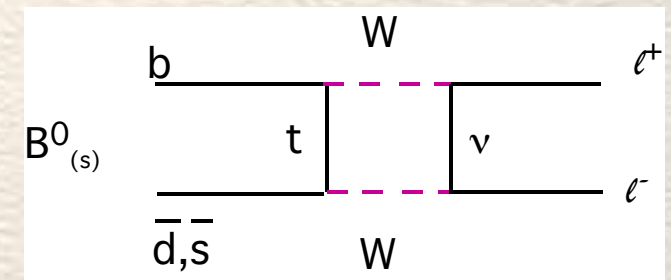
- The B^0_d and B^0_s mesons can also decay to $\mu^+ \mu^-$, which proceeds via a weak penguin or box diagram



- In the Standard Model, the branching fractions are predicted to be

$$\mathcal{B}(B^0_s \rightarrow \mu^+ \mu^-) = (3.25 \pm 0.17) \times 10^{-9}$$

$$\mathcal{B}(B^0_d \rightarrow \mu^+ \mu^-) = (0.107 \pm 0.010) \times 10^{-9}$$



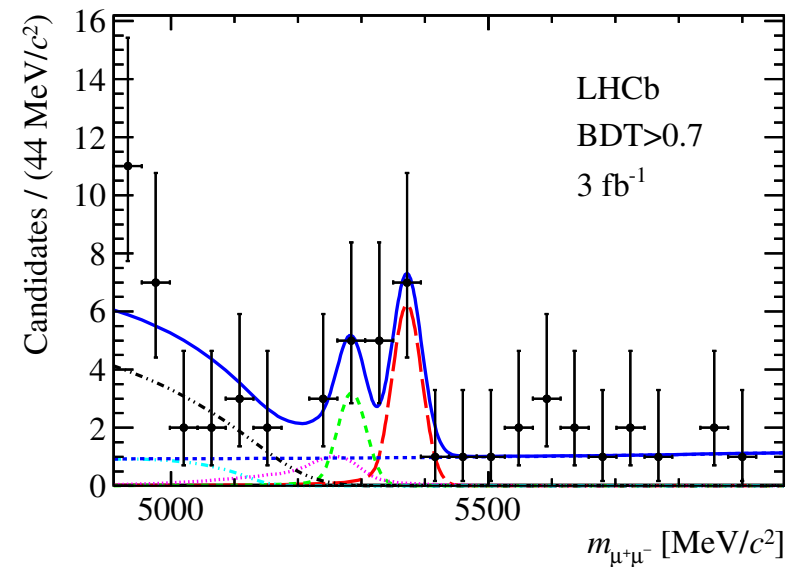
- LHCb found the first evidence for $B^0_s \rightarrow \mu^+ \mu^-$ at the 4.0σ level yielding

$$\mathcal{B}(B^0_s \rightarrow \mu^+ \mu^-) = (2.9^{+1.1}_{-1.0}(\text{stat})^{+0.3}_{-0.1}(\text{sys})) \times 10^{-9}$$

- CMS has confirmed the LHCb result at 4.3σ

$$\mathcal{B}(B^0_s \rightarrow \mu^+ \mu^-) = (3.0^{+0.9}_{-0.8}(\text{stat})^{+0.6}_{-0.4}(\text{sys})) \times 10^{-9}$$

- The measured branching fractions agree well with the SM prediction



WA

$$\mathcal{B}(B^0_s \rightarrow \mu^+ \mu^-) = (2.9 \pm 0.7) \times 10^{-9}$$



Implications of $B \rightarrow \mu^+ \mu^-$ Measurements

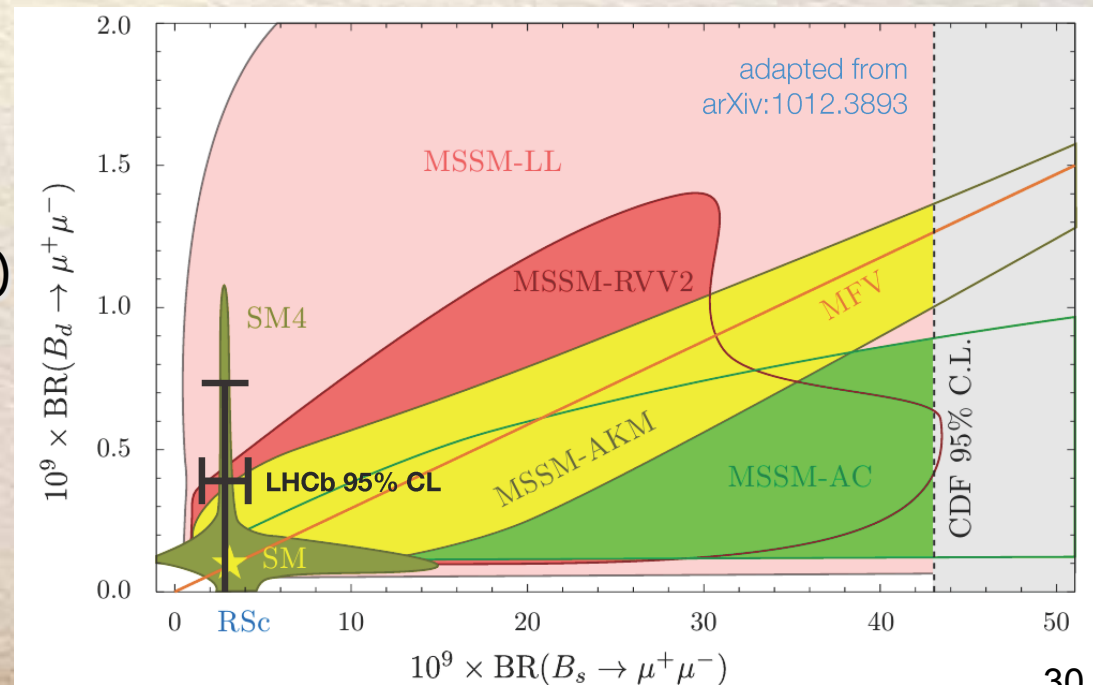
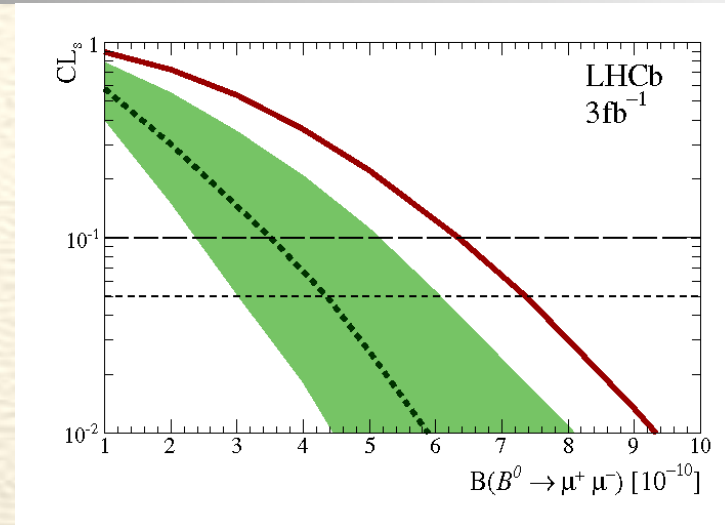
- For B_d^0 , LHCb has derived the lowest branching fraction upper limit

$$B(B_d^0 \rightarrow \mu^+ \mu^-) < 7.4 \times 10^{-10} \quad @ \quad 95\% \text{ CL}$$

- The LHCb $B(B_s^0 \rightarrow \mu^+ \mu^-)$ and $B(B_d^0 \rightarrow \mu^+ \mu^-)$ measurements are shown in comparison to predictions of

- The Standard Model
- Minimum Flavor Violation Model
- 4 SUSY models
 - Only left-handed currents (LL)
 - Agashe and Carone (AC)
 - Ross, Velasco-Sevilla, Vives (RVV2)
 - Antusch, King, Malinsky (AKM)

- The $B(B_s^0 \rightarrow \mu^+ \mu^-)$ measurements place strong constraints on the parameter space of New Physics Models

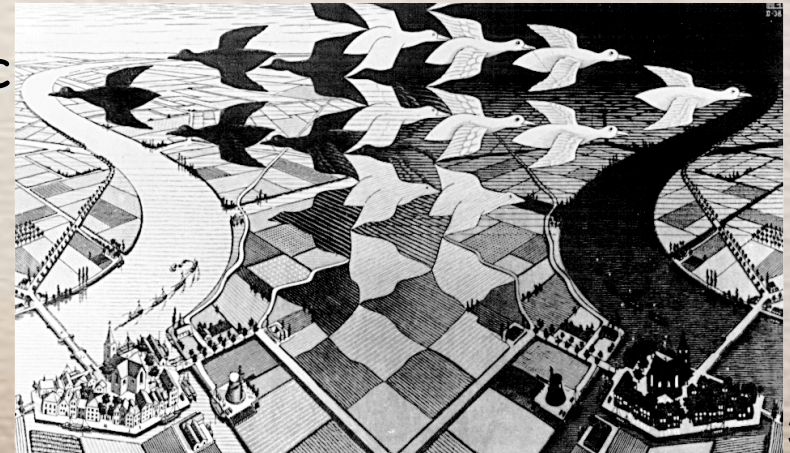
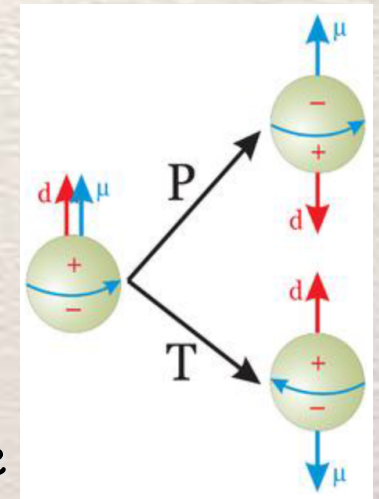
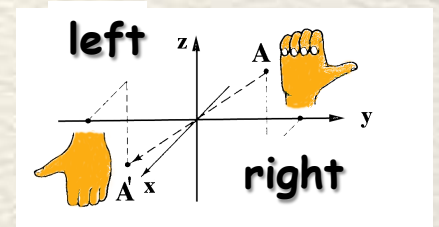


CP Violation in the B System

CP Violation

- Particle physics use many internal symmetries, e.g. C, P, T
- *P*: parity transforms:
left-handed state \leftrightarrow right-handed state
- *C*: C-parity transforms
particle \leftrightarrow antiparticle
- *T*: time reversal transforms:
forward moving time \leftrightarrow backward moving time
- *CP*: transforms:
left-handed particle \leftrightarrow right-handed antiparticle
- *CP* is conserved in strong and electromagnetic interactions
- *CP* is violated in weak processes

$$\vec{X} \longrightarrow -\vec{X}$$



CP Violation in the B System

- States produced in strong and electromagnetic interactions are the flavor eigenstates, e.g. $|B^0\rangle$ & $|\bar{B}^0\rangle$
- However, they decay weakly as $|B^0_H\rangle$ and $|B^0_L\rangle$ that can be expressed as linear combinations of the flavor eigenstates:

$$\begin{aligned} |B^0_L\rangle &= p |B^0_d\rangle + q |\bar{B}^0_d\rangle \\ |B^0_H\rangle &= p |B^0_d\rangle - q |\bar{B}^0_d\rangle \end{aligned} \quad \text{with} \quad |q|^2 + |p|^2 = 1 \quad (22)$$

- Since individual decay channels have small branching fractions $\mathcal{O}(10^{-3})$ that contribute with alternating signs, both states have the same lifetime: $\Gamma_H = \Gamma_L = \Gamma$
- $|B^0_H\rangle$ is the heavier state, $|B^0_L\rangle$ is the lighter state
- Their mass difference $\Delta M = M_H - M_L$ represents the $B^0_d - \bar{B}^0_d$ oscillation frequency
- The ratio q/p represents a phase factor for the mixing; in the SM $|q/p| = 1$
- The 2-particle system is described by a Schrödinger equation with mass matrix M and decay matrix Γ

B Decay Rate and λ

- We want to look at decays of $|B^0\rangle$ & $|\bar{B}^0\rangle$ into CP eigenstates f_{CP} as a function of time

- Thus, we define the amplitudes $A(t) \equiv \langle f_{CP} | H | B^0(t) \rangle$ and $\bar{A}(t) \equiv \langle f_{CP} | H | \bar{B}^0(t) \rangle$ (23)

- The figure of merit for CP violation is where η denotes the CP eigenvalue of the CP eigenstate

$$\lambda = \eta \frac{q \bar{A}}{p A} \quad (24)$$

- The time-dependent decay rates are

$$\Gamma \begin{pmatrix} B_{phys}^0(t) \\ \bar{B}_{phys}^0(t) \end{pmatrix} \rightarrow f_{CP} = |A|^2 \exp\{-\Gamma t\} \times \begin{bmatrix} \frac{1 + |\lambda|^2}{2} & + & \frac{1 - |\lambda|^2}{2} \cos(\Delta Mt) & - & \Im m \lambda \sin(\Delta Mt) \\ & - & & + & \end{bmatrix} \quad (25)$$

- We define the time-dependent CP asymmetry (26)

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{\Gamma(\bar{B}_{phys}^0(t) \rightarrow f_{CP}) - \Gamma(B_{phys}^0(t) \rightarrow f_{CP})}{\Gamma(\bar{B}_{phys}^0(t) \rightarrow f_{CP}) + \Gamma(B_{phys}^0(t) \rightarrow f_{CP})} \quad (26)$$

Time-dependent CP Asymmetry

- For the $B^0 \bar{B}^0$ system, we obtain

$$\mathcal{A}_{f_{CP}}(t) = \frac{-\left(1 - |\lambda|^2\right) \cos(\Delta Mt) + 2\Im m\lambda \sin(\Delta Mt)}{\left(1 + |\lambda|^2\right)} \quad (27)$$

- The first term represents direct CP violation; the second term results from the interference of decays with and without $B^0 \bar{B}^0$ mixing
- If in addition, $|\bar{A}/A|=1 \Rightarrow |\lambda|=1$ and the CP asymmetry reduces to

$$\mathcal{A}_{f_{CP}}(t) = +\Im m\lambda \sin(\Delta Mt) \quad (28)$$

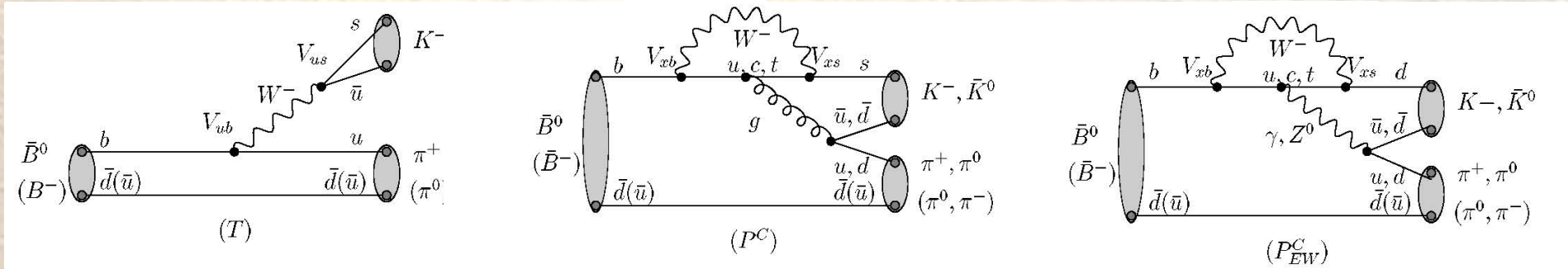
- Note that λ is directly related to CKM matrix elements
- At the $\Upsilon(4S)$, a $B^0 \bar{B}^0$ pair is produced that is entangled until one B decays \rightarrow however, the equations (27) and (28) still hold if t is replaced with $t_{f_{CP}} - t_{\text{tag}}$ where $t_{f_{CP}}$ is the decay time of one B meson into the CP eigenstate and t_{tag} is the decay time of the other B into a state that identifies the B flavor

Since B mesons produced at the $\Upsilon(4S)$ are nearly at rest ($p_B \approx 340 \text{ MeV}$), the $\Upsilon(4S)$ system needs to be boosted \rightarrow at PEP II boost is $\beta_\gamma = 0.56$



Direct CP Violation in $B^0 \rightarrow K^+ \pi^-$

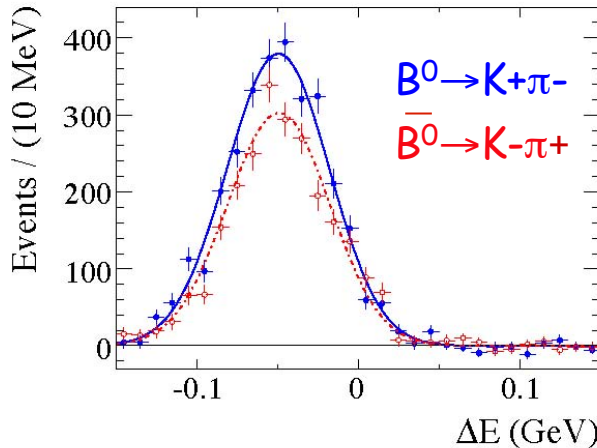
- We observe a huge decay asymmetry between $B^0 \rightarrow K^+ \pi^-$ & $\bar{B}^0 \rightarrow K^- \pi^+$



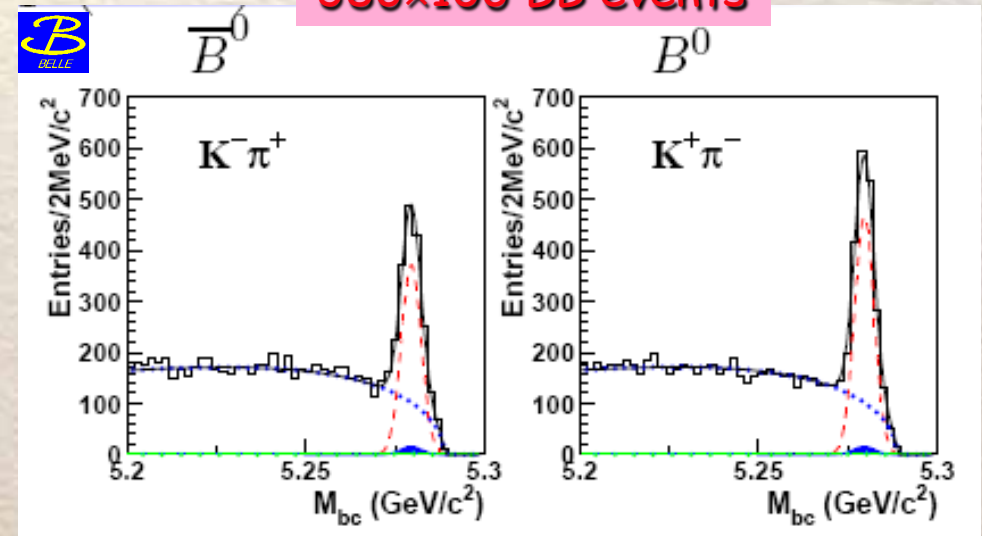
- The world average is $\mathcal{A}_{CP}(B^0 \rightarrow K^+ \pi^-) = -0.82 \pm 0.006$

- LHCb also observes large CP in $B^0_s \rightarrow K^+ \pi^-$ Decays

467 × 10⁶ $B\bar{B}$ events



386 × 10⁶ $B\bar{B}$ events



$$\mathcal{A}_{CP}(B^0_s \rightarrow K^+ \pi^-) = 0.27 \pm 0.04 \pm 0.01$$

$$\mathcal{A}_{CP}(B^+ \rightarrow K^+ \pi^0) = 0.040 \pm 0.021$$

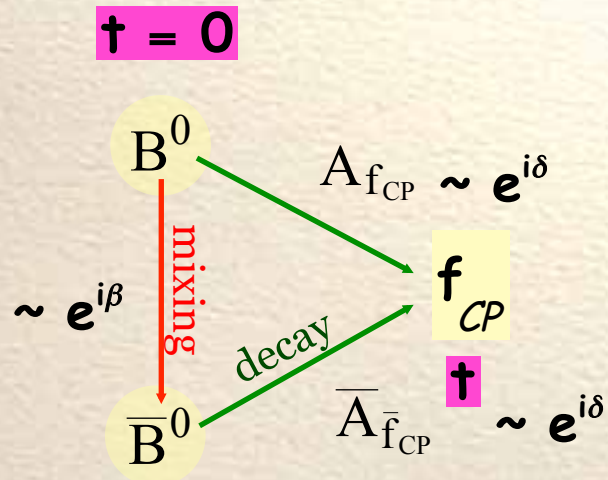


This effect is not seen in charged B decays

$$\mathcal{A}_{CP}(B^+ \rightarrow K^0 \pi^+) = -0.015 \pm 0.019$$

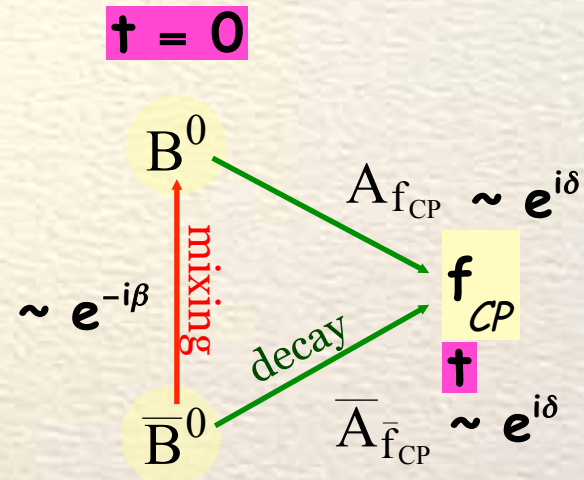
CP Violation caused by Interference of Decays with and without $B^0\bar{B}^0$ Mixing

Process



Decay rate

CP conjugated process



CP-conjugated decay rate



→ CP Violation is caused by the interference between mixing & decay

- Typically, we need to measure the time dependence of the CP asymmetry as the time-integrated asymmetry vanishes

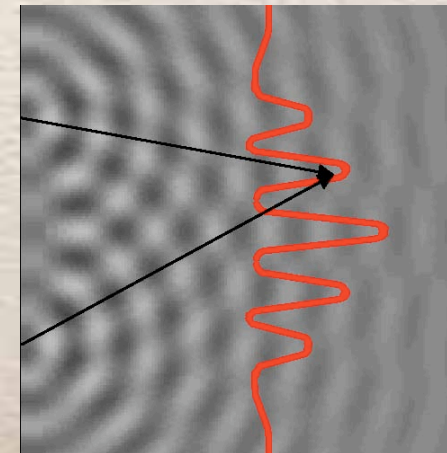
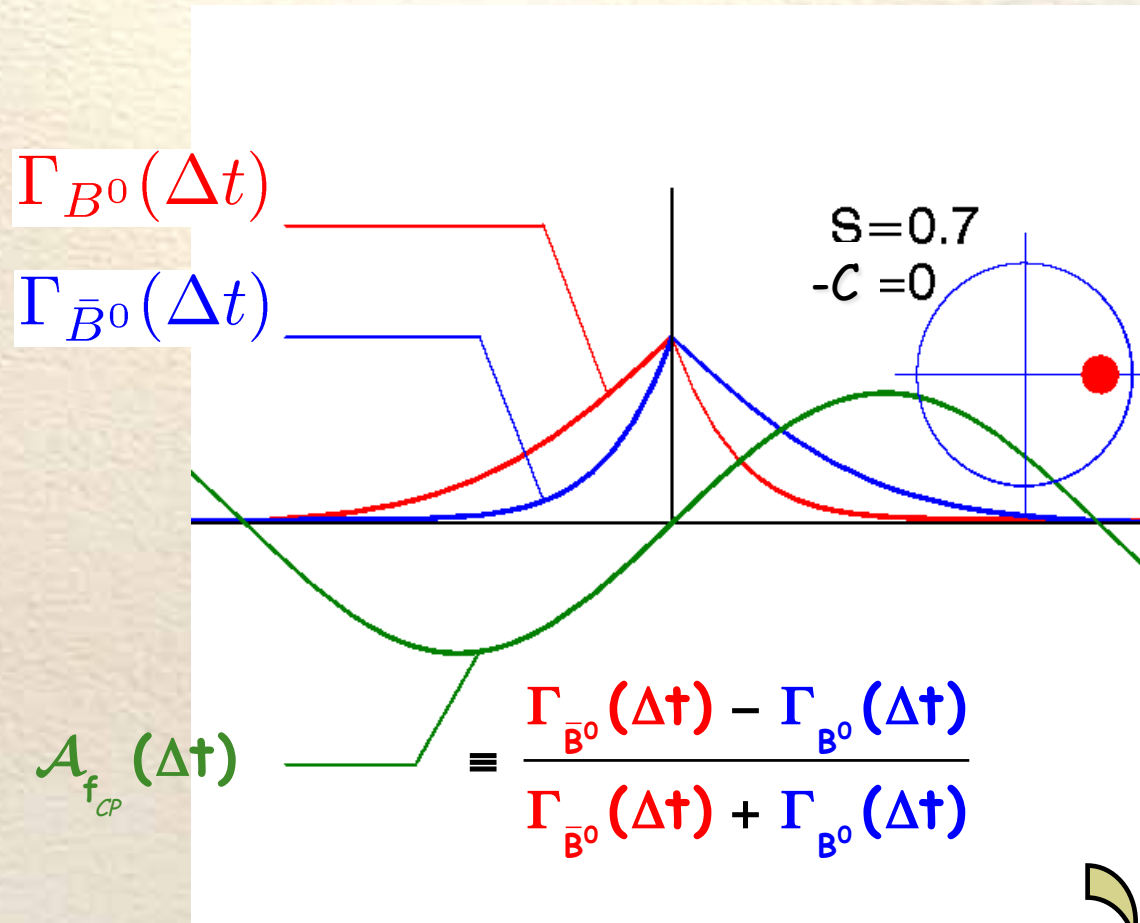


Illustration of time-dependent CP Violation

- Let's look at the time dependence of the B^0 and \bar{B}^0 decay rates and the CP asymmetry



$$A_{f_{CP}}(\Delta t) = S \cdot \sin \Delta m \cdot \Delta t - C \cdot \cos \Delta m \cdot \Delta t$$

Mixing-induced CP

Direct CPV

Unitarity Conditions of the CKM Matrix

- In the Standard Model, the CKM matrix produces CP violation
- The unitarity conditions of the CKM matrix yields 6 triangular relations
- Physics-wise most interesting is the relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad (29)$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ 1 & -\sin\theta_c & 1 \end{matrix}$

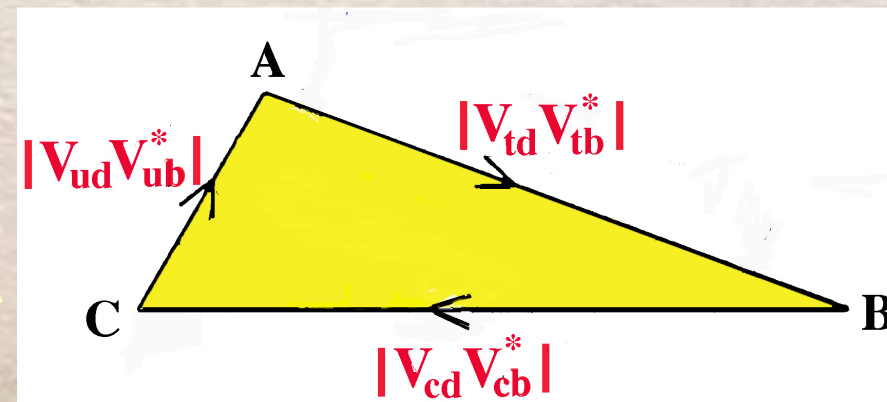
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$\begin{matrix} \downarrow & & \downarrow^* \\ \uparrow & & \uparrow \end{matrix}$

- The graphical representation of this relation yields the so-called Unitarity Triangle

- All 6 triangles have the same area
 $A \propto J = \Im(V_{ij}V_{kl}V_{il}^*V_{kj}^*) \neq 0$
 for any $i \neq k$ and $j \neq l$
 (Jarlskog determinant)
 $J = (3.01 \pm 0.19) \times 10^{-5}$

~~CP~~

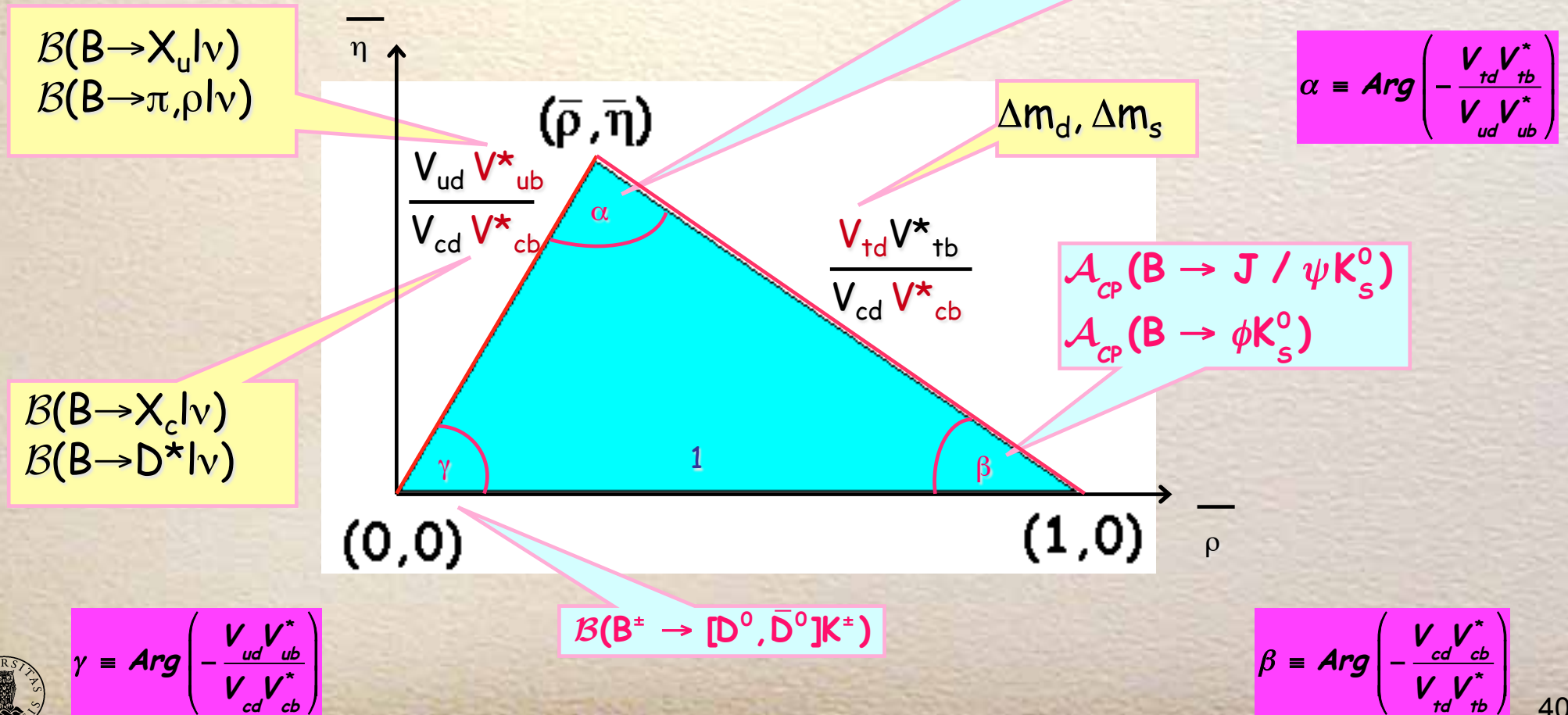


The Unitarity Triangle

● B factories measure 4 CKM elements & 3 phases

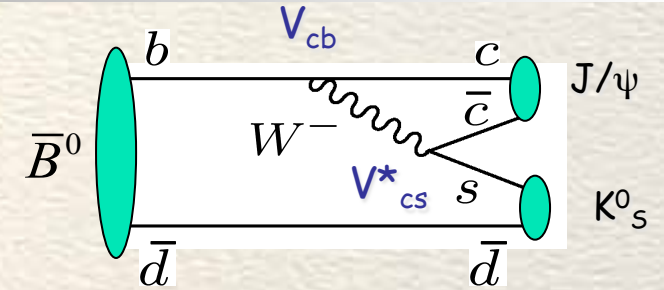
→ overconstrains unitarity triangle

→ test of Standard Model

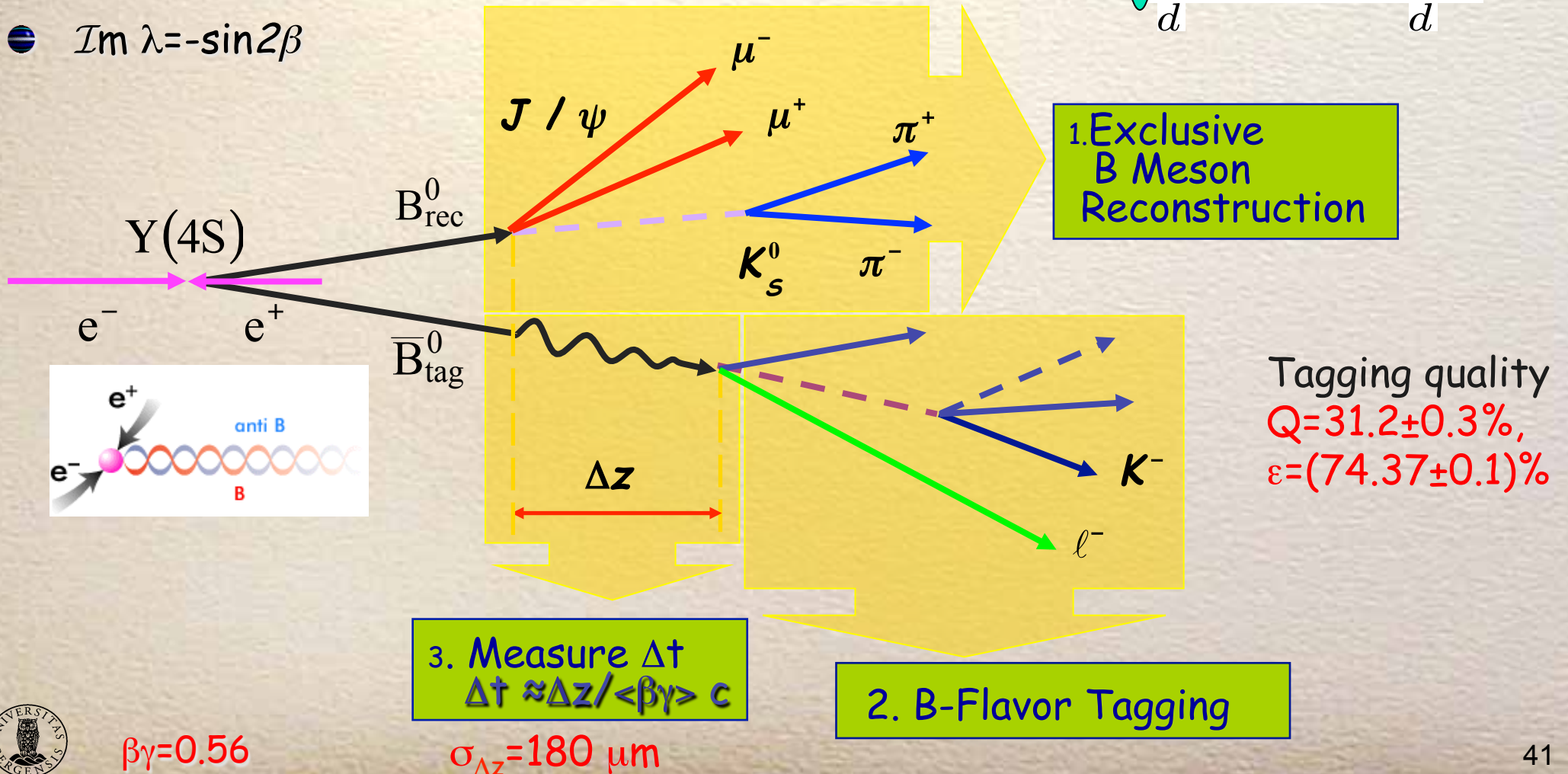


Measurement of $\sin 2\beta$ in $(c\bar{c})K^0$ Modes

- We need to measure 3 quantities to determine the CP asymmetry in $B^0 \rightarrow J/\psi K^0$

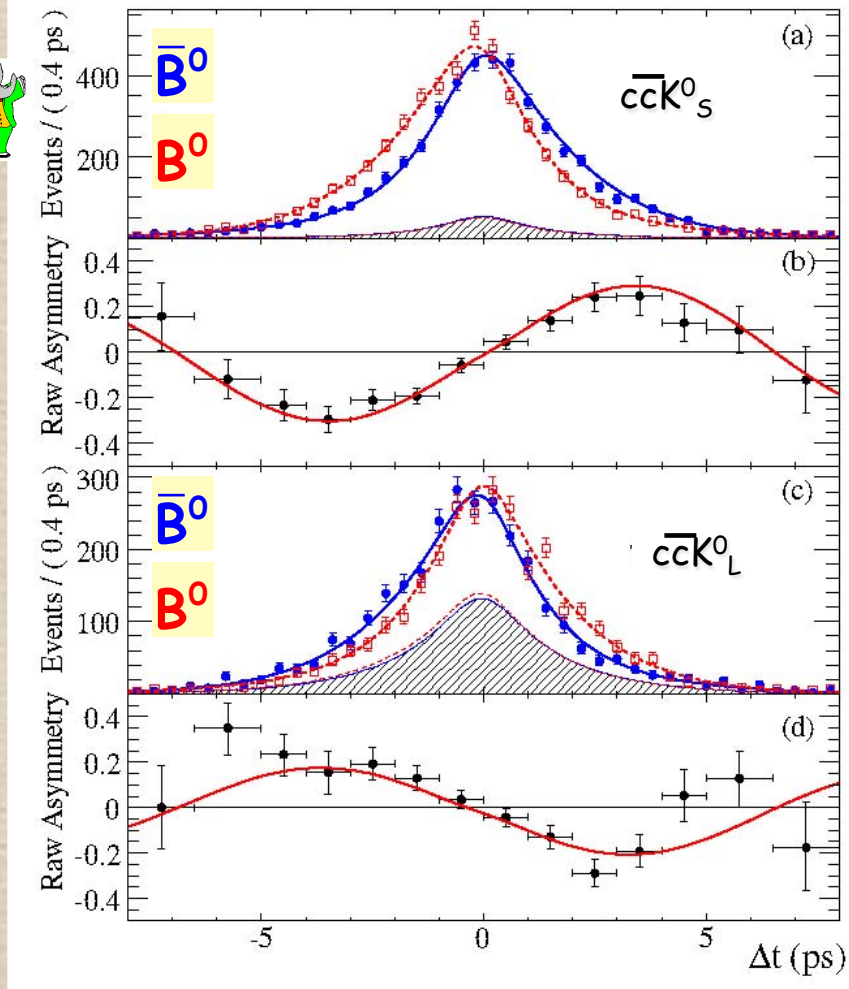


- $\text{Im } \lambda = -\sin 2\beta$



sin2β Measurements

- 15481 tagged events (465 M BB)

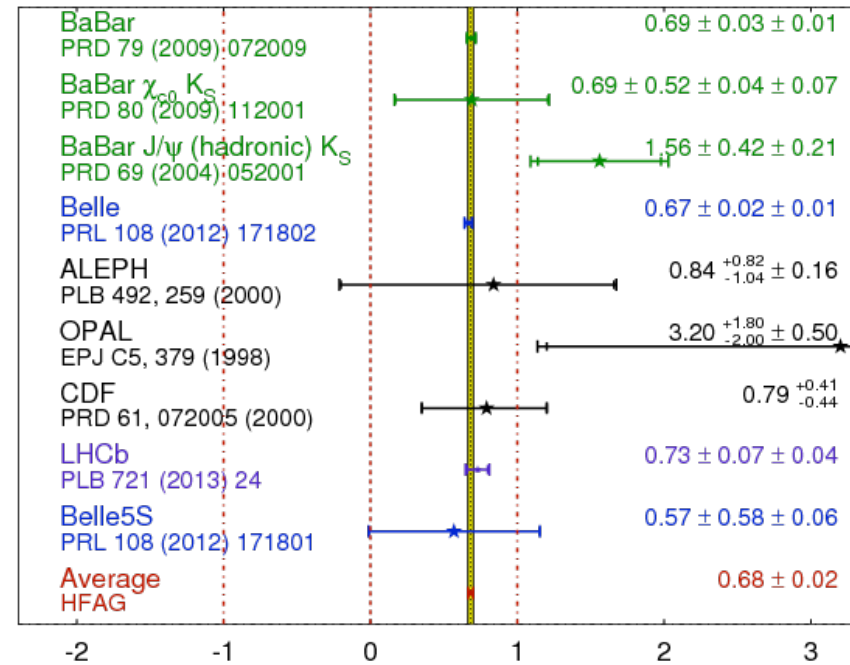


$$\sin 2\beta = 0.687 \pm 0.028 \pm 0.012$$

$$C = 0.024 \pm 0.020 \pm 0.016$$

$$\sin(2\beta) \equiv \sin(2\phi_1)$$

HFAG
Moriond 2014
PRELIMINARY



- $\sin 2\beta$ world average is 0.68 ± 0.02
 $\rightarrow \beta = (21.5^{+0.8}_{-0.7})^\circ$

- Cosine term is consistent with zero
 0.005 ± 0.017



Measurement of $\sin 2\beta$ in $B \rightarrow \phi K^0$

- In the *Standard Model*, *CP* violating asymmetries from $B \rightarrow J/\psi K^0$ & $B \rightarrow \phi K^0$ modes yield the same value of “ $\sin 2\beta$ ”

- Using 465×10^6 (383×10^6) $B\bar{B}$ events, BABAR observed 381 ± 23 ϕK^0_S & (151 ± 22 ϕK^0_L events)

- Using 535×10^6 $B\bar{B}$ events, Belle observed 307 ± 21 ϕK^0_S & 114 ± 17 ϕK^0_L events

- The *CP* asymmetry measurements from BABAR & Belle yield:

$$S_{\phi K^0} = +0.74^{+0.11}_{-0.13}$$

$$C_{\phi K^0} = 0.01 \pm 0.14$$

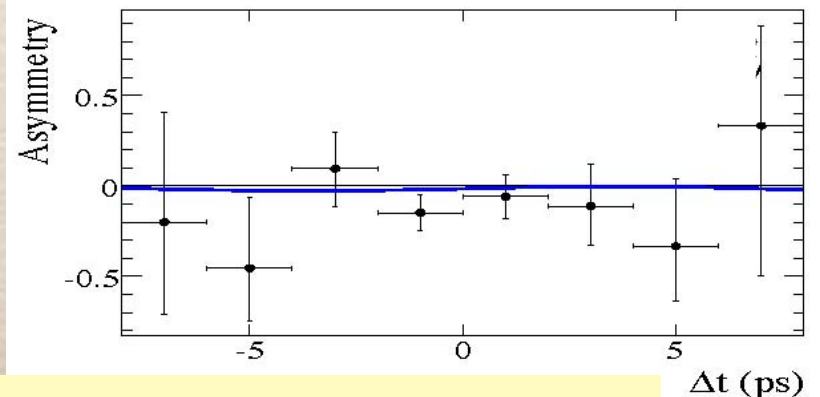
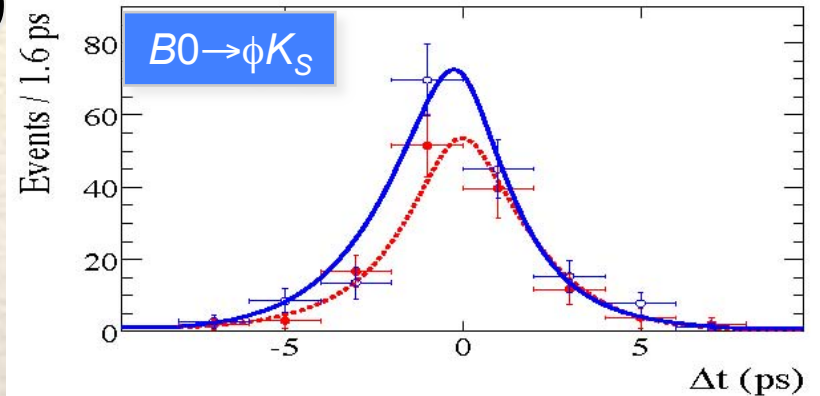
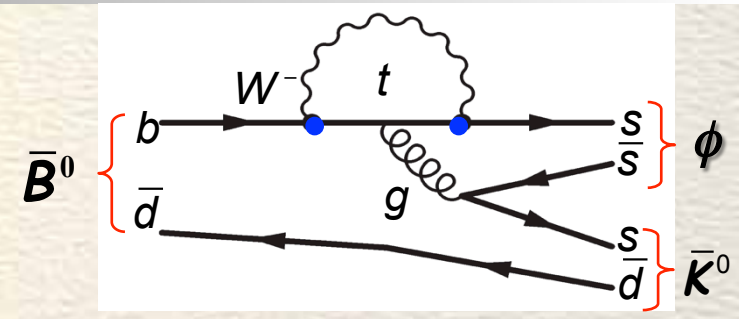
- This agrees well with the SM prediction

- Another related mode is $B \rightarrow \eta' K^0$ yielding

$$S_{\eta' K^0} = +0.59 \pm 0.07$$

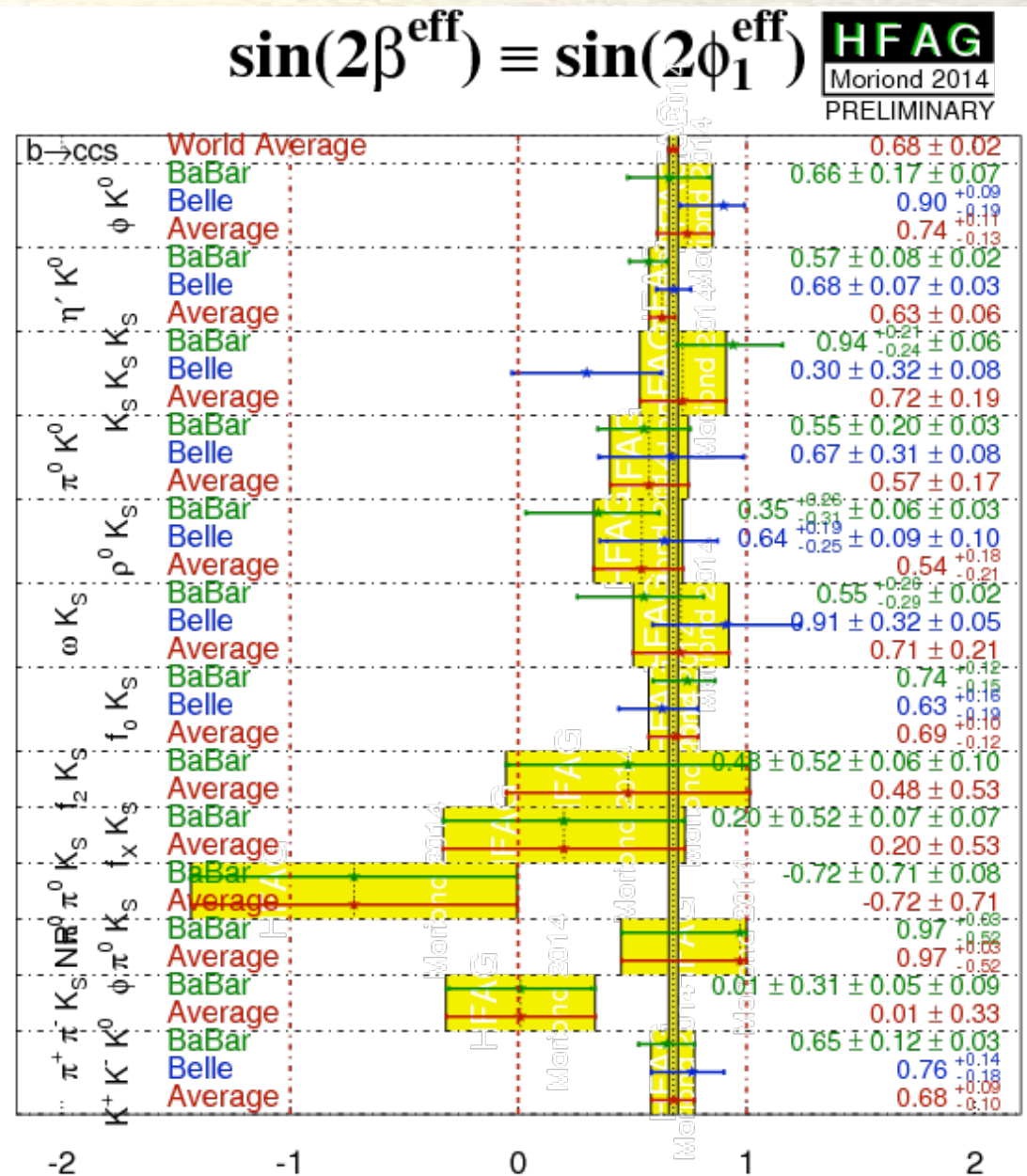
$$C_{\eta' K^0} = -0.05 \pm 0.05$$

$$A_{f_{CP}}(\Delta t) = S \cdot \sin \Delta m \cdot \Delta t - C \cdot \cos \Delta m \cdot \Delta t$$



Comparison of $\sin 2\beta$ in Penguin-Dominated Modes

- For the penguin-dominated rare hadronic decays BABAR and Belle yield consistent results
- For most penguin-dominated rare hadronic decays, the measured value of $\sin 2\beta^{\text{eff}}$ agrees with the $\sin 2\beta$ world average obtained from $B \rightarrow \bar{c}cK^0$ modes



CP Violation in the B_s System

- The phase difference between the B_s mixing amplitude and $b \rightarrow c\bar{c}s$ decay amplitude of B_s meson is $\phi_s = 2\beta_s$, where in SM

$$\beta_s = \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right) \sim 1^\circ \quad (35)$$

- This is defined in analogy to β for B_d decays

- CDF, D0, CMS ATLAS and LHCb have measured β_s using the decay $B_s \rightarrow J/\psi\phi$

- LHCb added $B_s \rightarrow J/\psi\pi^+\pi^-$ modes

- Perform a 2-D fit in the β_s - $\Delta\Gamma_s$ plane w & w/o other constraints

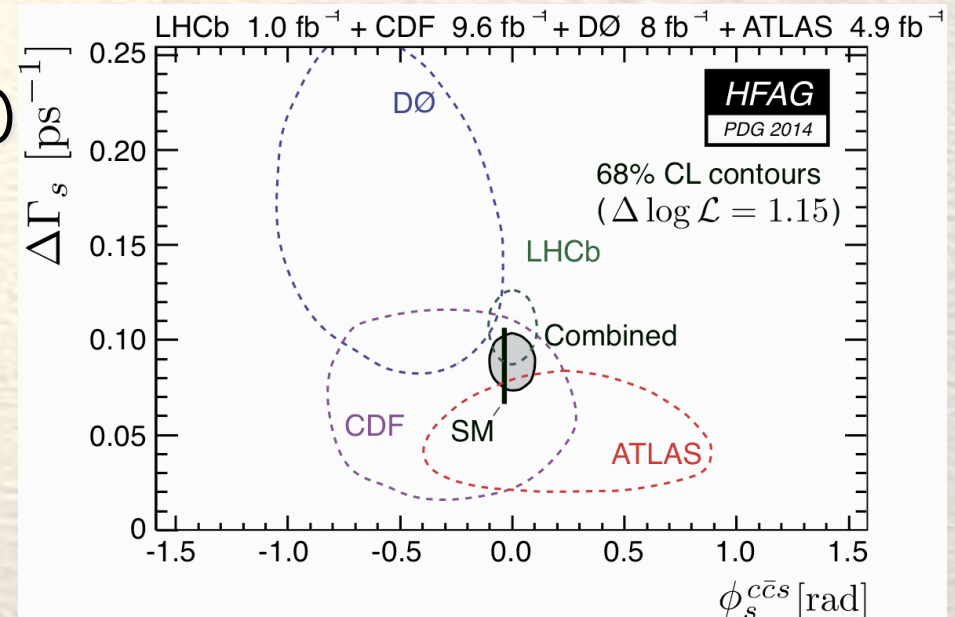
- Present world average is $\phi_s = 2\beta_s = (0.00 \pm 0.07) \text{ rad}$

- At ICHEP 2014, LHCb showed updated result with 3 fb⁻¹ of $B_s \rightarrow J/\psi\pi^+\pi^-$ data

→ LHCb average

$$\phi_s = (0.07 \pm 0.055) \text{ rad}$$

LHCb also measured ϕ_s in $B_s \rightarrow \phi\phi$ (3 fb⁻¹) yielding $\phi_s = (-0.17 \pm 0.15 \pm 0.03) \text{ rad}$



Study of the B_s Meson

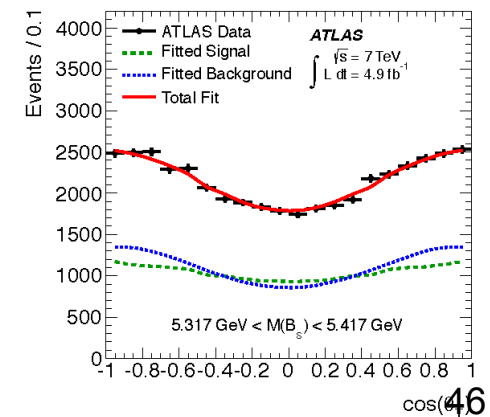
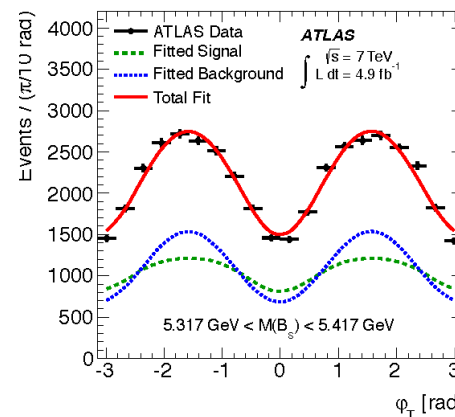
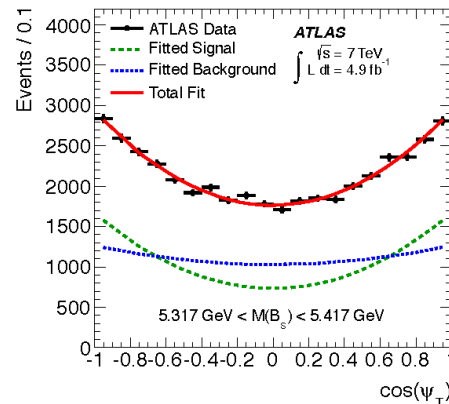
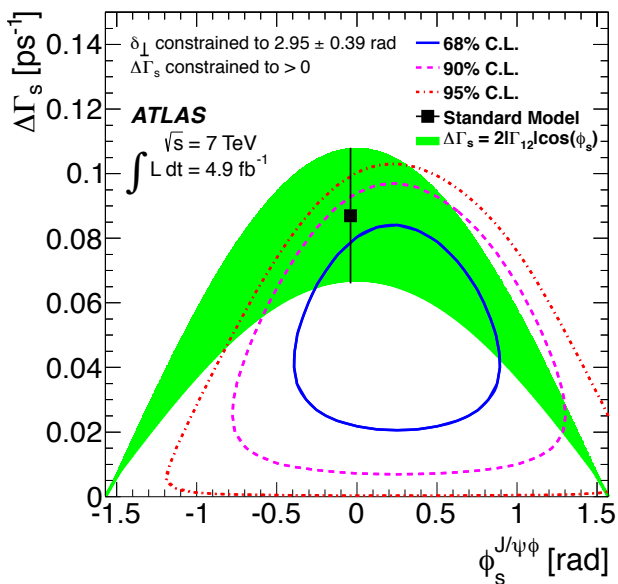
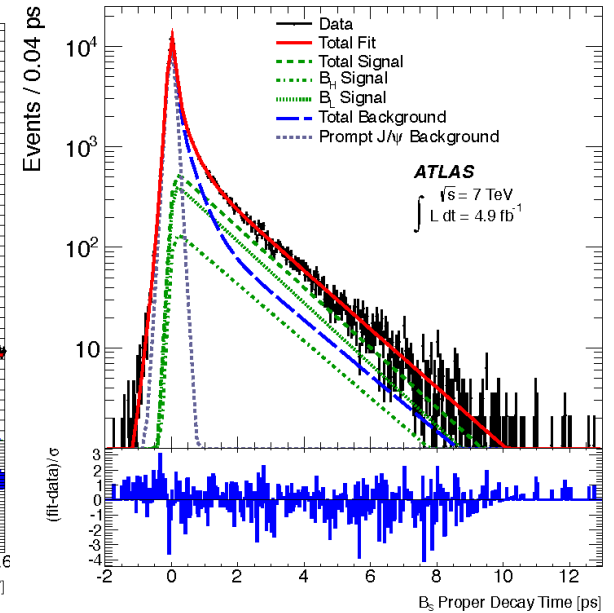
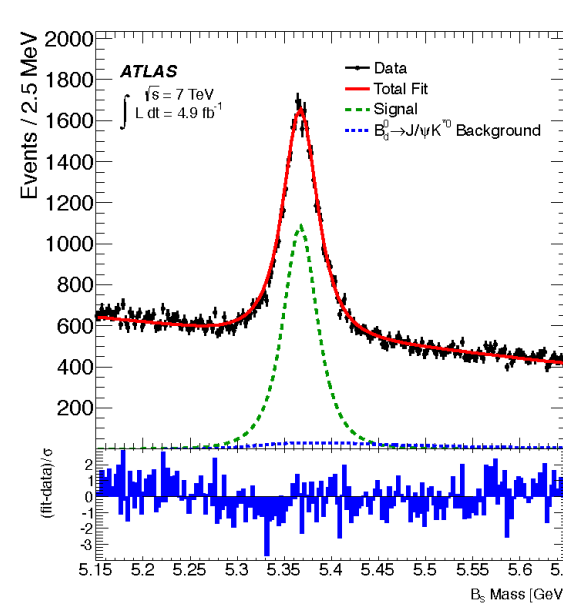
- ATLAS performed a time-dependent full angular analysis of $B_s \rightarrow J/\psi\phi$ (26 parameter ML fit) to extract amplitudes, strong phases, Γ_s , $\Delta\Gamma_s$ and ϕ_s

$$\phi_s = (0.22 \pm 0.41 \pm 0.10) \text{ rad}$$

$$\Delta\Gamma_s = (0.053 \pm 0.021 \pm 0.010) \text{ ps}^{-1}$$

$$\Gamma_s = (0.677 \pm 0.007 \pm 0.004) \text{ ps}^{-1}$$

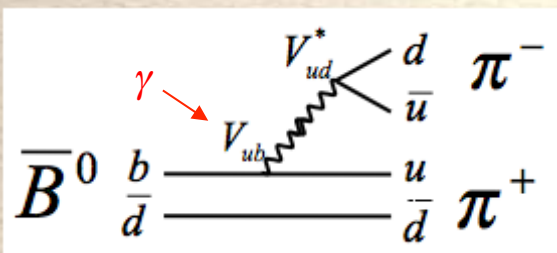
- This agrees with results from CMS and LHCb and the SM predictions



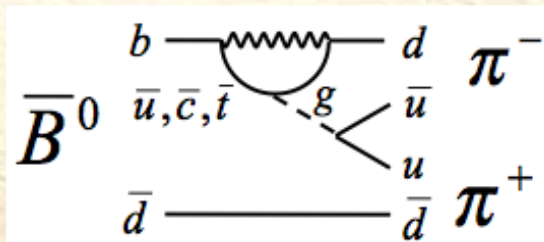
Measurement of $\sin 2\alpha$

- Interference of $b \rightarrow u\bar{u}d$ decay w&w/o $B^0\bar{B}^0$ mixing yields angle α

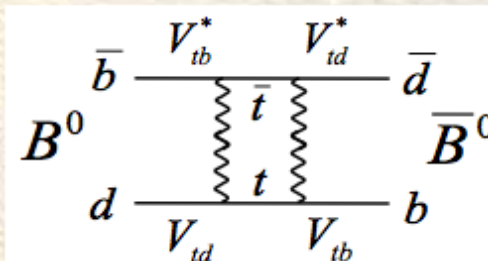
tree



penguin



$B^0\bar{B}^0$ mixing



- Thus we need to measure Δt -dependent CP asymmetries of $b \rightarrow u\bar{u}d$ processes, such as $B \rightarrow \pi\pi, \rho\pi, \rho\rho, \dots$

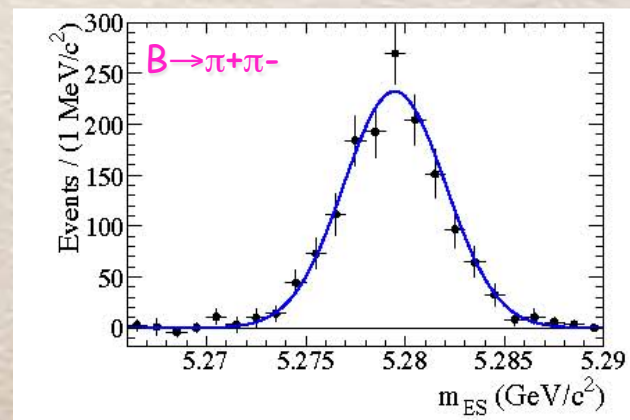
- However, penguin pollution complicates extraction of $\alpha \rightarrow$ measure $\sin 2\alpha_{eff}$

- Since $|\lambda| \neq 1$, CP asymmetry has form

$$A_{\pi\pi}(\Delta t) = -C_{\pi\pi} \cos \Delta m_d \Delta t + S_{\pi\pi} \sin \Delta m_d \Delta t$$

$$S_{\pi\pi} = 2\Im m \lambda = \sin 2\alpha \quad (36)$$

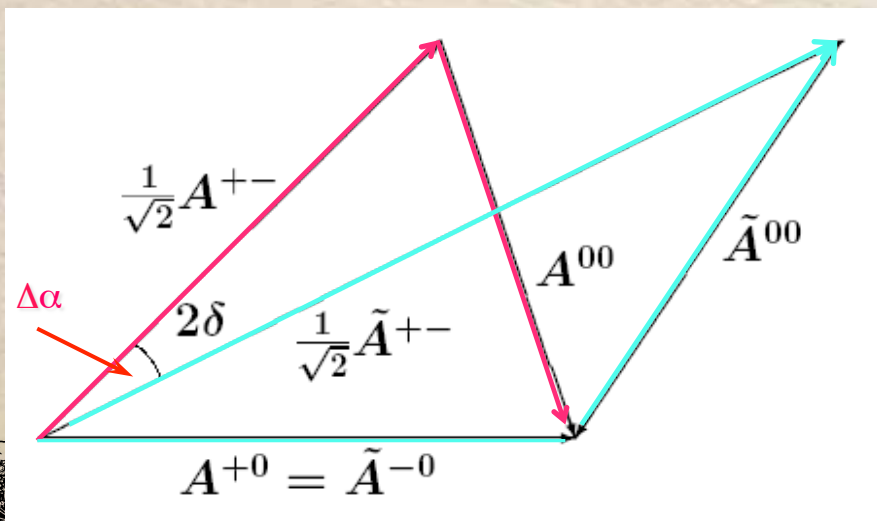
1394 \pm 54 events



The Gronau-London Method

- $S_{\pi\pi}$ measures $2\alpha_{eff} = 2\alpha + \Delta\alpha$, where $\Delta\alpha$ can be determined using the Gronau-London method
- The decays $B \rightarrow \pi^+ \pi^-, \pi^+ \pi^0, \pi^0 \pi^0$ are related by SU(2)
 - Have isospin relations between amplitudes A_{+-}, A_{+0}, A_{00}
- Central observation is that $\pi\pi$ states can have I=2 or 0, while gluonic penguins only contribute to I = 0 ($\Delta I = 1/2$ rule)
 - $\pi^+ \pi^0$ is pure I = 2, so only tree amplitude → $|A_{+0}| = |A_{-0}|$

$B \rightarrow \pi\pi$ isospin analysis

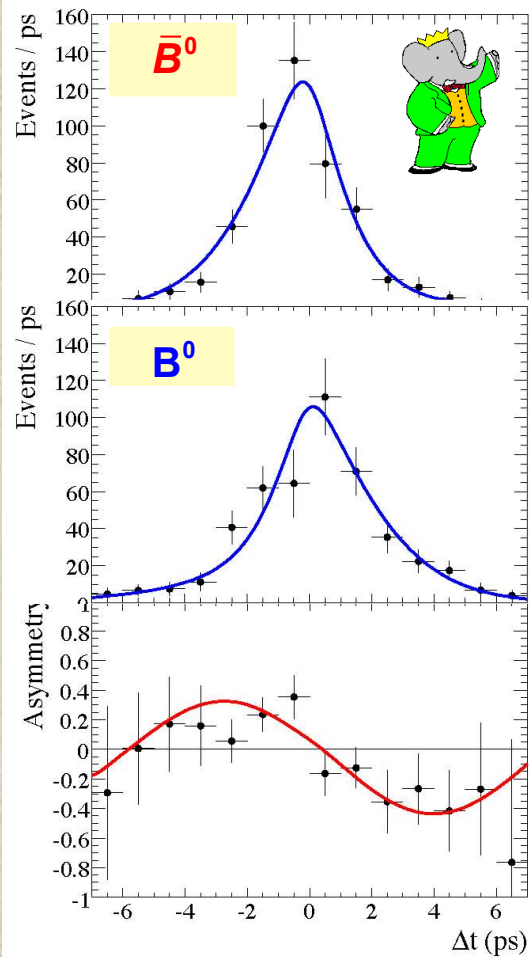


- Need to measure: C_{+-}, C_{00} (amplitudes in Cos terms for $\pi^+ \pi^-$ & $\pi^0 \pi^0$), & A_{00}, A_{+0}
- Effective isospin analysis requires A_{00} & A_{+0} very large, or very small!

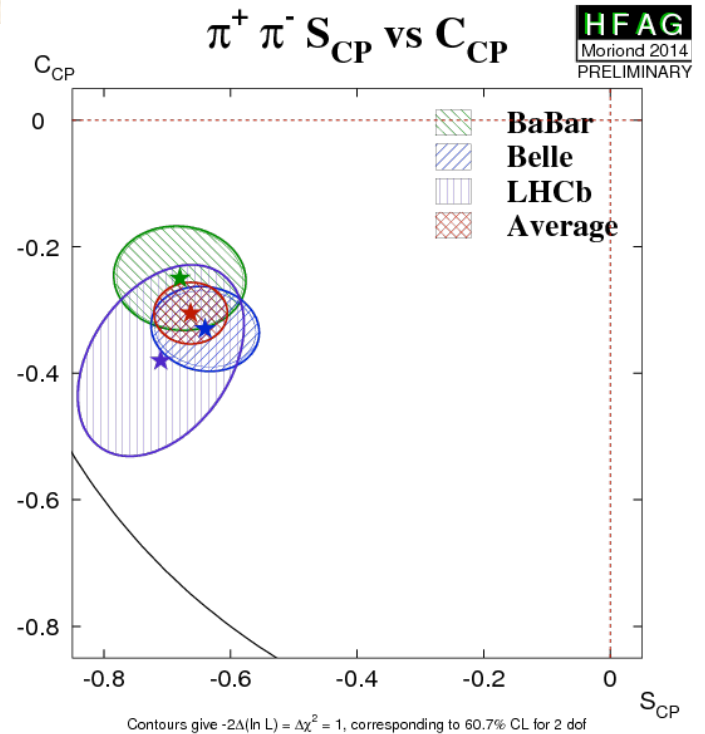
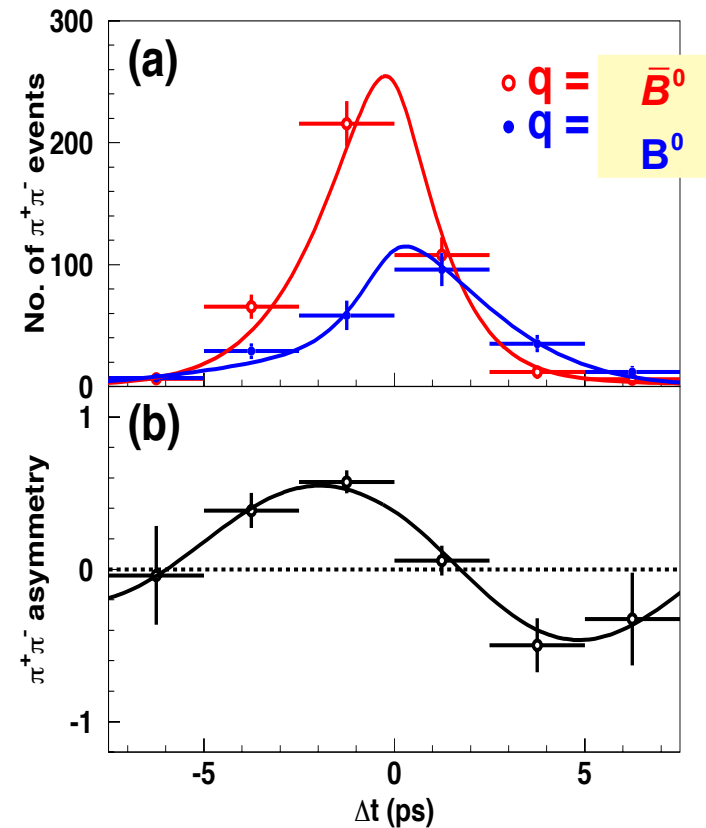
Measurement of $\sin 2\alpha$ in $B \rightarrow \pi\pi$

Compare results on $B \rightarrow \pi\pi$ from BABAR, Belle and LHCb

467×106 $B\bar{B}$ events



535×106 $B\bar{B}$ events



LHCb

$S_{\pi\pi} = -0.68 \pm 0.10 \pm 0.03$

$C_{\pi\pi} = -0.25 \pm 0.08 \pm 0.02$

$S_{\pi\pi} = -0.64 \pm 0.08 \pm 0.03$

$C_{\pi\pi} = -0.33 \pm 0.06 \pm 0.03$

$S_{\pi\pi} = -0.71 \pm 0.13 \pm 0.02$

$C_{\pi\pi} = -0.31 \pm 0.15 \pm 0.02$



Summary of α Measurements

- Summary on α measurements

- From combined $\pi\pi$, $\rho\pi$, $\rho\rho$ results constrain α to:

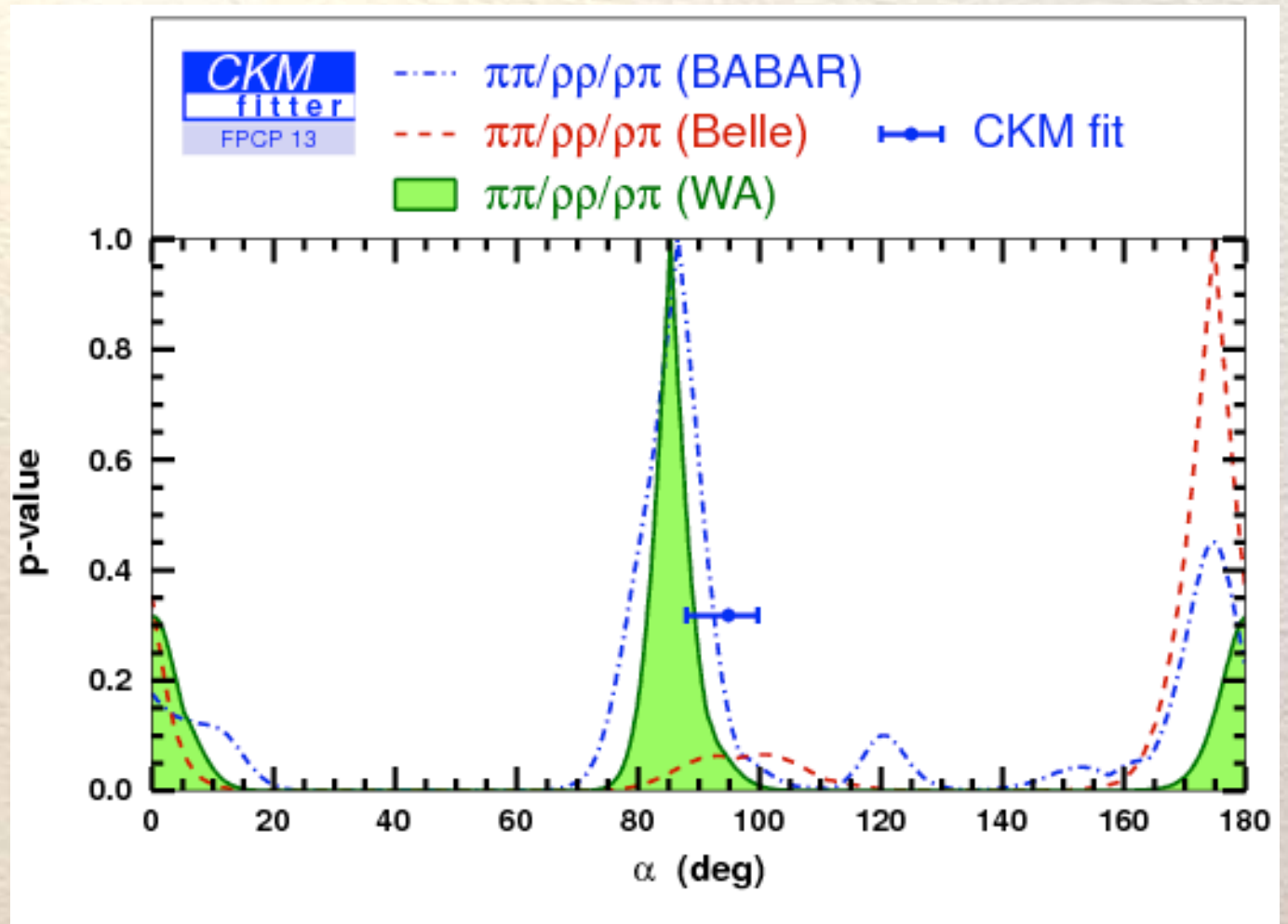
$$\alpha = [85.4^{+4.0}_{-3.9}]^\circ$$

- Best constraints from $B \rightarrow \rho\rho$

- Mirror solutions are disfavored

- CKM indirect constraint yields:

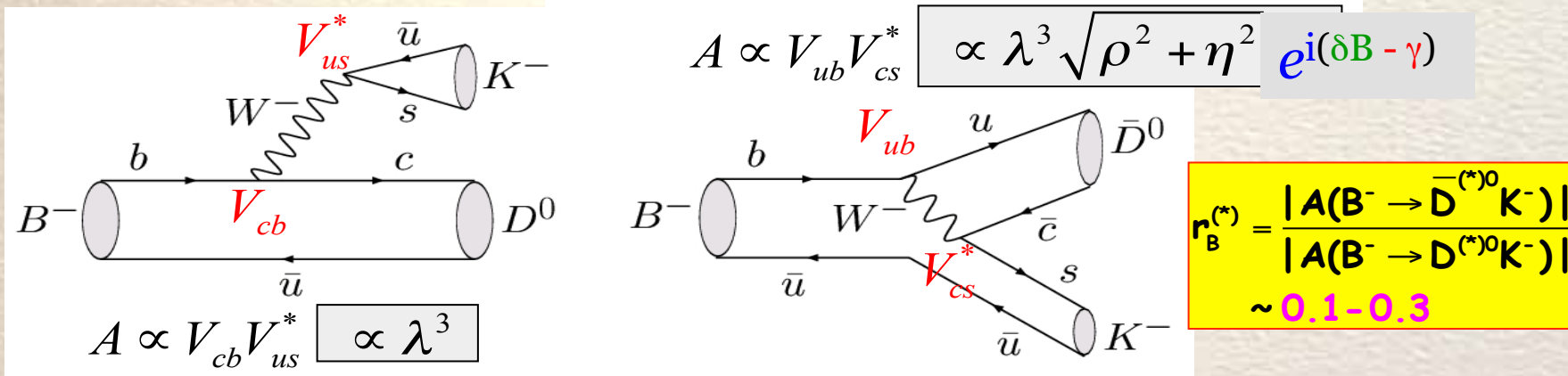
$$\alpha = [94.9^{+4.8}_{-6.8}]^\circ$$



Very good agreement

Measurement of Angle γ

- The angle γ is measurable via interference between $B^- \rightarrow D^0 K^-$ & $B^- \rightarrow \bar{D}^0 K^-$ decays, where the D^0 / \bar{D}^0 decay to common final state



- Use 3 different methods:
 - Gronau-London-Wyler (GLW 1991)

Use $B^- \rightarrow D_{CP^\pm}^0 K^-$ decays

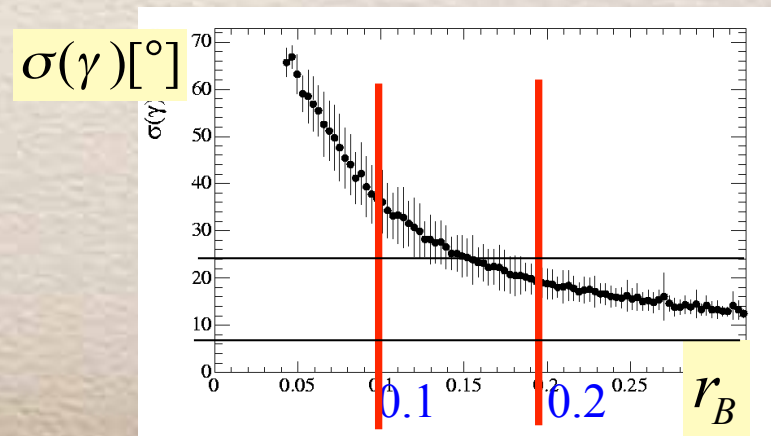
- Atwood-Dunietz-Soni (ADS 2001)

Use $B^- \rightarrow D^{(*)0} (K^- \pi^+) K^-$ decays

- Giri, Grossman, Soffer Zupan (GGSZ) D^0 Dalitz plot analysis

Use $B^- \rightarrow D^0 (K_S^0 \pi^+ \pi^-) K^-$ decays

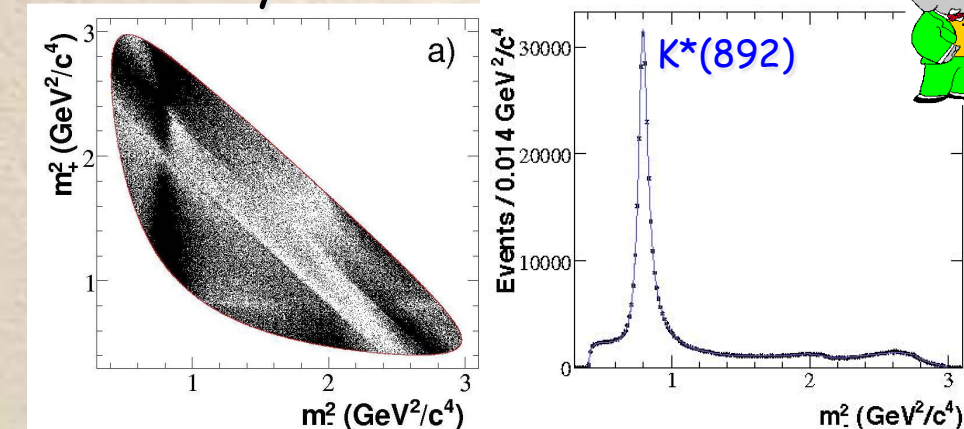
Size of the CP asymmetry & error on γ depends on $r_B^{(*)}$



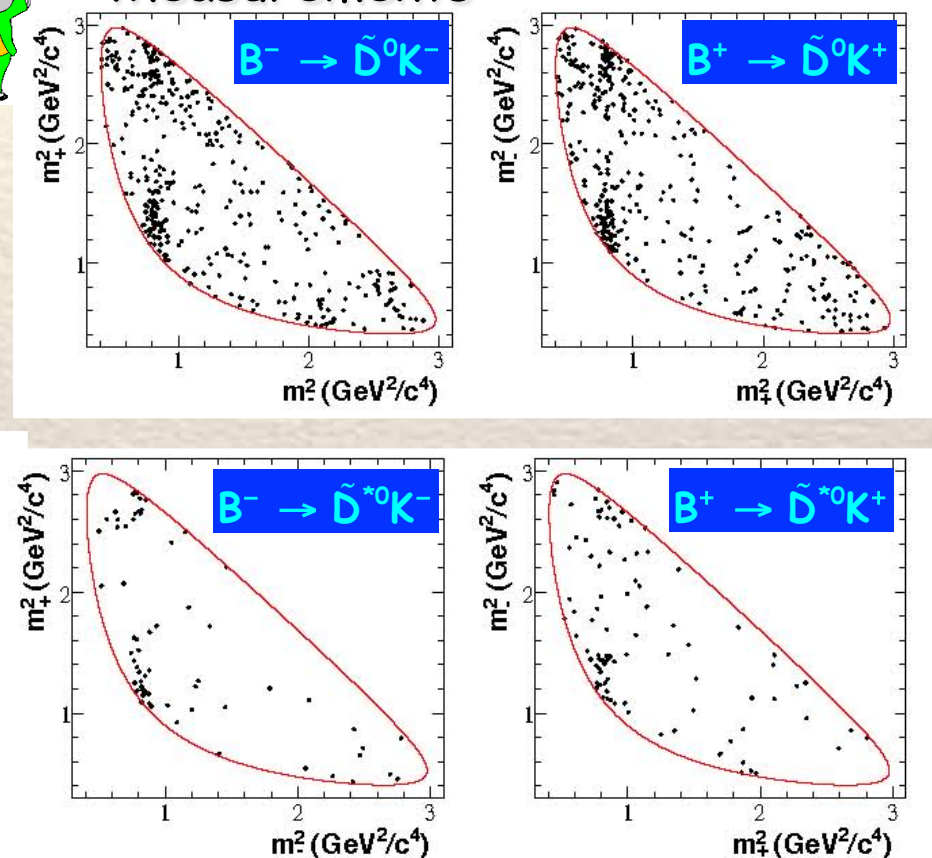
Dalitz Plot in $B^- \rightarrow D^0 (K^0_S \pi^+ \pi^-) K^-$

- Decay model of $K_S \pi^+ \pi^-$: coherent sum of 10 Breit-Wigner amplitudes:
 - 7 distinct resonances $K^*(892)$, $K^*_0(1430)$, $K^*_2(1430)$, $K^*(1680)$ (4 Cabibbo-allowed and 3 DCS), $\rho(770)$, $\omega(782)$, $f_2(1270)$ (CP-ES) plus 1 non-resonant term ($\pi\pi$ S-Wave)

Decay Model

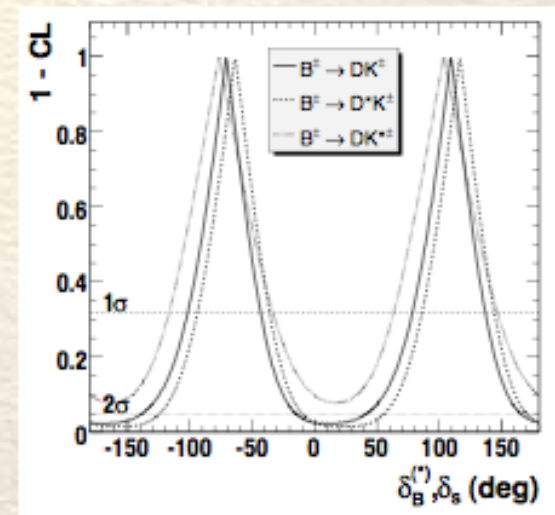
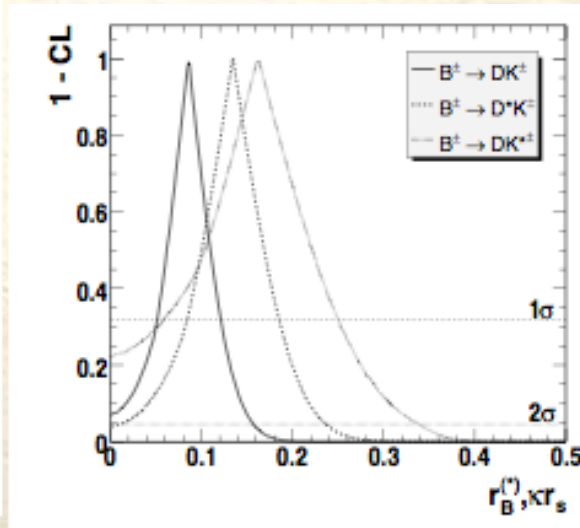
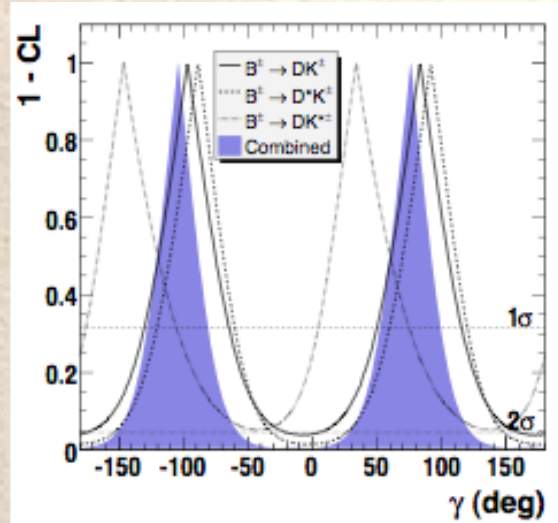


Measurements



Measurement of Angle γ in $B^- \rightarrow D^0 (K^0_s \pi^+ \pi^-) K^-$

- From the $B^- \rightarrow D^{(*)0} K^-$ & corresponding decays BABAR extracts γ , r_B and δ_B



- We get 2-fold ambiguity: $(\gamma, \delta^{(*)}_B) \rightarrow (\gamma + \pi, \delta^{(*)}_B + \pi)$

$$r_B = 0.085 \pm 0.035 \pm 0.010 \pm 0.011$$

$$r_B^* = 0.135 \pm 0.051 \pm 0.011 \pm 0.005$$

$$\delta_B = (109^{+28}_{-31 \text{stat}} \pm 4_{\text{sys}} \pm 7_{\text{Dalitz}})^\circ$$

$$\delta_B^* = (63^{+28}_{-30 \text{stat}} \pm 5_{\text{sys}} \pm 4_{\text{Dalitz}})^\circ$$

$$\gamma = (76 \pm 22_{\text{stat}} \pm 5_{\text{sys}} \pm 5_{\text{Dalitz model}})^\circ$$

$$\gamma = (76^{+12}_{-13} \pm 4_{\text{sys}} \pm 9_{\text{Dalitz model}})^\circ$$



Summary of γ Measurements

- Combining all 3 methods for $B^- \rightarrow D^{(*)0}K^{*-}$ yields first measurement of γ

- A few years ago this was thought to be impossible

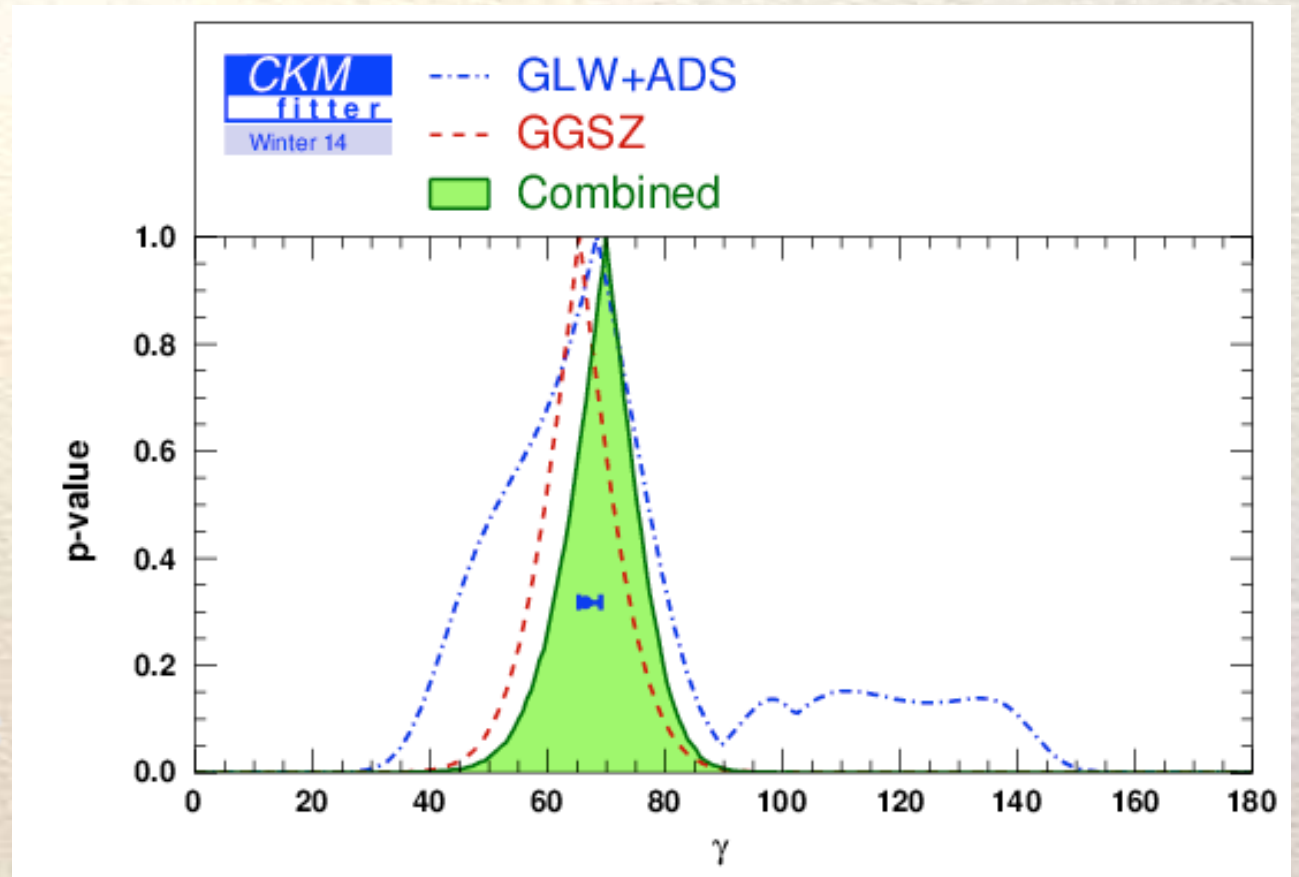
- From combined analysis

$$\gamma = [70.0^{+7.7}_{-9.0}]^\circ$$

- From indirect constraints

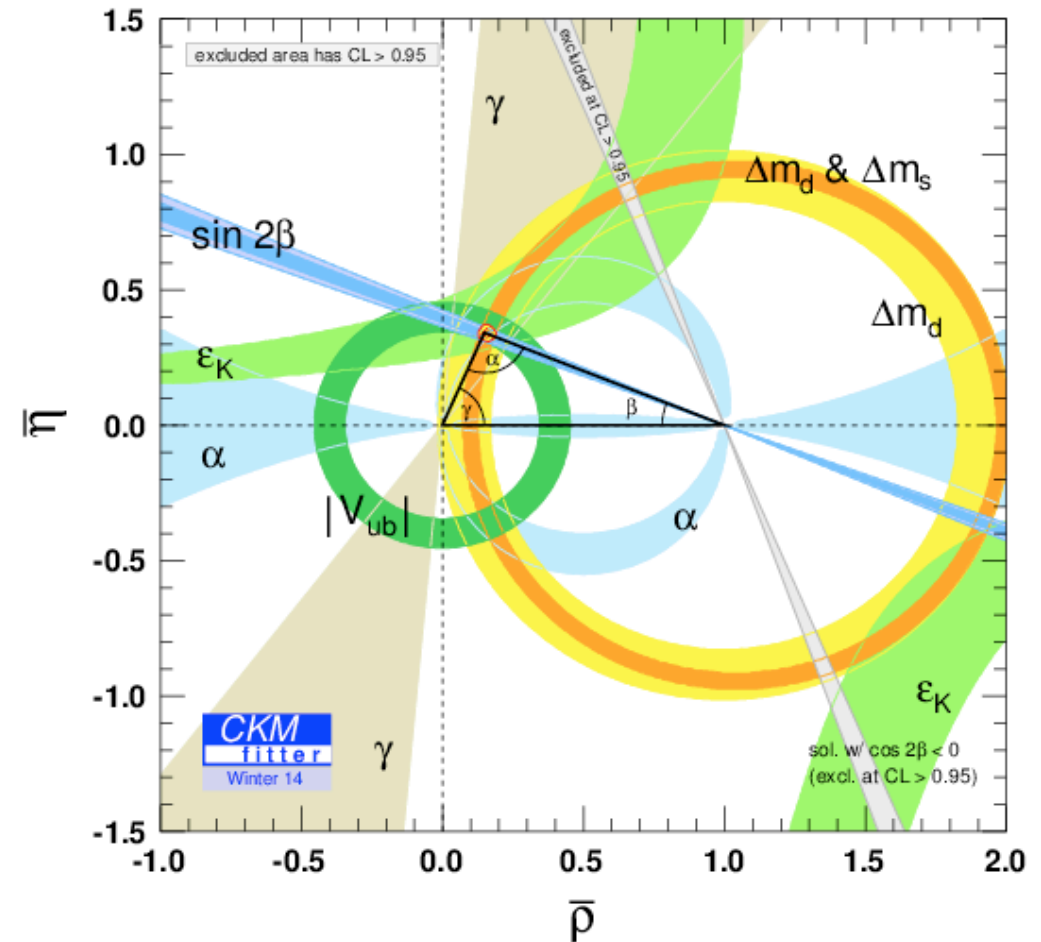
$$\gamma = [66.4^{+1.2}_{-3.3}]^\circ$$

- γ measurements obtained from 2 independent methods are in good agreement (errors are still large)



Present Status of Unitarity Triangle

- Global fit in ρ - η plane using V_{ub} , V_{cb} , Δm_d , Δm_s , ε_K , $\sin 2\beta$, $\cos 2\beta$, $\sin 2\alpha$, γ
- Measurements of 3 sides agree with measurements of the 3 angles
 - expect this in the *SM*
 - no $\mathcal{O}(1)$ New Physics effects
- Since the SM works so well, look for New Physics as a correction to the *SM* at $\mathcal{O}(0.1)$

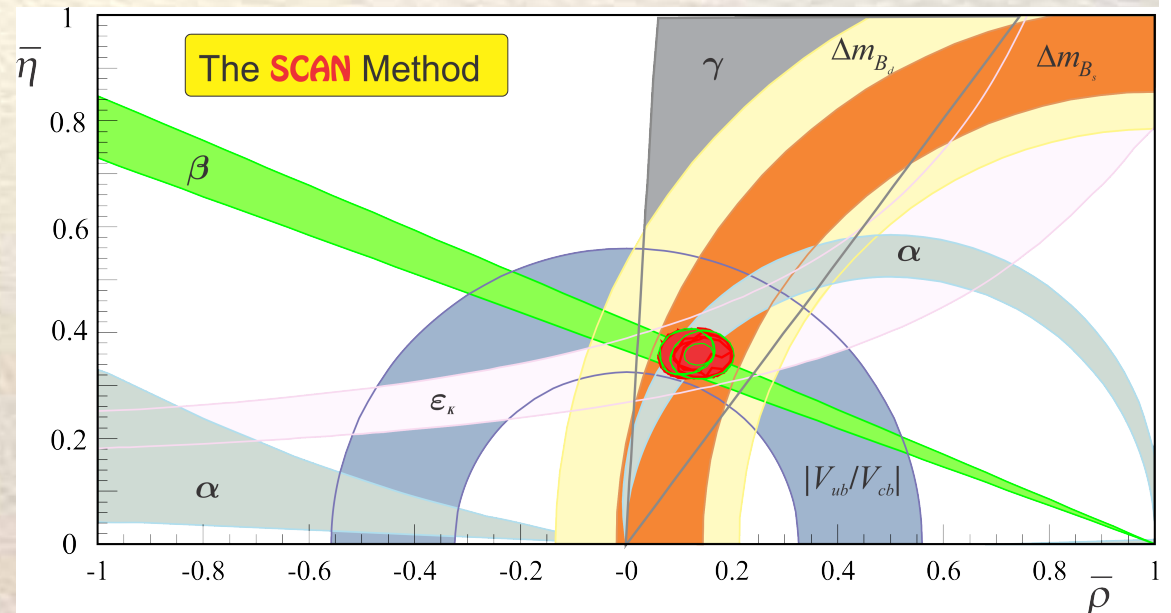


$88.9^\circ < \alpha < 97.3^\circ$ @ 95% CL
 $20.5^\circ < \beta < 23.5^\circ$ @ 95% CL
 $61.4^\circ < \gamma < 68.9^\circ$ @ 95% CL

$0.133 < \bar{\rho} < 0.182$ @ 95% CL
 $0.318 < \bar{\eta} < 0.366$ @ 95% CL

Present Status of Unitarity Triangle in Scan Method

- Perform a fit using 256 measurements and 114 parameters
 - use all available B and A_{CP} measurements in $B \rightarrow PP, PV, VV, a_1P$ for α
 - use B & A_{CP} measurements in $B^- \rightarrow D^{(*)0}K^-(\pi^-), B^- \rightarrow D^0K^{*-}(\rho^-)$ for γ
 - use $\varepsilon_K, \Delta m_d, \Delta m_s, \sin 2\beta, \mathcal{B}(B \rightarrow \tau^\pm \nu), m_t, m_c$ and CKM matrix elements
- Theory uncertainties δ in $V_{ub}, V_{cb}, f_{B_d}, B_{B_d}, f_{B_s}, B_{B_s}, B_K$ have a non-Gaussian part that we scan over
- So we select particular values within $\pm 1\delta$ and perform a fit
- For $P(\chi^2) > 5\%$, we plot contours in the $\bar{\rho}-\bar{\eta}$ plane
 - allowed range is envelope of all contours



$81.1^\circ < \alpha < 93.3^\circ$ @ 95% CL
 $19.0^\circ < \beta < 25.8^\circ$ @ 95% CL
 $62.8^\circ < \gamma < 79.8^\circ$ @ 95% CL

$0.058 < \bar{\rho} < 0.191$ @ 95% CL
 $0.314 < \bar{\eta} < 0.406$ @ 95% CL

T Violation in the B System

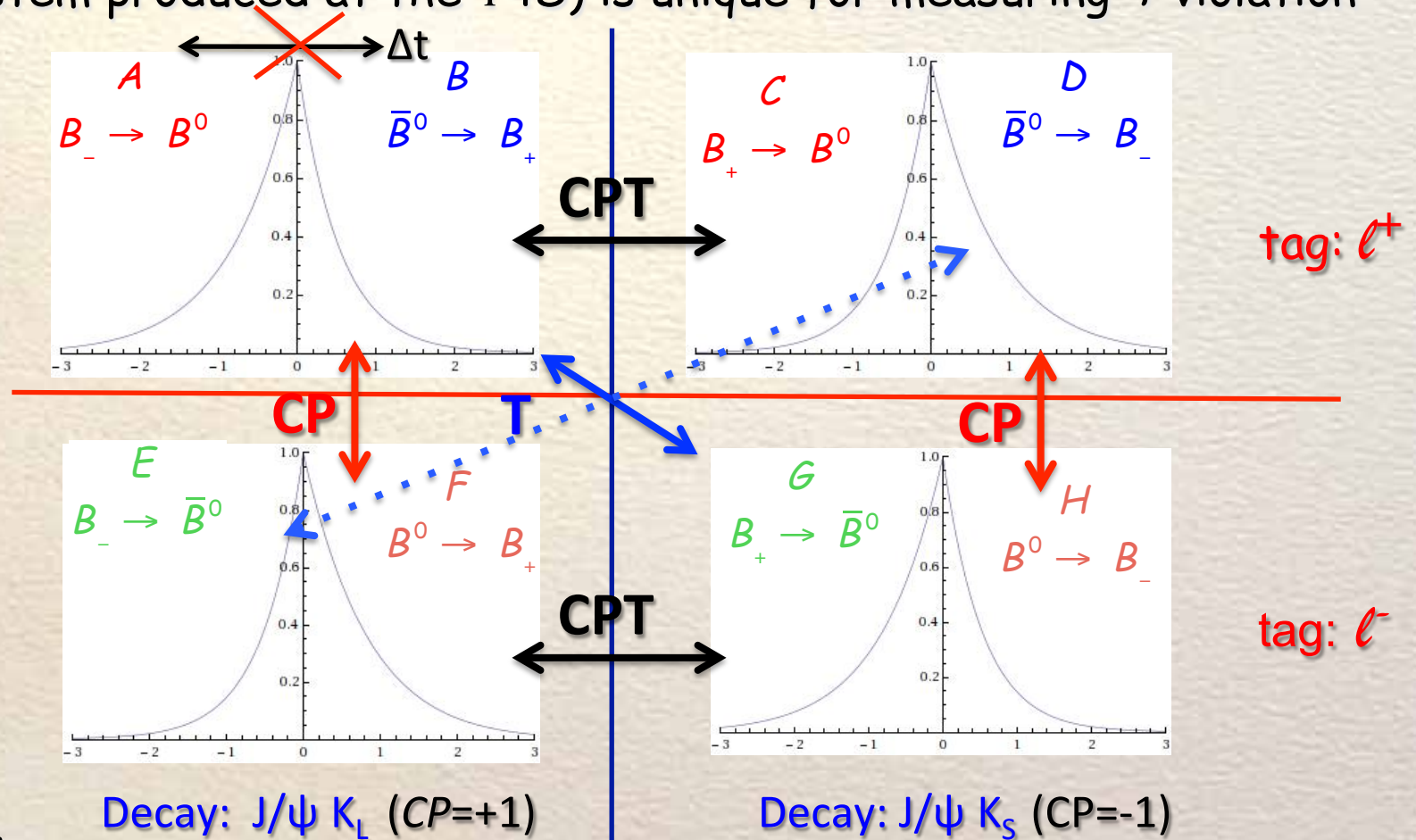
T Violation

- The $B^0\bar{B}^0$ system produced at the $\Upsilon(4S)$ is unique for measuring T violation

- Use the decay $B \rightarrow J/\psi K^0$ and explore time distribution

- Define B_+ : CP even
 B_- : CP odd

- We construct:



$$\Delta S_T^+ = S_{\ell^-, K_L^0}(\Delta t < 0) - S_{\ell^+, K_S^0}(\Delta t > 0)$$

$$\Delta S_T^- = S_{\ell^-, K_L^0}(\Delta t > 0) - S_{\ell^+, K_S^0}(\Delta t < 0)$$

$$\Delta C_T^+ = C_{\ell^-, K_L^0}(\Delta t < 0) - C_{\ell^+, K_S^0}(\Delta t > 0)$$

$$\Delta C_T^- = C_{\ell^-, K_L^0}(\Delta t > 0) - C_{\ell^+, K_S^0}(\Delta t < 0)$$

where $S_{X,Y}$ and $C_{X,Y}$ come from time-dep asymmetries as \mathcal{A}_{CP58}



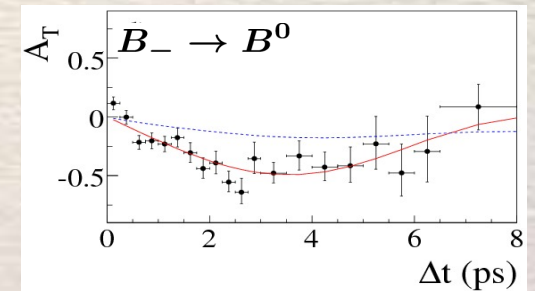
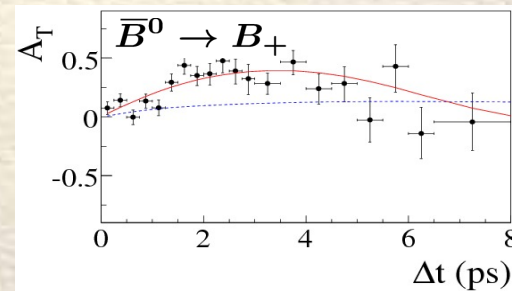
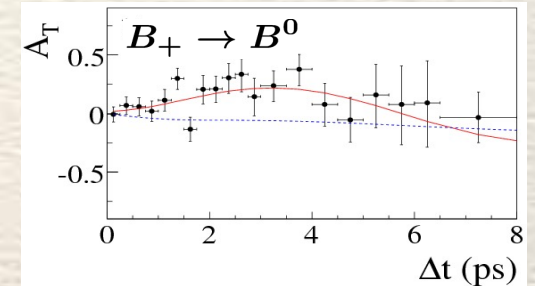
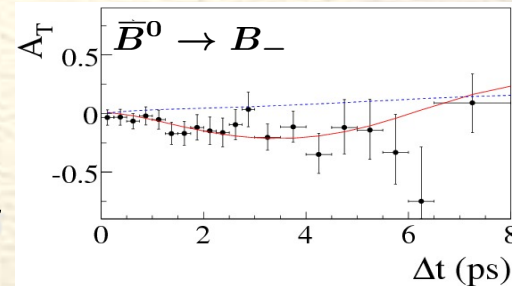
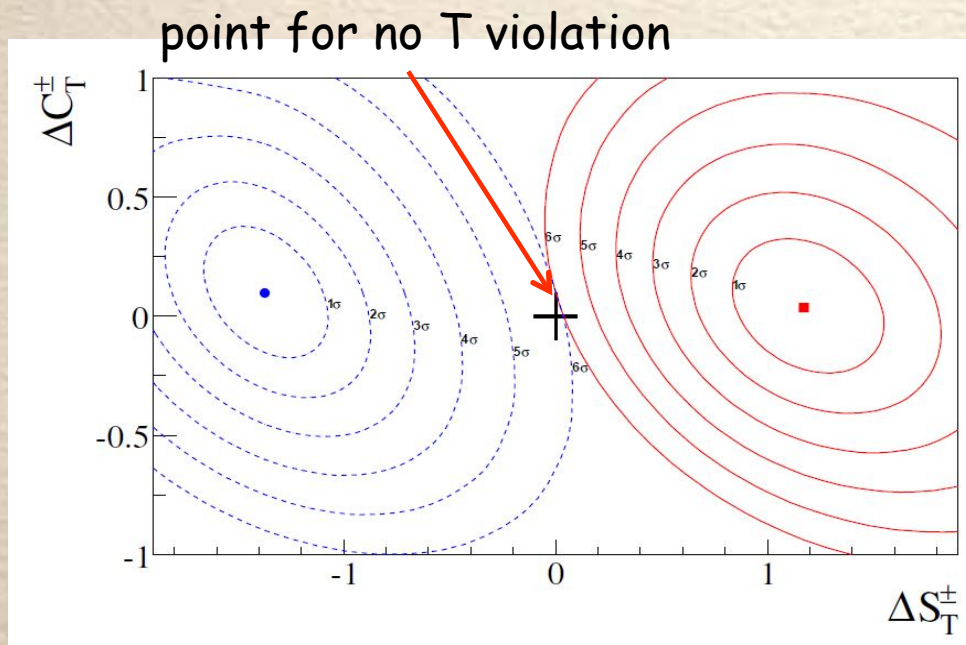
T Violation Asymmetries

- Define 4 asymmetries, e.g.

$$A_T = \frac{N(B_+ \rightarrow \bar{B}^0) - N(\bar{B}^0 \rightarrow B_+)}{N(B_+ \rightarrow \bar{B}^0) + N(\bar{B}^0 \rightarrow B_+)} \approx \frac{\Delta C_T^+}{2} \cos \Delta m \Delta t + \frac{\Delta S_T^+}{2} \sin \Delta m \Delta t \quad (37)$$

- The A_T asymmetries clearly show the expected T violation

- Plot of ΔC_T^\pm versus ΔS_T^\pm shows a 14σ effect

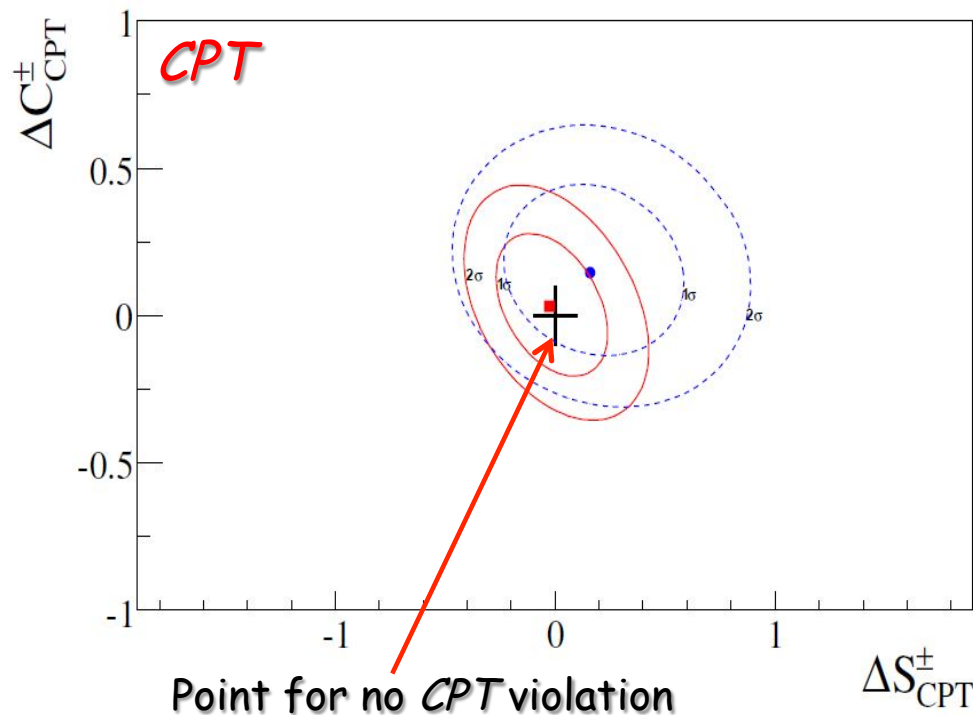
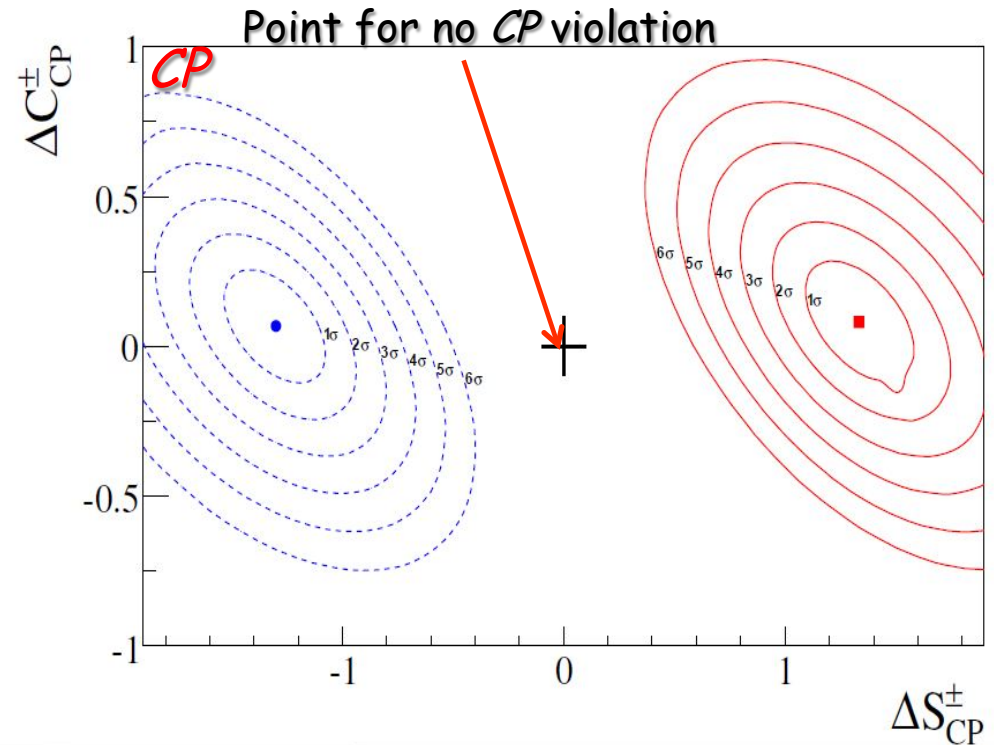


ΔS_T^+	$=$	$-1.37 \pm 0.14 \pm 0.06$
ΔS_T^-	$=$	$1.17 \pm 0.18 \pm 0.11$
ΔC_T^+	$=$	$0.10 \pm 0.16 \pm 0.08$
ΔC_T^-	$=$	$0.04 \pm 0.16 \pm 0.08$

Check for CP Violation and CPT Conservation

- Define in similar way $(\Delta C_{CP}^{\pm}, \Delta S_{CP}^{\pm})$ and $(\Delta C_{CPT}^{\pm}, \Delta S_{CPT}^{\pm})$ pairs

$$\begin{aligned} \Delta S_{CP}^+ &= -1.30 \pm 0.10 \pm 0.07 \\ \Delta S_{CP}^- &= 1.33 \pm 0.12 \pm 0.06 \\ \Delta C_{CP}^+ &= 0.07 \pm 0.10 \pm 0.03 \\ \Delta C_{CP}^- &= 0.08 \pm 0.09 \pm 0.04 \end{aligned}$$



$$\begin{aligned} \Delta S_{CPT}^+ &= 0.16 \pm 0.20 \pm 0.09 \\ \Delta S_{CPT}^- &= -0.03 \pm 0.13 \pm 0.06 \\ \Delta C_{CPT}^+ &= 0.15 \pm 0.17 \pm 0.07 \\ \Delta C_{CPT}^- &= 0.03 \pm 0.14 \pm 0.08 \end{aligned}$$

- Measure T and CP violation and CPT conservation in the same sample

Conclusion and Outlook

- B physics has produced many interesting results and is still a very active field
- The Belle II upgrade is in full progress → expect data taking 2016?
- LHCb is planning a full detector upgrade for the 2018 shutdown
- In the next decade, LHCb and Belle II will make many more precision measurements

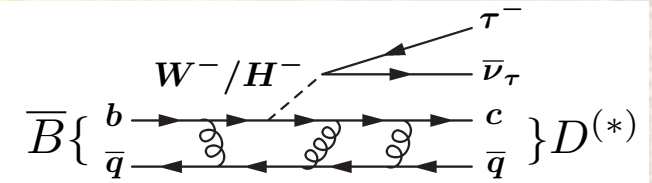
Type	Observable	Current precision (Spring 2012)	LHCb 2018 (7-8 fb ⁻¹)	Upgrade (50 fb ⁻¹)	Theory uncertainty
B_s^0 mixing	$2\beta_s(B_s^0 \rightarrow J/\psi\phi)$	0.10 [139]	0.025	0.008	~0.003
Gluonic penguins	$2\beta_s^{\text{eff}}(B_s^0 \rightarrow \phi\phi)$	–	0.17	0.03	0.02
	$2\beta_s^{\text{eff}}(B_s^0 \rightarrow K^{*0}\bar{K}^{*0})$	–	0.13	0.02	< 0.02
Right-handed currents	$2\beta_s^{\text{eff}}(B_s^0 \rightarrow \phi\gamma)$	–	0.09	0.02	< 0.01
Electroweak penguins	$S_3(B^0 \rightarrow K^{*0}\mu^+\mu^-; 1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.08 [68]	0.025	0.008	0.02
	$s_0 A_{\text{FB}}(B^0 \rightarrow K^{*0}\mu^+\mu^-)$	25 % [68]	6 %	2 %	7 %
Higgs penguins	$\mathcal{B}(B_s^0 \rightarrow \mu^+\mu^-)$	1.5×10^{-9} [13]	0.5×10^{-9}	0.15×10^{-9}	0.3×10^{-9}
	$\mathcal{B}(B^0 \rightarrow \mu^+\mu^-)/\mathcal{B}(B_c^0 \rightarrow \mu^+\mu^-)$	–	~100 %	~35 %	~5 %
Unitarity triangle angles	$\gamma(B \rightarrow D^{(*)}K^{(*)})$	~10–12° [252, 266]	4°	0.9°	negligible
	$\gamma(B_s^0 \rightarrow D_s K)$	–	11°	2.0°	negligible
	$\beta(B^0 \rightarrow J/\psi K_S^0)$	0.8° [44]	0.6°	0.2°	negligible

Backup Slides

$$B \rightarrow D^{(*)} \tau^+ \nu$$

B⁻ → Dτ⁻ν Decay Rate

- For B → D^(*)τν, the differential decay rate in the SM is



$$\frac{d\Gamma(B \rightarrow D^{(*)}\tau\nu)}{dq^2} = \frac{G_F^2 |V_{cb}|^2 p_{D^{(*)}}^* q^2 \left(1 - \frac{m_\tau^2}{q^2}\right)^2}{96\pi^3 m_B^2} \left[\left(|H_+|^2 + |H_-|^2 + |H_0|^2 \right) \left(1 - \frac{m_\tau^2}{q^2}\right) + \frac{3m_\tau^2}{2q^2} |H_s|^2 \right] \quad (12)$$

$$H_\pm(q^2) = (m_B + m_V) A_1(q^2) \mp \frac{2m_B |\vec{p}_V|}{m_B + m_V} V(q^2) \quad (13)$$

$$H_s(q^2) = \frac{2m_B |p_{D^{(*)}}^*|}{\sqrt{q^2}} A_0(q^2) \quad (15)$$

where H_i are the helicity amplitudes that are functions of formfactors

$$H_0(q^2) = \frac{m_B + m_V}{2m_V \sqrt{q^2}} \left[(m_B^2 - m_V^2 - q^2) A_1(q^2) - \frac{4m_B^2 |\vec{p}_V|^2}{(m_B + m_V)^2} A_2(q^2) \right] \quad (14)$$

- Charged Higgs decays may enhance the decay rate
- In the 2Higgs Doublet Model (2HDM) type II the decay rate is multiplied by

$$\left(1 - \frac{\tan^2 \beta}{m_{H^\pm}^2} \frac{q^2}{1 \mp \frac{m_c}{m_b}} \right)^2 \quad \text{where } \beta \text{ is the ratio of the VEVs of the 2 doublets} \quad (16)$$



B → Dτν Higgs Contribution

- B → D(*)τν may get sizable contributions from H⁻ compared to B → D(*)μν due to the large τ mass that also reduces the B → D(*)τν phase space

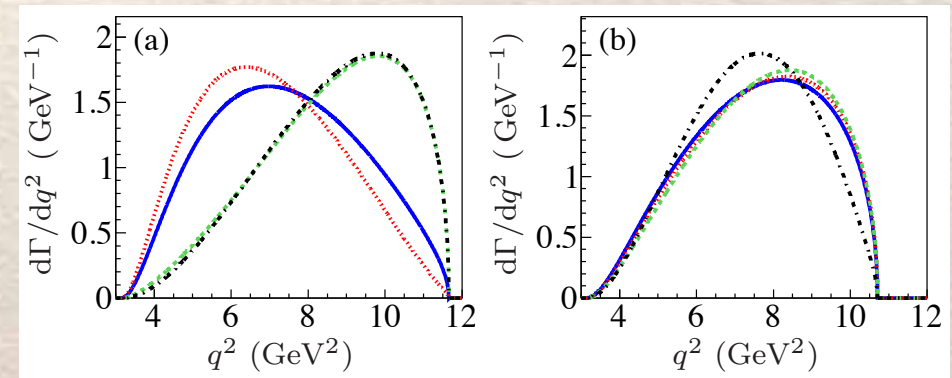
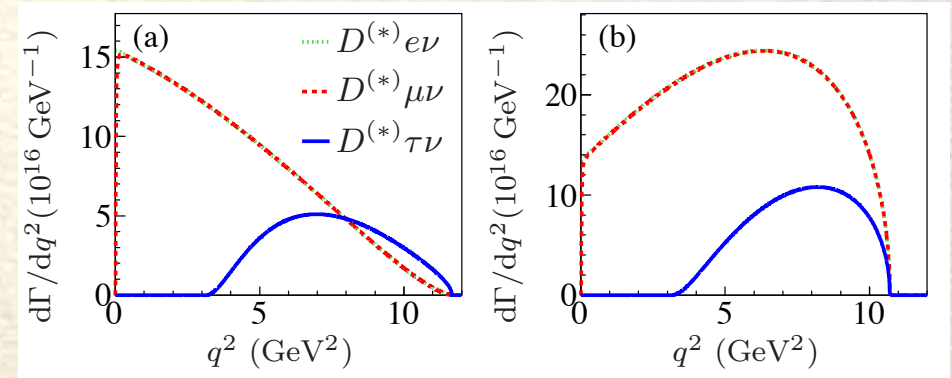
- The shape of the q² spectrum is modified by H[±] contributions

- The strategy is to measure the ratios

$$R(D) = \frac{\mathcal{B}(\bar{B} \rightarrow D\tau^{-}\bar{\nu})}{\mathcal{B}(\bar{B} \rightarrow D\ell^{-}\bar{\nu})} \quad \ell = e, \mu$$

$$R(D^*) = \frac{\mathcal{B}(\bar{B} \rightarrow D^*\tau^{-}\bar{\nu})}{\mathcal{B}(\bar{B} \rightarrow D^*\ell^{-}\bar{\nu})}$$

- The ratios are independent of |V_{cb}| & to a large extent to hadronic matrix elements



— SM
 tanβ/m_{H⁺} = 0.3 GeV⁻¹
 tanβ/m_{H⁺} = 0.5 GeV⁻¹
 tanβ/m_{H⁺} = 1 GeV⁻¹



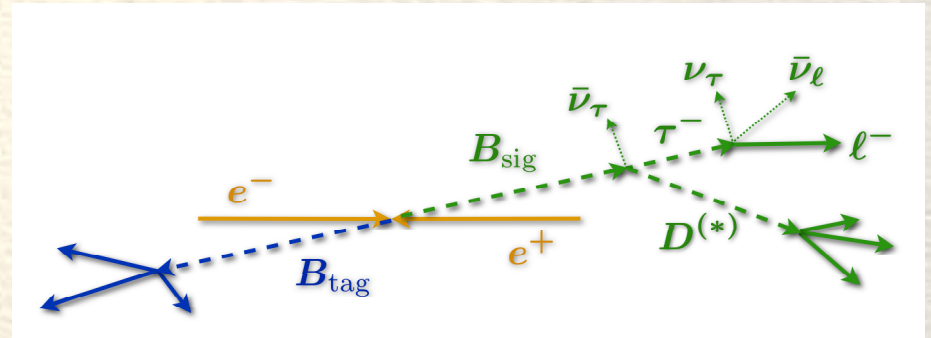
In the SM

$$R(D) = 0.297 \pm 0.017$$

$$R(D^*) = 0.252 \pm 0.003$$

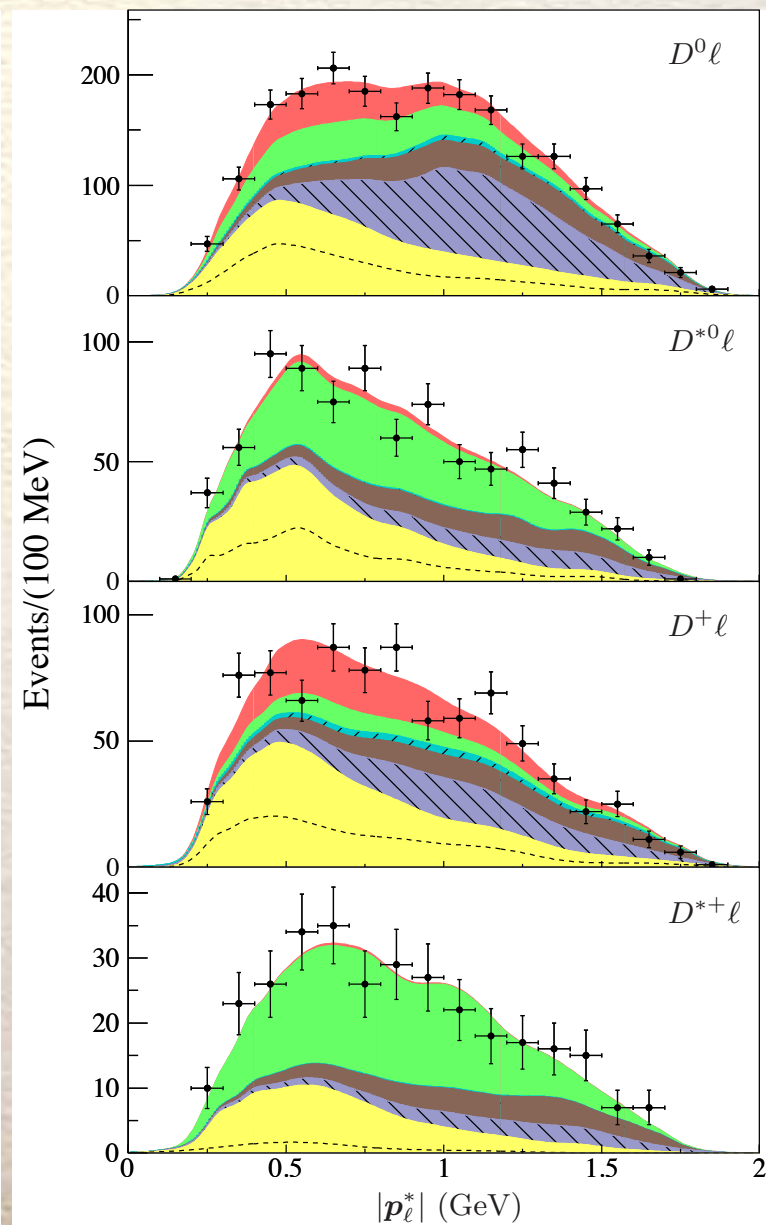
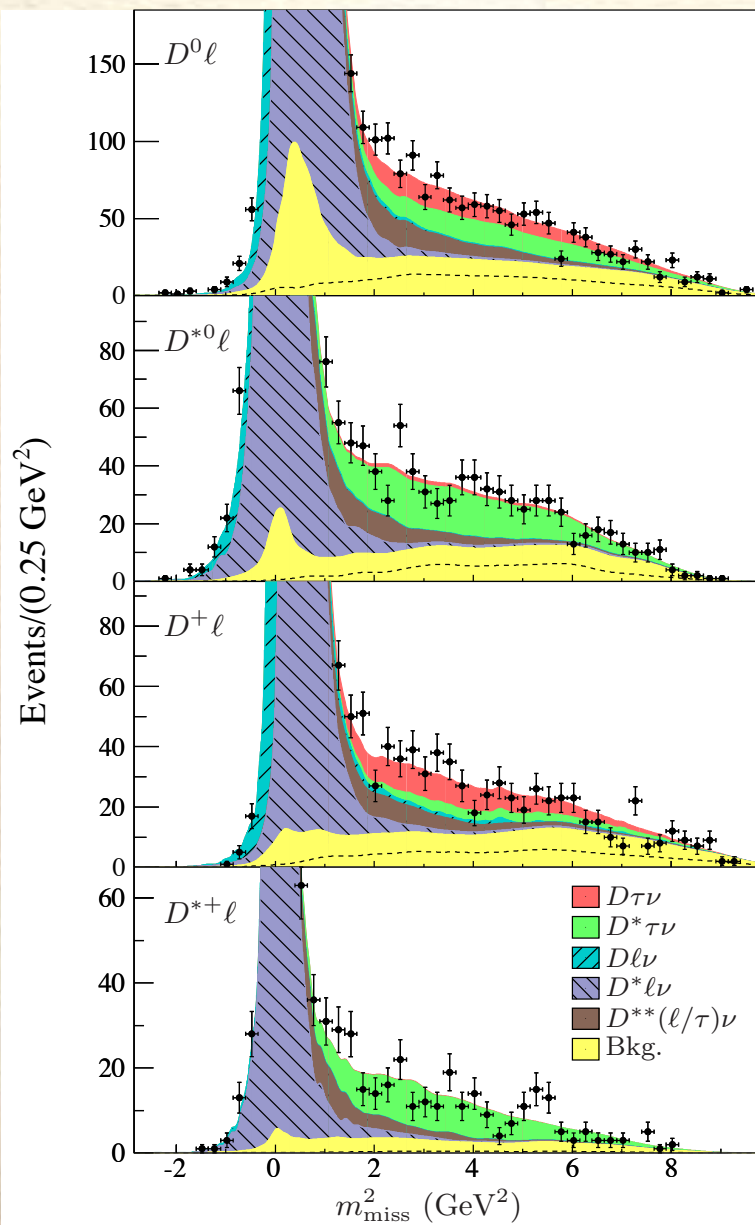
B → Dτν Analysis

- Experimentally, we reconstruct one B in a hadronic final state using 2968 individual modes, called B_{tag}
- For each B_{tag} , we look for a semileptonic decay, i.e. a lepton and a D or D^*
- We reconstruct the τ only in leptonic decays, → get signal and normalization modes
- The D^0 (D^+) is reconstructed in 5 (7) final states, the D^{*+} in $D^0 \pi^+$, $D^+ \pi^0$ and D^{*0} in $D^0 \pi^0$, $D^0 \gamma$
- Backgrounds originate from $B \rightarrow D^{**} \tau / \ell \nu$, generic BB and qq continuum
- For signal events, there are three neutrinos → we look at various kinematic observables: q^2 , p_l (lepton momentum), m_{miss}^2 (missing mass squared), $\Delta E = E_{\text{tag}} - E_{\text{beam}}$, $m_{\text{ES}} = (E_{\text{beam}}^2 - p_{\text{tag}}^2)^{1/2}$, E_{extra} (extra neutral E in calorimeter)



$B \rightarrow D\tau\nu$ m_{miss}^2 and p_ℓ Distributions

- 4 final states
- $D^0\ell$
- $D^{*0}\ell$
- $D^+\ell$
- $D^{*+}\ell$

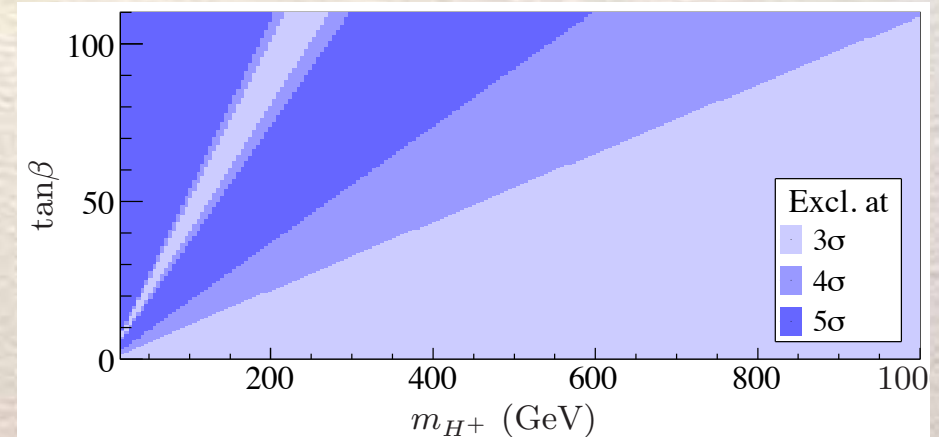
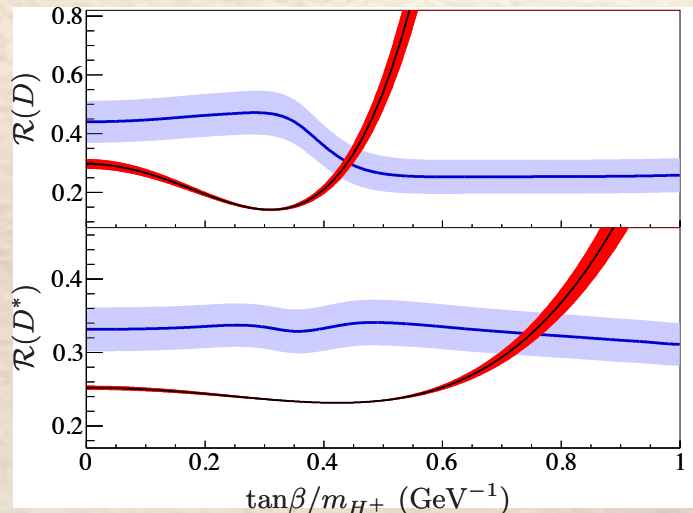
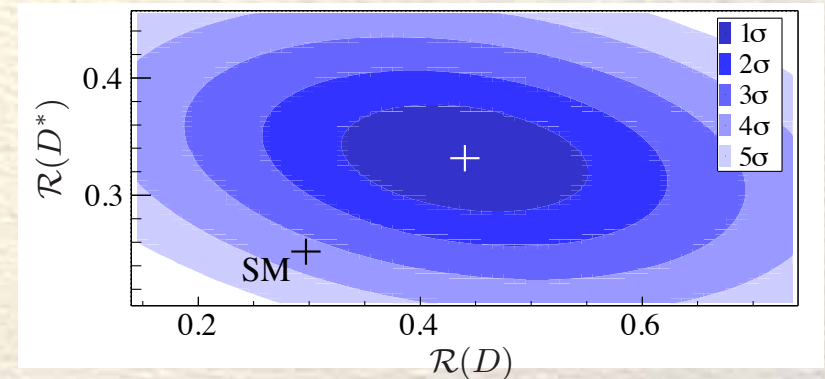


B → Dτν Implications on 2HDM

● Measure $\mathcal{R}(D) = 0.440 \pm 0.058 \pm 0.042$ $\mathcal{R}(D^*) = 0.332 \pm 0.024 \pm 0.018$

● This is much higher than the SM prediction, disagreement of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ with the SM is 3.4σ

● We can compare the measurements with the predictions of 2HDM



● The combination of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ excludes 2HDM, since allowed regions 0.44 ± 0.02 GeV⁻¹ and 0.75 ± 0.02 GeV⁻¹ do not overlap



The type III 2HDM still has two solutions

B → Dτν

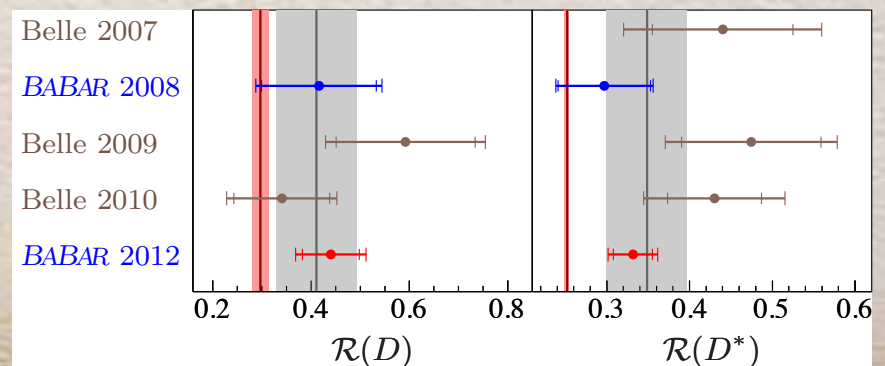
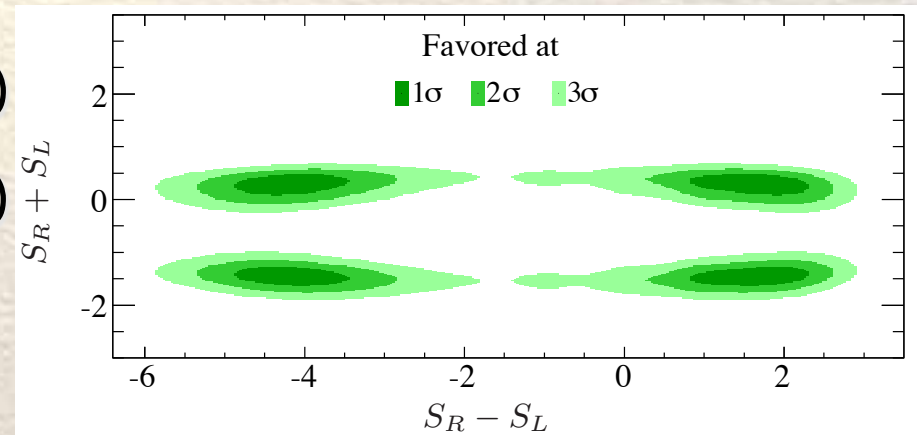
- Lets look at a type III 2HDM including scalar and pseudoscalar contributions
- This modifies $R(D)$ and $R(D^*)$

$$H_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}} \left[\left(\bar{c} \gamma_\mu P_L b \right) \left(\bar{\tau} \gamma^\mu P_L \nu_\tau \right) + S_L \left(\bar{c} P_L b \right) \left(\bar{\tau} P_L \nu_\tau \right) + S_R \left(\bar{c} P_R b \right) \left(\bar{\tau} P_R \nu_\tau \right) \right] \quad (17)$$

$$R(D) = R(D)_{SM} + A'_D \text{Re}(S_R + S_L) + B'_D |S_R + S_L|^2 \quad (18)$$

$$R(D^*) = R(D^*)_{SM} + A'_{D^*} \text{Re}(S_R - S_L) + B'_{D^*} |S_R - S_L|^2 \quad (19)$$

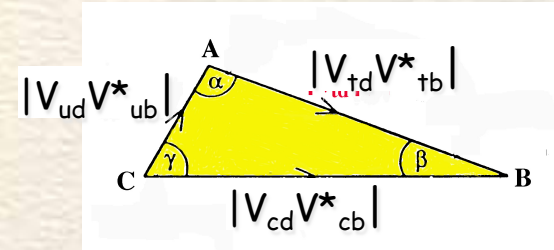
- There are 4 solutions, the q^2 spectra disfavor those at $S_R + S_L = -1.5$
- BABAR results are in good agreement with other measurements
- Is this a hint for new physics?



Unitarity Conditions of the CKM Matrix

- CP asymmetries in B^0 decays into CP eigenstates provide a good way to measure the 3 angles of the Unitarity Triangle, defined by

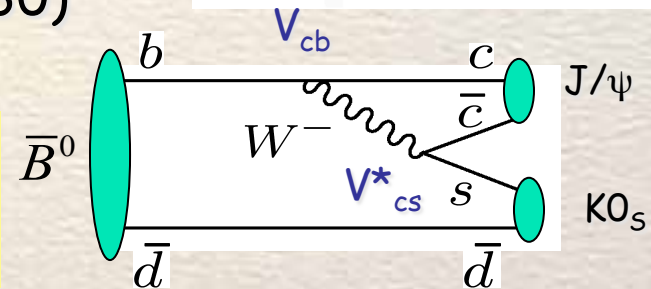
$$\alpha \equiv \text{Arg} \left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right), \quad \beta \equiv \text{Arg} \left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right), \quad \gamma \equiv \text{Arg} \left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right) \quad (30)$$



- Lets consider some examples

$B \rightarrow J / \psi K_s^0$:

$$\lambda_{\psi K_s^0} = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*} \right) \left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \right) \Rightarrow \Im m \lambda = -\sin 2\beta$$

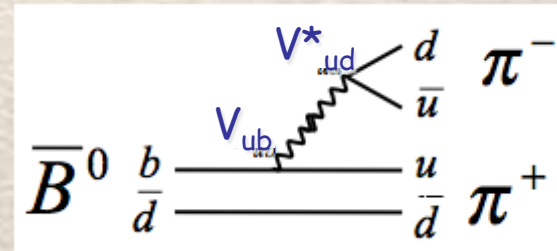


$B \rightarrow \pi^+ \pi^-$:

$$\lambda_{\pi\pi} = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left(\frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*} \right) \Rightarrow \Im m \lambda = \sin 2\alpha$$

(31)

(32)



- Measurement of γ is more complicated (see later)
- The goal is to make many independent measurements of both sides and angles of the Unitarity Triangle to overconstrain it \rightarrow test SM

CP Violation in the B System

- There are many useful constraints from B and K decays

$$R_u = \left| \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} \right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

→ small blue circle

$$R_t = \left| \frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}$$

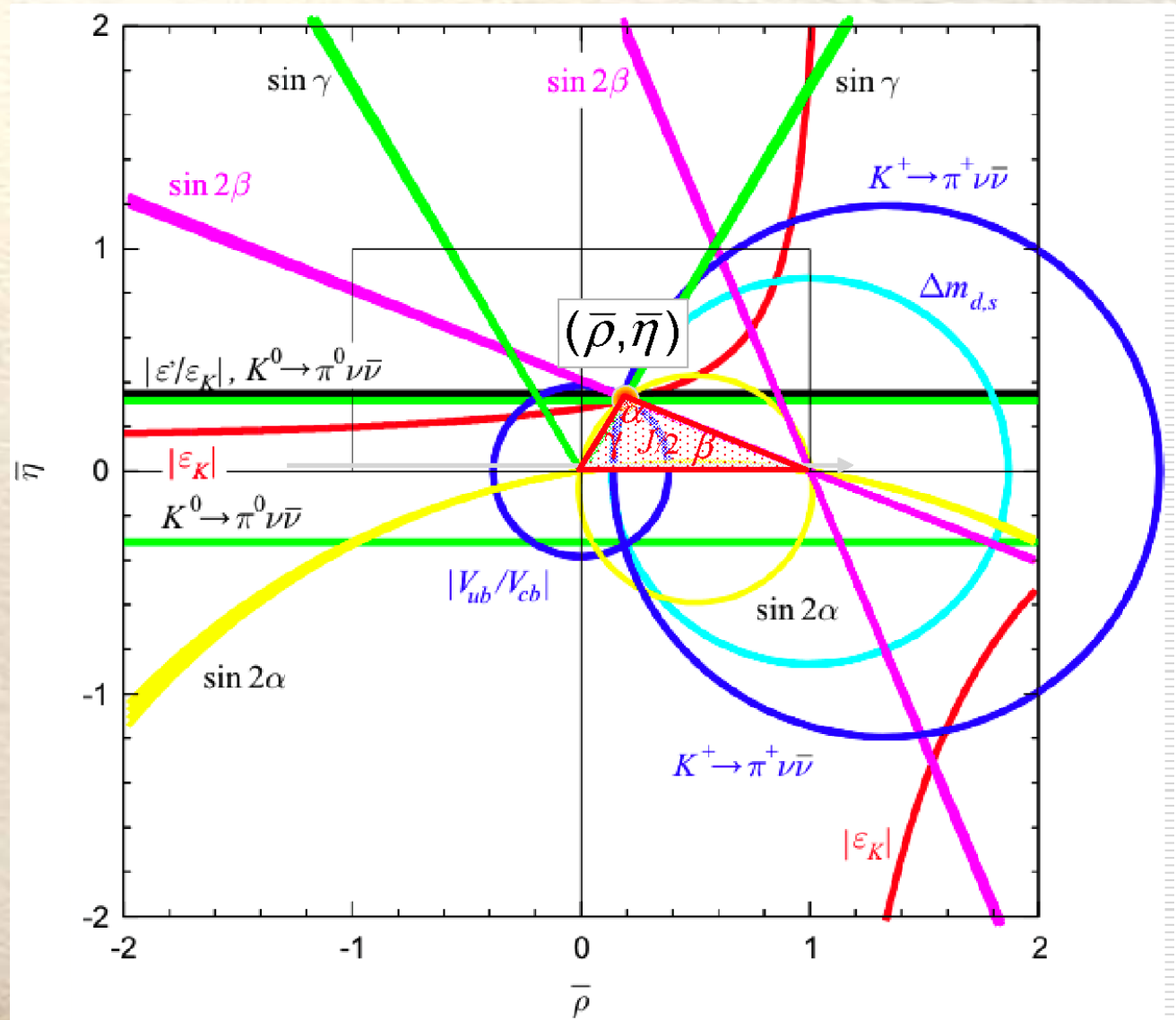
→ cyan circle

$$\sin 2\alpha = \frac{2\bar{\eta} [R_u^2 - \bar{\rho}]}{R_t^2 R_u^2}$$

→ yellow circles

$$\sin 2\beta = \frac{2\bar{\eta} [1 - \bar{\rho}]}{R_t^2}$$

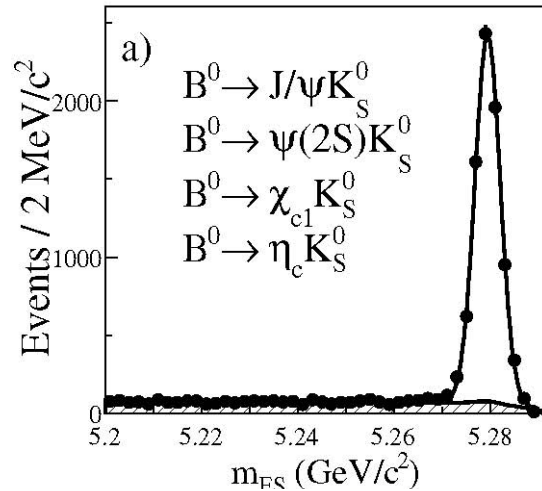
→ magenta rays



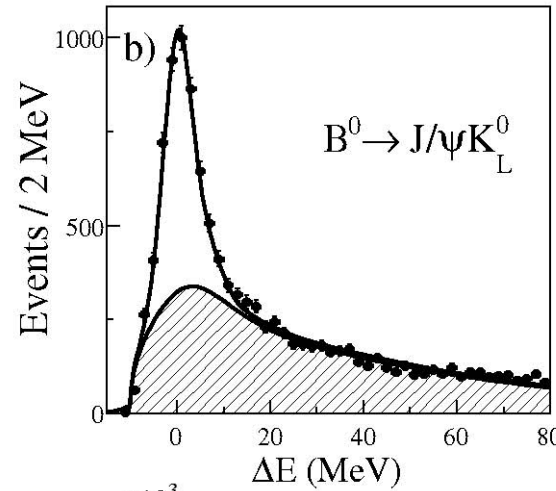
CP Violation in the B System

- We measured different samples

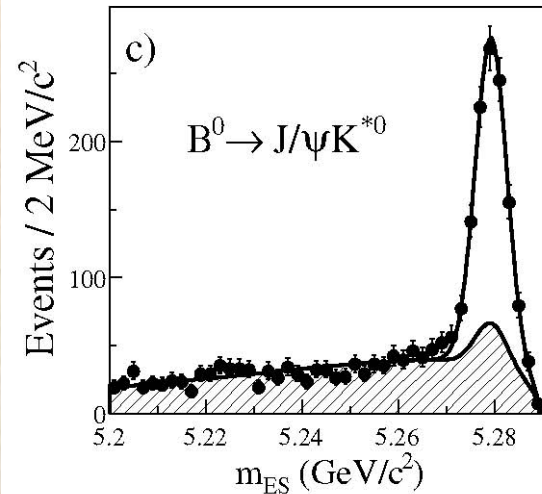
9377 tagged events



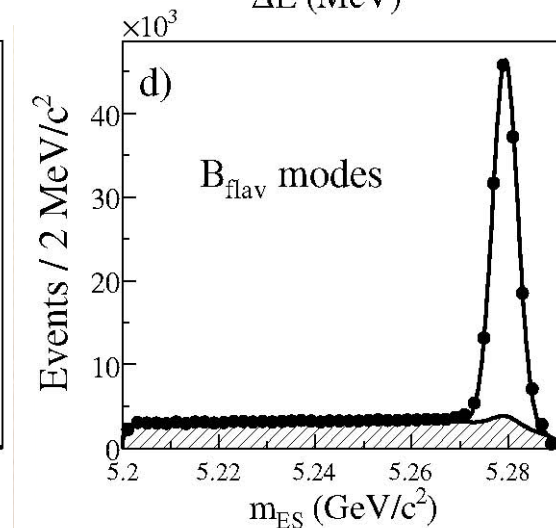
5813 tagged events



1291 tagged events



166276 flavor tags



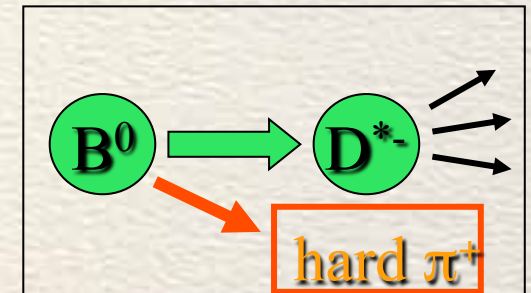
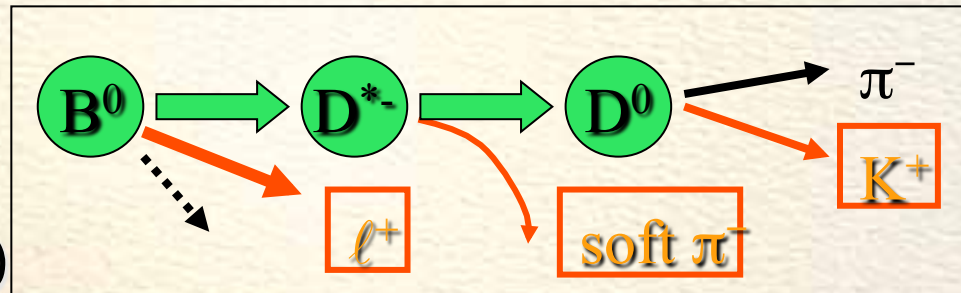
Control sample has the same mistag rate and Δt resolution as CP sample (Δt error dominated by recoil side) \rightarrow use in fit as well

Tagging and Δt Reconstruction

- Tag 2nd B by

- Tagging quality

$$Q = \varepsilon(1 - 2\omega)^2 \quad (33)$$



- Error on $\sin 2\beta$ depends on tagging quality

$$\sigma(\sin 2\beta) \propto \frac{1}{\sqrt{Q}} \quad (34)$$

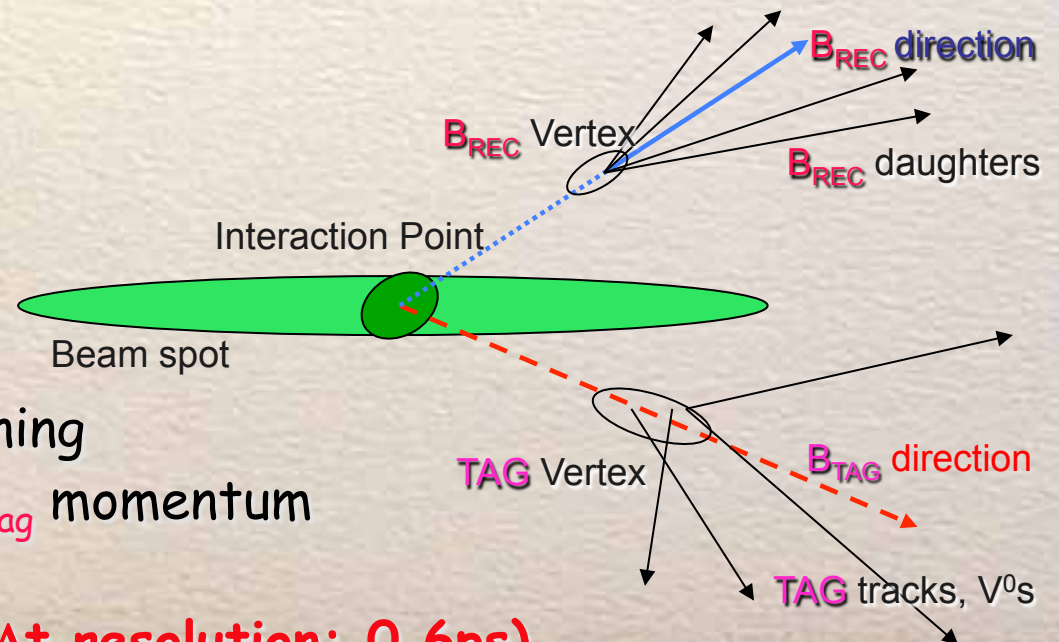
- Error on $\sin 2\beta$ depends on tagging quality: $Q = 31.2 \pm 0.3\%$, $\varepsilon = (74.37 \pm 0.1)\%$

- Δt reconstruction:

- Get B_{rec} vertex from charged daughter tracks

$$\sigma_z(B_{rec}) = 65 \mu\text{m} \Rightarrow B_{rec} \text{ momentum}$$

- Determine B_{tag} vertex from remaining charged tracks $\Rightarrow B_{tag}$ vertex & B_{tag} momentum



Average Δz resolution: $180 \mu\text{m}$ (Δt resolution: 0.6ps)

Extraction of α from $B \rightarrow \pi\pi$

- Use as inputs:

$$\mathcal{B}(B^0 \rightarrow \pi^+\pi^-)$$

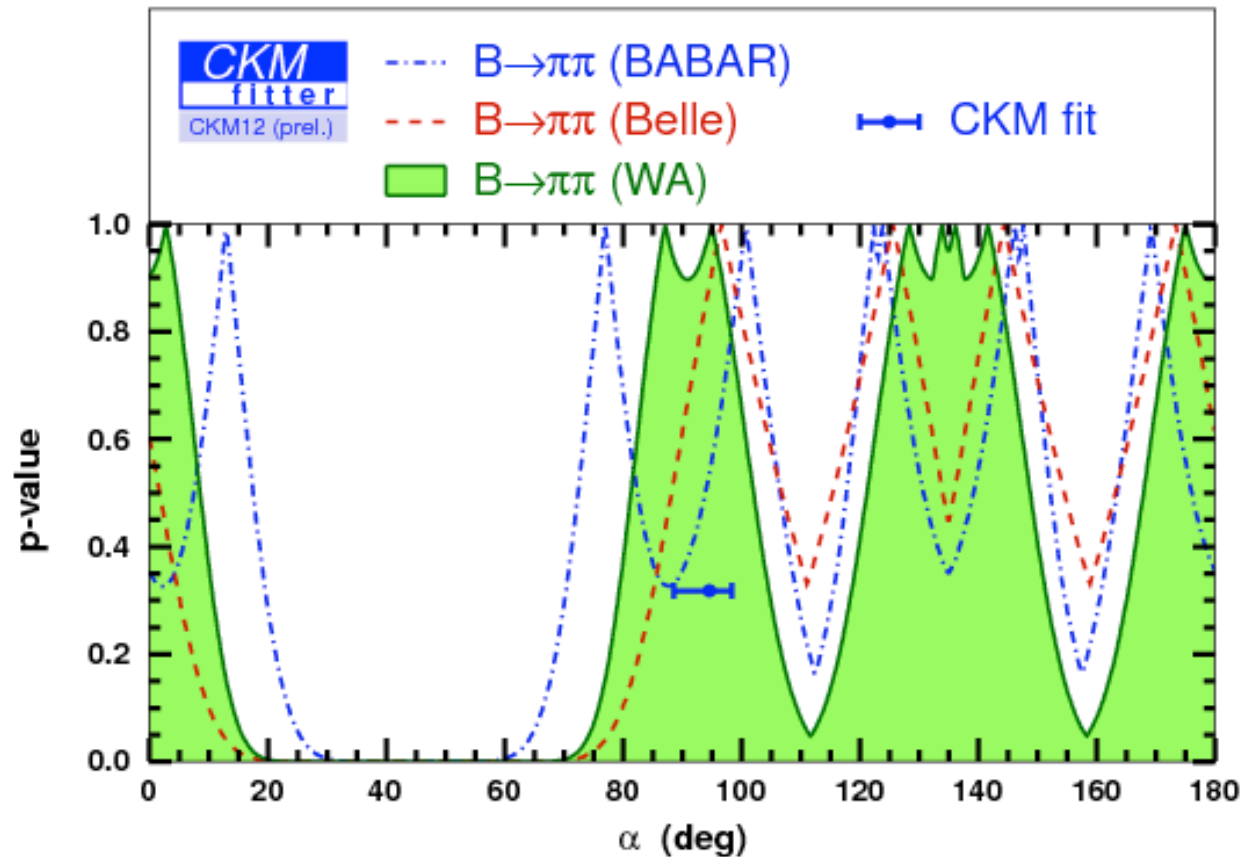
$$\mathcal{B}(B^0 \rightarrow \pi^0\pi^0)$$

$$\mathcal{B}(B_{\pm} \rightarrow \pi^{\pm}\pi^0)$$

$$S_{\pi^+\pi^-}$$

$$C_{\pi^+\pi^-}$$

$$C_{\pi^0\pi^0}$$



- For $B \rightarrow \pi\pi$ obtain no stringent constraint on α
 - \rightarrow Need either also measurements from $B \rightarrow \rho\pi$, $B \rightarrow \rho\rho$, & $B \rightarrow a_1\pi$
 - \rightarrow or need to combine all measured $B \rightarrow PP$ modes

Direct CP Violation in $B \rightarrow X_s \gamma$

- In the SM, direct CP violation in $B \rightarrow X_s \gamma$ is predicted to be small:
 $-0.6\% < \mathcal{A}_{CP}(X_s \gamma) < 2.8\%$

- Its measurement is performed by using exclusive self-tagging decays

- Using 16 such modes in a sample of $471 \times 10^6 \bar{B}B$ events, BABAR measured

$$\mathcal{A}_{CP}(X_s \gamma) = (1.7 \pm 1.9_{\text{stat}} \pm 1.0_{\text{sys}})\%$$

→ most precise \mathcal{A}_{CP} result
[arXiv:1406.0534](https://arxiv.org/abs/1406.0534)

- The difference for charged and neutral B decays is

$$\Delta \mathcal{A}_{CP}(X_s \gamma) = \mathcal{A}_{CP}(B^+ \rightarrow X_s^+ \gamma) - \mathcal{A}_{CP}(B^0 \rightarrow X_s^0 \gamma)$$

- $\Delta \mathcal{A}_{CP}(B \rightarrow X_s \gamma)$ depends on $C_{7\gamma}$ and C_{8g} :

$$\Delta \mathcal{A}_{CP}(X_s \gamma) \approx 4\pi^2 \alpha_s \frac{\bar{\Lambda}_{78}}{m_b} \Im m \frac{C_8^{\text{eff}}}{C_7^{\text{eff}}} \approx 0.12 \frac{\bar{\Lambda}_{78}}{100 \text{ MeV}} \Im m \frac{C_8^{\text{eff}}}{C_7^{\text{eff}}}$$

$$17 \text{ MeV} < \bar{\Lambda}_{78} < 190 \text{ MeV}$$

Benzke et al., PRL106, 141801 (2011)

- In the SM, C_7^{eff} and C_8^{eff} are real → $\Delta \mathcal{A}_{CP}(X_s \gamma) = 0$

- This may be modified by new physics contributions



BABAR measures

$$\Delta \mathcal{A}_{CP}(X_s \gamma) = (5.0 \pm 3.9_{\text{stat}} \pm 1.5_{\text{sys}})\%$$

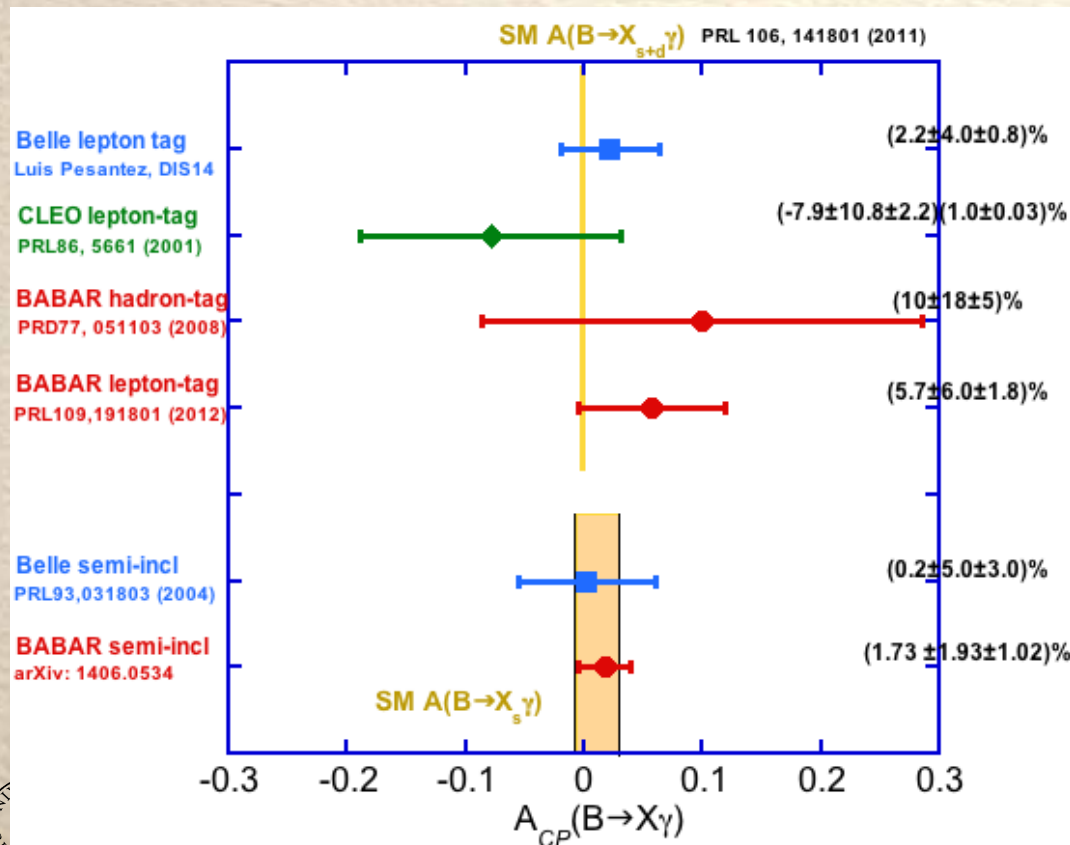
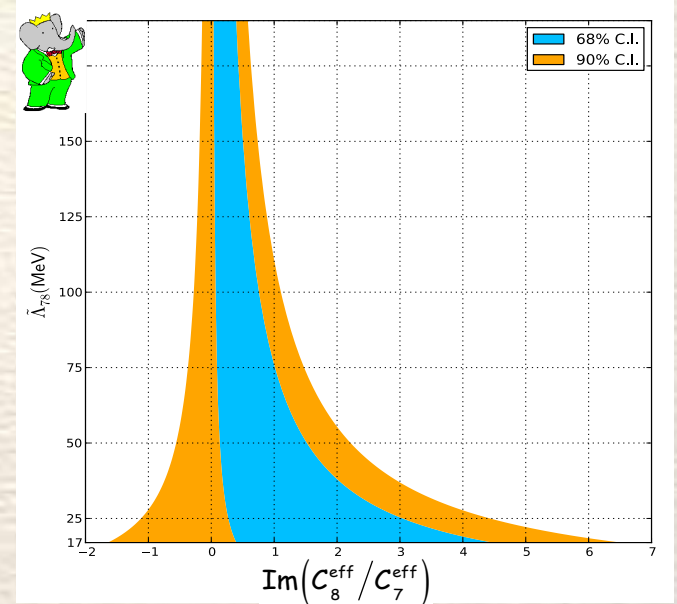
→ first $\Delta \mathcal{A}_{CP}(B \rightarrow X_s \gamma)$ result

Direct CP Violation in $B \rightarrow X_s \gamma$

- This yields a 90% CL constraints on $\text{Im}(C_8^{\text{eff}}/C_7^{\text{eff}})$

$$-1.64 < \text{Im}(C_8^{\text{eff}}/C_7^{\text{eff}}) < 6.52 \text{ @90\% CL}$$

- This is the first constraint on the ratio of Wilson coefficients $C_8^{\text{eff}}/C_7^{\text{eff}}$ in this process

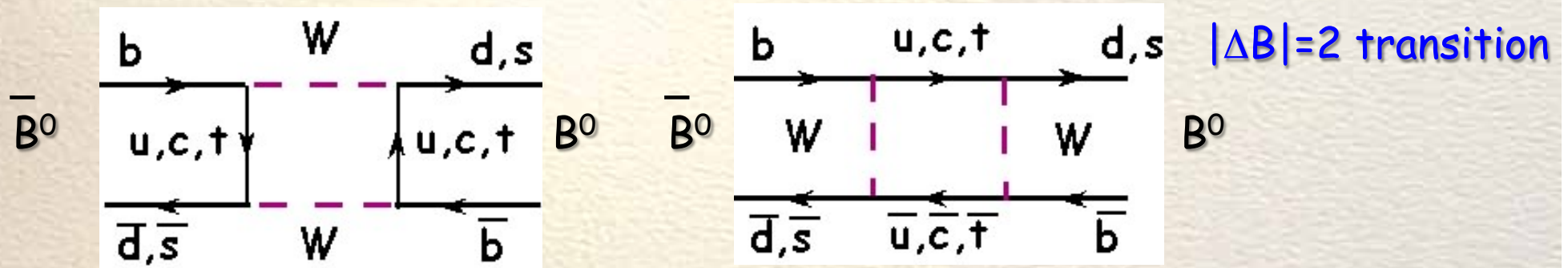


- In the fully inclusive analysis, A_{CP} has $b \rightarrow s \gamma$ & $b \rightarrow d \gamma$ contributions
- In the SM, $A_{CP}(B \rightarrow X_{s+d}) = 0$
- Determine B/\bar{B} from charge of tag
- All results agree well with the SM

$B^0-\bar{B}^0$ Mixing

B⁰ \bar{B}^0 Mixing

- B⁰_d and B⁰_s can oscillate into their antiparticles \bar{B}^0 _d and \bar{B}^0 _s, respectively



box diagrams

- The Hamiltonian of the $B^0\bar{B}^0$ system has a mass matrix M and a decay matrix Γ
 - M corresponds to a phase in the wave function while Γ corresponds to a probability density, which decays exponentially with time and has the characteristic decay width
- By diagonalizing the Hamiltonian, we obtain 2 CP eigenstates whose
- masses differ by
$$\Delta M = 2\Re e\sqrt{\left(M_{12} - \frac{1}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{1}{2}\Gamma_{12}^*\right)} \quad (20)$$

where M_{12} & Γ_{12} are the off diagonal matrix elements of M and Γ , respectively



For B^0 _d, $\Gamma_{12} \approx 0$

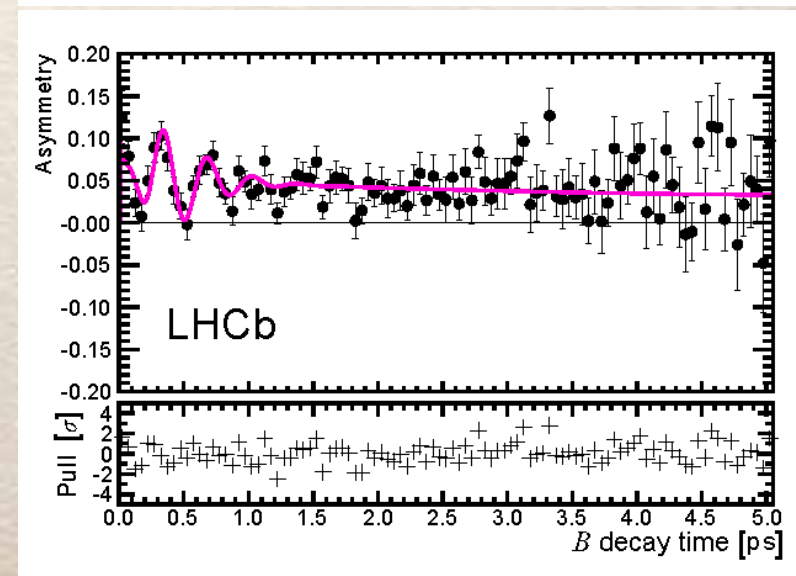
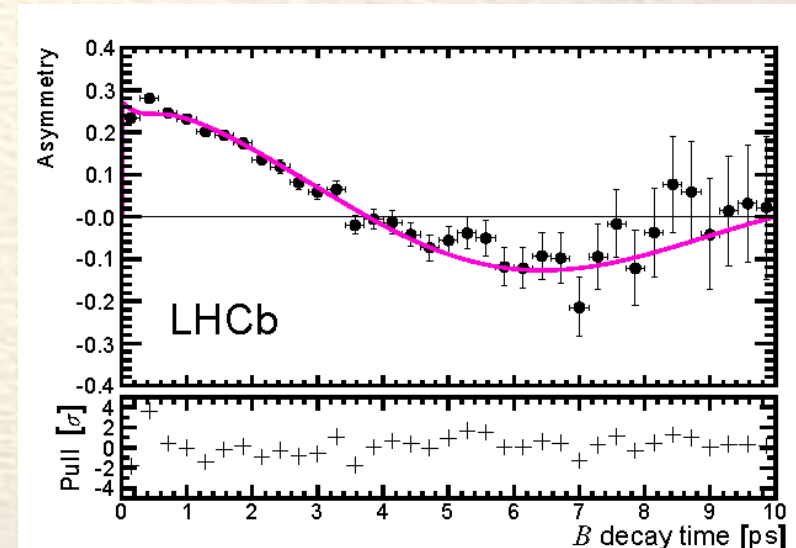
$B_d^0 \bar{B}_d^0$ and $B_s^0 \bar{B}_s^0$ Oscillation

- For a top quark in the box diagram, the $|\Delta B|=2$ weak Hamiltonian yields a mass difference of

$$\Delta M = \frac{G_F}{6\pi^2} B_B f_B^2 m_B |V_{td}^* V_{tb}|^2 M_W^2 S_0(x_t) \eta_B \quad (21)$$

where G_F is the Fermi constant, f_B is the B_d^0 decay constant, B_B is the bag factor, V_{tb} and V_{td} are CKM matrix elements, m_B is the B_d^0 mass, M_W is the W boson mass, $S_0(m_t^2/M_W^2)$ is the function of the box, m_t is the top quark mass and η_B is a QCD factor $\rightarrow f_B$ & B_B have theory uncertainties

- For B_s^0 , ΔM is gotten from eq (21) by replacing f_B , B_B , m_B , and $|V_{td}^* V_{tb}|^2$ with the corresponding B_s^0 quantities



$B^0\bar{B}^0$ Mixing Measurements

- For B^0_d , the world average for ΔM is

$$\Delta M_{B^0_d} = (0.51 \pm 0.004) \text{ ps}^{-1}$$

- For B^0_s , the world average for ΔM is

$$\Delta M_{B^0_s} = 17.671 \pm 0.022 \text{ ps}^{-1}$$

