On some Top Quark Physics Issues

S. Tokár, Comenius Univ., Bratislava

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Topics in This Talk

- Motivation for top physics
- Top quark mass – what do we measure?
- On role of the top quark mass in intrinsic consistency tests of the Standard model
- Stability of vacuum and the top mass

…

Top quark physics: Motivation

- Very high mass: near EWSB scale η

  Top Youkawa coupling $\lambda_t = \sqrt{2}m_{top}/\eta \approx 1$

- tt-bar production X-sections: test of QCD → to is produced at very small distances $1/m_t \Rightarrow \alpha_s(m_{top}) \approx 0.1$: pert. expansion converges rapidly

- Top decays before hadronization

$1/m_t$	<	$1/\Gamma_t$	<	$1/\Lambda$	<	m_t/Λ^2
Production time <		Lifetime	<	Hadronization time	<	Spin decorrelation time

→ study of spin characteristics (test of V-A)

- Cross sections sensitive to new physics

  → resonat production of $t\bar{t}$, decay: t →Hb

• Important background for Higgs studies

Top is special!

Stringent tests of SM + Search for New physics

η = 246 GeV

$\Lambda \approx$ 250 MeV

What is the Top Quark Mass?

The top-quark mass is presently inferred:

- ✓ by the kinematical reconstruction of the invariant mass of its decay products *via* the matrix element or the template method
- ✓ by its relation to the top-quark pair production cross section

1st case: more precise - not a well defined renormalization scheme $\Rightarrow$ a theoretical uncertainty in its interpretation - *top quark mass is reconstructed*

2nd case: not so precise – renormalization scheme is unambiguously defined – interpretation: *pole mass of the top quark is reconstructed*

On top quark mass

- World (LHC+Tevatron) top quak mass combination:

$$m_{\text{top}} = 173.34 \pm 0.76\ (0.27 \pm 0.24 \pm 0.67)\ \text{GeV}$$

- After *A. Hoang and I. Stewart, Nucl.Phys.Proc.Suppl. 185 (2008) 220,* the latest combinations of top mass measurements includes a statement like:

"In all measurements considered in the present combination, the analyses are calibrated to the MC top-quark mass definition. It is expected that the difference between the MC mass definition and the formal pole mass of the top quark is up to the order of 1 GeV."

- In *S. Moch et al., arXiv:1405.4781* is the suggestion:

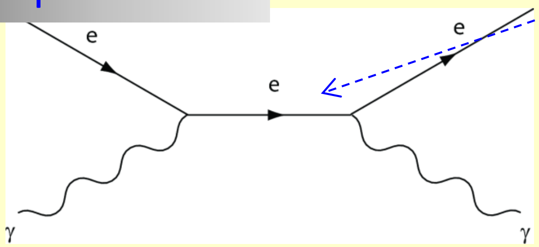
A more appropriate description of the content of Ref. [*Hoang and Stewart*] is: "The uncertainty on the translation from the MC mass definition to a theoretically well defined short distance mass definition at a low scale is currently estimated to be of the order of 1 GeV ."

> - My understanding is:
>
> Measured top mass (MC mass) $\neq$ top pole („true") mass by at least ~1 GeV

Particle mass – preliminary words

Let us start with electron

Compton effect



Electron propagator $\equiv \dfrac{i\left(\not p + m\right)}{p^2 - m^2 + i\varepsilon}, \quad \not p = p_\mu \gamma^\mu$

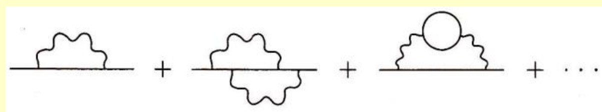
Electron mass = pole in electron propagator

→ it is free-field propagator

Propagator correction: 

→ divergent renormalization

Characterized by loop momentum μ or size 1/ μ

1PI≡ 

Propagator in general:

$$\frac{i}{\not p - m_{pole}} = \frac{i}{\not p - m_0} + \frac{i}{\not p - m_0}\left(\frac{\Sigma(\not p)}{\not p - m_0}\right) + \frac{i}{\not p - m_0}\left(\frac{\Sigma(\not p)}{\not p - m_0}\right)^2 \cdots = \frac{i}{\not p - m_0 - \Sigma(\not p)}$$

bare mass (m_0)

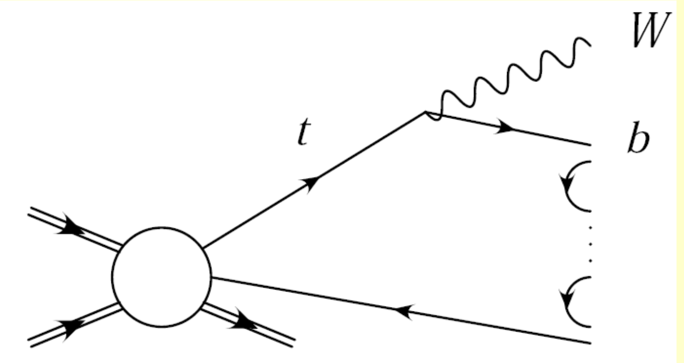
Other expansion: all bubles with size < 1/ μ_R are absorbed into mass

$$\Rightarrow \frac{i}{\not p - m_{pole}} = \frac{i}{\not p - m(\mu_R) - \Sigma(\mu_R, \not p)}$$

$\boldsymbol{m(\mu_R)}$ ≡ short distance mass, $\boldsymbol{\Sigma}$ ≡ self-energy: contribution of interactions to mass

What is the Top Quark Mass ?

Top quark pole mass: corresponds to pole in the full top quark propagator – Difference *wrt* electron:



- ✓ top is unstable – pole is complex: $m_{top} + i\Gamma_{top}$
- ✓ Top is colored object - due to confinement its mass cannot be determined with accuracy better than Λ_{QCD} (non-perturbative effects)

Pole mass is close to invariant mass of the top decay products.
Ambiguities: extra radiation, color reconnection and hadronization – at least one quark not coming from top decay is trapped by *b*-quark.

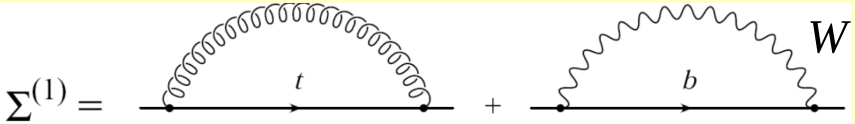
Pole mass vs short distance mass perturbatively (+ non-perturb. corrections):

$$m_{pole} = \bar{m}(\bar{m})\left(1 + \frac{4}{3}\frac{\bar{\alpha}_s(\bar{m})}{\pi} + 8.28\left(\frac{\bar{\alpha}_s(\bar{m})}{\pi}\right)^2 + \cdots\right) + O(\Lambda_{QCD})$$

Not present in electron case

Scale $\mu_R = \bar{m} \gg \Lambda_{QCD}$

top self energy Σ pert. expanded in α_s

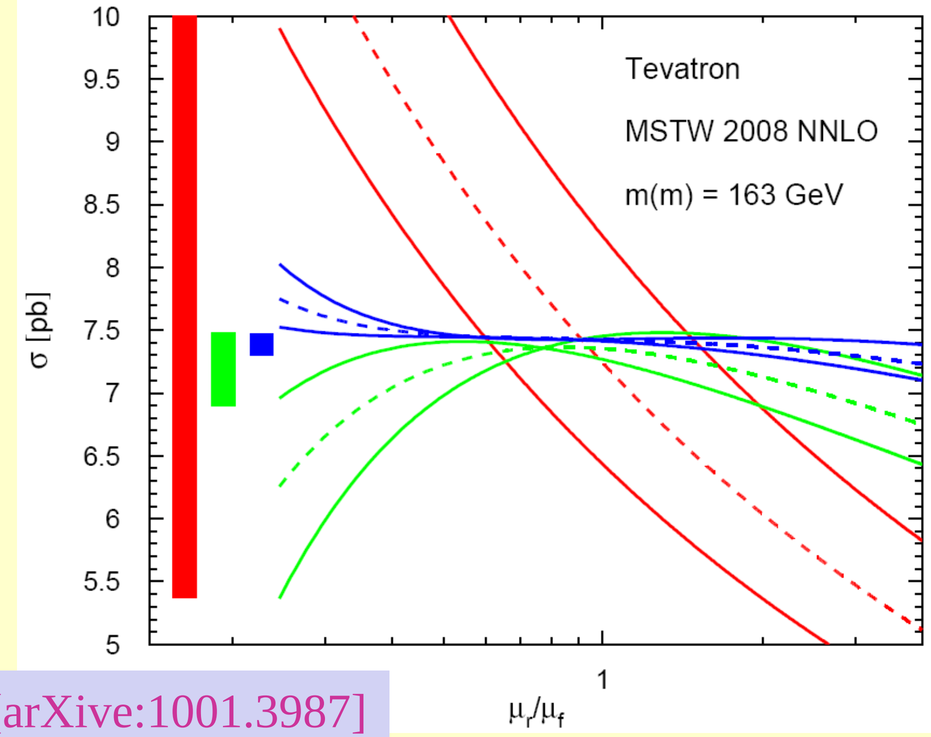


arXiv:hep-ph/9612329v1

Determination of short distance mass

Short distance mass ≡ $\overline{MS}$ mass

Determination of ≡ $\overline{MS}$ mass *via* the total cross section $\sigma_{pp\to t\bar{t}X}$



[arXive:1001.3987]

Total $t\bar{t}$ X sec for LO(red), NLO(green) and NNLO(blue) vs μ_R/μ_F (renormalization/factorization scales)

Total NLO Xsection vs top pole mass:

$$\sigma_{pp\to t\bar{t}X} = \alpha_S^2\sigma^{(0)}(m_{top}) + \alpha_S^3\sigma^{(1)}(m_{top}) + \ldots$$

Using the relation between top pole mass and running mass ($\overline{MS}$ scheme):

$$m_{top} = m(\mu_R)\left(1 + \alpha_S(\mu_R)d^{(1)}(\mu_R) + \ldots\right)$$

Coefficient $d^{(1)}$ is known to 3-loop order

Total $t\bar{t}$ X section vs top $\overline{MS}$ mass $\bar{m} = m(m)$:

$\mu_F = \bar{m}/2\,, \bar{m}\,, 2\bar{m}$

$$\sigma_{pp\to t\bar{t}X} = \alpha_S^2\sigma^{(0)}(\bar{m}) + \alpha_S^3\left(\sigma^{(1)}(\bar{m}) + \bar{m}d^{(1)}\,\partial_m\sigma^{(0)}(m)\Big|_{m=\bar{m}}\right) + \ldots$$

Measuring the total $t\bar{t}$ X section we can extract $\overline{MS}$ mass $\bar{m}$!

$$\bar{m} = 163.0 \pm 1.6^{+0.6}_{-0.3}\ \text{GeV}$$

Measured vs Top pole mass

Solution after the conclusion:

Measured top quark mass (m_t^{MC}) differs from pole top mass m_t^{pole} due to non-perturbative effects $\Rightarrow$

- ✓ the m_t^{MC} mass can be related to a scale-dependent short-distance mass: MSR mass $m_t^{MSR}(R)$ (see *Nucl.Phys.Proc.Suppl. 185 (2008) 220*)
- ✓ The analysis done in *Nucl.Phys.Proc.Suppl. 185 (2008) 220* gives:

$$m_t^{MC} = m_t^{MRS}\left(3_{-2}^{+6}\right)\mathrm{GeV}$$

For each choice of R the MSR mass $m_t^{MSR}(R)$ represents a different mass definition – we have R= 1, 3 and 9 GeV.

The quoted scale uncertainty is an estimate of the conceptual uncertainty that is currently contained in this relation - associated to

- unknown higher order corrections
- MC machinery - how the parton shower, shower cuts and hadronization model are implemented

From short distance to pole mass

Reconstruct. mass is identified with short distance mass at low scale O(1) GeV:
$m^{rec} \longrightarrow m^{MSR}(R)$ with R ≃ 1 . . . 9 GeV ⇒

Two options [*arXiv:1405.4781*]:

- ✓ evolve $m^{MSR}(R)$ from low scale $R{\sim}3$ GeV to $R = m(m)$ and convert from $m(m)$ to pole mass

1, 2 and 3 loops

$m^{MSR}(1)$	$m^{MSR}(2)$	$m^{MSR}(3)$	$m(m)$	$m^{pl}{}_{1lp}$	$m^{pl}{}_{2lp}$	$m^{pl}{}_{3lp}$
173.72	173.40	172.78	163.76	171.33	172.95	173.45

- ✓ convert from $m^{MSR}(R)$ at low scale directly to pole mass - nonperturbative method used (Effective HQ theory approach (Wilson coefficients))

$m^{MSR}(1)$	$m^{MSR}(2)$	$m^{MSR}(3)$	$m^{pl}{}_{1lp}$	$m^{pl}{}_{2lp}$	$m^{pl}{}_{3lp}$
173.72	173.40	172.78	173.72	173.87	173.98

Using 1st approach and the world top mass average:

$$m_{pole} = 173.34 \pm 0.76\text{GeV (exp)} + \Delta m(\text{th})$$

where $\Delta m(\text{th}) = \pm 0.7\text{GeV}$ ($m^{rec} \rightarrow m^{MSR}$(3GeV)) + 0.5GeV ($m(m) \rightarrow m_{pole}$)

Top pole mass from tt-bar jet

In [arXiv:1303.6415]→ suggestion to use on top pole mass determination:
Top-quark pairs in association with a hard jet .
From NLO calculations ⇒

$$\frac{\Delta\sigma_{t\bar{t}+\text{1-jet+X}}}{\sigma_{t\bar{t}+\text{1-jet+X}}} \approx -5\frac{\Delta m_t^{pole}}{m_t^{pole}}$$

For study: the dimensionless differential distribution

$$R\left(m_t^{pole},\rho_s\right)=\frac{1}{\sigma_{t\bar{t}+\text{1-jet+X}}}\frac{d\sigma_{t\bar{t}+\text{1-jet+X}}}{d\rho_s}\left(m_t^{pole},\rho_s\right),\quad \rho_s=\frac{2m_0}{\sqrt{s_{t\bar{t}j}}}$$

Due to the normalization many experimental and theoretical uncertainties cancel between numerator and denominator.

Sensitivity:

$$\mathcal{S}(\rho_s,\Delta m_t)=\sum_{\Delta=\pm\Delta m_t}\frac{|\mathcal{R}(\rho_s,170\ \text{GeV}/c^2)-\mathcal{R}(\rho_s,170\ \text{GeV}/c^2+\Delta)|}{2|\Delta|\mathcal{R}(170\ \text{GeV},\rho_s)} \Rightarrow \left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \approx (m_t\mathcal{S})\times\left|\frac{\Delta m_t}{m_t}\right|$$

For $\rho_s \approx 0.8$ a 1% change in m_t leads to a relative change of 17% of R.

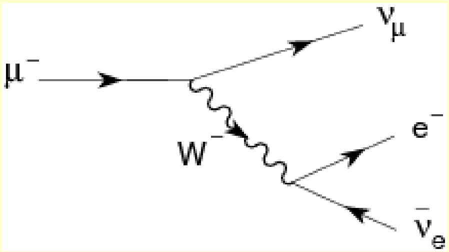
Uncertainties related to uncalculated higher order corrections or to PDFs are expected to affect the mass measurement by less than 1GeV.

Top quark *vs* EW precision measurements

Relation between
W boson, top quark and Higgs boson masses

Top mass and EW precision physics

Radiative corrections to W-boson propagator (e.g. for $\mu^- \rightarrow \nu_\mu W^- \rightarrow \nu_\mu e^- \bar{\nu}_e$):



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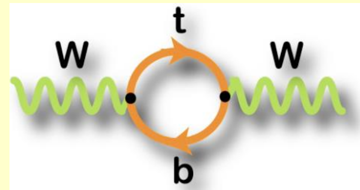

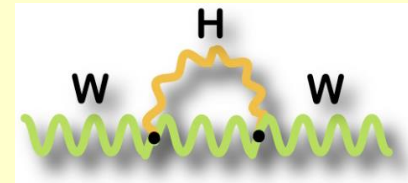

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$\Delta\rho \sim m_t^2 / M_W^2$ $\quad (\Delta r)_{nl} \propto \ln M_H^2 / M_Z^2$

Masses of top, W and Higgs are bounded by

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r), \quad \Delta r = \Delta\alpha + \frac{s_W}{c_W}\Delta\rho + (\Delta r)_{nl}$$

M_Z, G_F and α are known with high precision:

- $M_Z = 91.1876 \pm 0.0021$ GeV
- $G_F = 1.166367(5) \times 10^{-5}$ GeV^{-2}
- $\alpha = 1/137.035999679(94)$

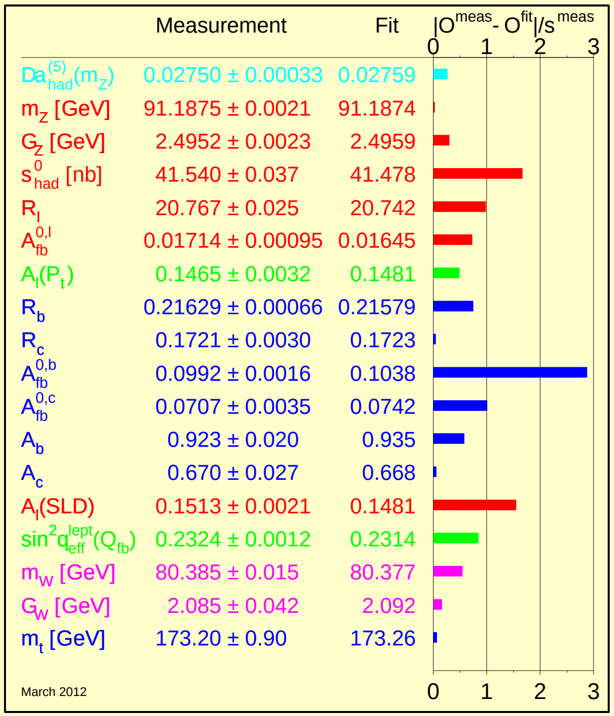
Measuring precisely masses m_t and M_W using (*) M_H can be extracted!

Drawback: dependence on Higgs mass is weak (logarithmic)

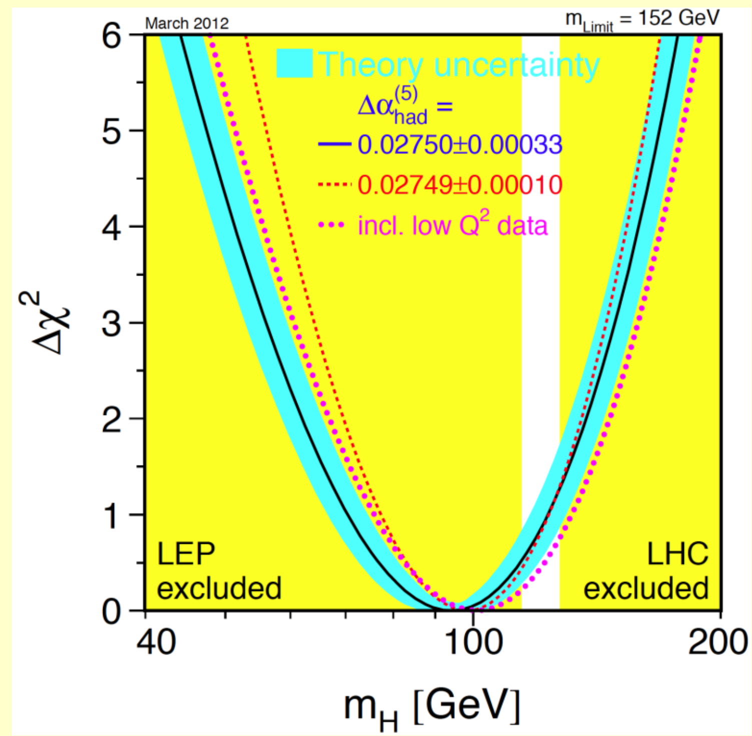
Global SM fit to EW precision data

Gfitter package used for the global fit (http://project-gfitter.web.cern.ch/project-gfitter/)

There is a set of N_{exp} precisely measured observables described by N_{exp} theoretical expressions – those are functions of N_{mod} model parameters



Fit gives for Higgs mass (measured M_H excluded)



From the global fit (measured m_t/M_W excluded):

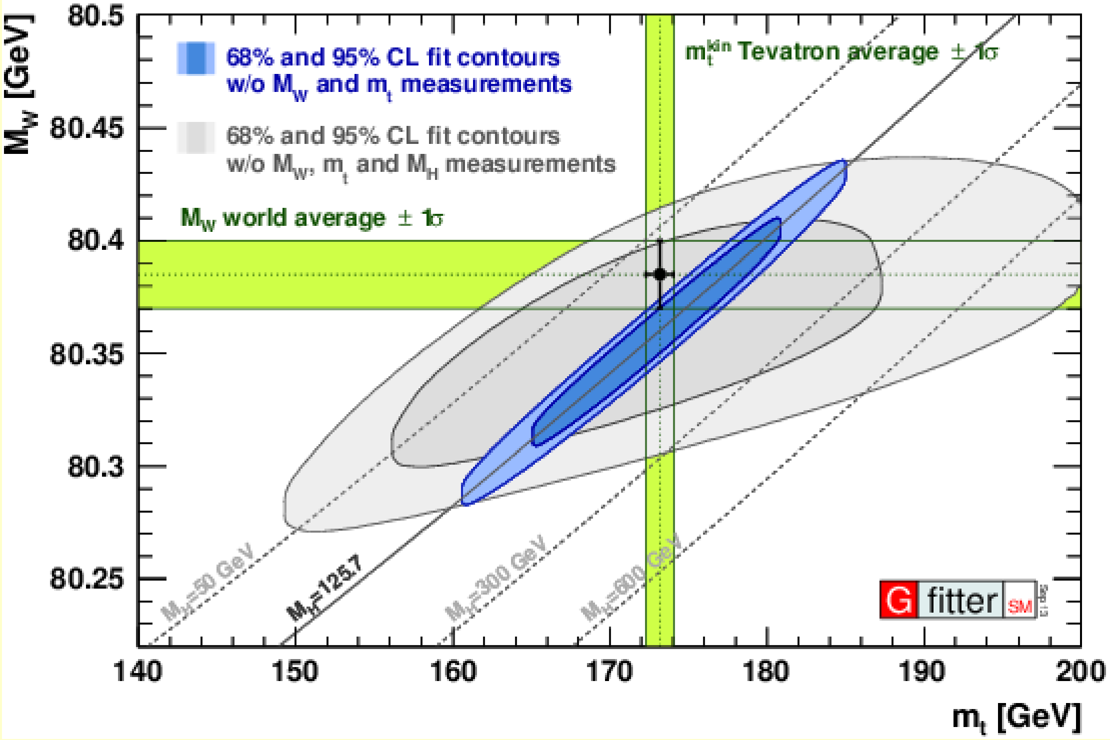
$$m_t = 175.8^{+2.8}_{-2.8}\,\text{GeV}, \quad M_W = 80.360 \pm 0.011\,\text{GeV}$$

$$M_H = 94^{+29}_{-24}\ \text{GeV}$$

$$M_H < 152\ \text{GeV at 95\% C.L.}$$

Top mass and EW precision physics

SM-like Higgs boson discovery at 125.7 ± 0.4 GeV is compatible with global EW data at 1.3 σ (p = 0.18)



Contours of 68% and 95% CL obtained from scans of fits with fixed variable pairs M_W vs m_t

Dependence of EW observables on M_H is only logarithmic!

The blue and grey allowed regions are the results of the fit including and excluding the M_H measurements, respectively.

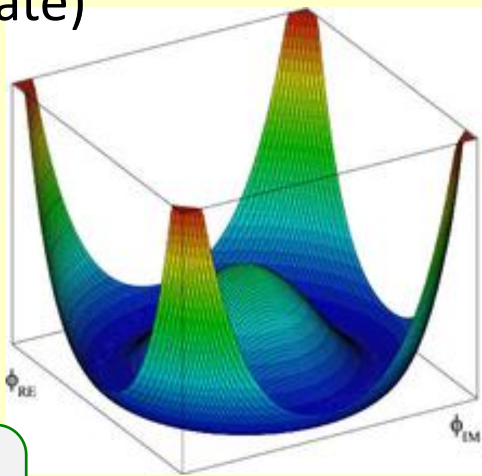
The horizontal and vertical bands indicate the 1σ regions of the M_W and m_t measurements.

Vacuum stability and top mass

Running quartic Higgs coupling

Higgs quartic coupling

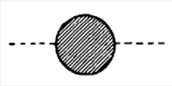
Higgs boson looks to be firmly established by LHC $\Rightarrow$
Vacuum has nonzero Higgs field component (Higgs condensate)
What can be said about its stability?



Higgs potential:

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2, \quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$$

For $\mu^2 < 0$ and $\lambda > 0$

Top loops mainly 

λ vs Higgs mass and Fermi constant

$\Rightarrow$ due to interactions λ is running constant – scale dependent (as mass):

$$\lambda(\mu_R) = \frac{G_F M_H}{\sqrt{2}} + \Delta\lambda(\mu_R)$$

$\Rightarrow$ What will happen if $\lambda < 0$?

$\Delta\lambda$ is calculated in two loop approximation – the most important contribution: due to QCD and top Yukawa interactions.

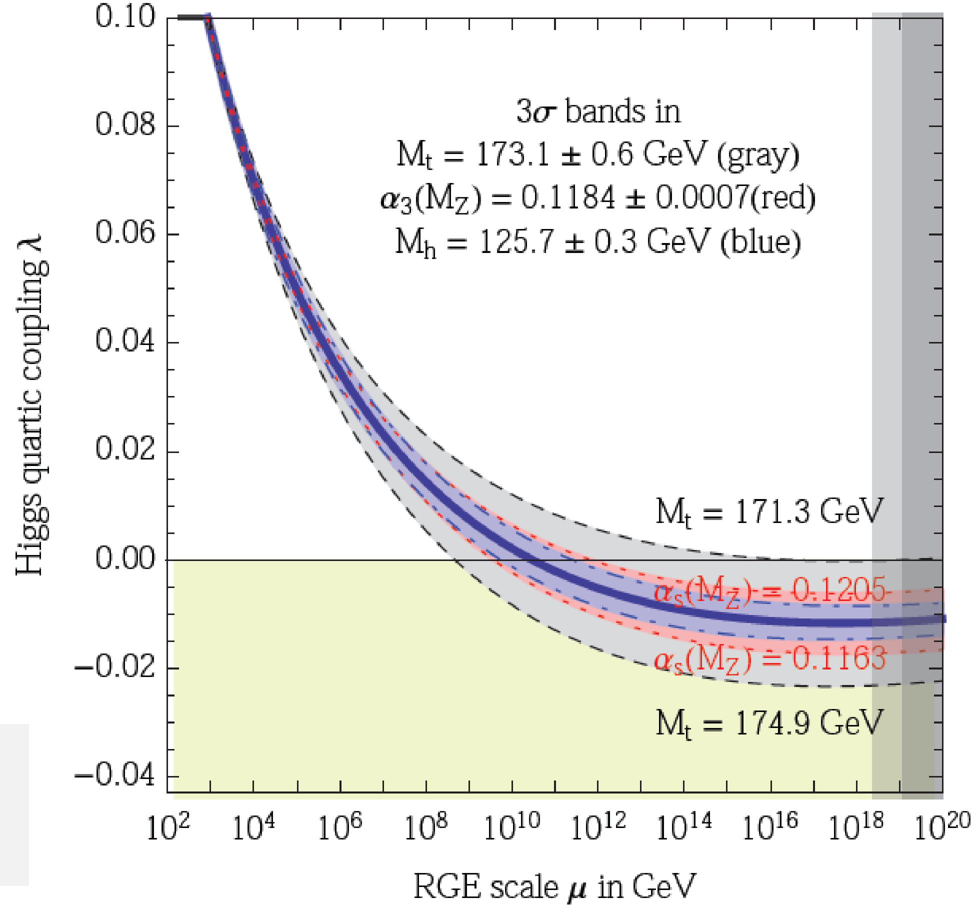
Top quark mass and instability of vacuum

High precision top mass is a fundamental input to the understanding of the SM, has cosmological implications.

Renorm group Higgs quartic coupling $\lambda(\mu)$ evolution – steep decrease due to 1-loop top corrections $\Rightarrow$ slope of λ strongly depends on m_t.

At μ =10^{10}-10^{11} GeV: change of λ sign
$\Rightarrow$ vacuum is only in a local minimum
$\Rightarrow$ meta-stability of vacuum.

SM assumed and evolution at NNLO



Dashed lines 3σ bands for m_t (gray), α_s (red) and M_H (blue)

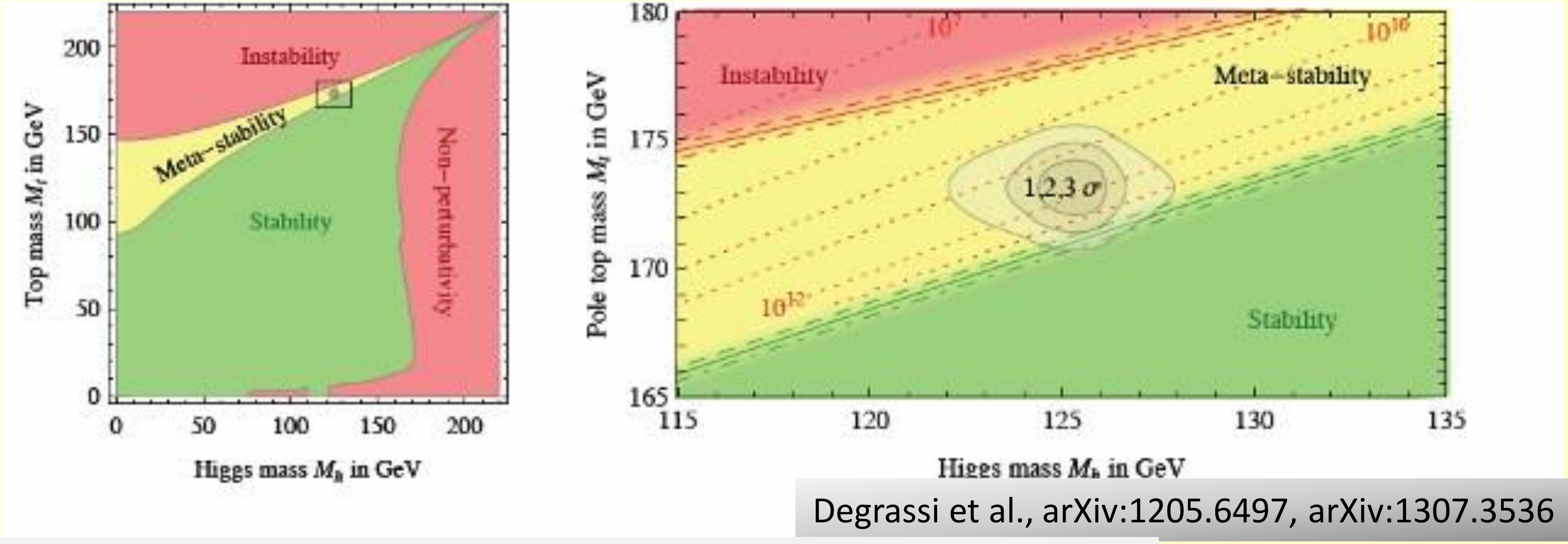
Coupling λ closed to 0 at Λ_P (Planck scale)!

Top quark mass and instability of vacuum

Assuming SM – no other scale between Λ_{EWSB} ~100 GeV and Λ_P ~10^{19} GeV

M_t =173.1 $\pm$ 0.6 GeV , $\alpha_S(M_Z)$= 0.1184$\pm$0.0007 and M_h =125.7 $\pm$ 0.3 GeV

It looks we live in the world with meta-stable vacuum!



Degrassi et al., arXiv:1205.6497, arXiv:1307.3536

The condition of absolute stability up to the Planck scale:

$$M_H\,[\mathrm{GeV}] > 129.4 + 1.4\left(\frac{m_t\,[\mathrm{GeV}] - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_S(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{th}$$

$$m_t < (171.36\ \pm\ 0.46)\,\mathrm{GeV}$$

What does it mean metastability?

From recent m_t and M_H measurements:

$\Rightarrow$ the SM Higgs potential develops an instability at scales well below M_P:

$$\log_{10}\frac{\Lambda_I}{\mathrm{GeV}} = 11.0 + 1.0\left(\frac{M_H}{\mathrm{GeV}} - 125.7\right) - 1.2\left(\frac{m_t}{\mathrm{GeV}} - 173.34\right) + 0.4\frac{\alpha_s(M_Z) - 0.1184}{0.0007}$$

$\Lambda_I \equiv$ the scale at which the effective Higgs quartic λ_{eff} becomes negative -

$\Lambda_I \approx 10^{11}$ GeV.

If it is true (Higgs potential is unstable) $\Rightarrow$

the Higgs field h can tunnel from the EW vacuum to the true vacuum at large field values

- lifetime for quantum tunneling turns out to be (much) larger than the age of the Universe, rendering our universe metastable.
- Question of M_H, m_t uncertainty is of a great importance ...

Implication for the inflation

Fluctuations in Higgs field during inflation are set by Hubble scale H:

$$\delta h = \frac{H}{2\pi}, \qquad H^2 = \frac{\pi}{16} M_P^2 \Delta_R^2 r$$

$\Delta_R \equiv$ amplitude of curvature perturbations measured by Planck ($\Delta_R^2 = 2.21\times10^{-9}$)
$r \equiv$ tensor-scalar ratio measured by BICEP2

– measurement of BICEP2 [***arXiv:1403.3985***] indicates:

$$H \simeq 1.0\times10^{14}\,\text{GeV}\sqrt{\frac{r}{0.16}}, \quad r \approx 0.2$$

When $H > \Lambda_I$ (instability scale), the likelihood that ***h*** fluctuates to the unstable region of the potential during inflation will be sizable [***arXiv:1404.5953***].

Fate of universe: different scenarios of the post-inflationary vacuum evolution – from "our universe is extremely improbable" to "the additional vacuum does not appear to preclude existence of our universe".

Conclusions

Top quark is a special fundamental particle – studying its properties we can:

- test the Standard model,
- to reveal a possible new physics,
- to understand better global questions concerning the Universe fate

Thank you!

Cross Section of Top Quark production

$t\,\bar{t}$ Production Cross Section

Top quark X-section: Experiment vs Theory

Factorization theorem:

$$\sigma = \sum_{i,j} \int dx_1 dx_2\, F_i^{(1)}(x_1,\mu_F) F_j^{(2)}(x_2,\mu_F)\, \hat{\sigma}_{ij}(s;\mu_F,\mu_R)$$

experiment (σ) — theory ($\hat{\sigma}_{ij}$)

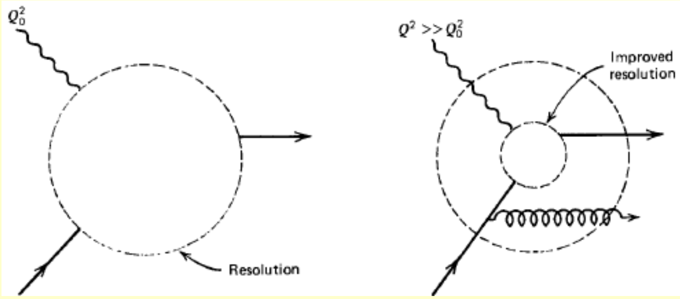
Parton Distribution Functions (PDFs)

$F_i^{(\lambda)}(x_\lambda,\mu_F)$ ≡ probability density to observe a parton ***i*** with longitudinal momentum fraction x_λ in incoming hadron λ, when probed at a scale μ_F

μ_F ≡ factorization scale (a free parameter) - it determines the proton structure if probed (by virtual photon or gluon) with $q^2 = -\mu_F^2$

μ_R ≡ renormalization scale – defines size of strong coupling constant

Usual choice: $\mu_F = \mu_R = \mu \in (m_t/2,\, 2m_t)$

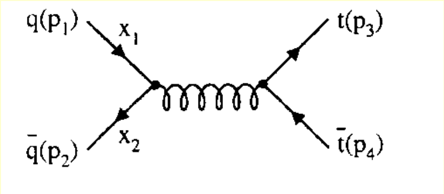


$t\,\bar{t}$ Production Cross Section

The LO top quark pairs cross section (Born term):

$$d\hat{\sigma} = \frac{1}{2(p_1+p_2)^2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\delta(p_1+p_2-p_3-p_4)\overline{|M|^2}$$

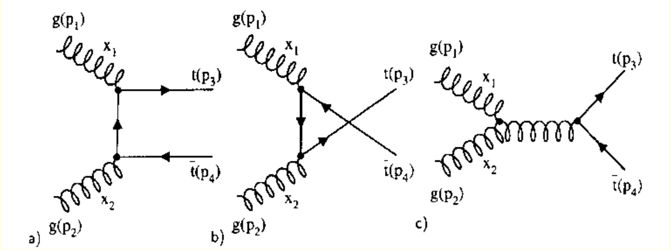
Quark –antiquark annihilation



$$\overline{|M|^2}(q\bar{q}\to t\bar{t}) = (4\pi\alpha_s)^2\frac{8}{9}\left(2\frac{(p_1\cdot p_3)^2+(p_2\cdot p_3)^2}{(p_1\cdot p_2)^2}+\frac{m_t^2}{(p_1+p_2)^2}\right)$$

Averaged over initial and summed over final color and spin state

Gluon fusion



$$\overline{|M|^2}(gg\to t\bar{t}) = (4\pi\alpha_s)^2\left(\frac{(p_1+p_2)^4}{24(p_1\cdot p_3)(p_2\cdot p_3)}-\frac{8}{9}\right)$$

$$\times\left(4\frac{(p_1\cdot p_3)^2+(p_2\cdot p_3)^2}{(p_1\cdot p_2)^4}+\frac{4m_t^2}{(p_1+p_2)^2}-\frac{m_t^4(p_1+p_2)^4}{(p_1\cdot p_3)^2(p_2\cdot p_3)^2}\right)$$

Experiment:

LO tt-bar Xsec is not sufficient!

Higher orders are needed

$t\bar{t}$ Production Cross Section

Theory for top X-section is at NNLO:

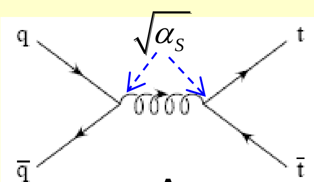
Xsec is expanded into series of strong coupling constant:

$$\sigma_{ij}\left(\beta,\frac{\mu^2}{m^2}\right)=\frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)}+\alpha_S\left[\sigma_{ij}^{(1)}+L\sigma_{ij}^{(1,1)}\right]+\alpha_S^2\left[\sigma_{ij}^{(2)}+L\sigma_{ij}^{(2,1)}+L^2\sigma_{ij}^{(2,2)}\right]+O\left(\alpha_S^3\right)\right\}$$

$LO \sim \alpha_S^2, \quad NLO \sim \alpha_S^3, \quad NNLO \sim \alpha_S^4 \quad \cdots \quad \beta=\sqrt{1-4m^2/s}$ $\qquad L \equiv$ big log term

NLO: virtual and real corrections are added to LO

Virtual corrections:



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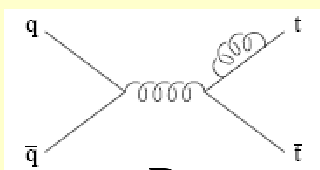

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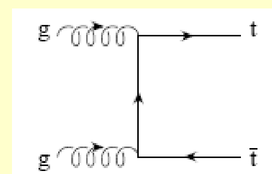

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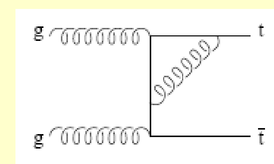
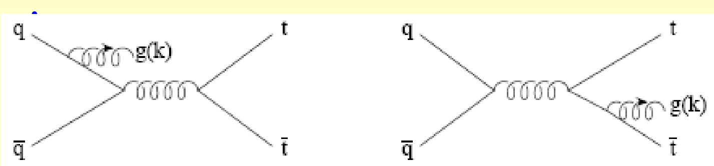
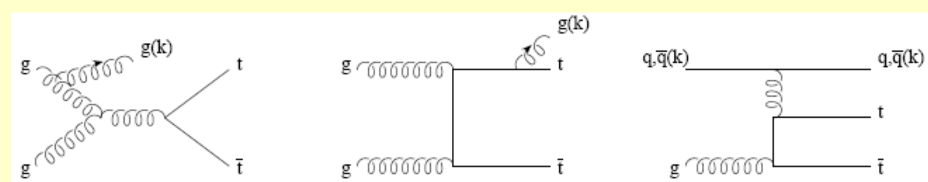


A B

Taking $|A+B|^2 = \ldots + AB^* + \ldots$, $AB^* \sim \alpha_S^3$

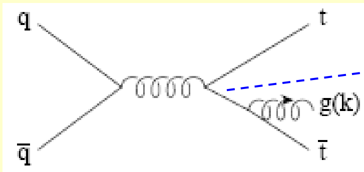
Real corrections – with real gluons ($\sim \alpha_S^3$):





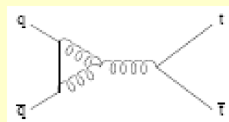
A few top Cross Section issues

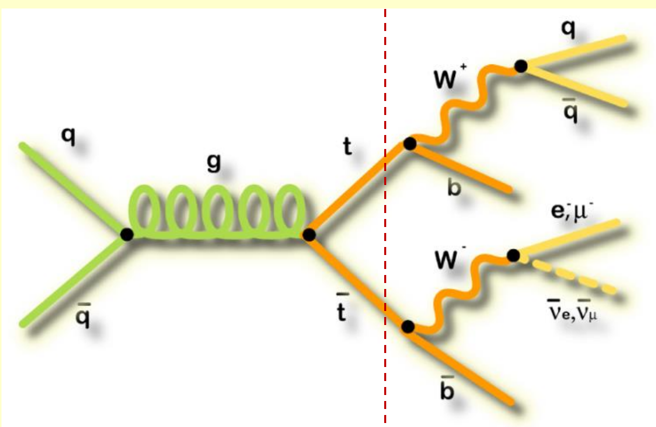
Higher order real and virtual corrections exhibit IR and UV divergences:

Example: 

$$\text{propagator} = \frac{1}{(p+k)^2} = \frac{1}{2E_p E_k} \cdot \frac{1}{1-\beta_p \cos\theta}, \quad \beta_p = \sqrt{1-m^2/E_p^2}$$

✓ IR singularity: $E_k \to 0$ and $1-\beta_p \cos\theta \to 0$ $\Rightarrow$ cancelled when Xsec of virtual and real emission are summed also mass singularities are cancelled $\Rightarrow$ Cancelation is not full $\Rightarrow$ presence of big logs (L) in Xsec terms !

✓ UV singularities in loops () are handled by renormalization.



In real we observe $t\bar{t}$ decay products not $t\bar{t}$

Factorization is used based on the narrow width approximation:

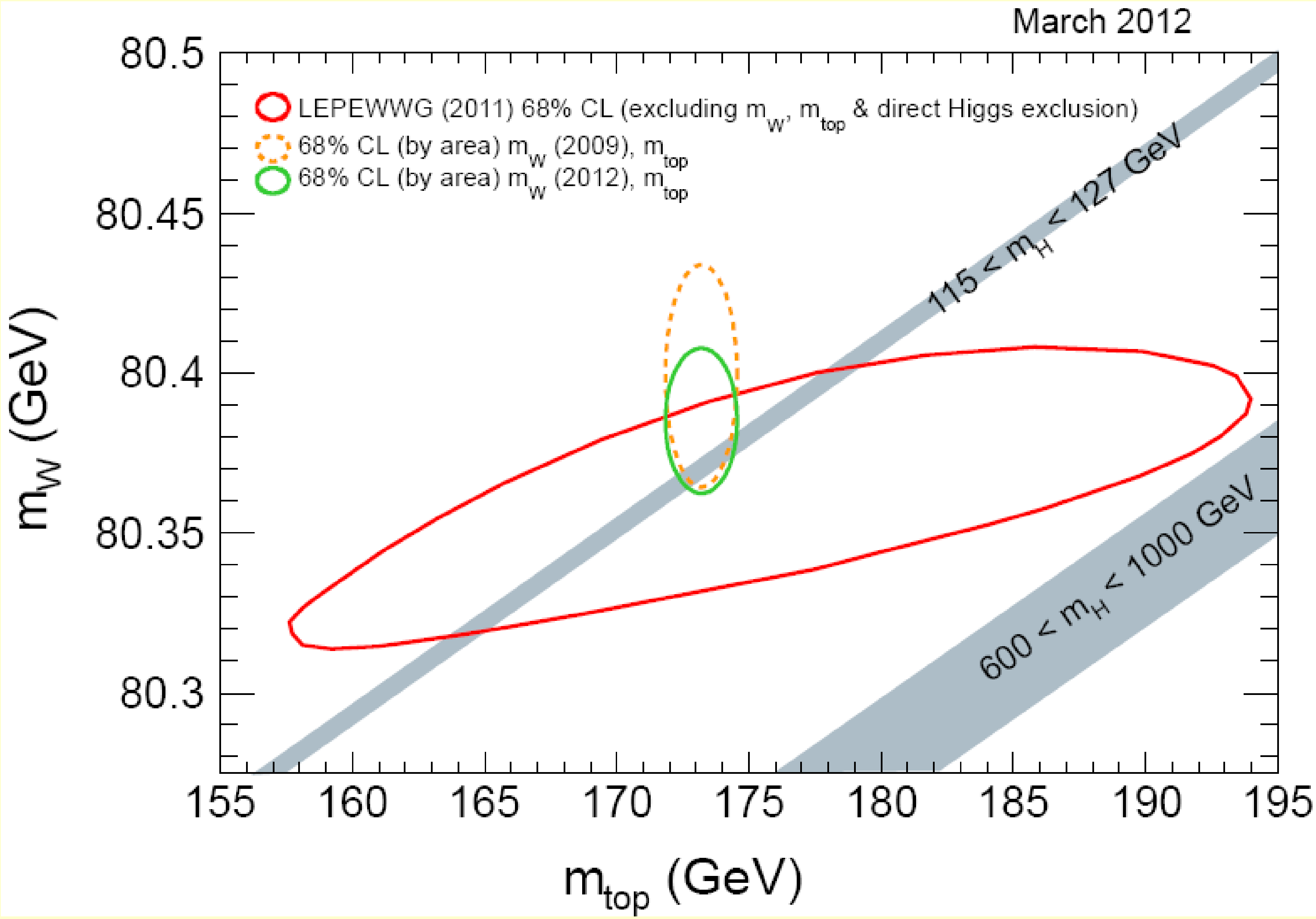
✓ polarized top quarks are produced on mass shell

✓ polarized on-shell top quarks decay

Narrow width app. vs direct $pp \to WWbb$:

For LHC 7TeV/DIL: Xsec(fb) 837 vs 841 also done for 14 and 1.96TeV

…an another Higgs restriction plot



What will be happen with the green ellipse after 20-30 fb^{-1}?…