SM@LHC Workshop
Madrid 2014

Higgs Production at N3LO

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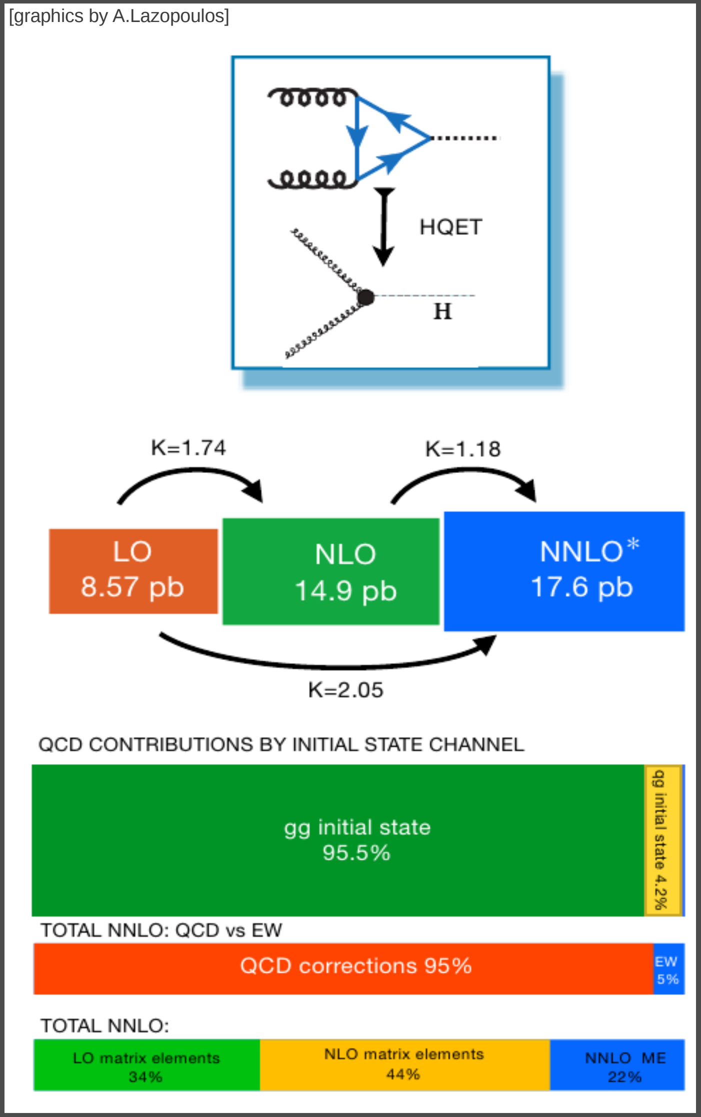
What we know about Higgs production



Perturbative Corrections:

- NLO QCD corrections known exactly (with top-bottom interference)
- NNLO QCD corrections (in HQET)
- subleading terms in the m_H/(2m_t) expansion
- EW corrections known
- mixed QCD EW corrections

Resummation:

- Soft gluon NNLL, Pi^2
- Transverse momentum resummation to NNLL (now also with exact top and bottom dependence)
- Jet transverse momentum and cone size R to NNLL
- Approximate N3LO
- Soft Virtual Approximation

Tools:

- mc@nlo, powheg, higlu, HNNLO, Hres, Hqt, Fehip, FehiPro, Hpro, Ihixs, **Ehixs** (soon!!)

NNLO Theory Uncertainty

IHixs@8TeV: [Anastasiou, Buehler, FH, Lazopoulos]
(all known perturb. corr.)

m_H(GeV)	MSTW08 $\sigma(pb)$	$\%\delta_{PDF}$	$\%\delta_{\mu_F}$
125	20.69	$^{+7.79}_{-7.53}$	$^{+8.37}_{-9.26}$

Central scale $\mu = \mu_R = \mu_F = \frac{m_H}{2}$

Scale variation $\frac{m_H}{4} \leq \mu \leq m_H$

The pdf uncertainty is computed at 90%CL.

De Florian & Grazzini @ 8TeV:
(all known pert. Corr. + NNLL soft resummation)

m_H (GeV)	σ (pb)	scale(%)	PDF+α_S(%)
125.0	19.31	+7.2 -7.8	+7.5 -6.9

Central scale $\mu = \mu_R = \mu_F = m_H$

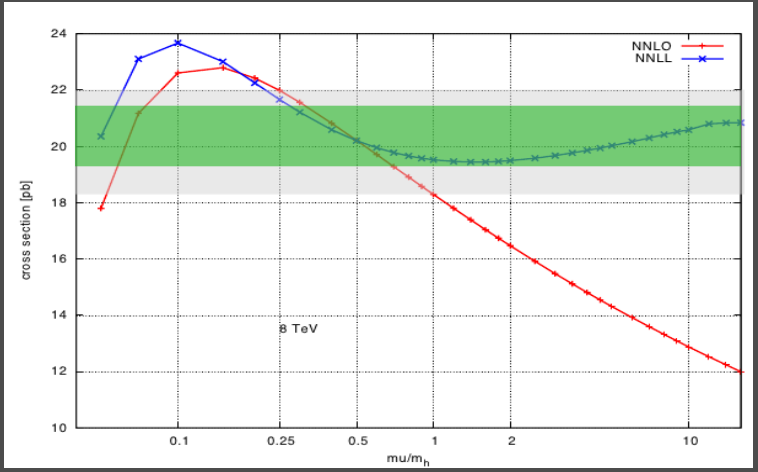
Scale variation $\frac{m_H}{2} \leq \mu_R, \mu_F \leq 2m_H$

The pdf uncertainty is computed at 68%CL. acc. To PDF4LHC

Note: dominant uncertainty is from μ_R variation

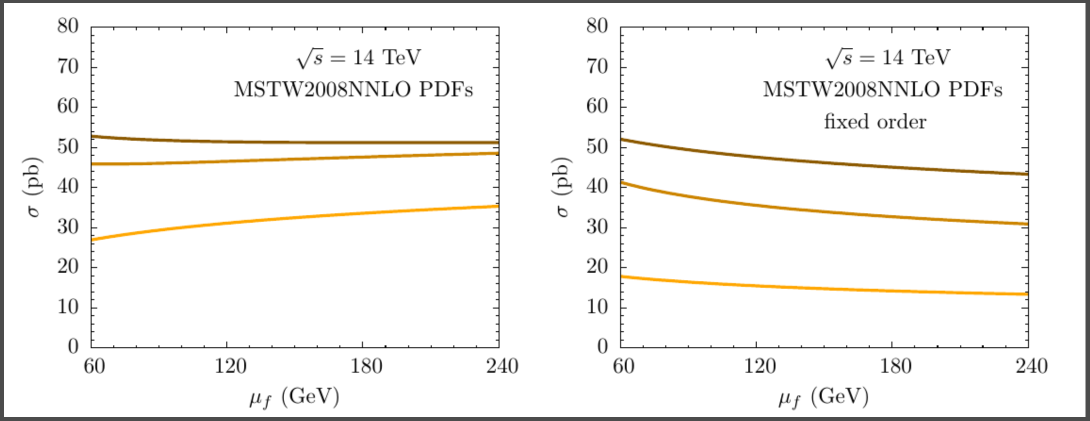
How can one reduce perturbative uncertainties?

Resummation may reduce scale uncertainties:



Treshhold resummation
[Grazzini's Online calculator]

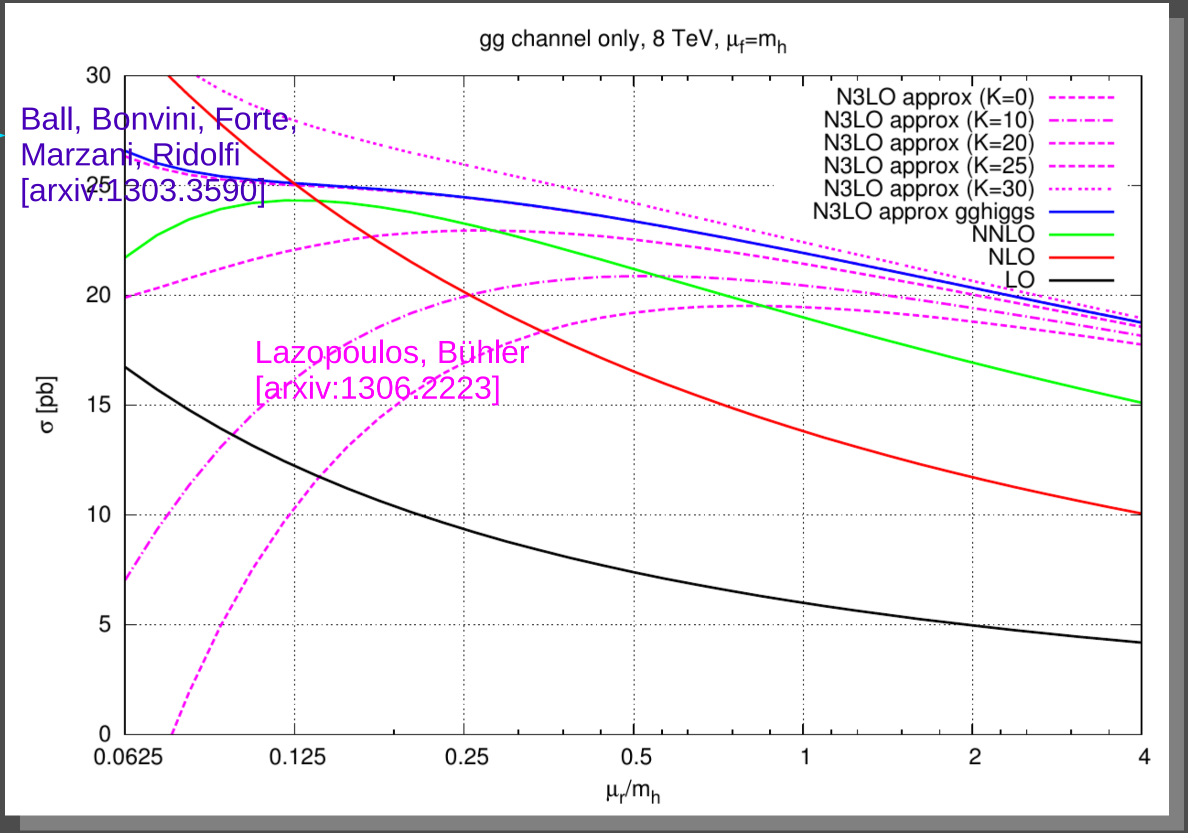
Treshhold with SCET and π² – resummation
[0809.4283]



Another way is to compute the next order:

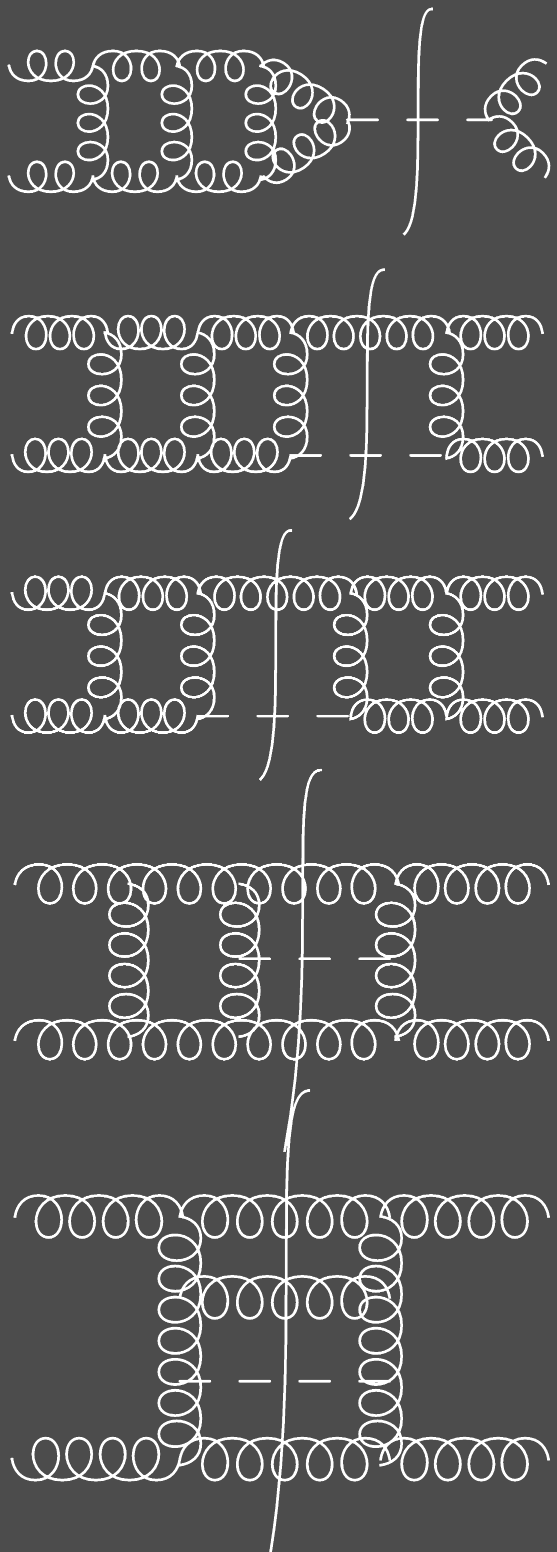
$$\sigma^{\text{N3LO}}_{PP\to H+X} = \alpha_s(\mu_R)^5 \left[K + f(\sigma^{\text{lower orders}}_{PP\to H+X}, \log \mu_R) \right]$$

N3LO approx
Soft + BFKL



The ultimate precision at N3LO

Order	Cross section [pb]	$\sigma/\sigma_{\mathrm{NNLO}}$	$\sigma/\sigma_{\mathrm{LO}}$
LO	$10.31\ ^{+26.9\%}_{-16.6\%}$	0.51	1.00
NLO	$17.41\ ^{+20.8\%}_{-12.7\%}$	0.86	1.69
NNLO	$20.27\ ^{+8.3\%}_{-7.1\%}$	1.00	1.97
$\mathrm{N^3LO}$ (K=0)	$18.53\ ^{+1.2\%}_{-7.9\%}$	0.91	1.80
$\mathrm{N^3LO}$ (K=5)	$19.23\ ^{+0.3\%}_{-5.1\%}$	0.95	1.87
$\mathrm{N^3LO}$ (K=10)	$19.92\ ^{+0.0\%}_{-2.6\%}$	0.98	1.93
$\mathrm{N^3LO}$ (K=15)	$20.62\ ^{+0.4\%}_{-2.2\%}$	1.02	2.00
$\mathrm{N^3LO}$ (K=20)	$21.31\ ^{+2.0\%}_{-3.1\%}$	1.05	2.07
$\mathrm{N^3LO}$ (K=30)	$22.70\ ^{+6.0\%}_{-4.9\%}$	1.12	2.20
$\mathrm{N^3LO}$ (K=40)	$24.09\ ^{+9.6\%}_{-6.5\%}$	1.19	2.34



Towards exact N3LO in HQET

Good News:

- Triple virtual was the first contribution to have been computed, so not the problem
- Integrals only depend on a single parameter

$$z = \frac{m_H^2}{\hat{s}}$$

Bad News:

- There is still a huge number of 3-loop diagrams to compute ~100 000
- Problem of infra-red divergences even more pronounced! Most singular limits unknown! IR poles up to $\frac{1}{\epsilon^6}$.
- Phase space integrals completely unknown from other processes

Can we tackle this giant with existing technology ?

Reverse Unitarity, IBPs and Differential Equations

Write cut-propagators as a difference of Feynman propagators

$$2\pi i \delta^{+}(q^2) \to \left(\frac{1}{q^2}\right)_c = \frac{i}{q^2 + i0} - \frac{i}{q^2 - i0}$$

to establish differentiation properties

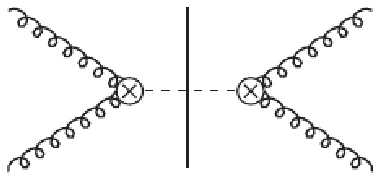
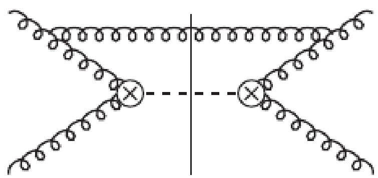
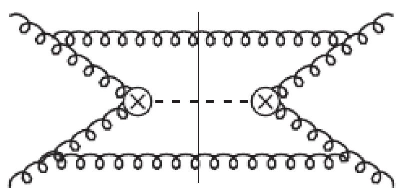
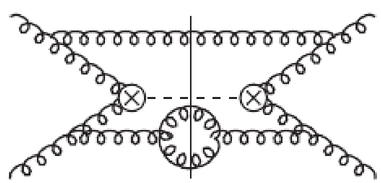
$$\frac{\partial}{\partial q^\mu}\left(\frac{1}{q^2}\right)_c = 2q^\mu \left(\frac{1}{q^2}\right)_c^2$$

This “trick” allows us derive IBP identities to find all linear relations among the Master Integrals. This then also allows us to set up a system of differential equations:

$$\frac{\partial}{\partial z} M_i(z,\epsilon) = \sum_j C_{ij}(z,\epsilon) M_j(z,\epsilon)$$

In principle (if we can triangulate the system), then all we need is a boundary condition

Growth of Complexity

LO		1 diagram	1 integral
NLO		10 diagrams	1 integral
NNLO		381 diagrams	18 integrals
N3LO		26565 diagrams	~200 integrals **Lower bound!**

The Soft Limit/Expansion

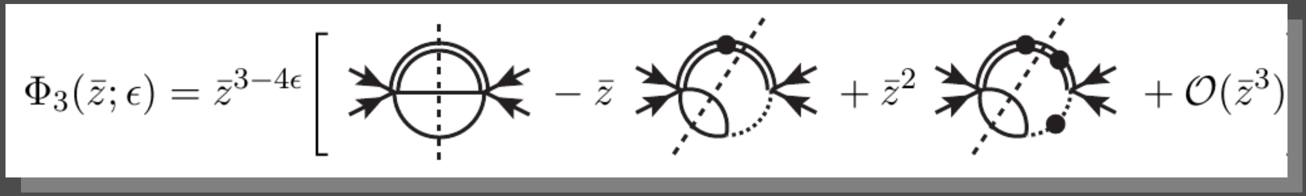
- The soft limit can can be found by rescaling appropriate momenta:

$$p_i^\mu \to (1-z)p_i^\mu$$

- For loop momenta consider soft, collinear and hard regions.

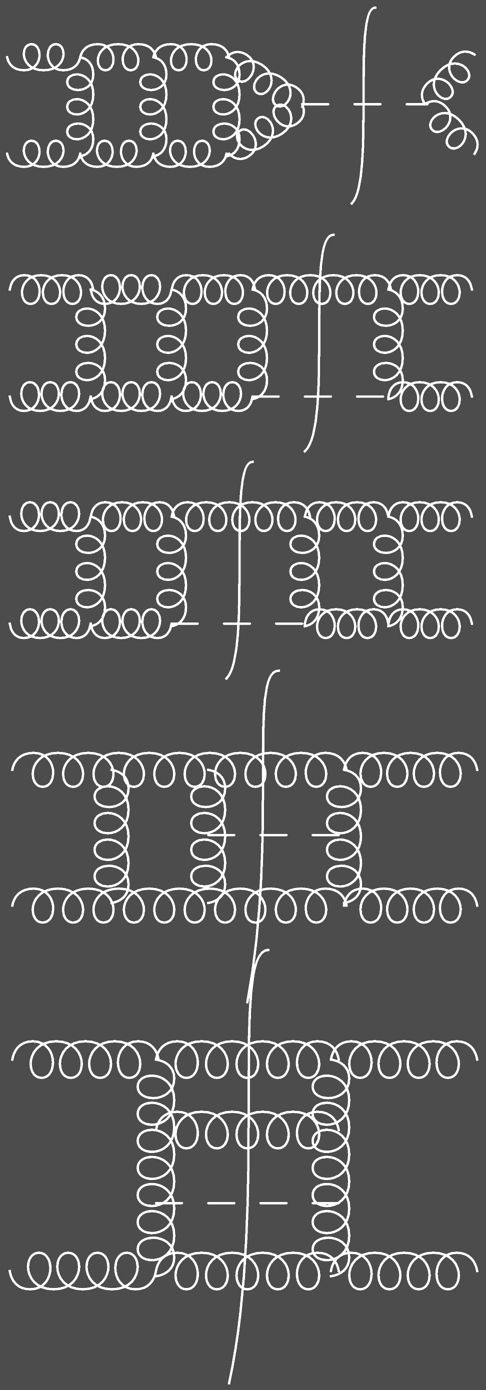
Bonus:

- Relations among soft masters can be found via IBPs
- Number of soft master integrals is **100** times smaller!!!
- Can use the soft master integrals to compute higher order coefficients in the soft expansion:



- Leads to a systematically improvable, fast convergent approximation to the full N3LO

N3LO Status



VVV:

- Known [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

RVV:

- 2-loop amplitude known up to O(ε) [Gehrmann, Jaquier, Glover, Koukoutsakis]
- One loop soft current known [Duhr, Gehrmann; Li, Zhu]
- Soft limit known
- Full calculation in progress

(RV)^2:

- Known [Anastasiou, Duhr, Dulat, FH, Mistlberger; Kilgore]

RRV:

- Soft limit known [Anastasiou, Duhr, Dulat, FH, Mistlberger]
- Next term / Full Calculation in progress

RRR:

- Know first two terms in soft expansion [Anasatsaiou, Duhr, Dulat, Mistlberger]

Collinear/UV counterterms:

- known [Pak, Rogal, Steinhauser; Anastasiou, Buehler, Duhr, FH; Höschele, Hoff, Pak, Steinhauser, Ueda; Buehler, Lazopoulos]

Higss Production at Threshold at N3LO

$$
\begin{aligned}
\hat{\eta}^{(3)}(z) = \delta(1-z)\Bigg\{ & C_A^3\left(-\frac{2003}{48}\zeta_6+\frac{413}{6}\zeta_3^2-\frac{7579}{144}\zeta_5+\frac{979}{24}\zeta_2\zeta_3-\frac{15257}{864}\zeta_4-\frac{819}{16}\zeta_3+\frac{16151}{1296}\zeta_2+\frac{215131}{5184}\right) \quad (1 \\
& + N_F\Bigg[C_A^2\left(\frac{869}{72}\zeta_5-\frac{125}{12}\zeta_3\zeta_2+\frac{2629}{432}\zeta_4+\frac{1231}{216}\zeta_3-\frac{70}{81}\zeta_2-\frac{98059}{5184}\right) \\
& \qquad + C_A C_F\left(\frac{5}{2}\zeta_5+3\zeta_3\zeta_2+\frac{11}{72}\zeta_4+\frac{13}{2}\zeta_3-\frac{71}{36}\zeta_2-\frac{63991}{5184}\right)+C_F^2\left(-5\zeta_5+\frac{37}{12}\zeta_3+\frac{19}{18}\right)\Bigg] \\
& + N_F^2\left[C_A\left(-\frac{19}{36}\zeta_4+\frac{43}{108}\zeta_3-\frac{133}{324}\zeta_2+\frac{2515}{1728}\right)+C_F\left(-\frac{1}{36}\zeta_4-\frac{7}{6}\zeta_3-\frac{23}{72}\zeta_2+\frac{4481}{2592}\right)\right]\Bigg\} \\
+ \left[\frac{1}{1-z}\right]_+ \Bigg\{ & C_A^3\left(186\,\zeta_5-\frac{725}{6}\zeta_3\zeta_2+\frac{253}{24}\zeta_4+\frac{8941}{108}\zeta_3+\frac{8563}{324}\zeta_2-\frac{297029}{23328}\right)+N_F^2 C_A\left(\frac{5}{27}\zeta_3+\frac{10}{27}\zeta_2-\frac{58}{729}\right) \\
& + N_F\left[C_A^2\left(-\frac{17}{12}\zeta_4-\frac{475}{36}\zeta_3-\frac{2173}{324}\zeta_2+\frac{31313}{11664}\right)+C_A C_F\left(-\frac{1}{2}\zeta_4-\frac{19}{18}\zeta_3-\frac{1}{2}\zeta_2+\frac{1711}{864}\right)\right]\Bigg\} \\
+ \left[\frac{\log(1-z)}{1-z}\right]_+ \Bigg\{ & C_A^3\left(-77\zeta_4-\frac{352}{3}\zeta_3-\frac{152}{3}\zeta_2+\frac{30569}{648}\right)+N_F^2 C_A\left(-\frac{4}{9}\zeta_2+\frac{25}{81}\right) \\
& + N_F\left[C_A^2\left(\frac{46}{3}\zeta_3+\frac{94}{9}\zeta_2-\frac{4211}{324}\right)+C_A C_F\left(6\,\zeta_3-\frac{63}{8}\right)\right]\Bigg\} \\
+ \left[\frac{\log^2(1-z)}{1-z}\right]_+ \Bigg\{ & C_A^3\left(181\,\zeta_3+\frac{187}{3}\zeta_2-\frac{1051}{27}\right)+N_F\left[C_A^2\left(-\frac{34}{3}\zeta_2+\frac{457}{54}\right)+\frac{1}{2}C_A C_F\right]-\frac{10}{27}N_F^2 C_A\Bigg\} \\
+ \left[\frac{\log^3(1-z)}{1-z}\right]_+ \Bigg\{ & C_A^3\left(-56\,\zeta_2+\frac{925}{27}\right)-\frac{164}{27}N_F C_A^2+\frac{4}{27}N_F^2 C_A\Bigg\} \\
+ \left[\frac{\log^4(1-z)}{1-z}\right]_+ & \left(\frac{20}{9}N_F C_A^2-\frac{110}{9}C_A^3\right)+\left[\frac{\log^5(1-z)}{1-z}\right]_+ 8\,C_A^3\,.
\end{aligned}
$$

The Soft Virtual Approximation

Keeping hard flux factor improves the approximation.

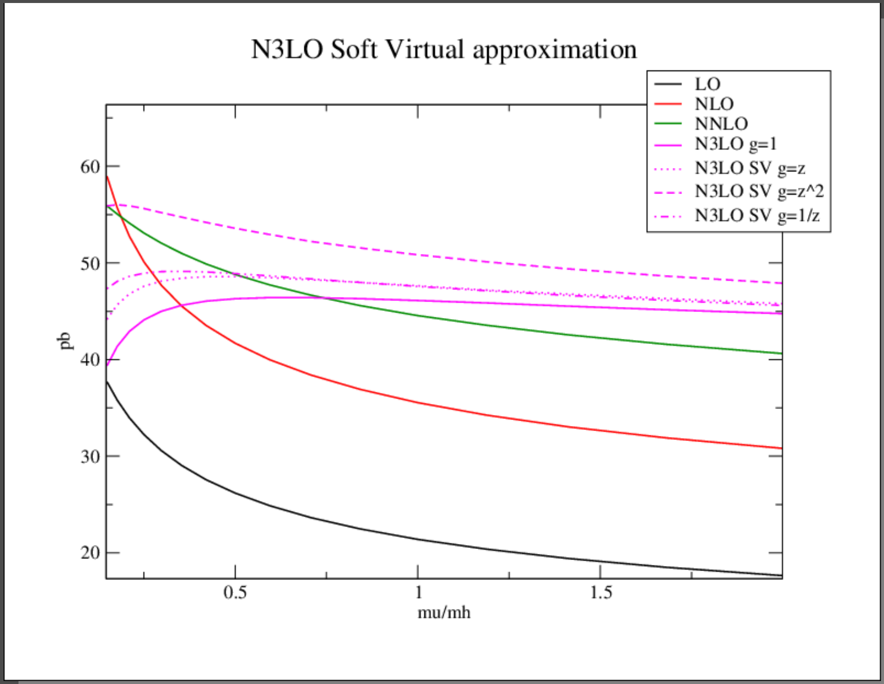
g(z) can be varid to estimate the accuracy of soft virtual approximation

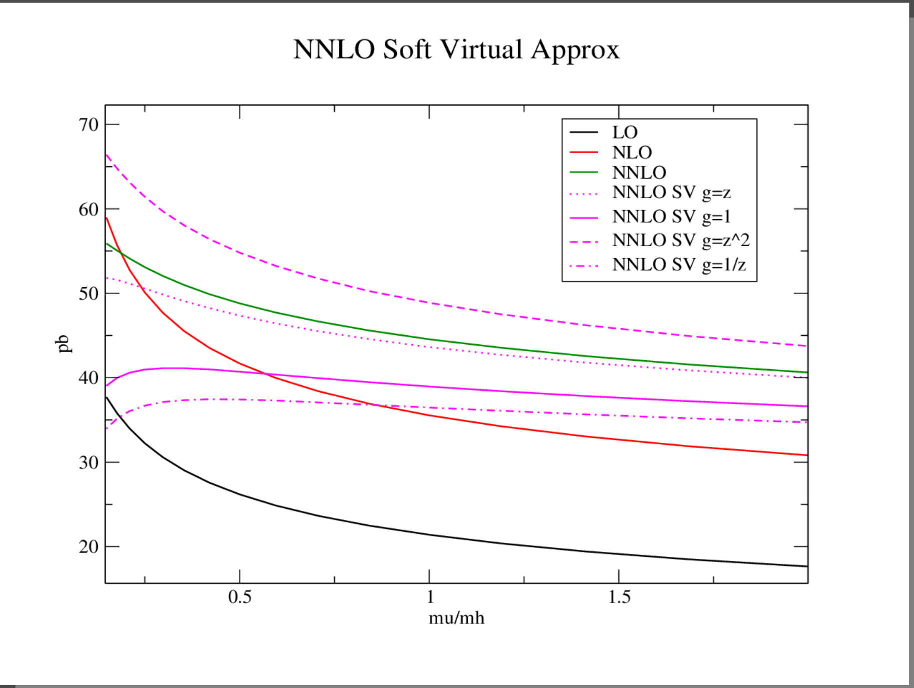
$$\sigma_{gg\to H}^{N^3LO,SV} = \int \mathrm{d}x_1 \mathrm{d}x_2 f_i(x_1) f_j(x_2) \int \mathrm{d}z \delta(z - \frac{\tau}{x_1 x_2}) \cdot z \cdot g(z) \cdot \left[\lim_{z\to 1} \frac{\sigma_{ij}^{N^3LO}(z)}{z g(z)} \right]$$

$$\tau = \frac{m_H^2}{S}$$

Having the exact limit at our disposal, we can get a first approximation to the N3LO which includes the purely virtual piece.

The Soft Virtual Approximation
NNLO vs N3LO





Besides Soft N3LO we also include full kinematic wilson coefficient and scale dependent N3LO contributions in all partonic channels. Scale variation is done using $\mu = \mu_R = \mu_F$

Conclusions

- We need to improve our understanding of the theoretical uncertainty on the gluon fusion cross section.
- Have presented the analytic result of the N3LO cross section in the soft limit, the first calculation done at N3LO for hadron colliders.
- Further coefficients of soft expansion of the N3LO are in close reach. The full result seems feasible.
- Have presented numerics for the soft virtual approximation at N3LO. The SV approximation gives an indication of the full N3LO, but has large uncertainties.

Backup

Shifting Logs and Collinear improved Soft-Virtual Approximation

$$\sigma^{N^3LO,SV}_{gg\to H} = \int \mathrm{d}x_1 \mathrm{d}x_2 f_i(x_1) f_j(x_2) \int \mathrm{d}z \delta(z - \frac{\tau}{x_1 x_2}) \cdot z \cdot g(z) \cdot \left[\lim_{z\to 1} \frac{\sigma^{N^3LO}_{ij}(z)}{z g(z)} \right]$$

The analytic structure of the partonic cross section can be written as

$$\frac{\sigma^{N^3LO}_{gg\to H}(z)}{z} = A\delta(1-z) + \sum_{n=1}^{5} B_n \left[\frac{\log^n(1-z)}{1-z}\right]_+ + C(z)$$

Can shift logs from plus to regular terms:

$$z\left[\frac{\log^n(1-z)}{1-z}\right]_+ = \left[\frac{\log^n(1-z)}{1-z}\right]_+ - \log^n(1-z)$$

Taking $g(z) = z$ reproduces the correct leading logarithm in the reguar part