

Effective theory approach in H-boson coupling determination

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on behalf of the HC (HiggsCharacterisation) collaboration

Artoisenet, de Aquino, Demartin, Frederix, Frixione, Maltoni,
Mandal, Mathews, Mawatari, Ravindran, Seth, Torrielli, Zaro

Alloul, Fuks, Sanz

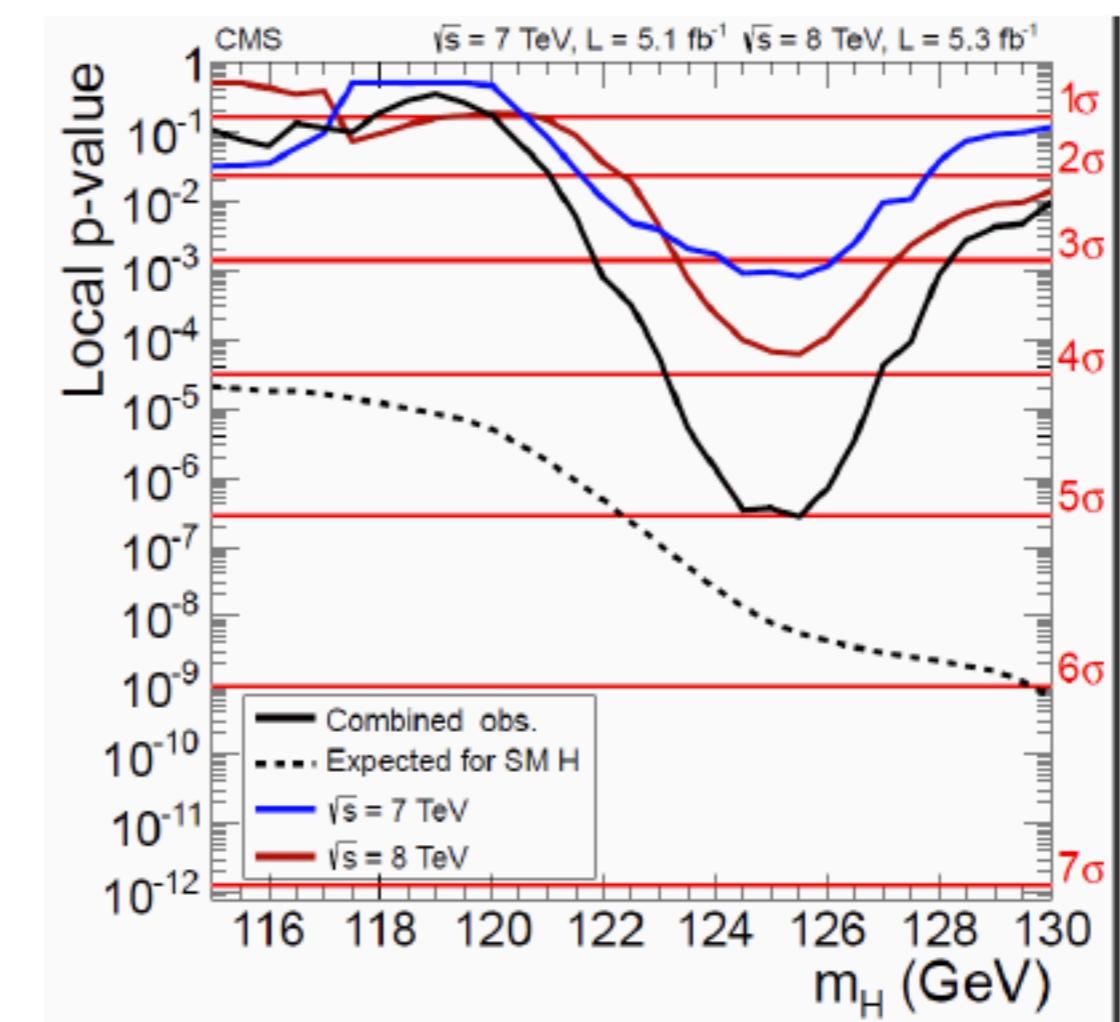
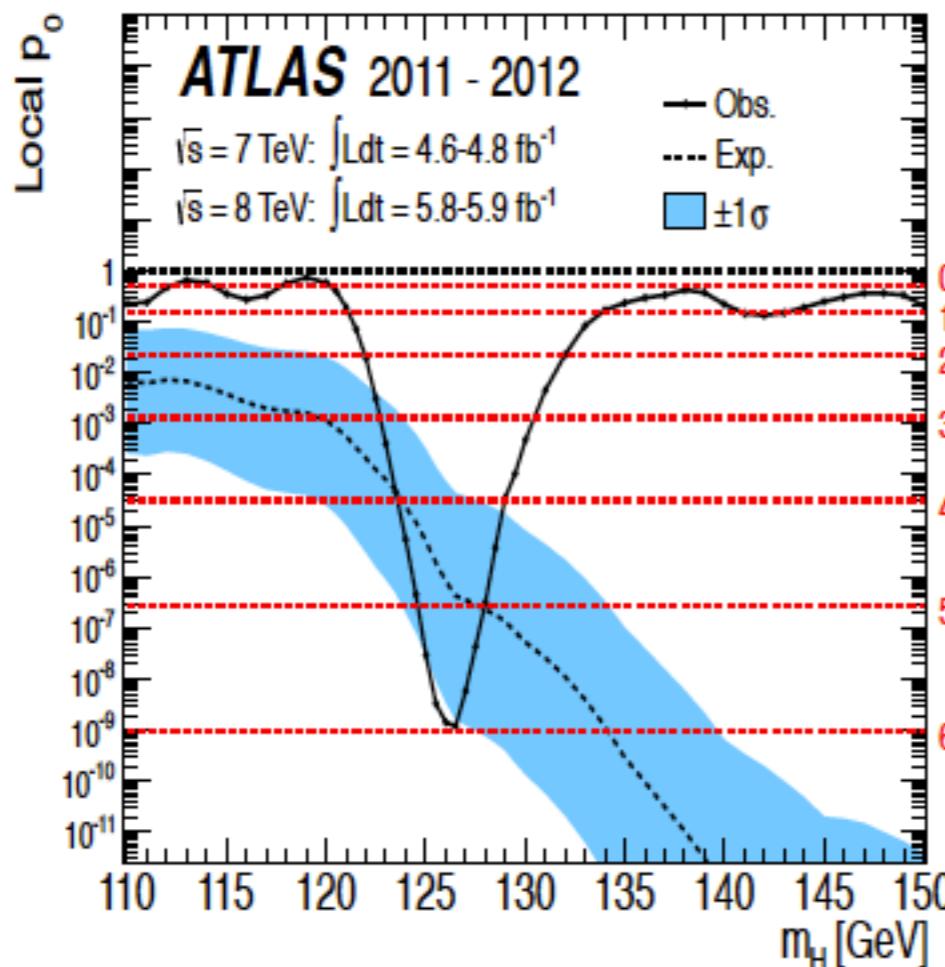
Contents

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 - Top-down approach vs. Bottom-up approach
 - Effective field theory
- Higgs characterisation framework
 - Effective Lagrangians -- $X(J=0,1,2)$
 - NLO QCD effects
- Summary

1. The discovered Higgs is NOT the SM Higgs

Fact I:

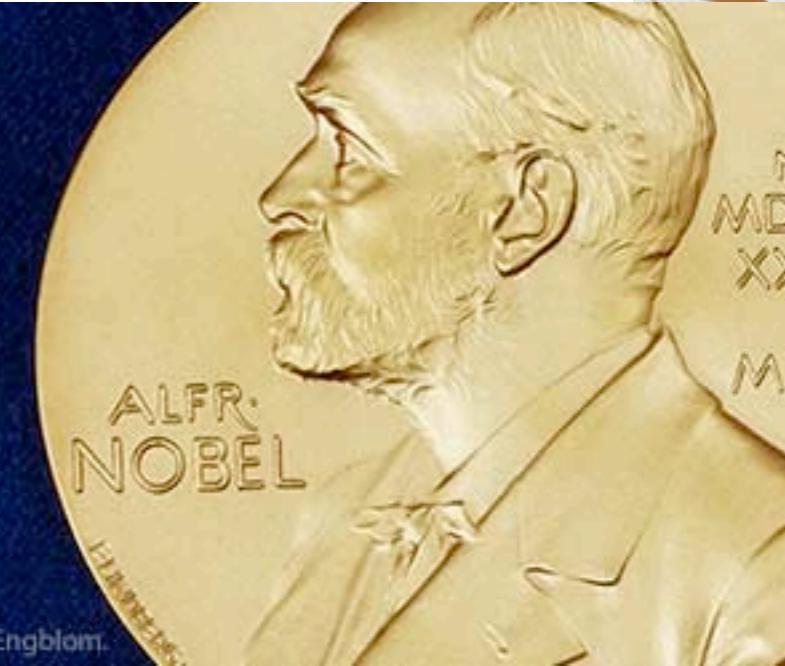
We have a discovery!



2013 NOBEL PRIZE IN PHYSICS

François Englert Peter W. Higgs

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BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout

Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium

(Received 26 June 1964)

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

P. W. HIGGS

Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

Received 27 July 1964

"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider" [<http://www.nobelprize.org>]



Fact I & II:

We have a discovery!

The SM cannot be the ultimate theory!

Q: Does the BSM physics have any (relevant) impact on the Higgs?

A: check changed properties in this talk

A: check for additional Higgs bosons

- heavier than 125.5 GeV
- lighter than 125.5 GeV \Rightarrow possible at the LHC?

Is this the Standard Model scalar boson?

- Higgs boson precision measurement
- determination of the Higgs boson Lagrangian
 - the structure of the operators, linked to the spin/parity of a Higgs boson
 - ▶ distributions
 - the coupling strength
 - ▶ rate
- How do we approach to get them?

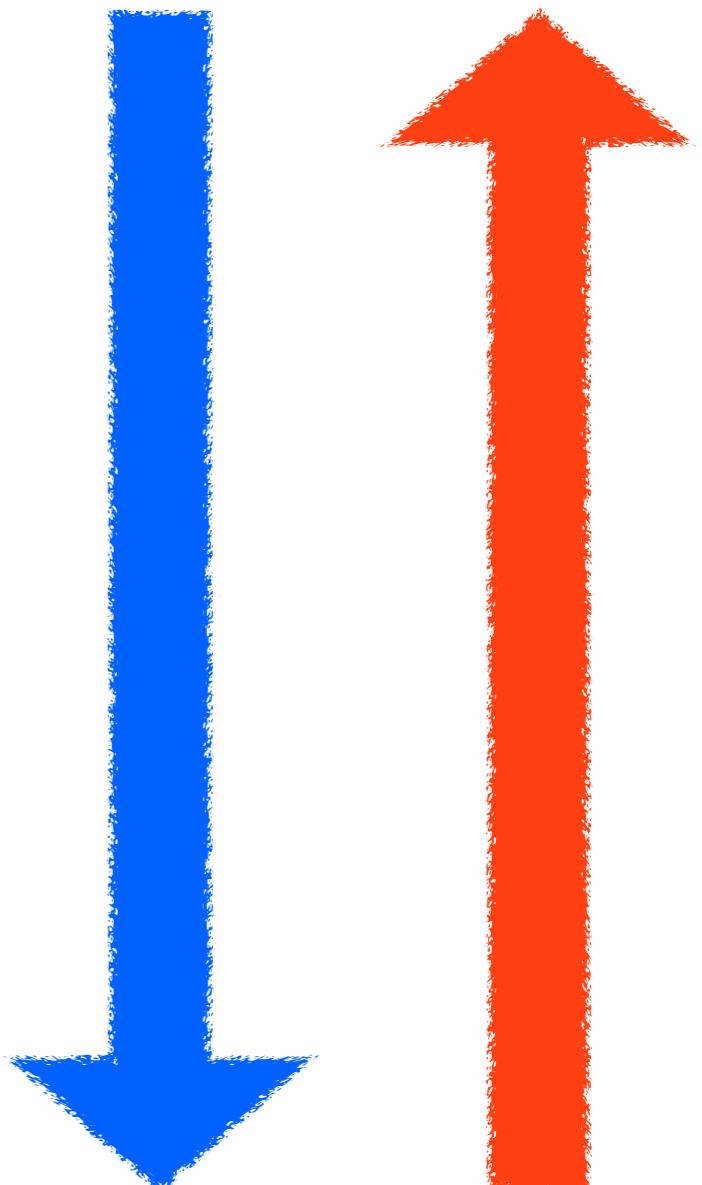
Physics beyond the SM:

Interesting (new) physics models :

- **2HDM:**
 - two Higgs doublets more natural than one
- **(N)MSSM:**
 - solves hierarchy problem, goes towards gravity
 - automatic electroweak symmetry breaking
 - gauge coupling unification
 - cold dark matter candidate
 - $(g - 2)_\mu$ solved easily
- **Little Higgs:**
 - (partially) solves the hierarchy problem
 - cold dark matter candidate
- **Extra dimensions:**
 - solves the hierarchy problem
 - cold dark matter candidate
- ...

⇒ pick your favorite model now – I pick the 2HDM, (N)MSSM

Top-down



bottom-up
in this talk

Effective field theory approach

- Given the fact that only a 125 GeV SM-like boson and nothing else so far, the effective field theory approach is one of the best way to explore BSM effects.
- All new particles and phenomena are assumed to appear at some scale Λ .
- Not predictive at scales larger than $\Lambda \rightarrow$ loss of unitarity
- Below Λ , all new physics effects are parametrized by higher dimensional gauge invariant operators made of SM fields. \rightarrow many parameters 59!
- No assumption on the form of new physics \rightarrow model independent
- Renormalisable order by order in the scale $\Lambda \rightarrow$ systematically improbable

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}_6 + \dots \quad \mathcal{L}_6 = \sum_i C_i Q_i$$

Buchmuller&Wyler 1986 ...
Grzadkowski et al. 2010

HEFT2013 - Higgs Effective Field theories

chaired by Michael Robert Trott (CERN), Gian Giudice (CERN), Roberto Contino (Sapienza Universita' di Roma), Riccardo Rattazzi (Ecole Polytechnique Federale de Lausanne (EPFL)), Christophe Grojean (ICREA - Institutio catalana de recerca estudis avancats (ES))

from Wednesday, 9 October 2013 at **08:00** to Friday, 11 October 2013 at **18:20** (Europe/Zurich)
at **CERN (4-3-006 - TH Conference Room)**

Description The purpose of this workshop is to gather experts to discuss the current state of the art in Effective Field Theory approaches to the Higgs sector. Workshop attendance is anticipated to be approximately 35 people and is initially by invitation. Details on the workshop program will be posted at a later date. Invitee registration is closed, public registration is now open.

Video Services Vidyo public room : HEFT2013_-_Higgs_Effective_Field_theories [More Info](#) | [Join Now!](#)

[Go to day ▾](#)

Wednesday, 9 October 2013

09:30 - 10:00 Welcome coffee/discussion ([4-2-011 - TH common room](#))
Welcome in common room

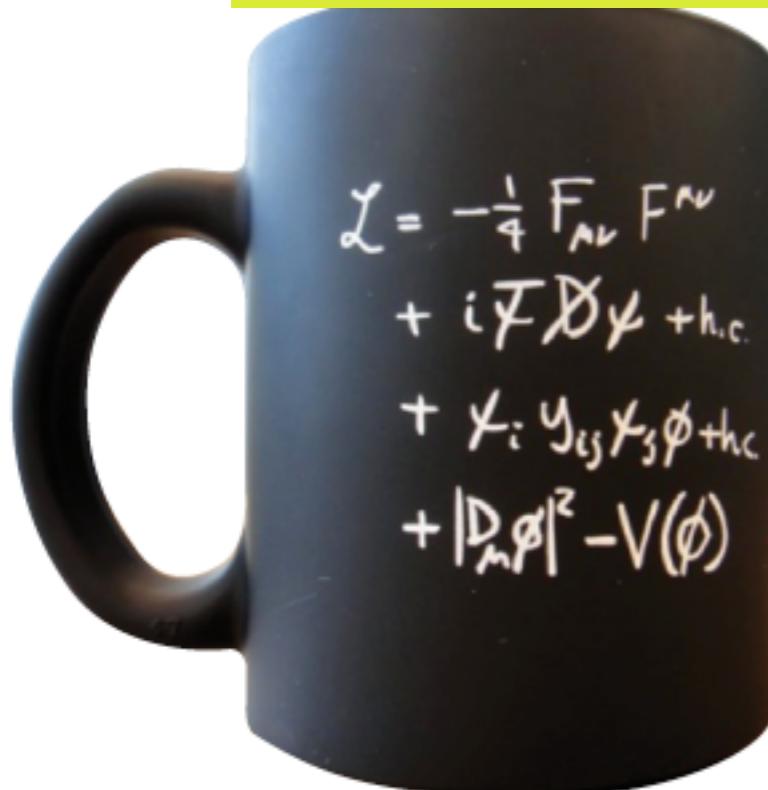
10:00 - 11:00 Higgs Effective Field Theories *1h0'*
Speaker: A. Manohar (UCSD)
Material: [Slides](#) [Video in CDS](#)

11:00 - 12:00 59 ways to leave the SM & implications for Higgs physics *1h0'*
Speaker: A Pomarol (IFAE)
Material: [Slides](#)

HEFT2014 (28 - 30 September 2014)

@ the Instituto de Física Teórica (UAM-CSIC), Madrid

Tool development (example)



FeynRules(v2.0)



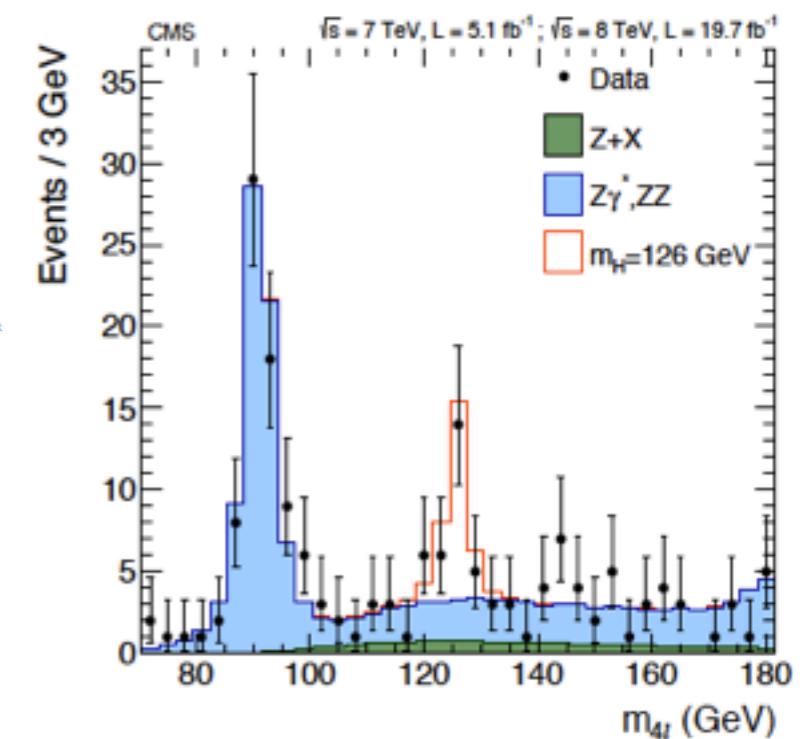
MadGraph5_aMC@NLO
(Pythia, Herwig)



Delphes



MadAnalysis5



Tools for Higgs Physics

Cross Section

ggF

- [HIGLU](#) (NNLO QCD+NLO EW)
- [iHixs](#) (NNLO QCD+NLO EW)
- [FeHiPro](#) (NNLO QCD+NLO EW)
- [HNNLO, HRes](#) (NNLO+NNLL QCD)
- [SusHi](#) (NNLO QCD)
- [RGHiggs](#) (NNLO+NNNLL QCD)
- [ggHiggs](#) (approx. NNNLO QCD)

VBF

- [VV2H](#) (NLO QCD)
- [VBFNLO](#) (NLO QCD)
- [HAWK](#) (NLO QCD+EW)
- [VBF@NNLO](#) (NNLO QCD)

WH/ZH

- [V2HV](#) (NLO QCD)
- [HAWK](#) (NLO QCD+EW)
- [VH@NNLO](#) (NNLO)

ttH

- [HQQ](#) (LO QCD)

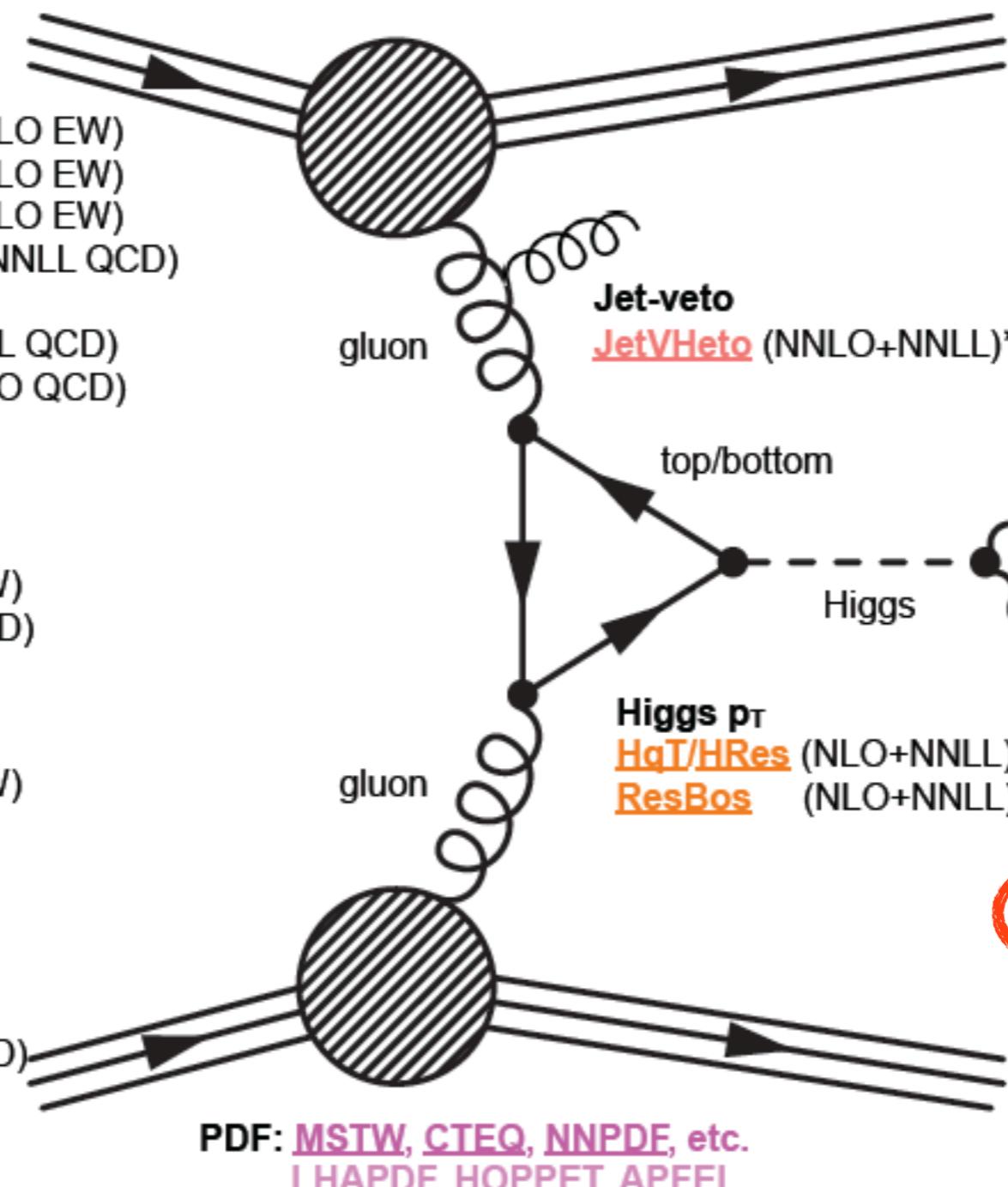
bbH

- [bbh@NNLO](#) (NNLO QCD)

HH

- [HPAIR](#) (NLO QCD)

+ private codes.



NLO MC

- [POWHEG MiNLO](#)
- [MadGraph5 aMC@NLO](#)
- [SHERPA MEPS@NLO](#)

LO MC

- [gg2VV](#)

NLO ME

- [MCFM](#), [MG5_aMC@NLO](#)

W/Z

- Higgs Decay**
- [HDECAY](#) (NLO++)
- [Prophecy4f](#) (NLO)

W/Z

Higgs Properties

- [MELA/JHU](#), [MEKD](#)
- [MG5_aMC@NLO \(HC\)](#)

in this talk

MSSM/2HDM

- [FeynHiggs](#), [CPSuperH](#)
- [SusHi+2HDMC](#)
- [HIGLU+HDECAY](#)

* NLO+NNLL in differential

Tools for Higgs properties

- The general scattering amplitude approach

$$A(X_{J=0} \rightarrow VV) = v^{-1} \left(g_1 m_V^2 \epsilon_1^* \epsilon_2^* + g_2 f_{\mu\nu}^{*(1)} f^{*(2),\mu\nu} + g_3 f^{*(1),\mu\nu} f_{\mu\alpha}^{*(2)} \frac{q_\nu q^\alpha}{\Lambda^2} + g_4 f_{\mu\nu}^{*(1)} \tilde{f}^{*(2),\mu\nu} \right)$$

- Couplings can be momentum dependent form factors and can be complex.
- Public code: e.g. JHUGen [1001.3396, 1208.4018, 1309.4819]

- The effective field theory approach

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}_6 + \dots \quad \mathcal{L}_6 = \sum_i C_i Q_i$$

- Public code: e.g. HEL (HiggsEffectiveLagrangian) [1310.5150]
HC (HiggsCharacterisation) [1306.6464, 1311.1829]

D6 Higgs Effective Lagrangian

[from Contino, Ghezzi, Grojean, Muhlleitner, Spira (JHEP '13)]

[Alloul, Fuks, Sanz (1310.5150)]

$$\begin{aligned} \mathcal{L}_{\text{SILH}} = & \frac{\bar{c}_H}{2v^2} \partial^\mu [\Phi^\dagger \Phi] \partial_\mu [\Phi^\dagger \Phi] + \frac{\bar{c}_T}{2v^2} [\Phi^\dagger \overleftrightarrow{D}^\mu \Phi] [\Phi^\dagger \overleftrightarrow{D}_\mu \Phi] - \frac{\bar{c}_6 \lambda}{v^2} [H^\dagger H]^3 \\ & - \left[\frac{\bar{c}_u}{v^2} y_u \Phi^\dagger \Phi \cdot \bar{Q}_L u_R + \frac{\bar{c}_d}{v^2} y_d \Phi^\dagger \Phi \bar{Q}_L d_R + \frac{\bar{c}_l}{v^2} y_\ell \Phi^\dagger \Phi \bar{L}_L e_R + \text{h.c.} \right] \\ & + \frac{ig \bar{c}_W}{m_W^2} [\Phi^\dagger T_{2k} \overleftrightarrow{D}^\mu \Phi] D^\nu W_{\mu\nu}^k + \frac{ig' \bar{c}_B}{2m_W^2} [\Phi^\dagger \overleftrightarrow{D}^\mu \Phi] \partial^\nu B_{\mu\nu} \\ & + \frac{2ig \bar{c}_{HW}}{m_W^2} [D^\mu \Phi^\dagger T_{2k} D^\nu \Phi] W_{\mu\nu}^k + \frac{ig' \bar{c}_{HB}}{m_W^2} [D^\mu \Phi^\dagger D^\nu \Phi] B_{\mu\nu} \\ & + \frac{\bar{g}'^2 c_\gamma}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu} + \frac{\bar{g}_s^2 c_g}{m_W^2} \Phi^\dagger \Phi G_{\mu\nu}^a G_a^{\mu\nu}, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{CP} = & \frac{ig \bar{c}_{HW}}{m_W^2} D^\mu \Phi^\dagger T_{2k} D^\nu \Phi \widetilde{W}_{\mu\nu}^k + \frac{ig' \bar{c}_{HB}}{m_W^2} D^\mu \Phi^\dagger D^\nu \Phi \widetilde{B}_{\mu\nu} + \frac{g'^2 \bar{c}_\gamma}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} \widetilde{B}^{\mu\nu} \\ & + \frac{g_s^2 \bar{c}_g}{m_W^2} \Phi^\dagger \Phi G_{\mu\nu}^a \widetilde{G}_a^{\mu\nu} + \frac{g^3 \bar{c}_{3W}}{m_W^2} \epsilon_{ijk} W_{\mu\nu}^i W_{\rho\sigma}^{jk} \widetilde{W}^{\rho\mu k} + \frac{g_s^3 \bar{c}_{3G}}{m_W^2} f_{abc} G_{\mu\nu}^a G_{\rho\sigma}^b \widetilde{G}_{\mu\nu}^{\rho\sigma} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_G = & \frac{g^3 \bar{c}_{3W}}{m_W^2} \epsilon_{ijk} W_{\mu\nu}^i W_{\rho\sigma}^{jk} W^{\rho\mu k} + \frac{g_s^3 \bar{c}_{3G}}{m_W^2} f_{abc} G_{\mu\nu}^a G_{\rho\sigma}^b G_{\mu\nu}^c + \frac{\bar{c}_{2W}}{m_W^2} D^\mu W_{\mu\nu}^k D_\rho W_k^{\rho\nu} \\ & + \frac{\bar{c}_{2B}}{m_W^2} \partial^\mu B_{\mu\nu} \partial_\rho B^{\rho\nu} + \frac{\bar{c}_{2G}}{m_W^2} D^\mu G_{\mu\nu}^a D_\rho G_a^{\rho\nu}, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{F_1} = & \frac{i\bar{c}_{HQ}}{v^2} [\bar{Q}_L \gamma^\mu Q_L] [\Phi^\dagger \overleftrightarrow{D}_\mu \Phi] + \frac{4i\bar{c}'_{HQ}}{v^2} [\bar{Q}_L \gamma^\mu T_{2k} Q_L] [\Phi^\dagger T_2^k \overleftrightarrow{D}_\mu \Phi] \\ & + \frac{i\bar{c}_{Hu}}{v^2} [\bar{u}_R \gamma^\mu u_R] [\Phi^\dagger \overleftrightarrow{D}_\mu \Phi] + \frac{i\bar{c}_{Hd}}{v^2} [\bar{d}_R \gamma^\mu d_R] [\Phi^\dagger \overleftrightarrow{D}_\mu \Phi] \\ & - \left[\frac{i\bar{c}_{Hud}}{v^2} [\bar{u}_R \gamma^\mu d_R] [\Phi \cdot \overleftrightarrow{D}_\mu \Phi] + \text{h.c.} \right] \\ & + \frac{i\bar{c}_{HL}}{v^2} [\bar{L}_L \gamma^\mu L_L] [\Phi^\dagger \overleftrightarrow{D}_\mu \Phi] + \frac{4i\bar{c}'_{HL}}{v^2} [\bar{L}_L \gamma^\mu T_{2k} L_L] [\Phi^\dagger T_2^k \overleftrightarrow{D}_\mu \Phi] \\ & + \frac{i\bar{c}_{He}}{v^2} [\bar{e}_R \gamma^\mu e_R] [\Phi^\dagger \overleftrightarrow{D}_\mu \Phi], \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{F_2} = & \left[- \frac{2g' \bar{c}_{uB}}{m_W^2} y_u \Phi^\dagger \cdot \bar{Q}_L \gamma^{\mu\nu} u_R B_{\mu\nu} - \frac{4g \bar{c}_{uW}}{m_W^2} y_u \Phi^\dagger \cdot (\bar{Q}_L T_{2k}) \gamma^{\mu\nu} u_R W_{\mu\nu}^k \right. \\ & - \frac{4g_s \bar{c}_{uG}}{m_W^2} y_u \Phi^\dagger \cdot \bar{Q}_L \gamma^{\mu\nu} T_a u_R G_{\mu\nu}^a + \frac{2g' \bar{c}_{dB}}{m_W^2} y_d \Phi \bar{Q}_L \gamma^{\mu\nu} d_R B_{\mu\nu} \\ & + \frac{4g \bar{c}_{dW}}{m_W^2} y_d \Phi (\bar{Q}_L T_{2k}) \gamma^{\mu\nu} d_R W_{\mu\nu}^k + \frac{4g_s \bar{c}_{dG}}{m_W^2} y_d \Phi \bar{Q}_L \gamma^{\mu\nu} T_a d_R G_{\mu\nu}^a \\ & \left. + \frac{2g' \bar{c}_{eB}}{m_W^2} y_\ell \Phi \bar{L}_L \gamma^{\mu\nu} e_R B_{\mu\nu} + \frac{4g \bar{c}_{eW}}{m_W^2} y_\ell \Phi (\bar{L}_L T_{2k}) \gamma^{\mu\nu} e_R W_{\mu\nu}^k + \text{h.c.} \right] \end{aligned}$$

◆ The model file is publicly available. (<https://feynrules.irmp.ucl.ac.be/wiki/HEL>)

before vs. after EW symmetry breaking

HC [arXiv: 1306.6464]

$$\begin{aligned} \mathcal{L}_0^f &= - \sum_{f=t,b,\tau} \bar{\psi}_f (c_\alpha \kappa_{Hff} g_{Hff} + i s_\alpha \kappa_{Aff} g_{Aff} \gamma_5) \psi_f X_0 \\ \mathcal{L}_0^V &= \left\{ c_\alpha \kappa_{SM} \left[\frac{1}{2} g_{HZZ} Z_\mu Z^\mu + g_{HWW} W_\mu^+ W^{-\mu} \right] \right. \\ &\quad - \frac{1}{4} \left[c_\alpha \kappa_{H\gamma\gamma} g_{H\gamma\gamma} A_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{A\gamma\gamma} g_{A\gamma\gamma} A_{\mu\nu} \tilde{A}^{\mu\nu} \right] \\ &\quad - \frac{1}{2} \left[c_\alpha \kappa_{HZ\gamma} g_{HZ\gamma} Z_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{AZ\gamma} g_{AZ\gamma} Z_{\mu\nu} \tilde{A}^{\mu\nu} \right] \\ &\quad - \frac{1}{4} \left[c_\alpha \kappa_{Hgg} g_{Hgg} G_{\mu\nu}^a G^{a,\mu\nu} + s_\alpha \kappa_{Agg} g_{Agg} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} \right] \\ &\quad \left. - \frac{1}{4} \frac{1}{\Lambda} \left[c_\alpha \kappa_{HZZ} Z_{\mu\nu} Z^{\mu\nu} + s_\alpha \kappa_{AZZ} Z_{\mu\nu} \tilde{Z}^{\mu\nu} \right] \right. \\ &\quad - \frac{1}{2} \frac{1}{\Lambda} \left[c_\alpha \kappa_{HWW} W_\mu^+ W^{-\mu\nu} + s_\alpha \kappa_{AWW} W_\mu^+ \tilde{W}^{-\mu\nu} \right] \\ &\quad - \frac{1}{\Lambda} c_\alpha \left[\kappa_{H\theta\gamma} Z_\nu \partial_\mu A^{\mu\nu} + \kappa_{H\theta Z} Z_\nu \partial_\mu Z^{\mu\nu} \right. \\ &\quad \left. + (\kappa_{H\theta W} W_\nu^+ \partial_\mu W^{-\mu\nu} + h.c.) \right] \right\} X_0 \end{aligned}$$

$$\begin{aligned} V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu \quad (V = A, Z, W^\pm), \quad \tilde{V}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} V^{\rho\sigma} \\ G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c, \end{aligned}$$

HEL [arXiv: 1310.5150]

Eq. (2.25)	Ref. [46]	Section 2.1
g_{hgg}	$c_\alpha \kappa_{Hgg} g_{Hgg}$	$g_H - \frac{4\bar{c}_g g_s^2 v}{m_W^2}$
\tilde{g}_{hgg}	$s_\alpha \kappa_{Agg} g_{Agg}$	$-\frac{4\bar{c}_g g_s^2 v}{m_W^2}$
$g_{h\gamma\gamma}$	$c_\alpha \kappa_{H\gamma\gamma} g_{H\gamma\gamma}$	$a_H - \frac{8\bar{c}_\gamma s_W^2}{m_W}$
$\tilde{g}_{h\gamma\gamma}$	$s_\alpha \kappa_{A\gamma\gamma} g_{A\gamma\gamma}$	$-\frac{8\bar{c}_\gamma s_W^2}{m_W}$
$g_{hz z}^{(1)}$	$\frac{1}{\Lambda} c_\alpha \kappa_{HZZ}$	$\frac{2g}{c_W^2 m_W} \left[\bar{c}_{HB} s_W^2 - 4\bar{c}_\gamma s_W^4 + c_W^2 \bar{c}_{HW} \right]$
$\tilde{g}_{hz z}$	$\frac{1}{\Lambda} s_\alpha \kappa_{AZZ}$	$\frac{2g}{c_W^2 m_W} \left[\bar{c}_{HB} s_W^2 - 4\bar{c}_\gamma s_W^4 + c_W^2 \bar{c}_{HW} \right]$
$g_{hz z}^{(2)}$	$\frac{1}{\Lambda} c_\alpha \kappa_{H\theta Z}$	$\frac{g}{c_W^2 m_W} \left[(\bar{c}_{HW} + \bar{c}_W) c_W^2 + (\bar{c}_B + \bar{c}_{HB}) s_W^2 \right]$
$g_{hz z}^{(3)}$	$c_\alpha \kappa_{SM} g_{HZZ}$	$\frac{g m_W}{c_W^2} \left[1 - \frac{1}{2} \bar{c}_H - 2\bar{c}_T + 8\bar{c}_\gamma \frac{s_W^4}{c_W^2} \right]$
$g_{haz}^{(1)}$	$c_\alpha \kappa_{HZ\gamma} g_{HZ\gamma}$	$\frac{gs_W}{c_W m_W} \left[\bar{c}_{HW} - \bar{c}_{HB} + 8\bar{c}_\gamma s_W^2 \right]$
\tilde{g}_{haz}	$s_\alpha \kappa_{AZ\gamma} g_{AZ\gamma}$	$\frac{gs_W}{c_W m_W} \left[\bar{c}_{HW} - \bar{c}_{HB} + 8\bar{c}_\gamma s_W^2 \right]$
$g_{haz}^{(2)}$	$\frac{1}{\Lambda} c_\alpha \kappa_{H\theta\gamma}$	$\frac{gs_W}{c_W m_W} \left[\bar{c}_{HW} - \bar{c}_{HB} - \bar{c}_B + \bar{c}_W \right]$
$g_{hww}^{(1)}$	$\frac{1}{\Lambda} c_\alpha \kappa_{HWW}$	$\frac{2g}{m_W} \bar{c}_{HW}$
\tilde{g}_{hww}	$\frac{1}{\Lambda} s_\alpha \kappa_{AWW}$	$\frac{2g}{m_W} \tilde{c}_{HW}$
$g_{hww}^{(2)}$	$\frac{1}{\Lambda} c_\alpha \kappa_{H\theta W}$	$\frac{g}{m_W} \left[\bar{c}_W + \bar{c}_{HW} \right]$

A framework for Higgs characterisation

JHEP11(2013)043 [arXiv: 1306.6464]
EPJC74(2014)2710 [arXiv: 1311.1829]

The FeynRules and MadGraph5_aMC@NLO framework

FeynRules model

P. de Aquino, K. Mawatari (Vrije U. Brussel)

MadWeight

P.Artoisenet (Nikhef)

aMC@NLO

F. Demartin, F. Maltoni (UC Louvain), M.Zaro (Paris)
R. Frederix, S. Frixione (CERN)
P.Torrielli (Zurich)

spin2 in aMC@NLO

M.K. Mandal (Harish-Chandra)
P. Mathews, S. Seth (Saha Inst.)
V. Ravindran (CIT)

Desiderata for a HC framework

- Study Higgs quantum numbers, coupling strengths and structures in a model independent way.
- Go beyond the “one-process, one-decay, one-observable” approach, i.e. go to “any-process, any-decay, any-observable” one.
- Equally useful for theorists (it can be systematically improved, changed - easily) and experimentalists (MC’s event generation directly in analyses).
- Adaptable to the present/future analyses and accuracy targets.

Higgs Characterisation model

- We implemented an effective Lagrangian featuring bosons $X(J^P=0^+, 0^-, 1^+, 1^-, 2^+)$ in FeynRules.
 - ▶ Effective field theory approach, valid up to a cutoff scale Λ
 - ▶ Only one new bosonic state $X(J^P)$ at the EW scale (No other state below the cutoff Λ)
 - ▶ Any new physics is described by the lowest dimensional operators.

The parametrization is based on the recent work [Englert, Goncalves-Netto, KM, Plehn (2013)].

Effective Lagrangian -- spin0

$$\mathcal{L}_0^f = - \sum_{f=t,b,\tau} \bar{\psi}_f (c_\alpha \kappa_{Hff} g_{Hff} + i s_\alpha \kappa_{Aff} g_{Aff} \gamma_5) \psi_f X_0$$

$$\begin{aligned} \mathcal{L}_0^V = & \left\{ c_\alpha \kappa_{\text{SM}} \left[\frac{1}{2} g_{HZZ} Z_\mu Z^\mu + g_{HWW} W_\mu^+ W^{-\mu} \right] \right. \\ & - \frac{1}{4} [c_\alpha \kappa_{H\gamma\gamma} g_{H\gamma\gamma} A_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{A\gamma\gamma} g_{A\gamma\gamma} A_{\mu\nu} \tilde{A}^{\mu\nu}] \\ & - \frac{1}{2} [c_\alpha \kappa_{HZ\gamma} g_{HZ\gamma} Z_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{AZ\gamma} g_{AZ\gamma} Z_{\mu\nu} \tilde{A}^{\mu\nu}] \\ & - \frac{1}{4} [c_\alpha \kappa_{Hgg} g_{Hgg} G_{\mu\nu}^a G^{a,\mu\nu} + s_\alpha \kappa_{Agg} g_{Agg} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}] \\ & - \frac{1}{4} \frac{1}{\Lambda} [c_\alpha \kappa_{HZZ} Z_{\mu\nu} Z^{\mu\nu} + s_\alpha \kappa_{AZZ} Z_{\mu\nu} \tilde{Z}^{\mu\nu}] \\ & - \frac{1}{2} \frac{1}{\Lambda} [c_\alpha \kappa_{HWW} W_{\mu\nu}^+ W^{-\mu\nu} + s_\alpha \kappa_{AWW} W_{\mu\nu}^+ \tilde{W}^{-\mu\nu}] \\ & - \frac{1}{\Lambda} c_\alpha [\kappa_{H\theta\gamma} Z_\nu \partial_\mu A^{\mu\nu} + \kappa_{H\theta Z} Z_\nu \partial_\mu Z^{\mu\nu} \\ & \quad + (\kappa_{H\theta W} W_\nu^+ \partial_\mu W^{-\mu\nu} + h.c.)] \Big\} X_0 \end{aligned}$$

parameter	description
Λ [GeV]	cutoff scale
c_α ($\equiv \cos \alpha$)	mixing between 0^+ and 0^-
κ_i	dimensionless coupling parameter

```
#####
## INFORMATION FOR FRBLOCK
#####
Block frblock
1 1.000000e+03 # Lambda
2 1.000000e+00 # ca
3 1.000000e+00 # kSM
4 1.000000e+00 # kHtt
5 1.000000e+00 # kAtt
6 1.000000e+00 # kHbb
7 1.000000e+00 # kAbb
8 1.000000e+00 # kHll
9 1.000000e+00 # kAll
10 1.000000e+00 # kHaa
11 1.000000e+00 # kAaa
12 1.000000e+00 # kHz
13 1.000000e+00 # kAza
14 1.000000e+00 # kHgg
15 1.000000e+00 # kAgg
16 0.000000e+00 # kHzz
17 0.000000e+00 # kAzz
18 0.000000e+00 # kHww
19 0.000000e+00 # kAww
20 0.000000e+00 # kHda
21 0.000000e+00 # kHdz
22 0.000000e+00 # kHdwR
23 0.000000e+00 # kHdwI
```

Effective Lagrangian -- spin0

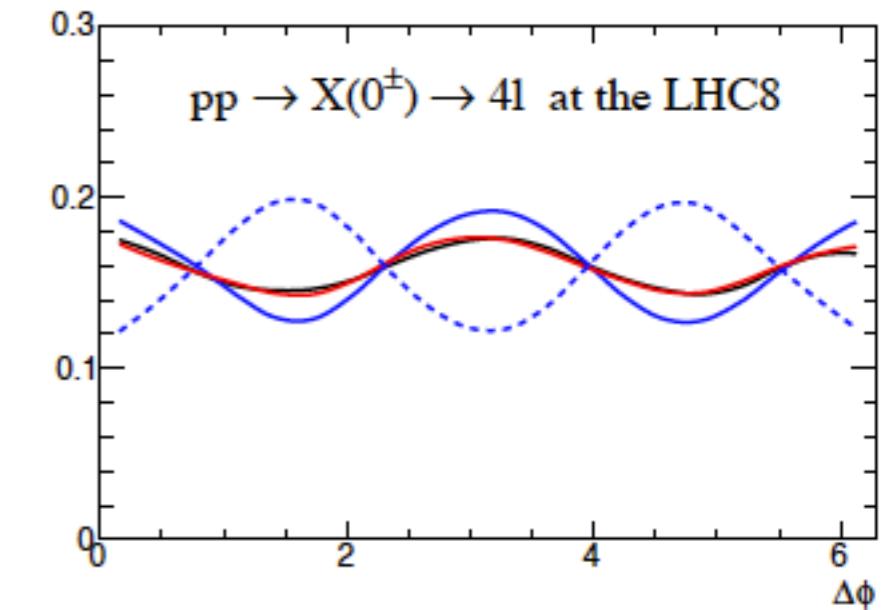
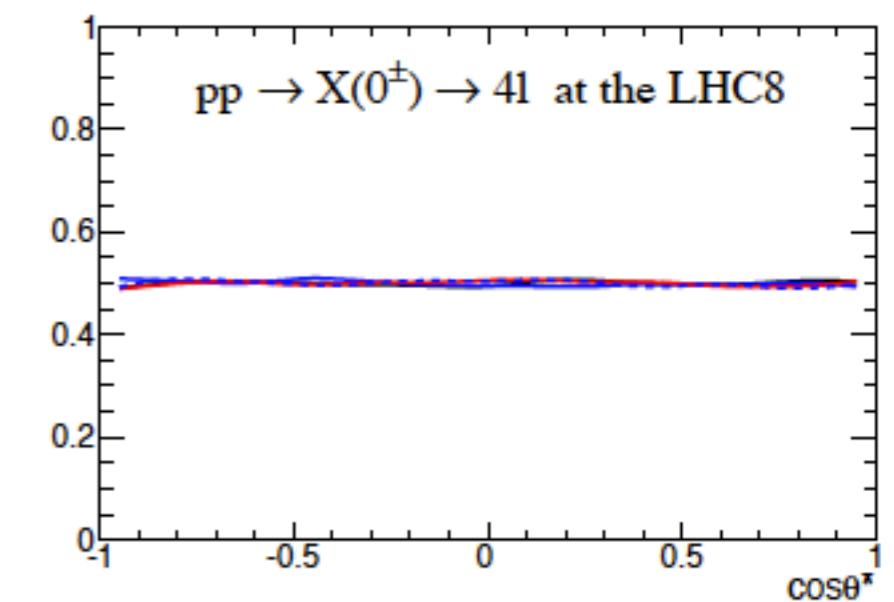
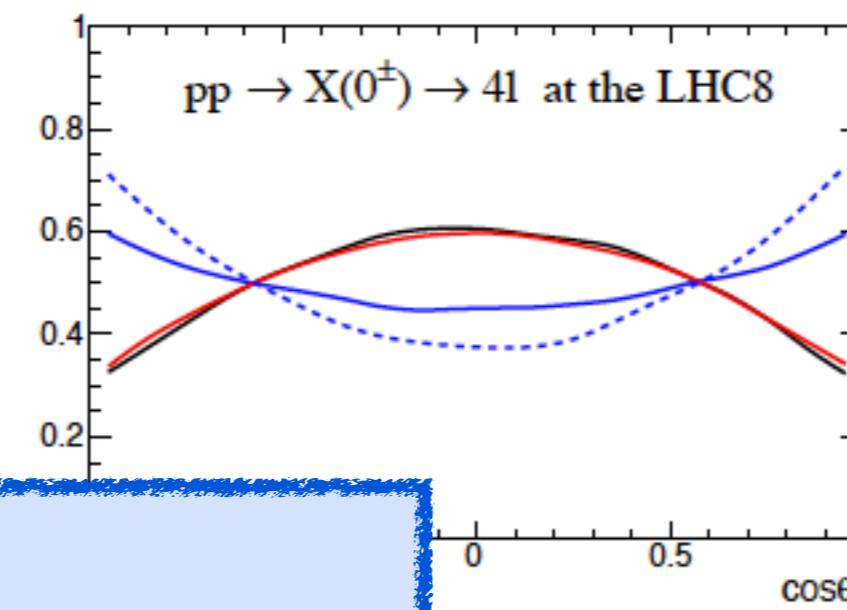
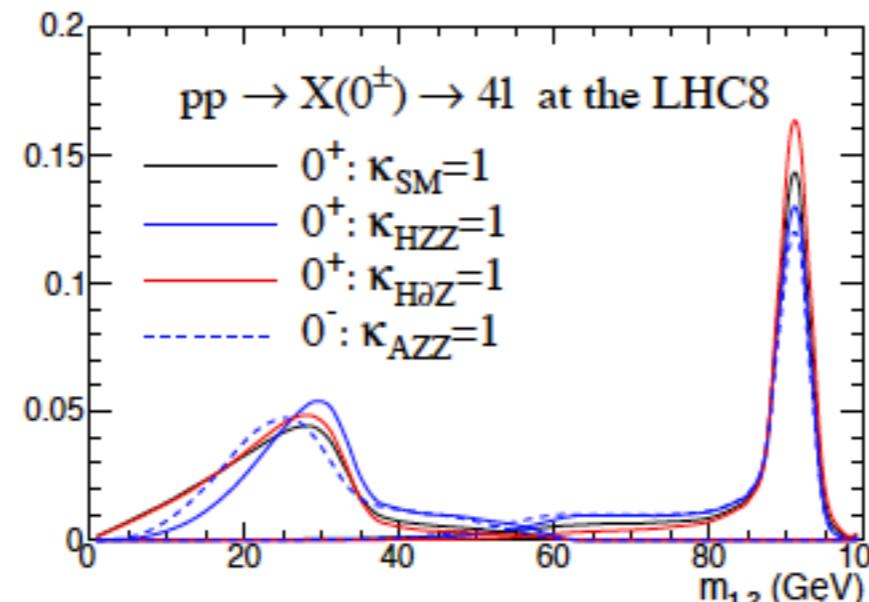
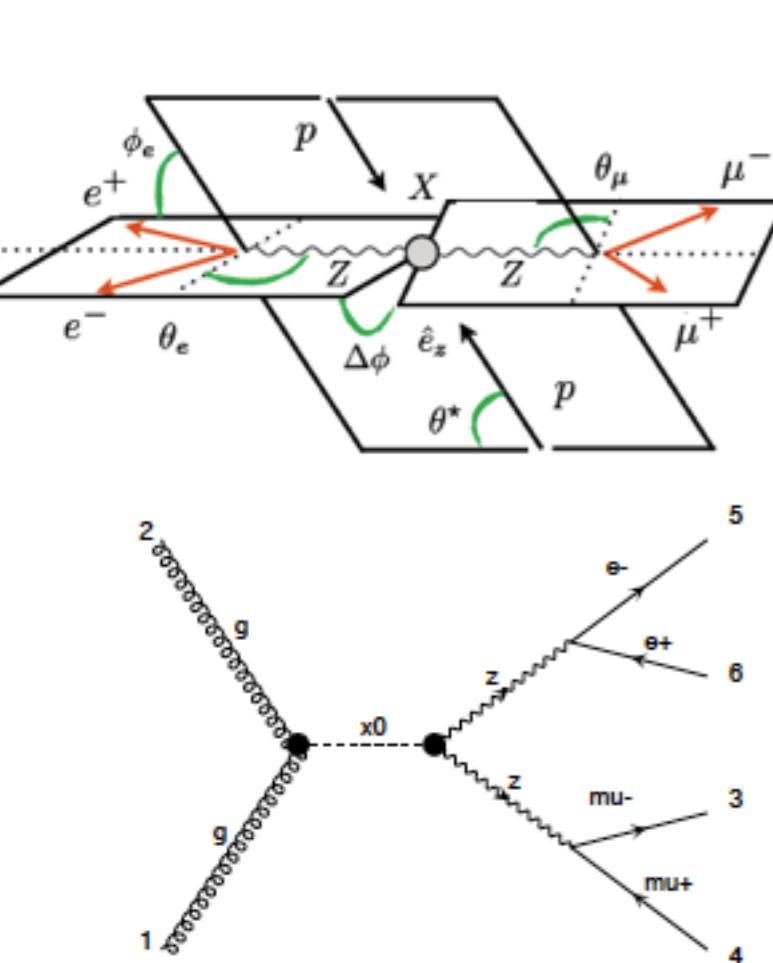
$$\mathcal{L}_0^f = - \sum_{f=t,b,\tau} \bar{\psi}_f (c_\alpha \kappa_{Hff} g_{Hff} + i s_\alpha \kappa_{Aff} g_{Aff} \gamma_5) \psi_f X_0$$

$$\begin{aligned} \mathcal{L}_0^V = & \left\{ c_\alpha \kappa_{SM} \left[\frac{1}{2} g_{HZZ} Z_\mu Z^\mu + g_{HWW} W_\mu^+ W^{-\mu} \right] \right. \\ & - \frac{1}{4} [c_\alpha \kappa_{H\gamma\gamma} g_{H\gamma\gamma} A_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{A\gamma\gamma} g_{A\gamma\gamma} A_{\mu\nu} \tilde{A}^{\mu\nu}] \\ & - \frac{1}{2} [c_\alpha \kappa_{HZ\gamma} g_{HZ\gamma} Z_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{AZ\gamma} g_{AZ\gamma} Z_{\mu\nu} \tilde{A}^{\mu\nu}] \\ & - \frac{1}{4} [c_\alpha \kappa_{Hgg} g_{Hgg} G_{\mu\nu}^a G^{a,\mu\nu} + s_\alpha \kappa_{Agg} g_{Agg} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}] \\ & - \frac{1}{4} \frac{1}{\Lambda} [c_\alpha \kappa_{HZZ} Z_{\mu\nu} Z^{\mu\nu} + s_\alpha \kappa_{AZZ} Z_{\mu\nu} \tilde{Z}^{\mu\nu}] \\ & - \frac{1}{2} \frac{1}{\Lambda} [c_\alpha \kappa_{HWW} W_\mu^+ W^{-\mu\nu} + s_\alpha \kappa_{AWW} W_\mu^+ \tilde{W}^{-\mu\nu}] \\ & - \frac{1}{\Lambda} c_\alpha [\kappa_{H\theta\gamma} Z_\nu \partial_\mu A^{\mu\nu} + \kappa_{H\theta Z} Z_\nu \partial_\mu Z^{\mu\nu} \\ & \quad + (\kappa_{H\theta W} W_\nu^+ \partial_\mu W^{-\mu\nu} + h.c.)] \Big\} X_0 \end{aligned}$$

Dimensionful **couplings g** are set as internal parameters so as to reproduce a **SM Higgs** for $\kappa=1$.

$g_{Xyy'} \times v$	ff	ZZ/WW	$\gamma\gamma$	$Z\gamma$	gg
H	m_f	$2m_{Z/W}^2$	$47\alpha_{EM}/18\pi$	$C(94\cos^2\theta_W - 13)/9\pi$	$-\alpha_s/3\pi$
A	m_f	0	$4\alpha_{EM}/3\pi$	$2C(8\cos^2\theta_W - 5)/3\pi$	$\alpha_s/2\pi$

Mass and angular distributions -- spin0



```
./bin/mg5_aMC
>import model HC
>generate p p > x0, x0 > mu- mu+ e- e+
>launch
```

Effective Lagrangian -- spin 1

- The most general interactions at the lowest canonical dimension:

$$\mathcal{L}_1^f = \sum_{f=q,\ell} \bar{\psi}_f \gamma_\mu (\kappa_{f_a} a_f - \kappa_{f_b} b_f \gamma_5) \psi_f X_1^\mu$$

$$\begin{aligned} \mathcal{L}_1^W = & i\kappa_{W_1} g_{WWZ} (W_{\mu\nu}^+ W^{-\mu} - W_{\mu\nu}^- W^{+\mu}) X_1^\nu + i\kappa_{W_2} g_{WWZ} W_\mu^+ W_\nu^- X_1^{\mu\nu} \\ & - \kappa_{W_3} W_\mu^+ W_\nu^- (\partial^\mu X_1^\nu + \partial^\nu X_1^\mu) \\ & + i\kappa_{W_4} W_\mu^+ W_\nu^- \tilde{X}_1^{\mu\nu} - \kappa_{W_5} \epsilon_{\mu\nu\rho\sigma} [W^{+\mu} (\partial^\rho W^{-\nu}) - (\partial^\rho W^{+\mu}) W^{-\nu}] X_1^\sigma \end{aligned}$$

$$\mathcal{L}_1^Z = -\kappa_{Z_1} Z_{\mu\nu} Z^\mu X_1^\nu - \kappa_{Z_3} X_1^\mu (\partial^\nu Z_\mu) Z_\nu - \kappa_{Z_5} \epsilon_{\mu\nu\rho\sigma} X_1^\mu Z^\nu (\partial^\rho Z^\sigma)$$

- Parity conservation implies that
 - ▶ for X_1^- $\kappa_{f_b} = \kappa_{V_4} = \kappa_{V_5} = 0$
 - ▶ for X_1^+ $\kappa_{f_a} = \kappa_{V_1} = \kappa_{V_2} = \kappa_{V_3} = 0$

Effective Lagrangian -- spin2

- via the energy-momentum tensor of the SM fields, starting from D5:

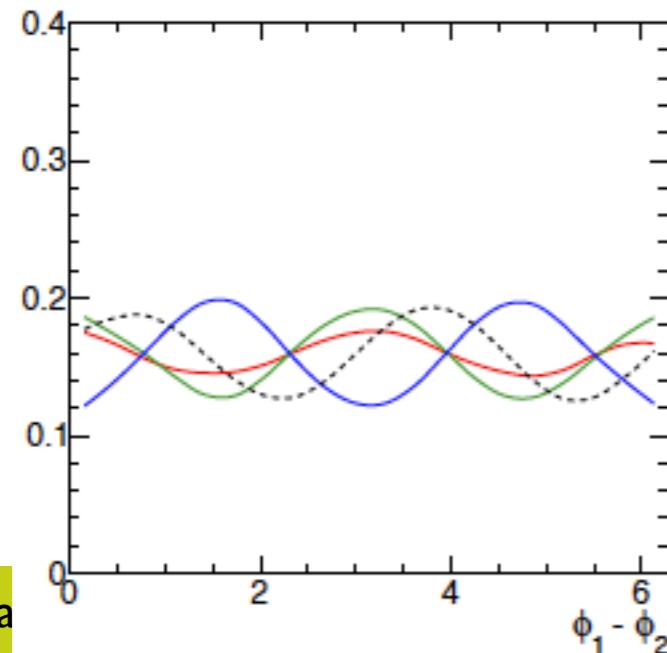
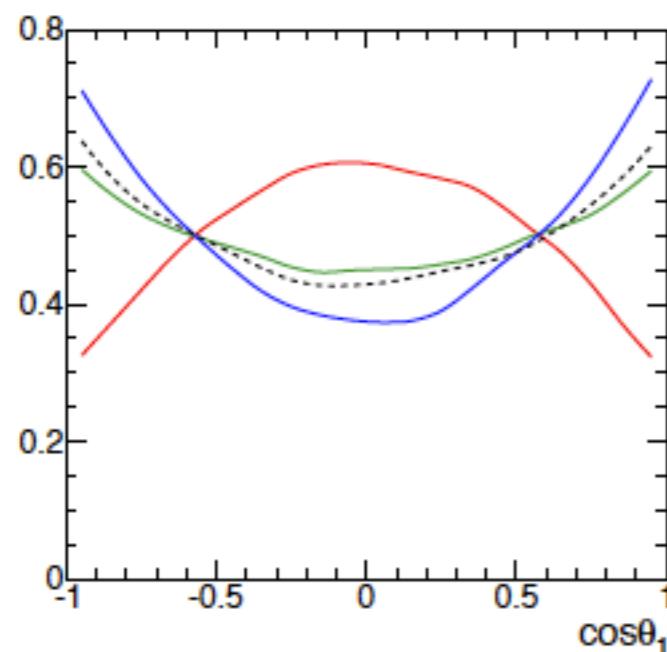
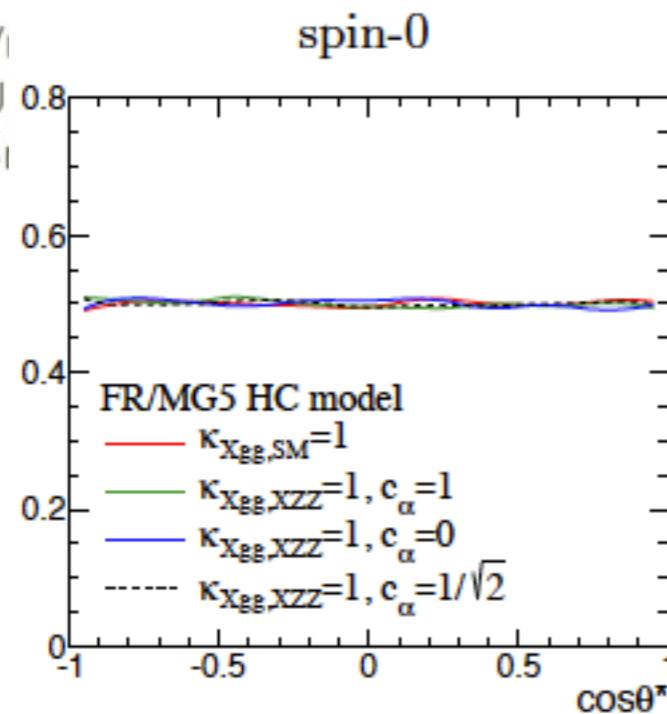
$$\mathcal{L}_2^f = -\frac{1}{\Lambda} \sum_{f=q,\ell} \kappa_f T_{\mu\nu}^f X_2^{\mu\nu}$$
$$\mathcal{L}_2^V = -\frac{1}{\Lambda} \sum_{V=Z,W,\gamma,g} \kappa_V T_{\mu\nu}^V X_2^{\mu\nu}$$

► The E-M tensor for QED:

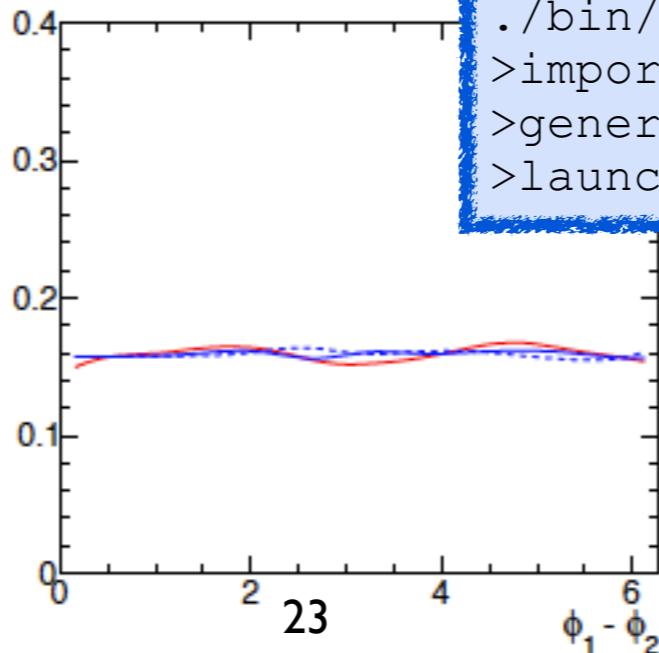
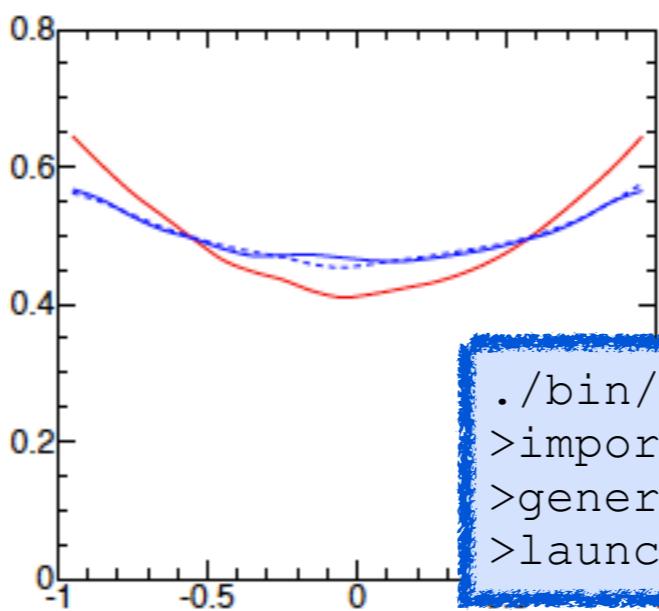
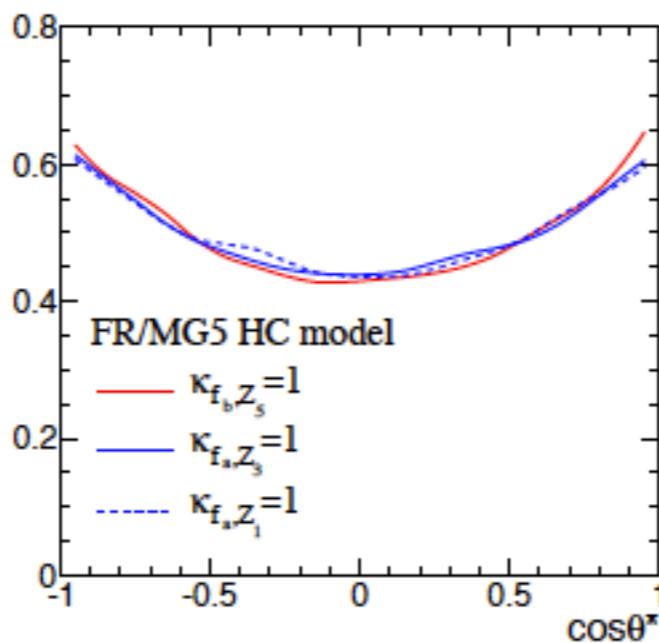
$$T_{\mu\nu}^f = -g_{\mu\nu} \left[\bar{\psi}_f (i\gamma^\rho D_\rho - m_f) \psi_f - \frac{1}{2} \partial^\rho (\bar{\psi}_f i\gamma_\rho \psi_f) \right] + \left[\frac{1}{2} \bar{\psi}_f i\gamma_\mu D_\nu \psi_f - \frac{1}{4} \partial_\mu (\bar{\psi}_f i\gamma_\nu \psi_f) + (\mu \leftrightarrow \nu) \right],$$
$$T_{\mu\nu}^\gamma = -g_{\mu\nu} \left[-\frac{1}{4} A^{\rho\sigma} A_{\rho\sigma} + \partial^\rho \partial^\sigma A_\sigma A_\rho + \frac{1}{2} (\partial^\rho A_\rho)^2 \right] - A_\mu^\rho A_{\nu\rho} + \partial_\mu \partial^\rho A_\rho A_\nu + \partial_\nu \partial^\rho A_\rho A_\mu,$$



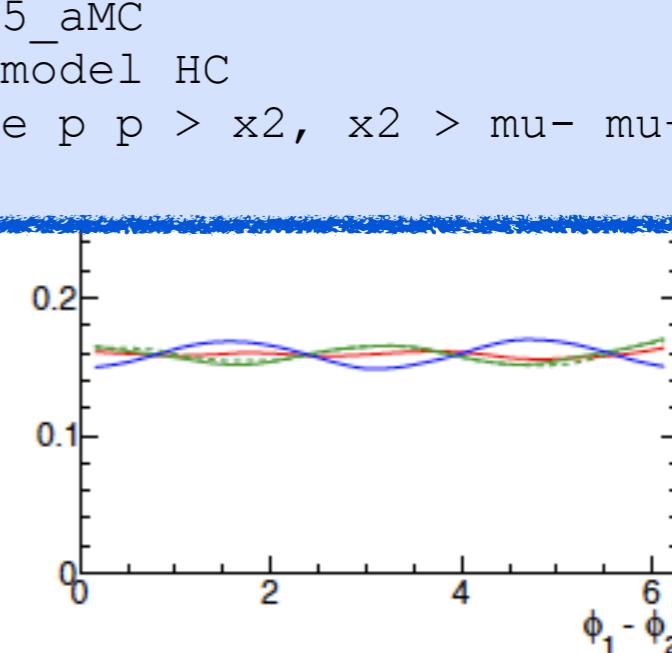
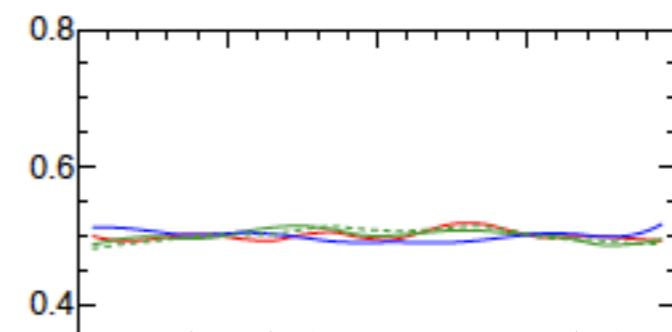
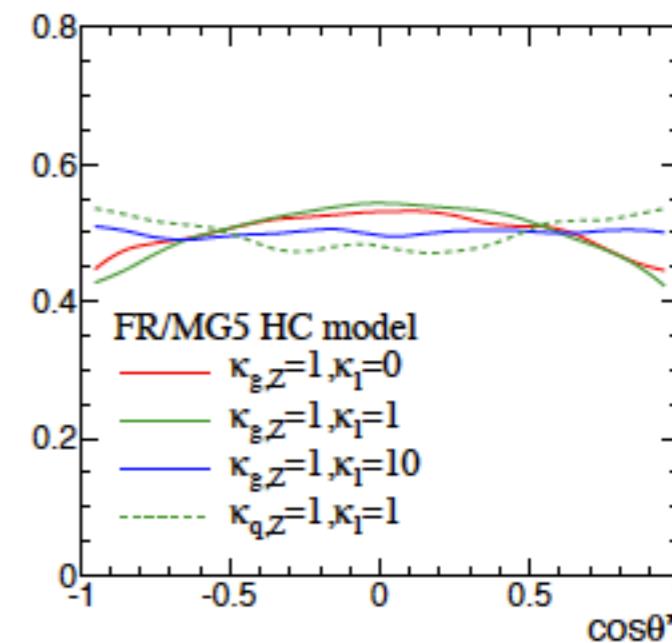
spin-0



spin-1



spin-2

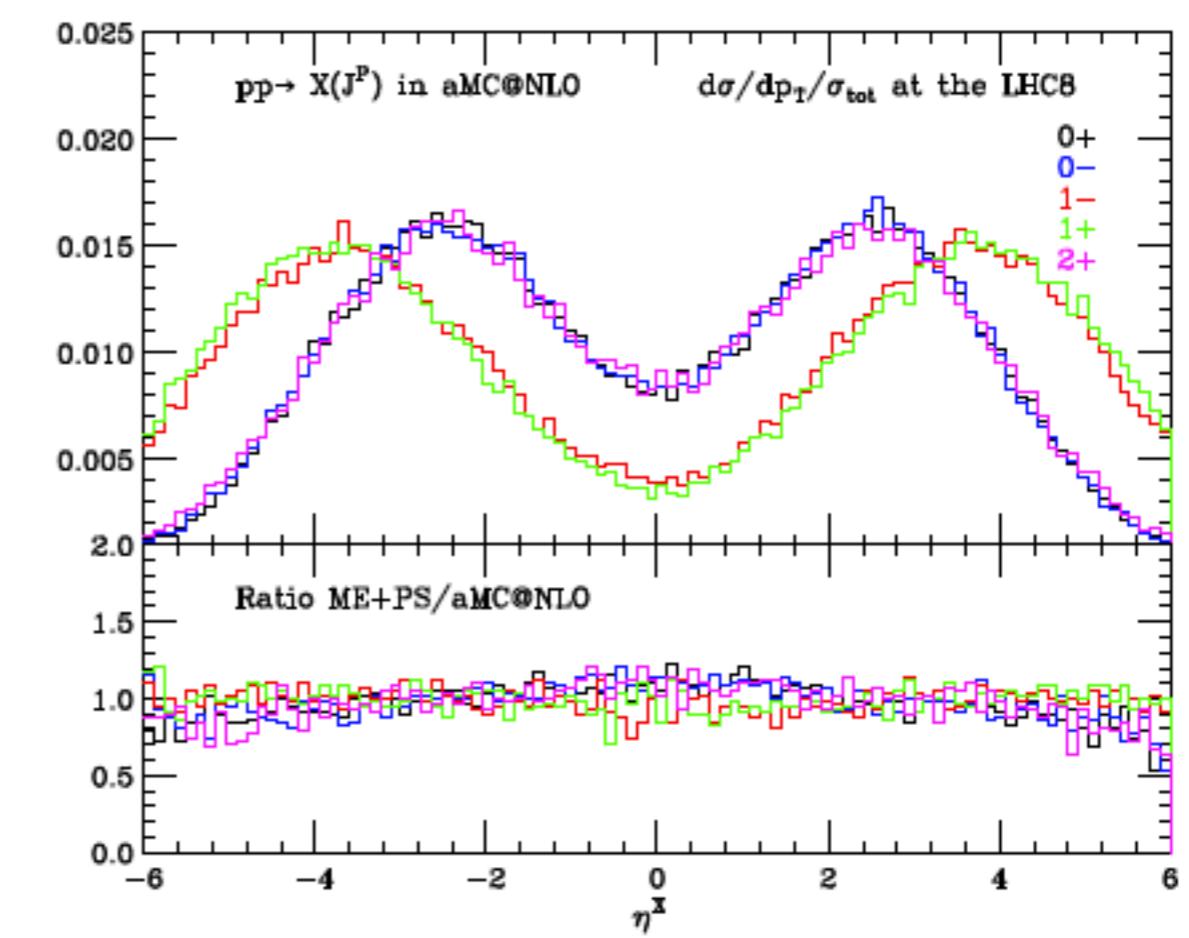
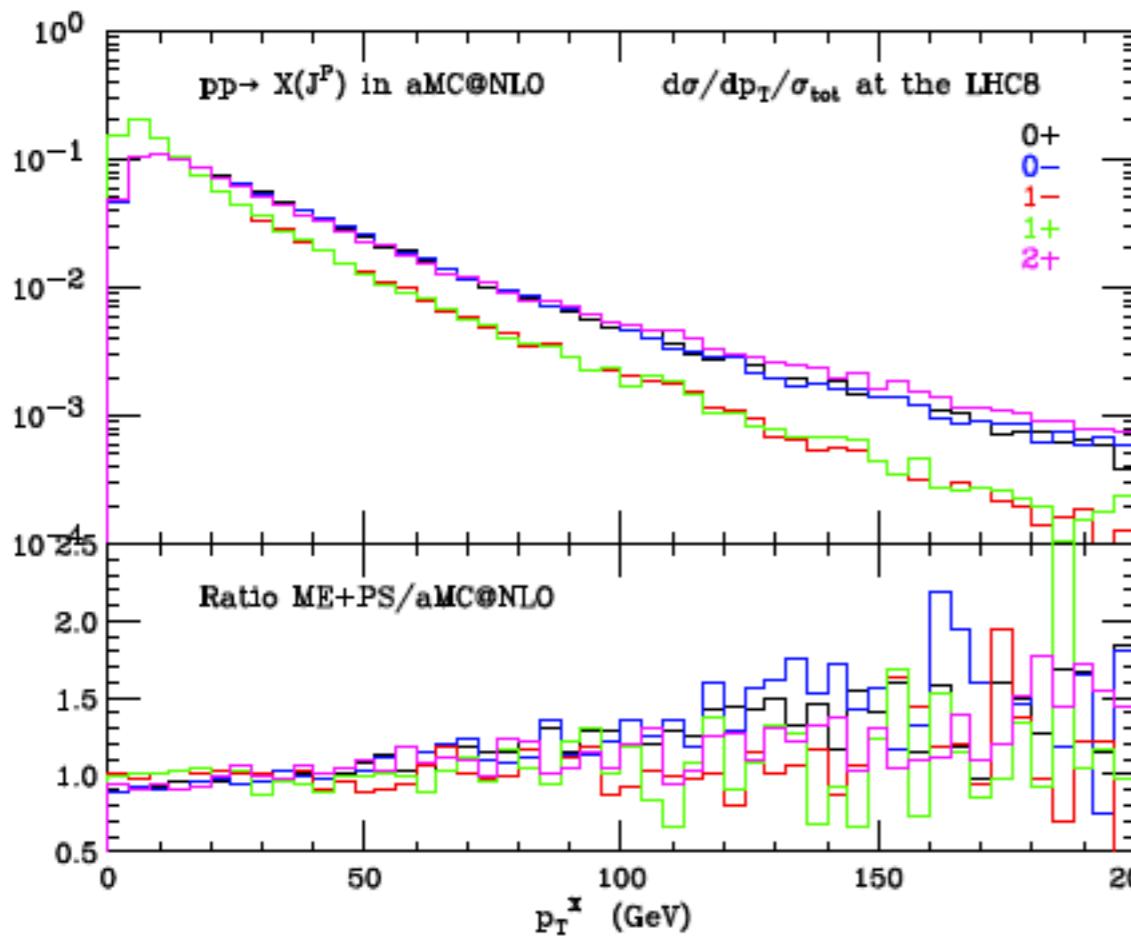


```
./bin/mg5_aMC  
>import model HC  
>generate p p > x1, x1 > mu- mu+ e- e+  
>launch
```

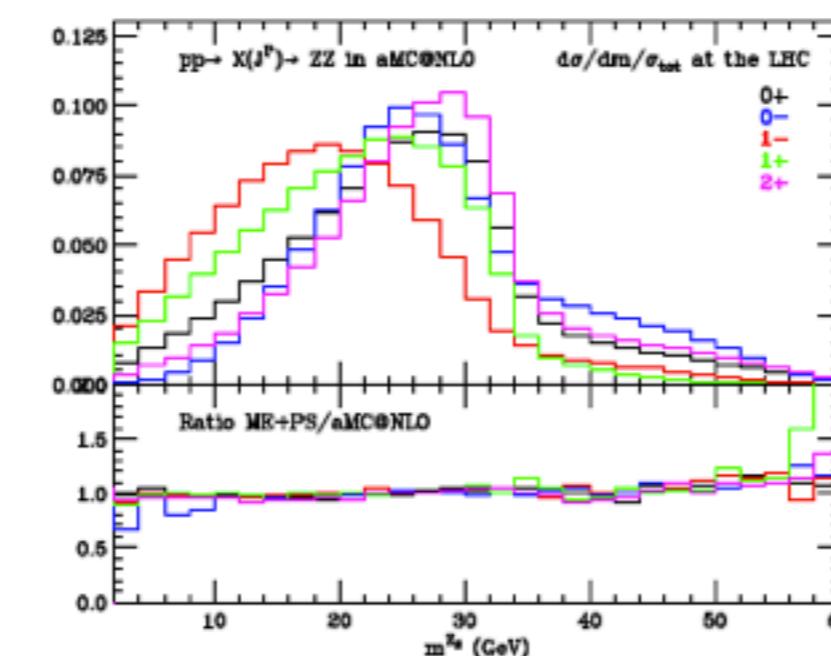
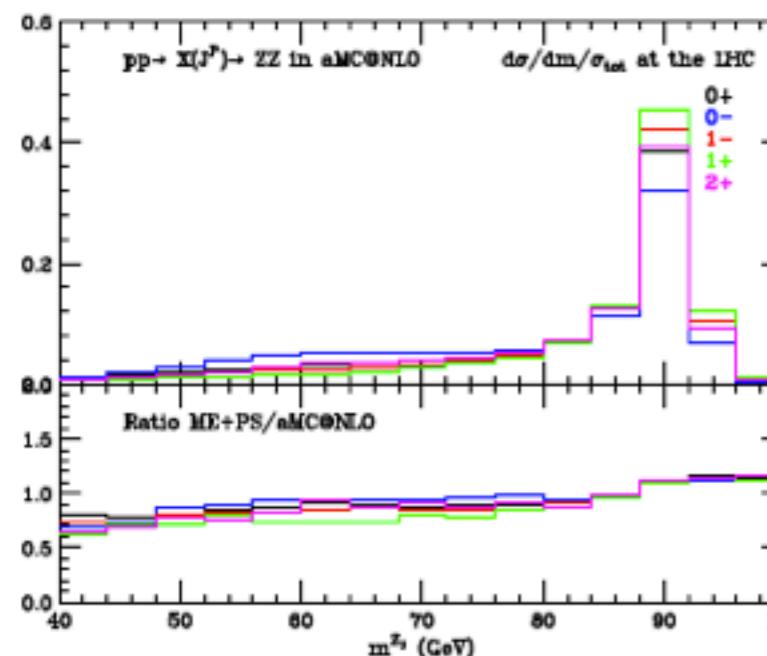
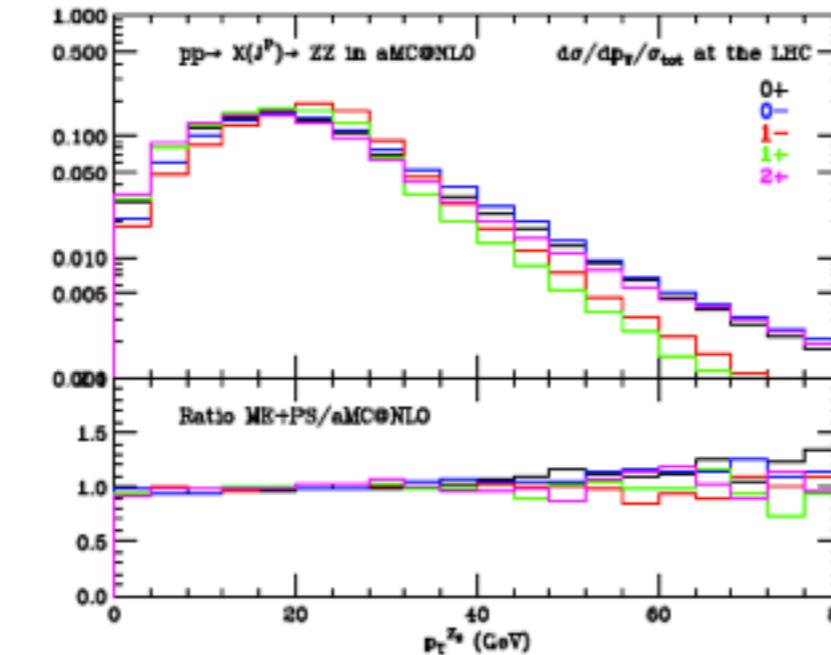
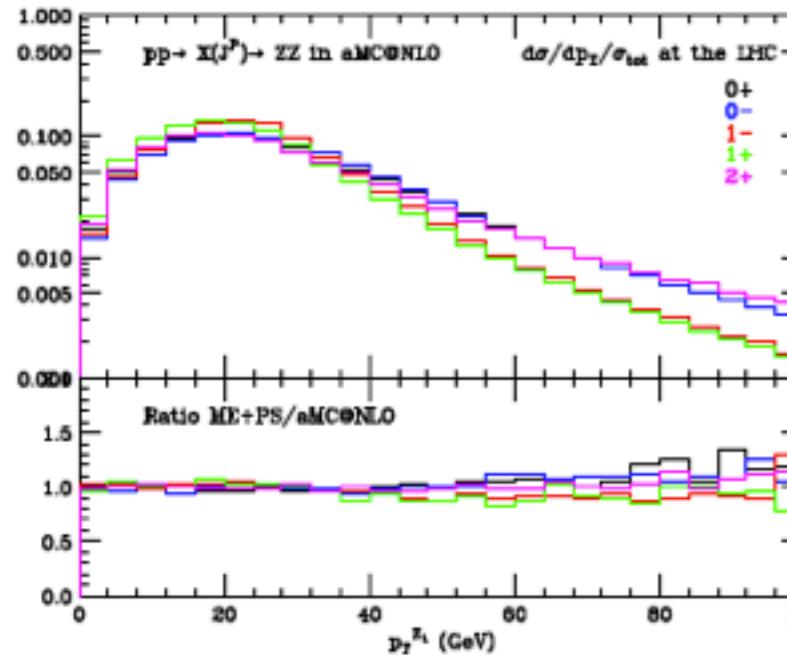
```
./bin/mg5_aMC  
>import model HC  
>generate p p > x2, x2 > mu- mu+ e- e+  
>launch
```

Higher order effects in QCD

- The LO predictions can be systematically improved by including the effects due to the emission of QCD partons.
 - ▶ LO Matrix-Element/Parton-Shower merging [ME+PS]
 - ▶ full-NLO matrix element with parton-shower [aMC@NLO+Pythia/Herwig]



aMC@NLO vs. ME+PS



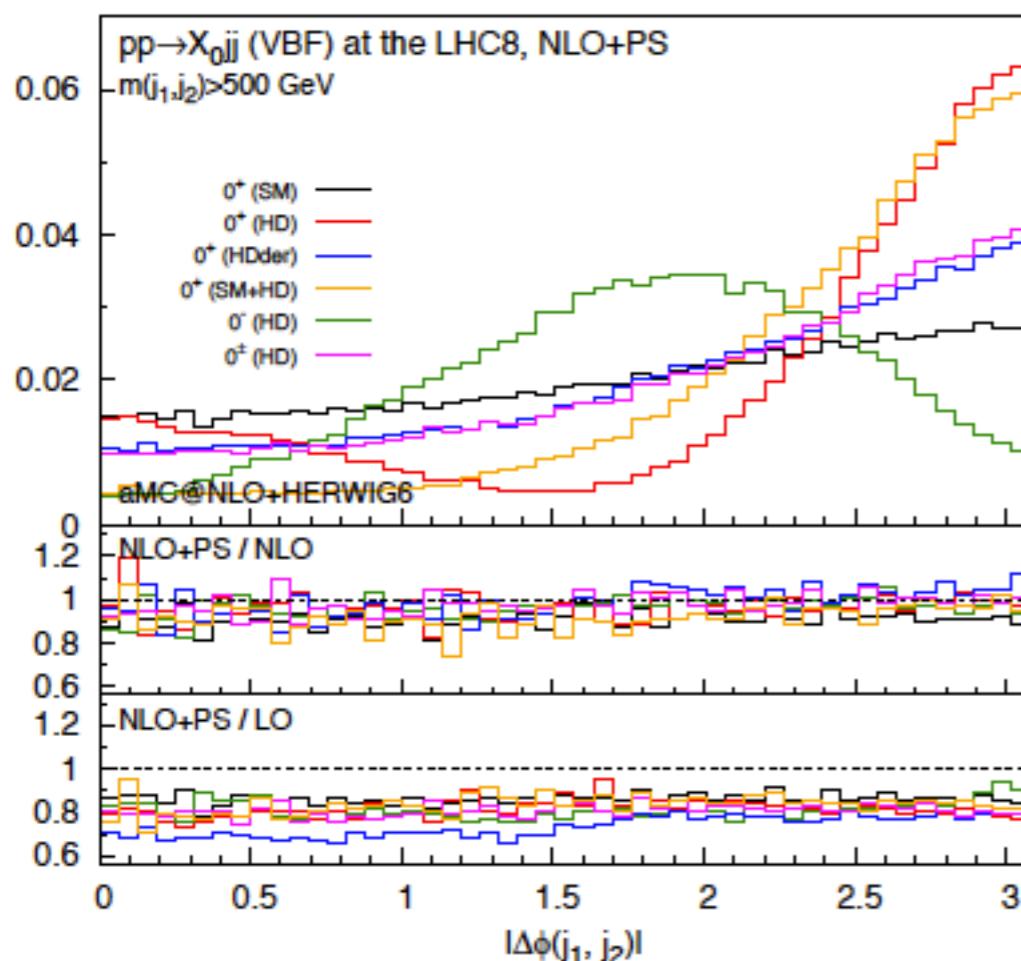
Good agreement between the ME+PS and aMC@NLO predictions for most observables.

For spin 0, the production and decay factorize, for spin 1 and 2 this does not happen and the full $2 \rightarrow 4,5$ matrix elements need to be used.

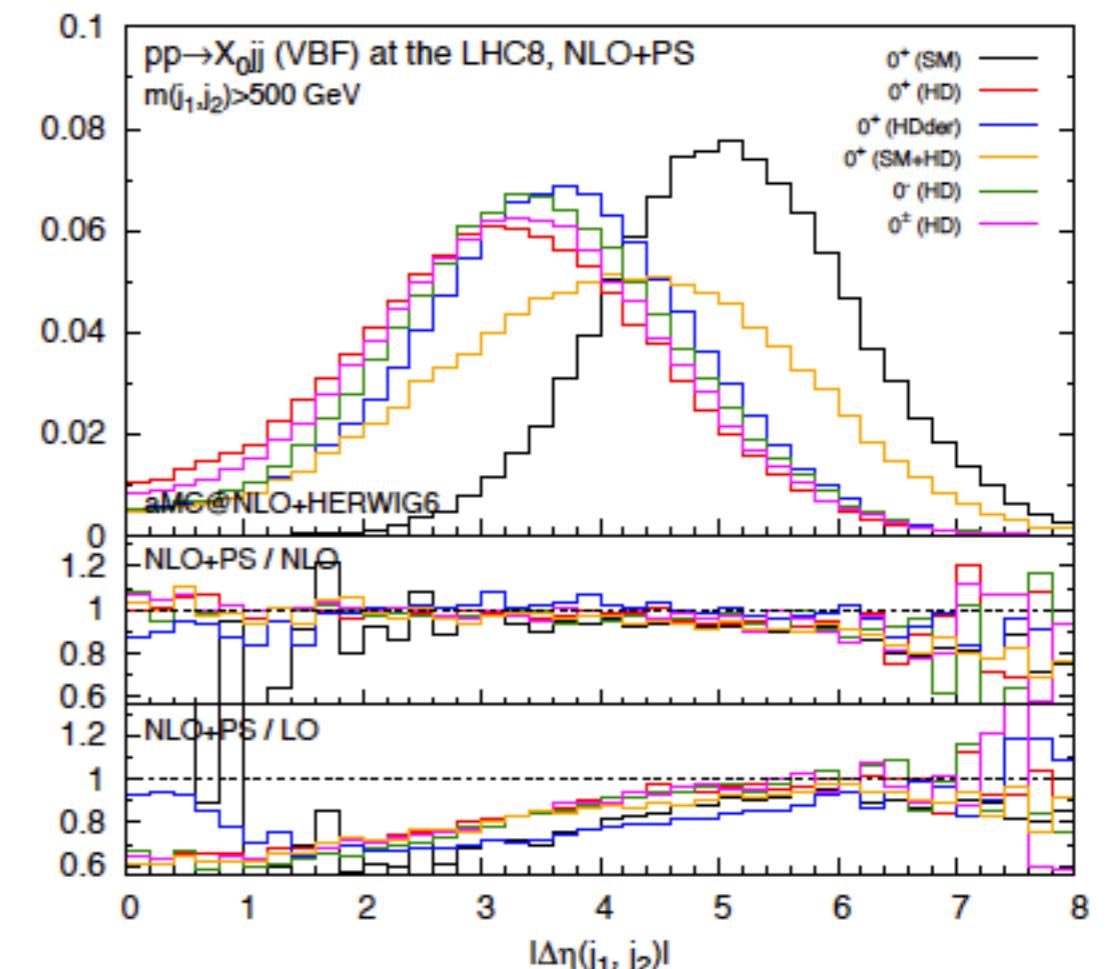
VBF

F.Maltoni, KM, M.Zaro [arXiv:1311.1829]

```
./bin/mg5_aMC
> import model HC_NLO
> generate p p > x0 j j QCD=0 [QCD]
> launch
```



scenario	σ_{LO} (fb)	σ_{NLO} (fb)	K		
0^+ (SM)	1509(1)	+4.7% -4.4%	1633(2) -1.5%	+2.0%	1.08
0^+ (HD)	69.66(6)	+7.5% -6.6%	67.08(13)	+2.2% -2.3%	0.96
0^+ (HDder)	721.9(6)	+11.0% -9.0%	684.9(1.5)	+2.3% -2.8%	0.95
0^+ (SM+HD)	3065(2)	+5.6% -5.1%	3144(5)	+1.6% -1.1%	1.03
0^- (HD)	57.10(4)	+7.7% -6.7%	55.24(11)	+2.1% -2.5%	0.97
0^\pm (HD)	63.46(5)	+7.6% -6.7%	61.07(13)	+2.3% -2.0%	0.96

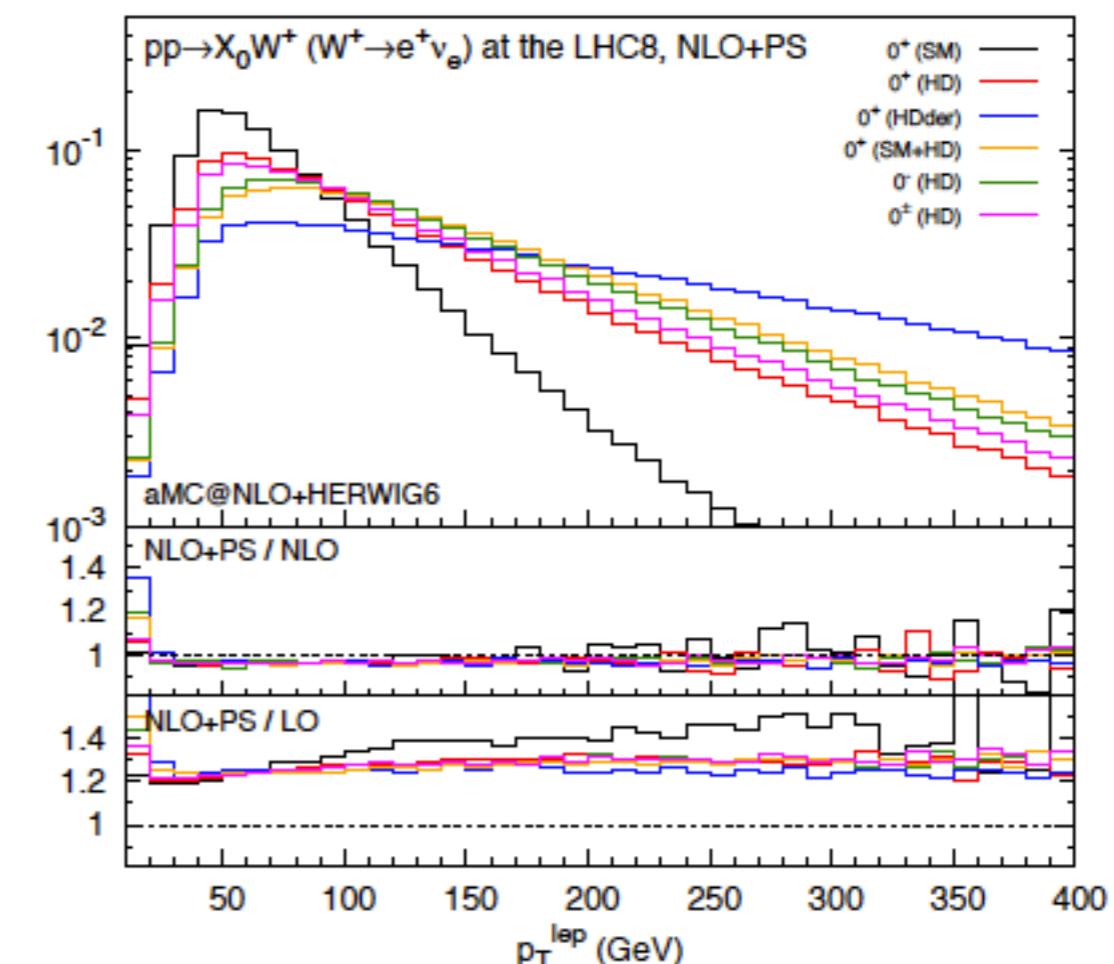
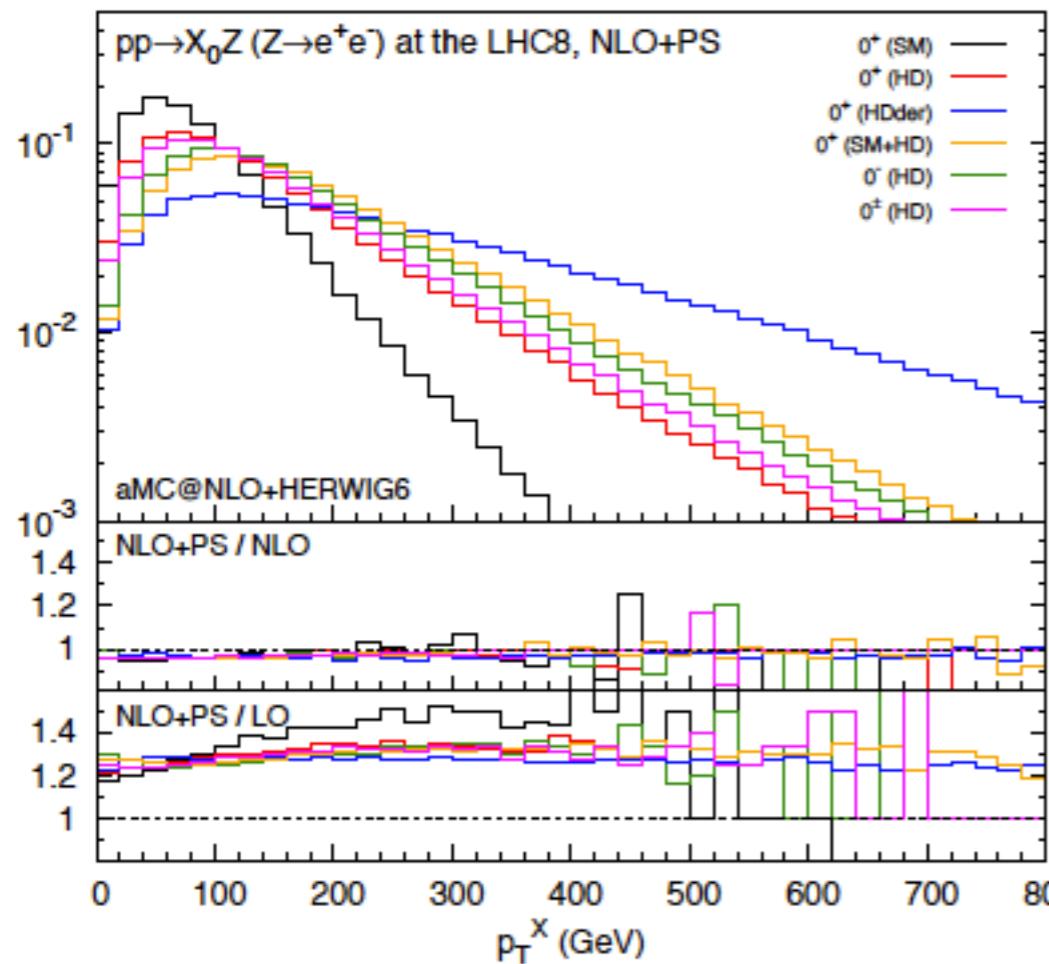


VH

F.Maltoni, KM, M.Zaro [arXiv:1311.1829]

```
./bin/mg5_aMC
> import model HC_NLO
> generate p p > x0 e+ e- [QCD]
> launch
```

scenario	σ_{LO} (fb)	σ_{NLO} (fb)	K
0^+ (SM)	39.58(3)	$+0.1\%$ -0.6%	51.22(5) $+2.2\%$ -1.8%
0^+ (HD)	13.51(1)	$+1.5\%$ -1.7%	17.51(1) $+1.9\%$ -1.3%
0^+ (HDder)	324.2(2)	$+4.7\%$ -4.3%	416.1(4) $+2.3\%$ -2.1%
0^+ (SM+HD)	118.8(1)	$+3.0\%$ -2.9%	154.2(1) $+1.8\%$ -1.6%
0^- (HD)	8.386(7)	$+2.6\%$ -2.6%	10.89(1) $+1.8\%$ -1.5%
0^\pm (HD)	10.96(1)	$+1.9\%$ -2.1%	14.22(1) $+1.8\%$ -1.3%



Summary

- After the discovery of a Higgs-like resonance at the LHC, the main focus of the analyses now is **the determination of the Higgs Lagrangian.**
- This includes
 - **the structure of the operators**, linked to the spin/parity of a ‘Higgs’ boson.
 - **the coupling strength.**
- MC tools to study the property of the SM-like boson are publicly available, e.g. JHUGen, HC, HEL.
- Event generation at NLO is possible for (several) spin 0,1,2 hypothesis and can be used to validate merged samples.

HiggsCharacterisation - FeynRules

http://feynrules.irmp.ucl.ac.be/wiki/HiggsCharacterisation

Google

gmail KEKmail VUBintra VUBmail raptools mawatari IIHE CP3 cluster translate inspire Wolfram workshop alc mac install Apple Wiki

HiggsCharacterisation - FeynRules

The Higgs Characterisation model

Authors

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- NLO
 - Federico Demartin (Université Catholique de Louvain) & Marco Zaro (LPTHE, Paris)
 - Emails: federico.demartin @ uclouvain.be

Description of the model

This is a model file for the characterisation of the boson recently discovered at the LHC. Our effective lagrangian consists of the SM (except for the Higgs itself), expressed through the physical degrees of freedom present below the EWSB scale, plus a new bosonic state $X(J^P)$ with spin/parity assignments $J^P = 0^+, 0^-, 1^+, 1^-,$ or 2^+ . The new state can couple to SM particles via interactions of the lowest possible dimensions. In addition, the state 0^+ is allowed to mix with the 0^- one, and can interact with SM particles with higher-dimensional operators beyond those of the SM. See more details in

- ↗[1306.6464](#) : P. Artoisenet, P. de Aquino, F. Demartin, R. Frederix, S. Frixione, F. Maltoni, M. K. Mandal, P. Mathews, K. Mawatari, V. Ravindran, S. Seth, P. Torrielli, M. Zaro, "A framework for Higgs characterisation" (*JHEP11(2013)043*).
- ↗[1307.5607](#) : P. de Aquino, K. Mawatari, "Characterising a Higgs-like resonance at the LHC" (*Proceedings for HPNP2013*).
- ↗[1311.1829](#) : F. Maltoni, K. Mawatari, M. Zaro, "Higgs characterisation via vector-boson fusion and associated production: NLO and parton-shower effects" (*EPJC74(2014)2710*).

Model files for LO

- [HC.fr](#) : the main model file.
- [SM_HC.fr](#) : This model requires the modified Standard Model implementation of [FeynRules](#).
- [Massless.rst](#), [Cabibbo.rst](#) : SM restriction files.
- [HC.nb](#) : this is an example Mathematica notebook that loads the model, calculates the Feynman rules and extract the model files within the UFO format.
- [HC_UFO.zip](#) : The model files in UFO format (for MadGraph5).

Model files for NLO (only for the $J=0$ case)

A few remarks before use; see the README file in the model.

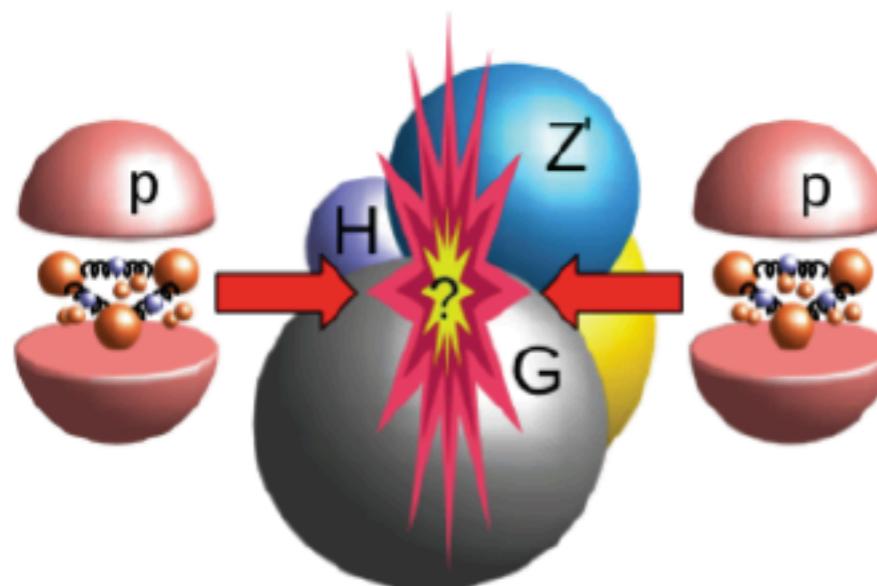
- [HC_NLO_X0_UFO.zip](#) : The model files in UFO format (for MadGraph5).

back-up

FeynRules(v2.0) in a nutshell

Alloul, Christensen, Degrande, Duhr, Fuks, <http://feynrules.irmp.ucl.ac.be>

- a Mathematica package that allows to derive Feynman rules from a Lagrangian.
- allows to export the Feynman rules to various matrix element generators, e.g. MadGraph, Herwig, Sherpa, ...
- The only requirements on the Lagrangian are:
 - ✓ Locality, Lorentz and gauge invariance **new!**
 - ✓ Supported field types: spin-0, 1/2, 1, **3/2.** and 2 (as well as superfields). [Christensen, de Aquino, Deutschmann, Duhr, Fuks, Garcia-Cely, Mattelaer, Mawatari, Oexl, Takaesu, EPJC(2013)]



MC Generator based on the papers:

"Spin-Determination of Single-Produced Resonances at Hadron Colliders"

Yanyan Gao, Andrei V. Gritsan, Zijin Guo, Kirill Melnikov, Markus Schulze and Nhan V. Tran
<http://arxiv.org/abs/1001.3396>

"On the Spin and Parity of a Single-Produced Resonance at the LHC"

Sara Bolognesi, Yanyan Gao, Andrei V. Gritsan, Kirill Melnikov, Markus Schulze, Nhan V. Tran and Andrew Whitbeck
<http://arxiv.org/abs/1208.4018>

"Constraining anomalous HVV interactions at proton and lepton colliders"

Ian Anderson, Sara Bolognesi, Fabrizio Caola, Yanyan Gao, Andrei V. Gritsan, Christopher B. Martin, Kirill Melnikov, Markus Schulze, Nhan V. Tran, Andrew Whitbeck, Yaofu Zhou
<http://arxiv.org/abs/1309.4819>

contacts: Markus Schulze, Nhan Tran ([email us](#))

[Home](#) [Download \(requires registration\)](#) [Manual](#)

References:

Please reference the above papers and refer to "JHU generator" when using the simulation program and "MELA" when using the matrix element likelihood analysis technique. The latter was also introduced in [1, 2]. The matrix element package (MELA) also depends on MCFM libraries for background parameterization which should be referenced [3] when used.

Generator for $pp \rightarrow X \rightarrow VV, VBF, X+JJ, pp \rightarrow VX, ee \rightarrow VX$

Description:

A generator giving parton-level information including full spin and polarization correlations for the processes $ab \rightarrow X \rightarrow VV$ ($V=Z,W,\gamma$), VBF, $X+JJ$, $pp \rightarrow VX$, $ee \rightarrow VX$. The Fortran program produces a single-produced X resonance via either the gluon fusion, $q\bar{q}$, VBF, or VH processes for either the Tevatron, the LHC, or an e^+e^- collider. The resonance X can be a spin-zero, -one, or -two particle with general couplings defined in the study. The output is in the LHE format and can be input to any parton showering program. Please see the manual file for further technical details.

Requirements:

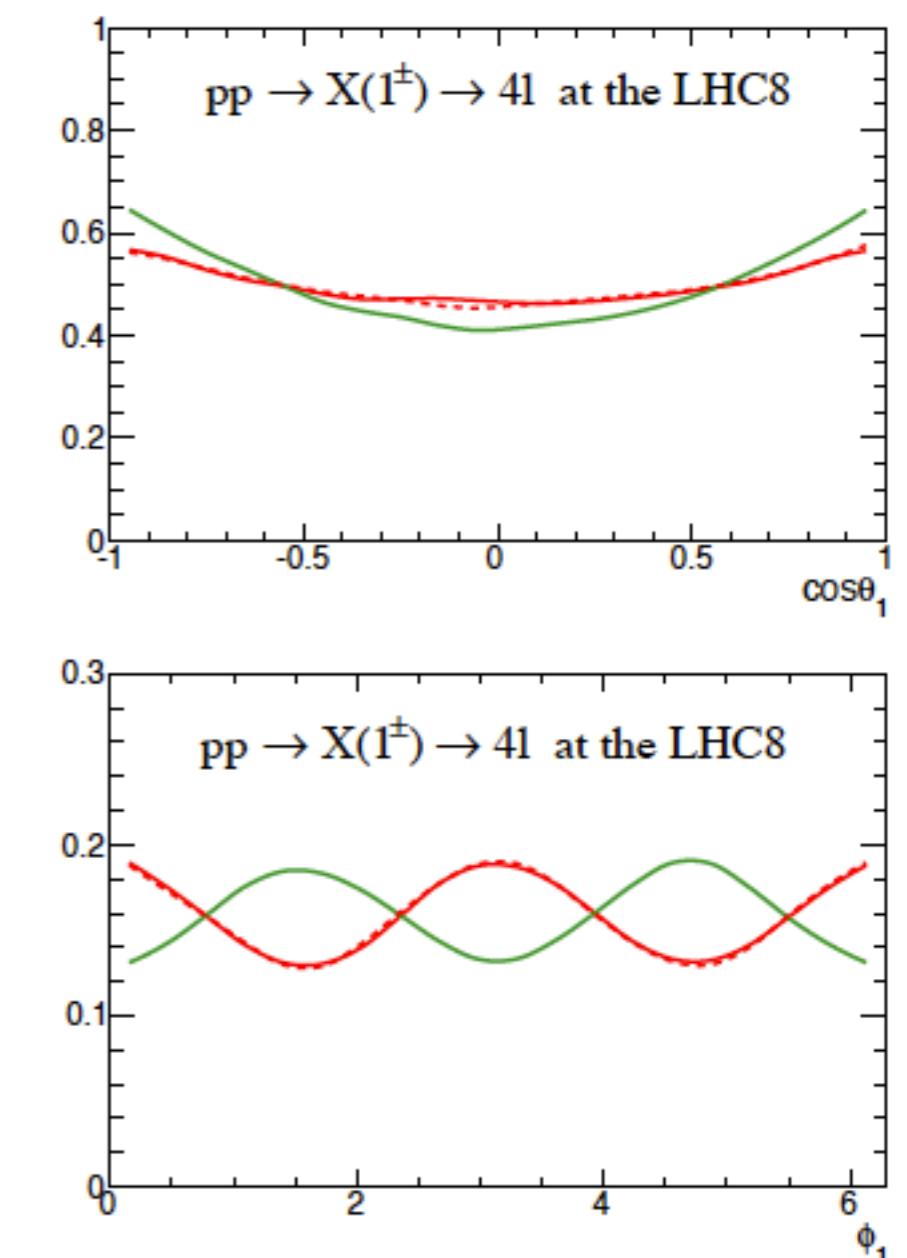
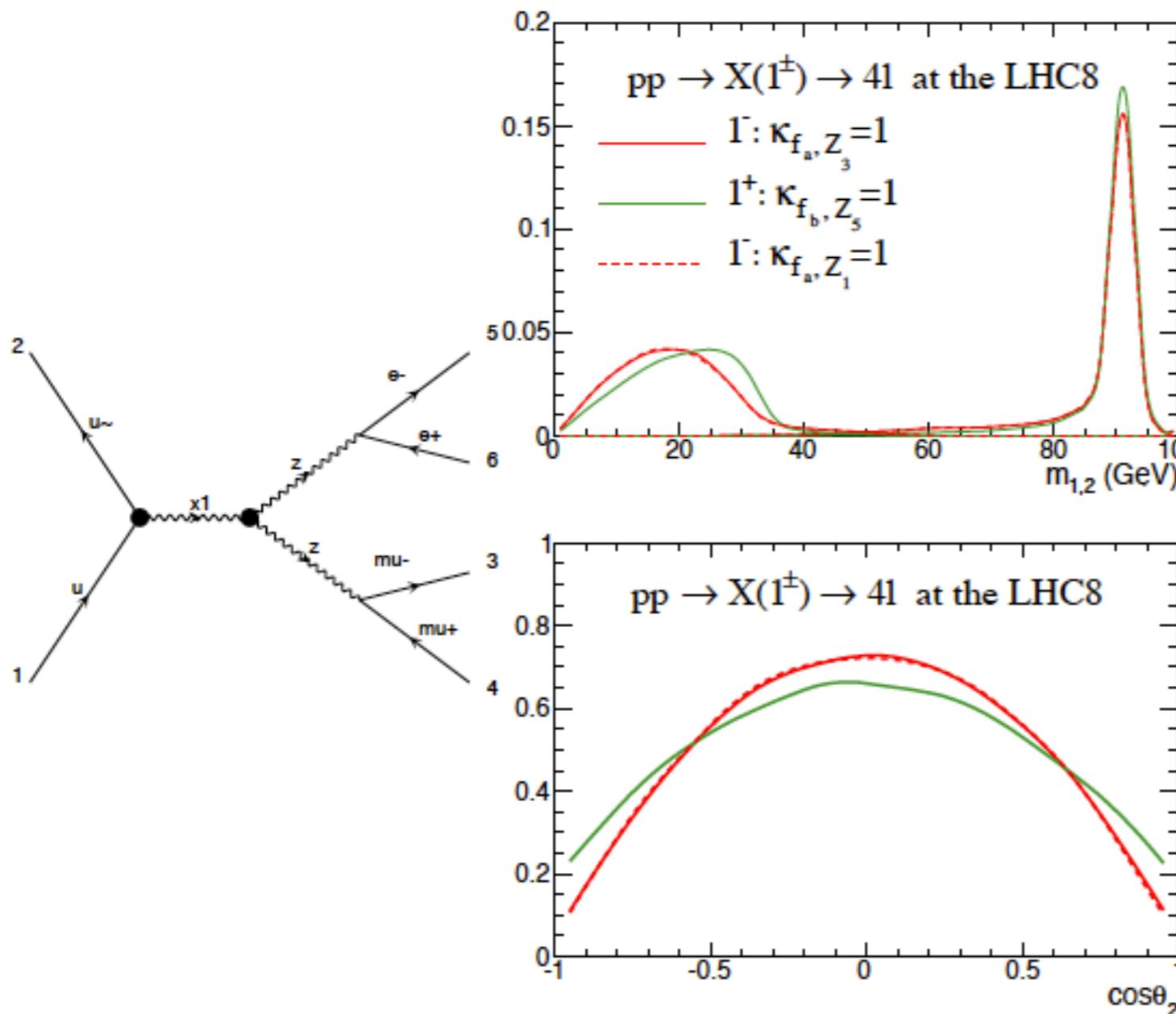
`ifort` (Intel Fortran compiler) or `gfortran` (part of `gcc`)

Effective Lagrangian → Amplitudes

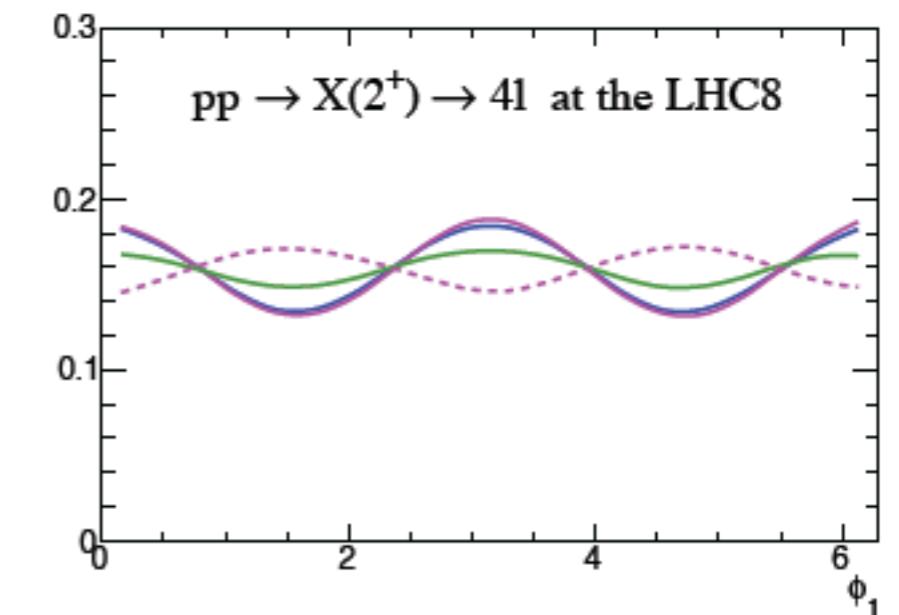
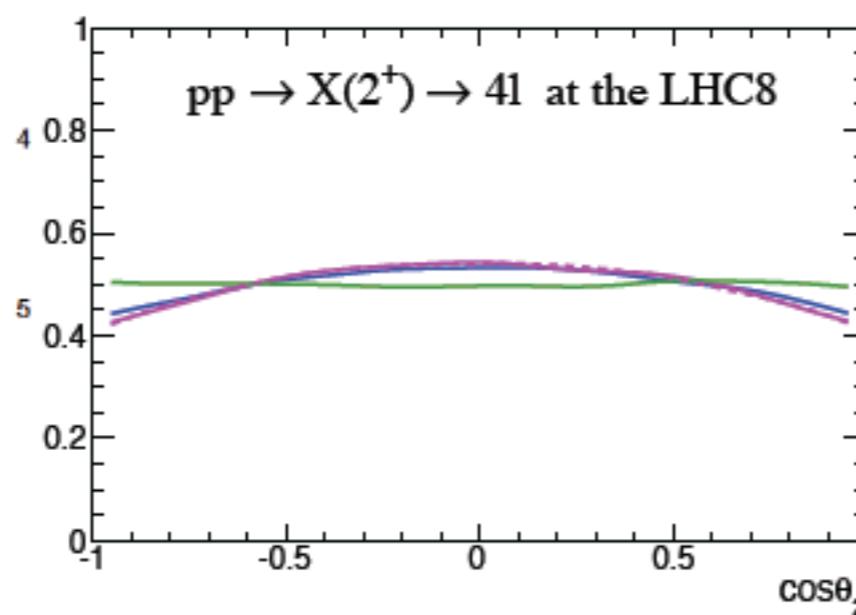
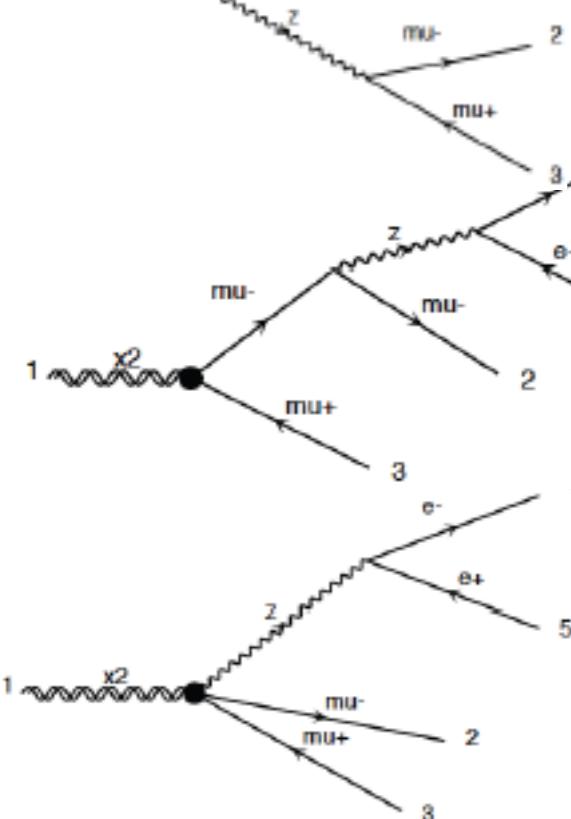
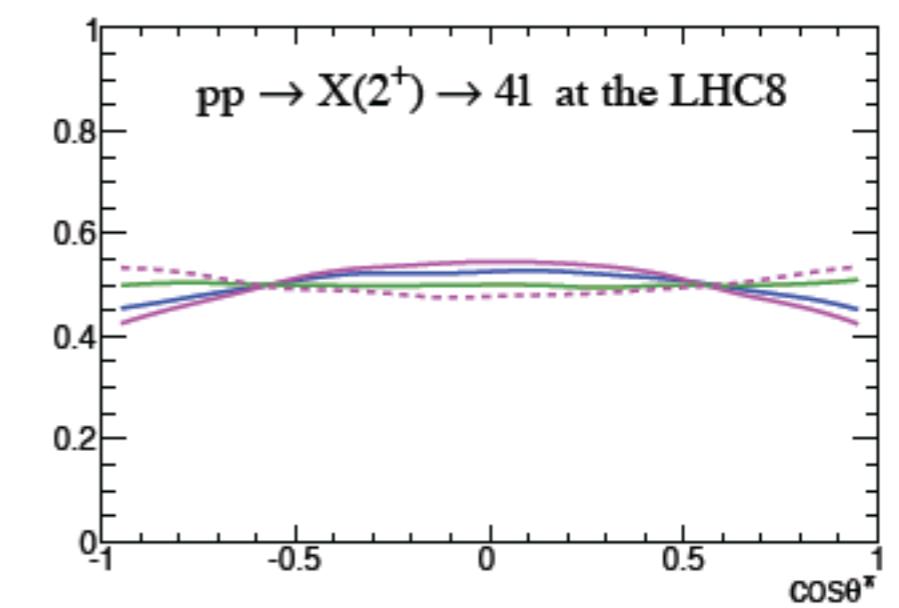
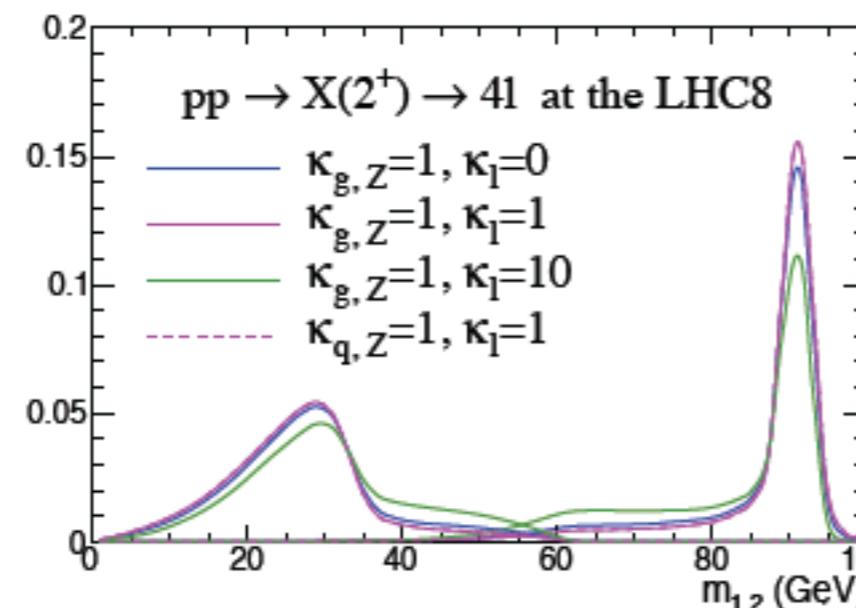
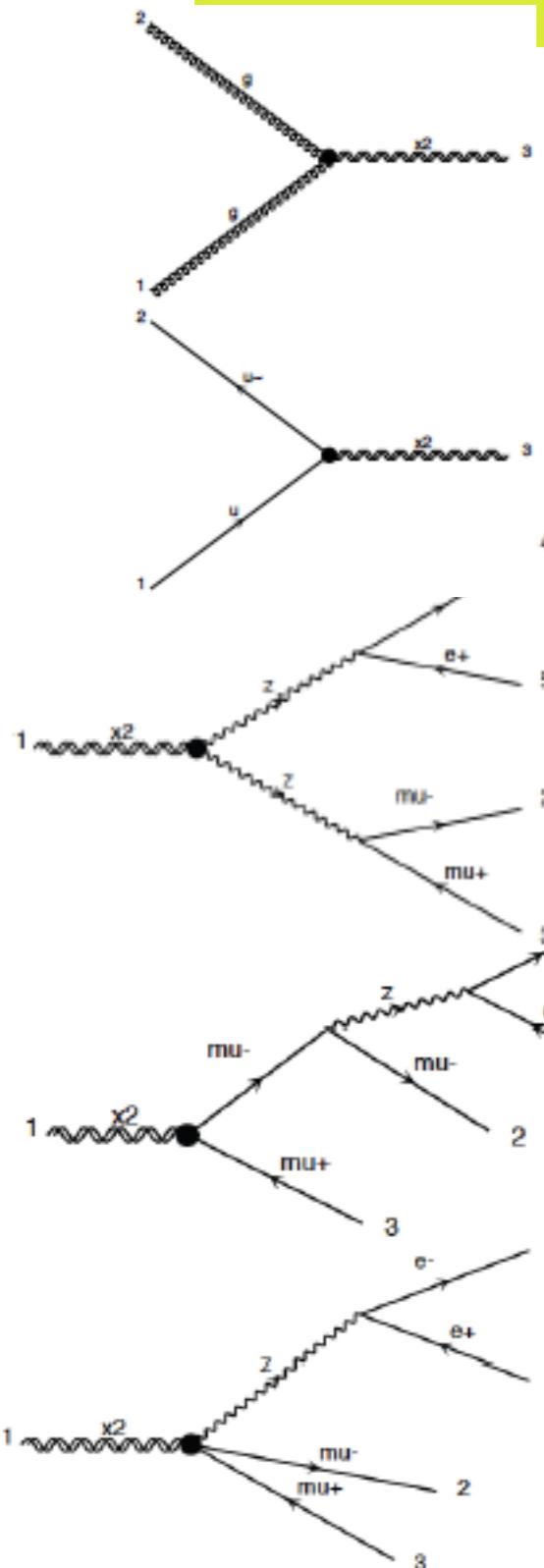
$$\begin{aligned}
 \mathcal{L} &= \frac{1}{2} c_\alpha \kappa_{\text{SM}} g_{HZZ} Z_\mu Z^\mu X_0 \longrightarrow i c_\alpha \kappa_{\text{SM}} g_{HZZ} g_{\mu\nu} && (\text{g}_1 \text{ term in JHU}) \\
 -\frac{1}{4} \frac{1}{\Lambda} c_\alpha \kappa_{HZZ} Z_{\mu\nu} Z^{\mu\nu} &\longrightarrow i c_\alpha \frac{\kappa_{HZZ}}{\Lambda} (g_{\mu\nu} q_1 \cdot q_2 - q_{2\mu} q_{1\nu}) && (\text{g}_2 \text{ term in JHU}) \\
 -\frac{1}{4} \frac{1}{\Lambda} s_\alpha \kappa_{AZZ} Z_{\mu\nu} \tilde{Z}^{\mu\nu} &\longrightarrow i s_\alpha \frac{\kappa_{AZZ}}{\Lambda} \epsilon_{\mu\nu\rho\sigma} q_2^\rho q_1^\sigma && (\text{g}_4 \text{ term in JHU}) \\
 -\frac{1}{\Lambda} c_\alpha \kappa_{H\partial Z} Z_\nu \partial_\mu Z^{\mu\nu} &\longrightarrow i c_\alpha \frac{\kappa_{H\partial Z}}{\Lambda} [g_{\mu\nu} (q_1 \cdot q_1 + q_2 \cdot q_2) - q_{1\mu} q_{1\nu} - q_{2\mu} q_{2\nu}]
 \end{aligned}$$

JHU scenario	HC parameter choice	
	X production	X decay
0_m^+	$\kappa_{Hgg} \neq 0$	$\kappa_{\text{SM}} \neq 0$ ($c_\alpha = 1$)
0_h^+	$\kappa_{Hgg} \neq 0$	$\kappa_{H\gamma\gamma, HZZ, HWW} \neq 0$ ($c_\alpha = 1$)
0^-	$\kappa_{Agg} \neq 0$	$\kappa_{A\gamma\gamma, AZZ, AWW} \neq 0$ ($c_\alpha = 0$)
1^+	$\kappa_{fa, f_b} \neq 0$	$\kappa_{Z_5, W_5} \neq 0$
1^-	$\kappa_{fa, f_b} \neq 0$	$\kappa_{Z_3, W_3} \neq 0$
2_m^+	$\kappa_q \neq 0$	$\kappa_{\gamma, Z, W} \neq 0$

Mass and angular distributions -- spin I

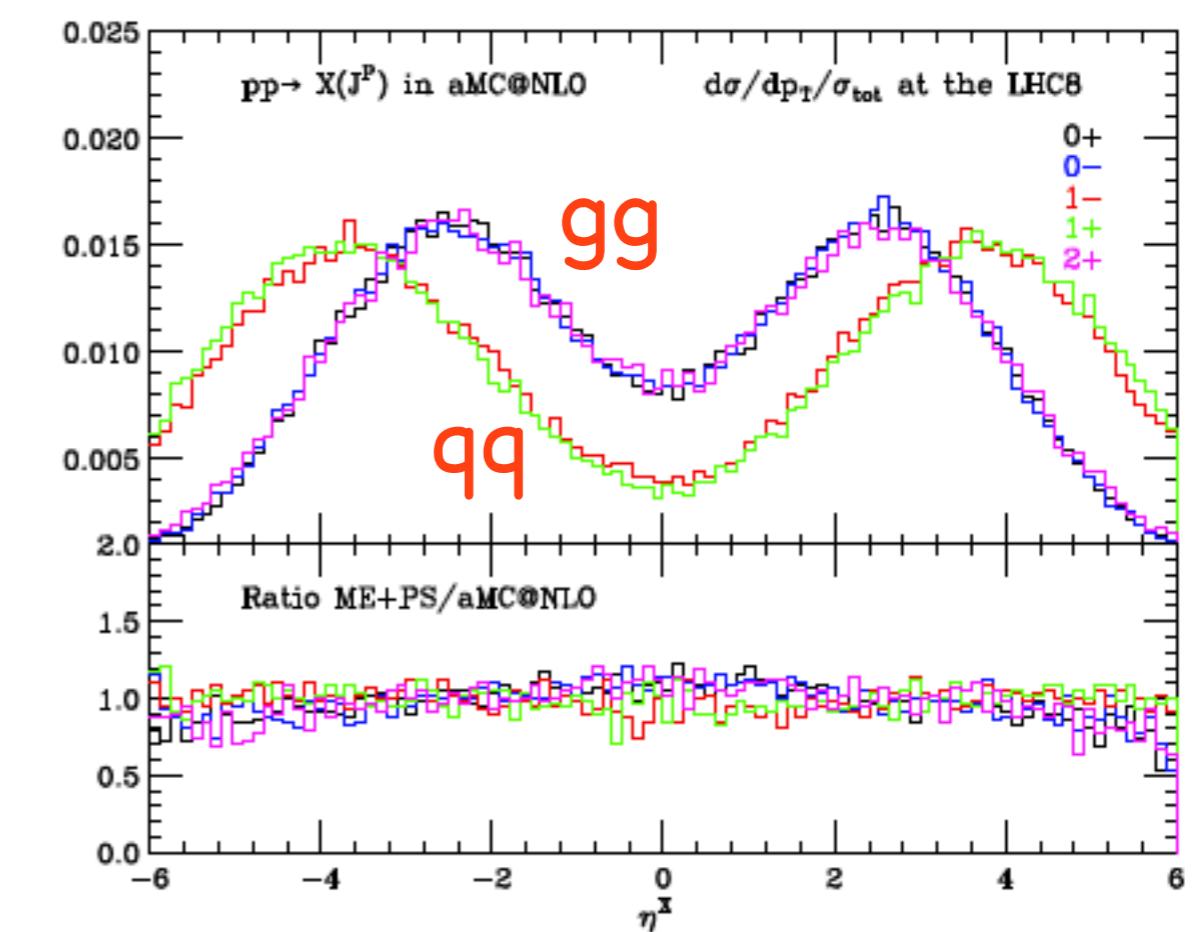
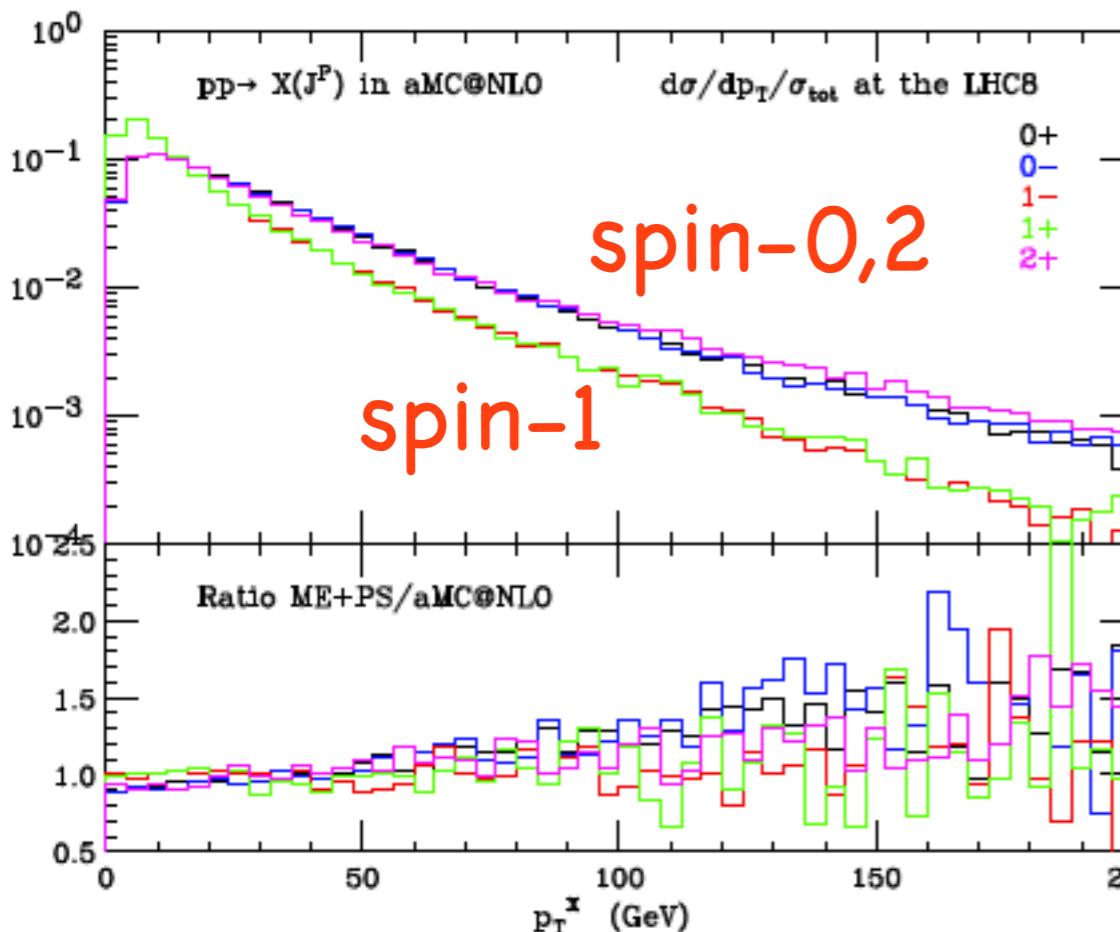


Mass and angular distributions -- spin2



Higher order effects in QCD (I)

inclusive production in $\text{pp} \rightarrow X(J^P)$



The matched sample is harder than aMC@NLO at large pT due to the extra 2 ME patrons in the matched sample.

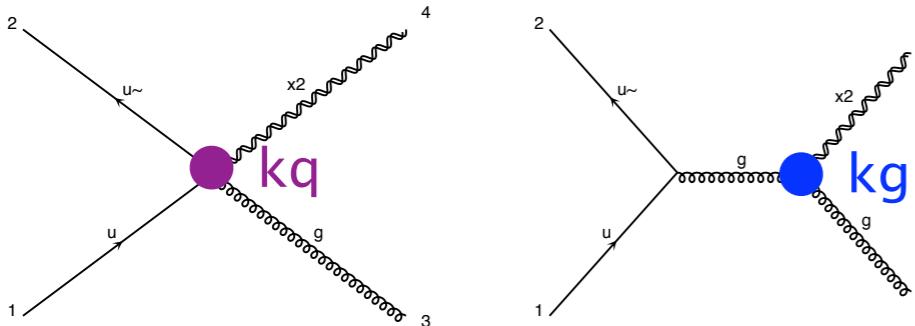
excellent agreement between
ME+PS and aMC@NLO

The different shapes are due to the different initial state.

Higher order effects in QCD (II)

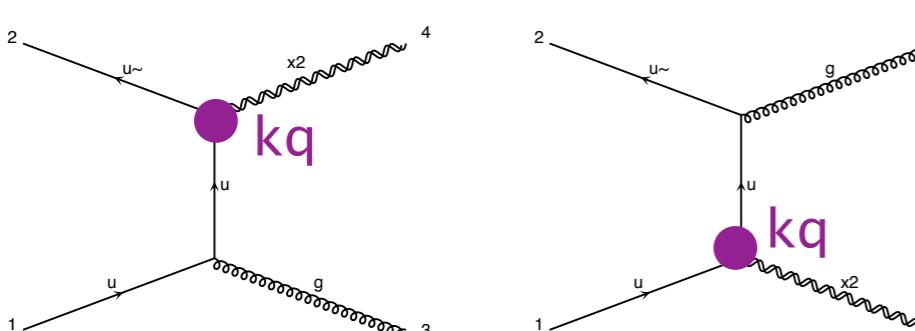
unitarity-violating behavior of models with a spin-2 state

$$\mathcal{L} = -\frac{1}{\Lambda} \kappa_q T_{\mu\nu}^q X_2^{\mu\nu} - \frac{1}{\Lambda} \kappa_g T_{\mu\nu}^g X_2^{\mu\nu}$$



$$|\mathcal{M}|^2 \propto s/\Lambda^2 \quad \text{for } \kappa_q = \kappa_g$$

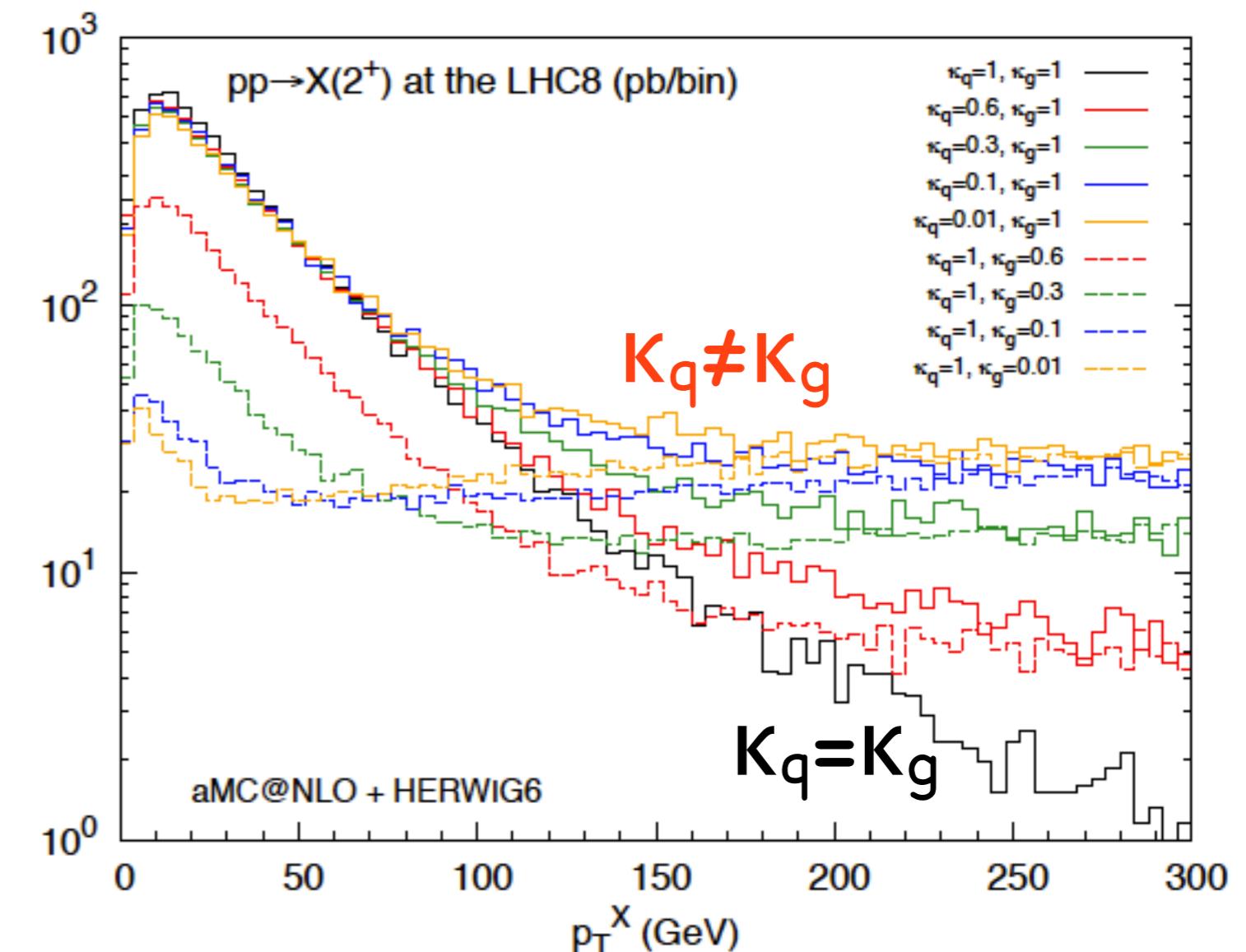
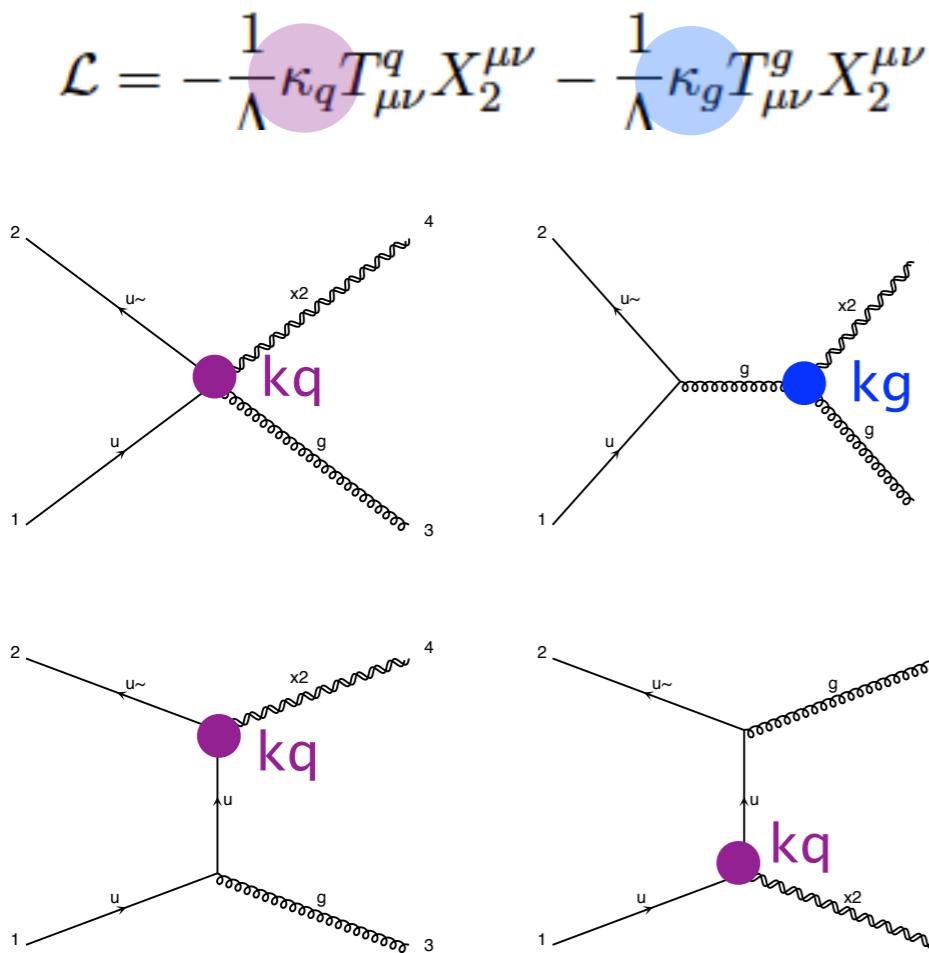
$$|\mathcal{M}|^2 \propto s^3/m^4 \Lambda^2 \quad \text{for } \kappa_q \neq \kappa_g$$



$$|\mathcal{M}|^2 = \frac{N}{\Lambda^2 s t u m^4} \left\{ 3 \kappa_g^2 m^4 [2m^4 - 2m^2(t+u) + t^2 + u^2] [m^4 - m^2(t+u) + 4tu] \right. \\ + (\kappa_q - \kappa_g) 6 \kappa_g m^4 s [m^6 + m^2 s(s+2u) - 2su(s+u)] \\ + (\kappa_q - \kappa_g)^2 s [6m^{10} - 6m^8(t+u) + 3m^6(t^2 + u^2) - 12m^4 tu(t+u) \\ \left. + 2m^2 tu(t^2 + 12tu + u^2) - 2tu(t^3 + t^2u + tu^2 + u^3)] \right\}, \quad (4.2)$$

Higher order effects in QCD (II)

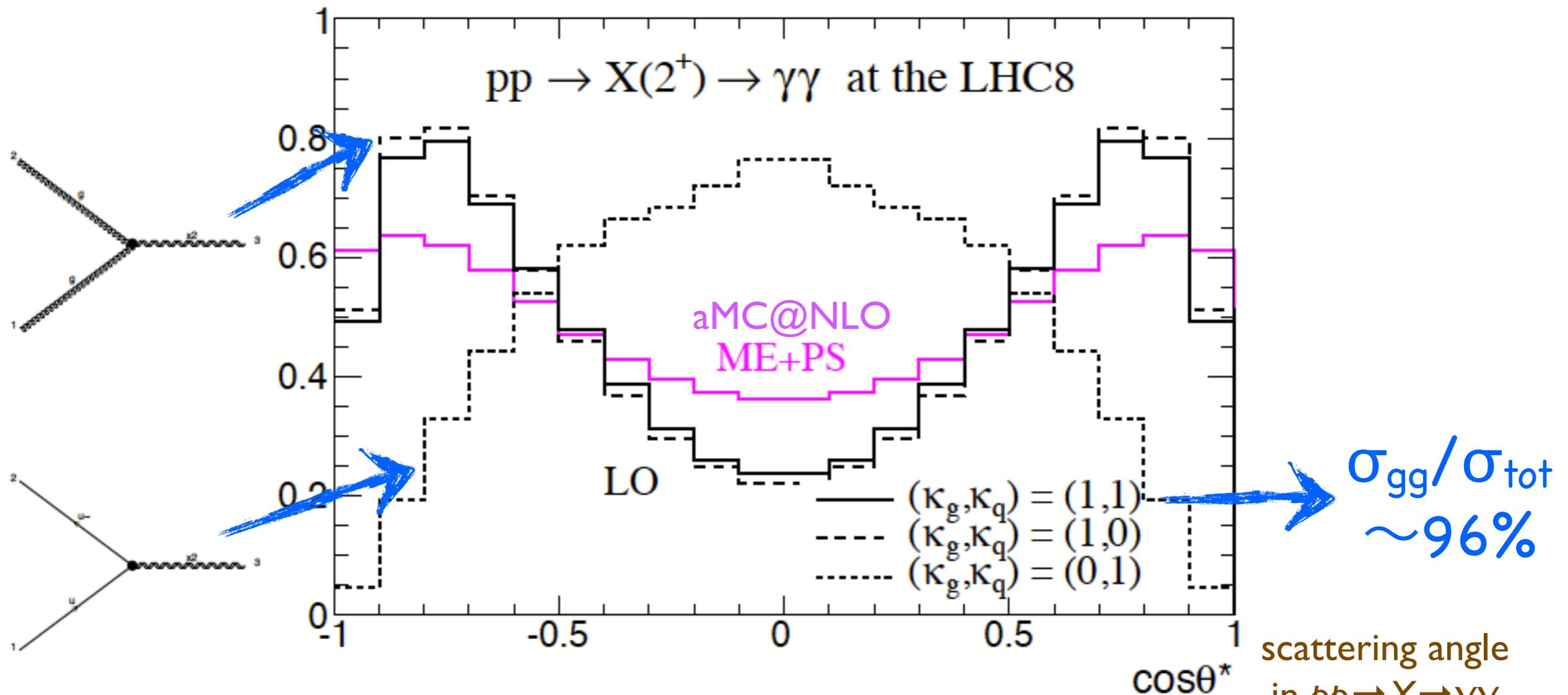
unitarity-violating behavior of models with a spin-2 state



A model with non-universal couplings
dramatically changes the $p_T(X)$ spectrum.

Higher order effects in QCD (III)

on spin observables for a spin-2 state



$$\frac{d\sigma(gg)}{d\cos\theta^*} \propto |d_{22}^2(\theta^*)|^2 + |d_{2-2}^2(\theta^*)|^2 = \frac{1}{8}(1 + 6\cos^2\theta^* + \cos^4\theta^*),$$

$$\frac{d\sigma(q\bar{q})}{d\cos\theta^*} \propto |d_{12}^2(\theta^*)|^2 + |d_{1-2}^2(\theta^*)|^2 = \frac{1}{2}(1 - \cos^4\theta^*).$$