Status for the calculation of theory errors in B decays

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El Dorado of a theorist and of an experimentalist is to find NP in observables where main hadronic uncertainties factor out, like $B_s \to \mu^+\mu^-$, that allows you to construct clean ratios like $S_{\mu\mu}$ or $\mathcal{A}_{\Delta\Gamma}^{\mu\mu}$. [Fleischer, Buras et al'13]

HOWEVER

- El Dorado does not exists...
- Nature decides by itself where NP could enter. Moreover, the more precision you need the less clean you discover that those channels are.

Conclusion: we have to work and find strategies to reduce as much as possible the sensitivity to hadronic physics.

In some cases like $\Delta_{CP}^{dir} = A_{CP}^{dir}(D^0 \to K^+K^-) - A_{CP}^{dir}(D^0 \to \pi^+\pi^-)$ you are strongly dominated by long distance such that the SM prediction is far from calling it a "prediction".

There is a long list of much better controlled modes like [G. Ricciardi'14]

$$R_{\tau/\ell}^{(*)} = \mathcal{B}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B} \to D^{(*)}l^-\bar{\nu}_\ell)$$

that slightly exceeds SM by 2σ that would require breakings of lepton-flavour universality.

I will focus here in: *A very promising/exciting case the angular observables of $B \to K^*(\to K\pi)\mu^+\mu^-$ that after including factorizable/non-factorizable (α_s, power) corrections allows for a precision analysis.*

⇒ In the short term the best paradigm to unveil **New Physics** in Flavour will be an accurate determination of Wilson coefficients. In particular those associated to operators:

$$\mathcal{O}_{\mathbf{7}} = \frac{e}{16\pi^2} m_b(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}, \quad \mathcal{O}_{\mathbf{9}} = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell), \quad \mathcal{O}_{\mathbf{10}} = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\gamma_5 \ell),$$

and chiral counterparts $\mathcal{O}'_{7,9,10}$ (L $\leftrightarrow$ R) looks particularly promising.

- Wilson Coefficients are tested $C_i = C_i^{SM} + \mathbf{C_i^{NP}}$ $\left\{ \begin{array}{l} \text{different levels of accuracy} \\ \text{allow different ranges of NP} \end{array} \right.$

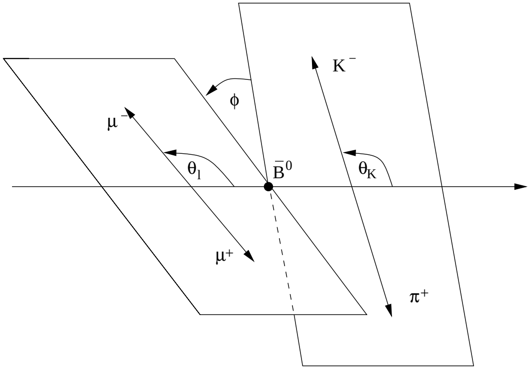
Wilson coefficients [$\mu_b = \mathcal{O}(m_b)$]	Observables	SM values
$\mathbf{C_7^{eff}}(\mu_\mathbf{b})$	$\mathcal{B}(\bar{B} \to X_s\gamma), A_I(B \to K^*\gamma), S_{K^*\gamma}, A_{FB}, F_L,$	-0.292
$\mathbf{C_9}(\mu_\mathbf{b})$	$\mathcal{B}(B \to X_s\ell\ell), A_{FB}, F_L,$	4.075
$\mathbf{C_{10}}(\mu_\mathbf{b})$	$\mathcal{B}(B_s \to \mu^+\mu^-), \mathcal{B}(B \to X_s\ell\ell), A_{FB}, F_L,$	-4.308
$\mathbf{C'_7}(\mu_\mathbf{b})$	$\mathcal{B}(\bar{B} \to X_s\gamma), A_I(B \to K^*\gamma), S_{K^*\gamma}, A_{FB}, F_L$	-0.006
$\mathbf{C'_9}(\mu_\mathbf{b})$	$\mathcal{B}(B \to X_s\ell\ell), A_{FB}, F_L$	0
$\mathbf{C'_{10}}(\mu_\mathbf{b})$	$\mathcal{B}(B_s \to \mu^+\mu^-), A_{FB}, F_L,$	0

More Precision Observables are necessary to overconstrain the deviations $\mathbf{C_i^{NP}}$

⇒ $B \to K^*(\to K\pi)\mu^+\mu^-$ can fulfill this requirement providing a set of large-recoil clean observables $\mathbf{P_{1,2,3}}, \mathbf{P'_{4,5,6,8}}$ and the corresponding CP observables $\mathbf{P^{CP}_{1,2,3}}, \mathbf{P^{CP\prime}_{4,5,6,8}}$

The angular distribution $\bar{\mathbf{B}}_{\mathbf{d}} \to \bar{\mathbf{K}}^{*0}(\to \mathbf{K}^-\pi^+)\mathbf{l}^+\mathbf{l}^-$ with the K^{*0} on the mass shell. It is described by $\mathbf{s} = \mathbf{q^2}$ and three angles θ_ℓ, $\theta_{\mathbf{K}}$ and ϕ

$$\frac{d^4\Gamma(\bar{B}_d)}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi}\mathbf{J}(\mathbf{q^2}, \theta_\ell, \theta_K, \phi) = \frac{9}{32\pi}\sum_i J_i(q^2) f_i(\theta_\ell, \theta_K, \phi)$$



θ_ℓ: Angle of emission between $\bar{K}^{*0}$ and μ^- in di-lepton rest frame.
$\theta_{\mathbf{K}}$: Angle of emission between $\bar{K}^{*0}$ and K^- in di-meson rest frame.
ϕ: Angle between the two planes.

$\mathbf{q^2}$: dilepton invariant mass square.

Notice LHCb uses $\theta_\ell^{LHCb} = \pi - \theta_\ell^{us}$

Three regions in q^2:

- **large recoil for** K^*: $E_{K^*} \gg \Lambda_{QCD}$ or $4m_\ell^2 \leq q^2 < 9$ GeV^2
- resonance region ($q^2 = m^2_{J/\Psi}, \ldots$) betwen $9 < q^2 < 14$ GeV^2.
- **low-recoil for** K^*: $E_{K^*} \sim \Lambda_{QCD}$ or $14 < q^2 \leq (m_B - m_{K^*})^2$.

Relation between J_i and P_j, P'_k observables

The coefficients $\mathbf{J_i}$ of the distribution can be reexpressed now in terms of this basis of clean observables:

Correspondence $\mathbf{J_i} \leftrightarrow \mathbf{P_i^{(\prime)}}$:

BROWN: LO FF-dependent observables (F_L Longitudinal Polarization Fraction of K^*)

RED: LO FF-independent observables at large-recoil (defined from these eqs.)

Here for simplicity ($m_\ell = 0$).
See [J.M'12] for $m_\ell \neq 0$.

$$(\mathbf{J_{2s}} + \mathbf{\bar{J}_{2s}}) = \frac{1}{4}\mathbf{F_T}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}} \qquad (\mathbf{J_{2c}} + \mathbf{\bar{J}_{2c}}) = -\mathbf{F_L}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}}$$

$$\mathbf{J_3} + \mathbf{\bar{J}_3} = \frac{1}{2}\mathbf{P_1}\mathbf{F_T}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}} \qquad \mathbf{J_3} - \mathbf{\bar{J}_3} = \frac{1}{2}\mathbf{P_1^{CP}}\mathbf{F_T}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}}$$

$$\mathbf{J_{6s}} + \mathbf{\bar{J}_{6s}} = 2\mathbf{P_2}\mathbf{F_T}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}} \qquad \mathbf{J_{6s}} - \mathbf{\bar{J}_{6s}} = 2\mathbf{P_2^{CP}}\mathbf{F_T}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}}$$

$$\mathbf{J_9} + \mathbf{\bar{J}_9} = -\mathbf{P_3}\mathbf{F_T}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}} \qquad \mathbf{J_9} - \mathbf{\bar{J}_9} = -\mathbf{P_3^{CP}}\mathbf{F_T}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}}$$

$$\mathbf{J_4} + \mathbf{\bar{J}_4} = \frac{1}{2}\mathbf{P'_4}\sqrt{\mathbf{F_T F_L}}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}} \qquad \mathbf{J_4} - \mathbf{\bar{J}_4} = \frac{1}{2}\mathbf{P_4^{\prime CP}}\sqrt{\mathbf{F_T F_L}}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}}$$

$$\mathbf{J_5} + \mathbf{\bar{J}_5} = \mathbf{P'_5}\sqrt{\mathbf{F_T F_L}}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}} \qquad \mathbf{J_5} - \mathbf{\bar{J}_5} = \mathbf{P_5^{\prime CP}}\sqrt{\mathbf{F_T F_L}}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}}$$

$$\mathbf{J_7} + \mathbf{\bar{J}_7} = -\mathbf{P'_6}\sqrt{\mathbf{F_T F_L}}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}} \qquad \mathbf{J_7} - \mathbf{\bar{J}_7} = -\mathbf{P_6^{\prime CP}}\sqrt{\mathbf{F_T F_L}}\frac{\mathbf{d\Gamma + d\bar{\Gamma}}}{\mathbf{dq^2}}$$

P_i, P_i' defines an **Optimal Basis** of observables, a compromise between:

- *I. Excellent experimental accessibility and simplicity of the fit.*
- *II. Reduced FF dependence (in the large-recoil region:* $0.1 \leq q^2 \leq 8\ \mathrm{GeV}^2$).

Our proposal for **CP-conserving basis**:

$$\left\{ \frac{\mathbf{d\Gamma}}{\mathbf{dq^2}}, \mathbf{A_{FB}}, \mathbf{P_1}, \mathbf{P_2}, \mathbf{P_3}, \mathbf{P_4'}, \mathbf{P_5'}, \mathbf{P_6'} \right\} \text{ or } \mathbf{P_3} \leftrightarrow \mathbf{P_8'} \text{ and } \mathbf{A_{FB}} \leftrightarrow \mathbf{F_L}$$

where $P_1 = A_T^2$ [Kruger, J.M'05],
$P_2 = \frac{1}{2} A_T^{\rm re}$, $P_3 = -\frac{1}{2} A_T^{\rm im}$ [Becirevic, Schneider'12]
$P'_{4,5,6}$ [Descotes, JM, Ramon, Virto'13]).

The corresponding **CP-violating basis** ($J_i + \bar{J}_i \to J_i - \bar{J}_i$ in numerators):

$$\{\mathbf{A_{CP}}, \mathbf{A_{FB}^{CP}}, \mathbf{P_1^{CP}}, \mathbf{P_2^{CP}}, \mathbf{P_3^{CP}}, \mathbf{P_4'^{CP}}, \mathbf{P_5'^{CP}}, \mathbf{P_6'^{CP}}\} \text{ or } \mathbf{P_3^{CP}} \leftrightarrow \mathbf{P_8'^{CP}} \text{ and } \mathbf{A_{FB}^{CP}} \leftrightarrow \mathbf{F_L^{CP}}$$

- There is another basis of observables $\mathbf{S_i = (J_i + \bar{J}_i)/(d\Gamma/dq^2 + d\bar{\Gamma}/dq^2)}$, however, it fails in II.

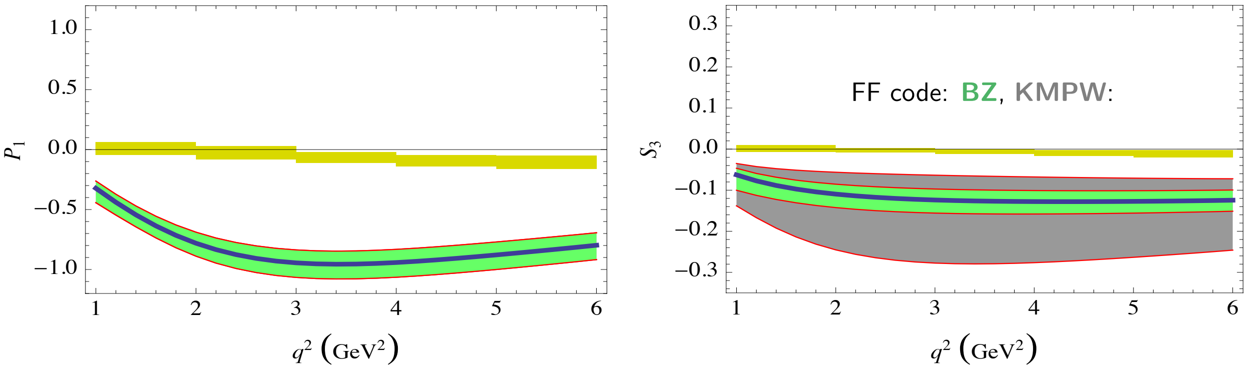
In presence of NP, S_i observables suffer an enhanced sensitivity to FF choice.

Main difference between an analysis using P_i, P'_j observables or S_i observables

Simple illustration of the difference in sensitivity between a P versus an S observable using two different FF parametrizations. Comparison between P_1 and S_3 in presence of New Physics:

- P_1 error band does not change using [A. Khodjamirian et al. '10]-KMPW (green) or [P. Ball and R. Zwicky,'05]-BZ (gray).
- S_3 band changes **substantially** when using KMPW or BZ.

The wide spread of different errors in literature associated to FF:



The discrimination between different NP cases and hadronic uncertainties is significantly lower in the S_i

Even though the lower sensitivity of the S_i it is positive to use them as a **cross check** of a FFI analysis using P_i.

Analysis of new LHCb data on

$$B \to K^* \mu^+ \mu^-$$

Experimental evidence: EPS+ Beauty

Present bins: [0.1,2], [2,4.3], [4.3,8.68], [1,6], [14.18,16], [16,19] GeV^2.

Observable	Experiment	SM prediction	Pull
$\langle P_1\rangle_{[0.1,2]}$	$-0.19^{+0.40}_{-0.35}$	$0.007^{+0.043}_{-0.044}$	-0.5
$\langle P_1\rangle_{[2,4.3]}$	$-0.29^{+0.65}_{-0.46}$	$-0.051^{+0.046}_{-0.046}$	-0.4
$\langle P_1\rangle_{[4.3,8.68]}$	$0.36^{+0.30}_{-0.31}$	$-0.117^{+0.056}_{-0.052}$	$+1.5$
$\langle P_1\rangle_{[1,6]}$	$0.15^{+0.39}_{-0.41}$	$-0.055^{+0.041}_{-0.043}$	$+0.5$
$\langle P_2\rangle_{[0.1,2]}$	$0.03^{+0.14}_{-0.15}$	$0.172^{+0.020}_{-0.021}$	-1.0
$\langle P_2\rangle_{[2,4.3]}$	$0.50^{+0.00}_{-0.07}$	$0.234^{+0.060}_{-0.086}$	**$+2.9$**
$\langle P_2\rangle_{[4.3,8.68]}$	$-0.25^{+0.07}_{-0.08}$	$-0.407^{+0.049}_{-0.037}$	**$+1.7$**
$\langle P_2\rangle_{[1,6]}$	$0.33^{+0.11}_{-0.12}$	$0.084^{+0.060}_{-0.078}$	$+1.8$
$\langle A_{\rm FB}\rangle_{[0.1,2]}$	$-0.02^{+0.13}_{-0.13}$	$-0.136^{+0.051}_{-0.048}$	$+0.8$
$\langle A_{\rm FB}\rangle_{[2,4.3]}$	$-0.20^{+0.08}_{-0.08}$	$-0.081^{+0.055}_{-0.069}$	-1.1
$\langle A_{\rm FB}\rangle_{[4.3,8.68]}$	$0.16^{+0.06}_{-0.05}$	$0.220^{+0.138}_{-0.113}$	-0.5
$\langle A_{\rm FB}\rangle_{[1,6]}$	$-0.17^{+0.06}_{-0.06}$	$-0.035^{+0.037}_{-0.034}$	-2.0

- $\mathbf{P_1}$: No substantial deviation (large error bars).
- $\mathbf{A_{\rm FB}}$-$\mathbf{P_2}$: A slight tendency for a lower value of the second and third bins of $A_{\rm FB}$ is consistent with a $2.9\,\sigma$ ($1.7\,\sigma$) deviation in the second (third) bin of P_2.
- **Zero**: Preference for a slightly higher q^2-value for the zero of $A_{\rm FB}$ (same as the zero of P_2).

Both effects can be accommodated with $\mathcal{C}_7^{\rm NP} < 0$ and/or $\mathcal{C}_9^{\rm NP} < 0$.

Observable	Experiment	SM prediction	Pull
$\langle P_4' \rangle_{[0.1,2]}$	$0.00^{+0.52}_{-0.52}$	$-0.342^{+0.031}_{-0.026}$	$+0.7$
$\langle P_4' \rangle_{[2,4.3]}$	$0.74^{+0.54}_{-0.60}$	$0.569^{+0.073}_{-0.063}$	$+0.3$
$\langle P_4' \rangle_{[4.3,8.68]}$	$1.18^{+0.26}_{-0.32}$	$1.003^{+0.028}_{-0.032}$	$+0.6$
$\langle P_4' \rangle_{[1,6]}$	$0.58^{+0.32}_{-0.36}$	$0.555^{+0.067}_{-0.058}$	$+0.1$
$\langle P_5' \rangle_{[0.1,2]}$	$0.45^{+0.21}_{-0.24}$	$0.533^{+0.033}_{-0.041}$	-0.4
$\langle P_5' \rangle_{[2,4.3]}$	$0.29^{+0.40}_{-0.39}$	$-0.334^{+0.097}_{-0.113}$	$\mathbf{+1.6}$
$\langle P_5' \rangle_{[4.3,8.68]}$	$-0.19^{+0.16}_{-0.16}$	$-0.872^{+0.053}_{-0.041}$	$\mathbf{+4.0}$
$\langle P_5' \rangle_{[1,6]}$	$0.21^{+0.20}_{-0.21}$	$-0.349^{+0.088}_{-0.100}$	$+2.5$
$\langle P_4' \rangle_{[14.18,16]}$	$-0.18^{+0.54}_{-0.70}$	$1.161^{+0.190}_{-0.332}$	-2.1
$\langle P_4' \rangle_{[16,19]}$	$0.70^{+0.44}_{-0.52}$	$1.263^{+0.119}_{-0.248}$	-1.1
$\langle P_5' \rangle_{[14.18,16]}$	$\mathbf{-0.79^{+0.27}_{-0.22}}$	$\mathbf{-0.779^{+0.328}_{-0.363}}$	$+0.0$
$\langle P_5' \rangle_{[16,19]}$	$\mathbf{-0.60^{+0.21}_{-0.18}}$	$\mathbf{-0.601^{+0.282}_{-0.367}}$	$+0.0$

Definition of the anomaly:

- $\mathbf{P_5'}$: There is a striking $4.0\,\sigma$ ($1.6\,\sigma$) deviation in the third (second) bin of P_5'.

  Consistent with large negative contributions in $\mathcal{C}_7^{\rm NP}$ and/or $\mathcal{C}_9^{\rm NP}$.

- $\mathbf{P_4'}$: in agreement with the SM, but within large uncertainties, and it has future potential to determine the sign of $\mathcal{C}_{10}^{\rm NP}$.

- $\mathbf{P_6'}$ and $\mathbf{P_8'}$: show small deviations with respect to the SM, but such effect would require complex phases in $\mathcal{C}_9^{\rm NP}$ and/or $\mathcal{C}_{10}^{\rm NP}$.

Us: $(-0.19-(-0.872))/\sqrt{0.16^2+0.053^2}=4.05$ and **Exp**: $(-0.19-(-0.872+0.053))/\sqrt{0.16^2+0.053^2}=3.73$

Our SM predictions+LHCb data

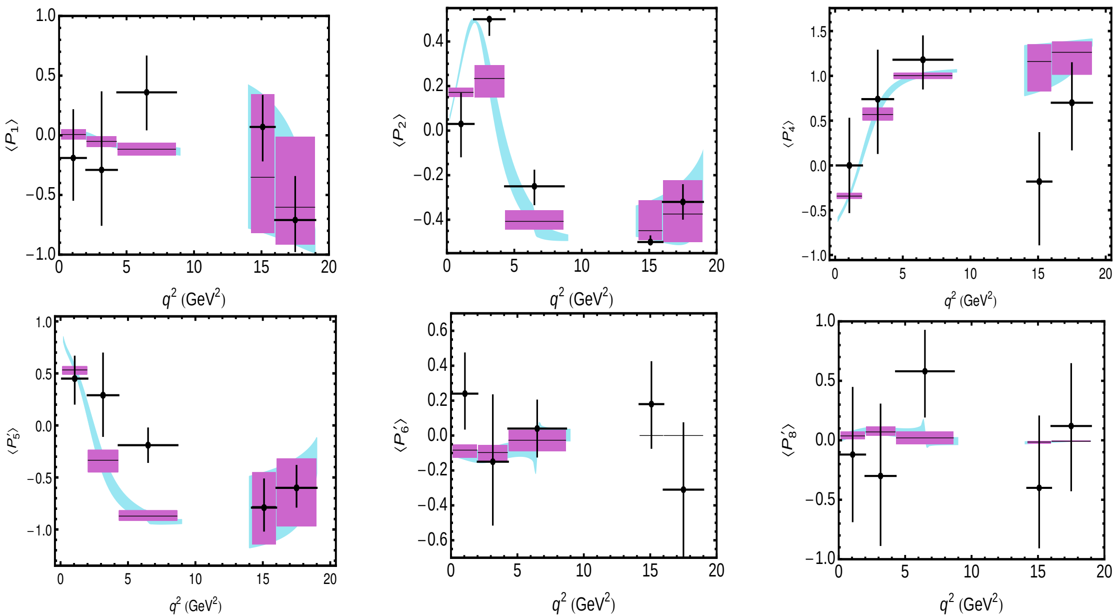


Figure : Experimental measurements and SM predictions for some $B \to K^* \mu^+ \mu^-$ observables. The black crosses are the experimental LHCb data. The blue band corresponds to the SM predictions for the differential quantities, whereas the purple boxes indicate the corresponding binned observables.

Goal: Determine the Wilson coefficients $\mathcal{C}_{7,9,10}$, $\mathcal{C}'_{7,9,10}$: $\mathcal{C}_i = \mathcal{C}_i^{SM} + \mathcal{C}_i^{NP}$

Standard χ^2 frequentist approach: Asymmetric errors included, estimate theory uncertainties for each set of $\mathcal{C}_i^{NP}$ and all uncertainties are combined in quadrature.

IMPORTANT: *Experimental correlations are included in the updated plot*

We do three analysis: a) large-recoil data b) large+low-recoil data c) [1-6] bin

Observables:

- $B \to K^* \mu^+ \mu^-$: We take observables P_1, P_2, P_4', P_5', P_6' and P_8' in the following binning:
  -**large-recoil**: $[0.1, 2], [2, 4.3], [4.3, 8.68]$ GeV^2.
  -**low recoil**: [14.18,16], [16,19] GeV^2
  -**wide large-recoil bin**: $[1, 6]$ GeV^2.

- Radiative and dileptonic B decays: $\mathcal{B}(B \to X_s \gamma)_{E_\gamma > 1.6\mathrm{GeV}}$, $\mathcal{B}(B \to X_s \mu^+ \mu^-)_{[1,6]}$ and $\mathcal{B}(B_s \to \mu^+ \mu^-)$, $A_I(B \to K^* \gamma)$ and the $B \to K^* \gamma$ time-dependent CP asymmetry $S_{K^*\gamma}$

General case all WC free

Result of our analysis (large+low recoil data+rad) if we allow **all Wilson coefficients** to vary freely:

Coefficient	$1\,\sigma$	$2\,\sigma$	$3\,\sigma$
$\mathcal{C}_7^{\rm NP}$	$[-0.05, -0.01]$	$[\mathbf{-0.06, 0.01}]$	$[-0.08, 0.03]$
$\mathcal{C}_9^{\rm NP}$	$[-1.6, -0.9]$	$[-1.8, -0.6]$	$[\mathbf{-2.1, -0.2}]$
$\mathcal{C}_{10}^{\rm NP}$	$[-\mathbf{0.4}, \mathbf{1.0}]$	$[-1.2, 2.0]$	$[-2.0, 3.0]$
$\mathcal{C}_{7'}^{\rm NP}$	$[-\mathbf{0.04}, \mathbf{0.02}]$	$[-0.09, 0.06]$	$[-0.14, 0.10]$
$\mathcal{C}_{9'}^{\rm NP}$	$[\mathbf{-0.2, 0.8}]$	$[-0.8, 1.4]$	$[-1.2, 1.8]$
$\mathcal{C}_{10'}^{\rm NP}$	$[-\mathbf{0.4}, \mathbf{0.4}]$	$[-1.0, 0.8]$	$[-1.4, 1.2]$

Table : 68.3% ($1\,\sigma$), 95.5% ($2\,\sigma$) and 99.7% ($3\,\sigma$) confidence intervals for the NP contributions to WC.

- This table tells you again that there is **strong evidence for a $\mathcal{C}_9^{\rm NP} < 0$, preference for $\mathcal{C}_7^{\rm NP} < 0$** and **no clear-cut evidence** for $\mathcal{C}_{10,7',9',10'}^{\rm NP} \neq 0$.
- *This does not imply that they will be at the end zero but that* **present data** *does not point clearly for a positive or negative value.*

General case all WC free

In conclusion our pattern of [PRD88 (2013) 074002] obtained from an $\mathcal{H}_{eff}$ approach is

$$\mathbf{C_9^{NP}} \sim [-\mathbf{1.6}, -\mathbf{0.9}], \quad \mathbf{C_7^{NP}} \sim [-\mathbf{0.05}, -\mathbf{0.01}], \quad \mathbf{C_9'} \sim \pm\delta \quad \mathbf{C_{10}}, \mathbf{C'_{7,10}} \sim \pm\epsilon$$

where δ is small (at maximum half $|\mathbf{C_9^{NP}}|$) and ϵ is smaller.

A simplified version is $C_9^{NP} = -1.5$ where C_9^{SM} includes all em corrections.

Other best fit points:

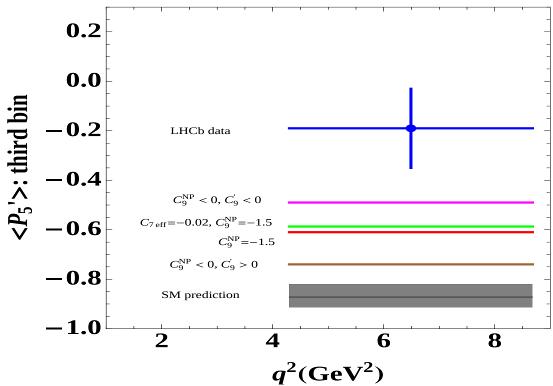
Large recoil: $\mathbf{C_9^{NP}} = -\mathbf{1.5}$, $C_{7eff}^{\rm NP} = -0.02$

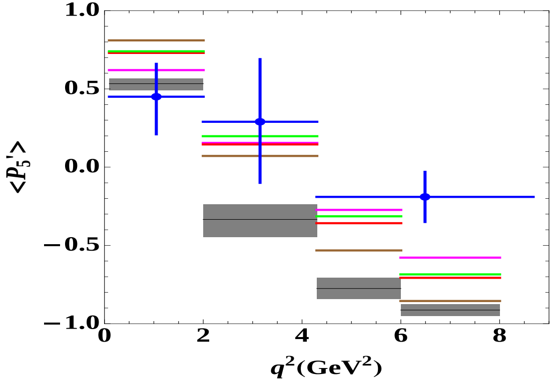
Large recoil: $\mathbf{C_9^{NP}} = -\mathbf{1.6}$, $C_{7eff}^{\rm NP} = -0.02$, $C_{10}^{\rm NP} > 0$, $\mathbf{C_{9\prime}^{NP}} < \mathbf{0}$, $C_{7\prime}^{\rm NP} > 0$, $C_{10\prime}^{\rm NP} < 0$.

Large+Low: $\mathbf{C_9^{NP}} = -\mathbf{1.2}$, $C_{7eff}^{\rm NP} = -0.03$, $C_{10}^{\rm NP} > 0$, $\mathbf{C_{9\prime}^{NP}} > \mathbf{0}$, $C_{7\prime}^{\rm NP} < 0$, $C_{10\prime}^{\rm NP} < 0$

A second solution was proposed $C_9^{NP} \sim -1$ and $C_9' \sim 1$ [Altmannshofer-Straub]. **However:**

- It strongly relies on low recoil in $B \to K\mu^+\mu^-$ where the size of duality violations is probably underestimated.
- Large recoil region of $B \to K^{(*)}\mu^+\mu^-$ shows a strong preference for $C_9^{NP} = -1.5$ and $C_9' \leq 0$.
  - Other branching ratios confirm preference for $C_9^{NP} = -1.5$ solution (updated WC with em corrections).
  - Using lattice FF (S. Meinel, private) extrapolating at low-q^2 confirms preference for $C_9^{NP} = -1.5$

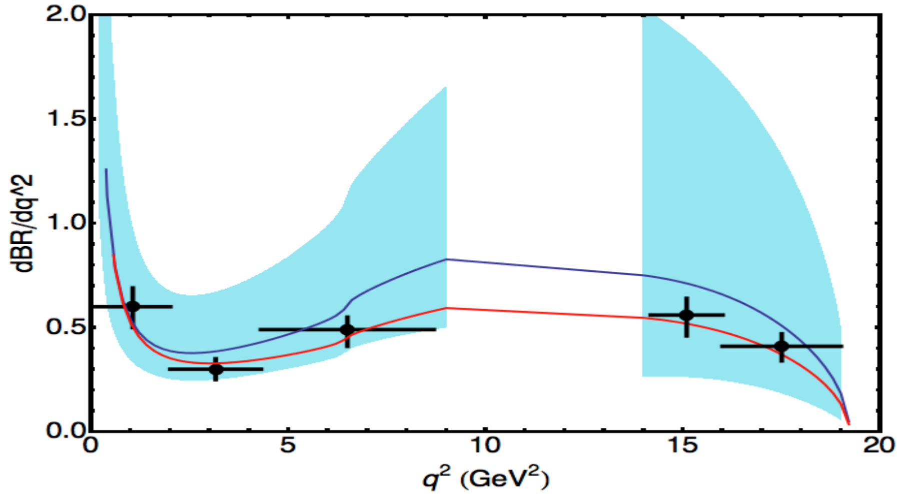




Solution with $C_9^{NP} \sim -1$ and $C_9' \sim 1$ (taking best fit point of A&S) (brown line) is strongly disfavored at large-recoil.

Branching Ratio of $B \to K^* \mu^+ \mu^-$

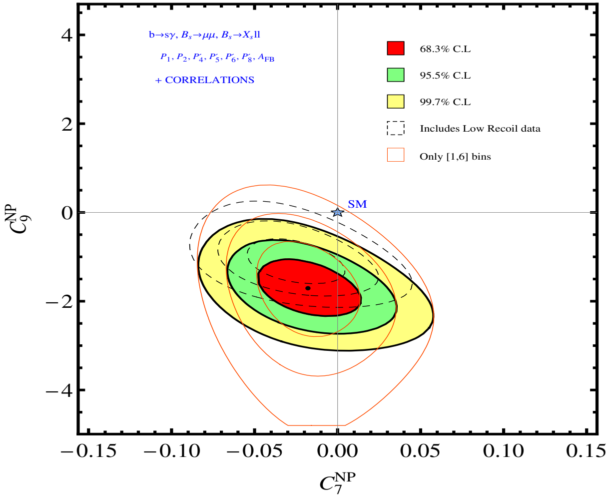
Let's look now to one observables strongly dependent on FF the BR ($x10^7$) that we use as cross check.



where the blue curve is SM and the red curve corresponds to $C_9^{NP} = -1.5$. Interestingly the central value goes in the right direction, but given the error bars all is consistent with data.

Updated result with experimental correlations

Updated result using $\mathbf{P_i}, \mathbf{P'_i}, \mathbf{A}_{\rm FB}$ and **experimental correlations**.



From the analysis of the set $\mathbf{P_i}, \mathbf{P'_i}, \mathbf{A}_{\rm FB}+$ **BR** $+$ exp. correlations we get:

4.3σ (large-recoil)
3.6σ (large + low recoil)
2.8σ for [1-6] bin.

Colored: large-recoil and
dashed: large+low recoil
orange: [1-6] bin

- We checked (for completeness) that we find **same significance** using $\mathbf{P_i}, \mathbf{P'_i}, \mathbf{F_L}$ instead of $\mathbf{A}_{\rm FB}$. **Positive:** **Our SM $\mathbf{F_L}$** fully compatible with all data (not only LHCb) and less correlated. **Negative:** Result using F_L is less solid than using A_{FB} since it depends on choice of FF.

We are further refining the theoretical analysis, in couple of months we will update it.

SM theory uncertainties

Large recoil region

There are basically two approaches:

- **OPTION 1: soft form factors + NLO QCDF** (factorizable+non-factorizable+weak annihilation) + factorizable power corrections

[S. Descotes, J. Matias, J. Virto]

All factorizable +non-factorizable NLO QCDF $\mathcal{O}(\alpha_s)$ correction given by [Beneke, Feldmann, Seidel]

- **OPTION 2: naive factorization (full form factors) + non-factorizable QCDF** (some weak annihilation neglected)

[Altmannshoffer, Ball, Barucha, Buras, Straub, Wick]

A *third complementary approach* for a future analysis of data of second round of LHCb data

Amplitude analysis

- Use symmetries to fix 4 real/imaginary parts of amplitudes to zero.
- Parametrize amplitudes by $A_i = \alpha_i + \beta_i q^2 + \gamma_i / q^2$. How to provide this info to theorists?

Option 1: Soft Form Factor decomposition (LO)

For q^2 in [0.1-9] GeV^2 region (large-recoil for K*) we have a set of 7 full form factors that describes the distribution.

- At LO in α_s and Λ/m_b:
  Set of FF reduces to two independent soft form factors
  $$\{V, A_0, A_1, A_2, T_1, T_2, T_3\} \to \{\xi_\perp, \xi_\|\}$$

- this implies relations among full FFs, e.g.

$$\frac{m_B(m_B + m_{K^*})A_1 - 2E(m_B - m_{K^*})A_2}{m_B^2 T_2 - 2Em_B T_3} = 1 + \mathcal{O}(\alpha_s, \Lambda/m_b)$$

- construct observables involving such ratios
  $\to$ **soft form factors cancel at LO** $\Rightarrow$ clean observables $P_i^{(\prime)}$ (S_i does not fulfill this condition)

Option1: Soft Form Factor decomposition (NLO)

$$
\begin{aligned}
V(q^2) &= \frac{m_B + m_{K^*}}{m_B}\,\xi_\perp(q^2) + \Delta V^{\alpha_s}(q^2) + \Delta V^{\Lambda}(q^2)\,,\\
A_1(q^2) &= \frac{2E}{m_B + m_{K^*}}\,\xi_\perp(q^2) + \Delta A_1^{\alpha_s}(q^2) + \Delta A_1^{\Lambda}(q^2)\,,\\
A_2(q^2) &= \frac{m_B}{m_B + m_{K^*}}\,\left[\xi_\perp(q^2) - \xi_\parallel(q^2)\right] + \Delta A_2^{\alpha_s}(q^2) + \Delta A_2^{\Lambda}(q^2)\,,\\
A_0(q^2) &= \frac{E}{m_{K^*}}\,\xi_\perp(q^2) + \Delta A_0^{\alpha_s}(q^2) + \Delta A_0^{\Lambda}(q^2)\,,\\
T_1(q^2) &= \xi_\perp(q^2) + \Delta T_1^{\alpha_s}(q^2) + \Delta T_1^{\Lambda}(q^2)\,,\\
T_2(q^2) &= \frac{2E}{m_B}\,\xi_\perp(q^2) + \Delta T_2^{\alpha_s}(q^2) + \Delta T_2^{\Lambda}(q^2)\,,\\
T_3(q^2) &= \left[\xi_\perp(q^2) - \xi_\parallel(q^2)\right] + \Delta T_3^{\alpha_s}(q^2) + \Delta T_3^{\Lambda}(q^2)
\end{aligned}
$$

where Blue is LO,

Red is NLO which include higher orders of α_s (ΔF^{α_s}) and Λ/m_b (ΔF^{Λ}) (factorizable p.c.)

Two possible ways to add factorizable effects beyond LO:

- OPTION 1 Add F^{α_s} and F^{Λ} to soft form factors: $F = F^{\rm soft}(\xi_\perp, \xi_\|) + \Delta F^{\alpha_s} + \Delta F^{\Lambda}$
- OPTION 2 Choose one set of full FF from LCSR, QCDSR, ... which includes α_s and Λ/m_b.

PROS and CONS:

OPTION 1:

++ All **correlations** among FF **included** automatically **by symmetry** relations.

++ Result is **FF parametrization independent at LO**, and only mild dependence at NLO appear. You are less dependent to all the inherent hypothesis attached to the calculation of FF. (crucial for NP)

- + Need to estimate size of factorizable power corrections. If correlations among FF included then p.c. become correlated and smaller providing much reliable results and less sensitive to errors in correlation matrices.

OPTION 2:

- - You have to rely on a perfect knowledge of correlations between FF parameters within a certain parametrization (with all inherent hypothesis built in), parametric uncertainties easy to obtain, but full correlation (Borel parameter and how to fix them) is very subtle/delicate. Difficult cross check.

\+ **Power corrections are built in** inside full FF but some non-factorizable not included (weak annihilation).

Factorizable power corrections and scheme dependence

Decomposition

$$F = F^{\rm soft}(\xi_\perp, \xi_\|) + \Delta F^{\alpha_s} + a_F + b_F \frac{q^2}{m_B^2}$$

depends on input scheme for soft FF $\xi_\perp, \xi_\|$

$\Rightarrow$ impact of power corrections depends on choice of scheme

1 scheme used by Jäger,Camalich:

$$\xi_\perp(q^2) \equiv T_1(q^2), \qquad \xi_\|(q^2) \equiv \frac{m_{K^*}}{E} A_0(q^2)\,.$$

$\to$ power corrections eliminated in T_1 and A_0

But: A_0-contribution suppressed by small lepton mass

2 scheme used by Beneke,Feldmann,Seidel and by us:

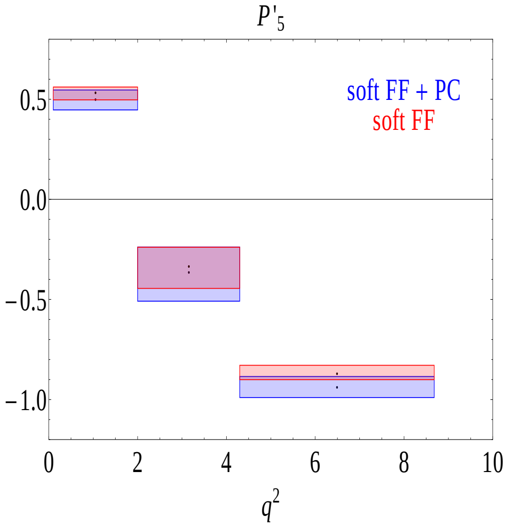
$$\xi_\perp(q^2) \equiv \frac{m_B}{m_B + m_{K^*}} V(q^2)\,,$$

$$\xi_\|(q^2) \equiv \frac{m_B + m_{K^*}}{2E} A_1(q^2) + \frac{m_B - m_{K^*}}{m_B} A_2(q^2)$$

$\to$ power corrections eliminated in V and minimized in A_1, A_2

Result for P_5' (preliminary)

The impact of factorizable PC is indeed smaller than previous estimates [Jaeger-Camalich].



We use KMPW Form Factors.
Similar results using Ball-Zwicky.

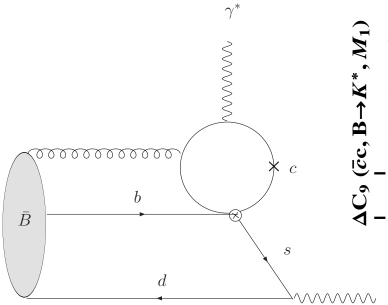
Our method: a more accurated analysis

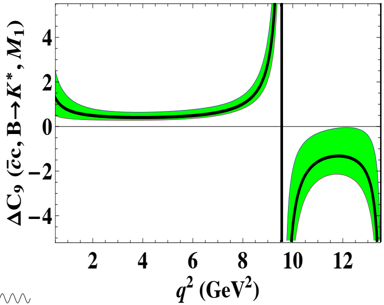
I Fit a_F, b_F to reproduce exactly full FF

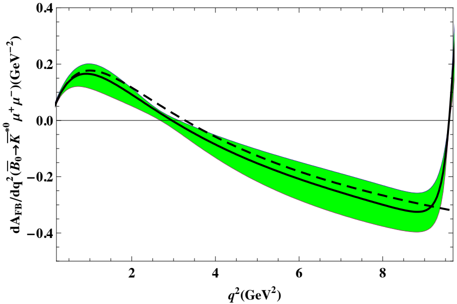
II Take $\Delta a_F = \Delta b_F = \mathcal{O}(\Lambda/m_B) \times F(0)$ corresponding to $> 100\%$ error w.r. to a_F, b_F.

Main improvements:

- Adequate choice of scheme to define $\xi_{\perp,\|}$ allows to partly absorb PC in soft FF.
- Use of all 6 correlations among a_i, b_i:
  - depend on scheme choice
  - exact relations of FF at $q^2 = 0$.
- In JC approach:
  - dangerous mixing of q^2 dependence for soft FF $\xi_\perp(q^2) = \frac{\xi_\perp(0)}{(1-\frac{q^2}{m_b^2})^2}$ and $\xi_\perp(q^2) = T_1(q^2)$
  - averaging different FF parametrization inconsistent.







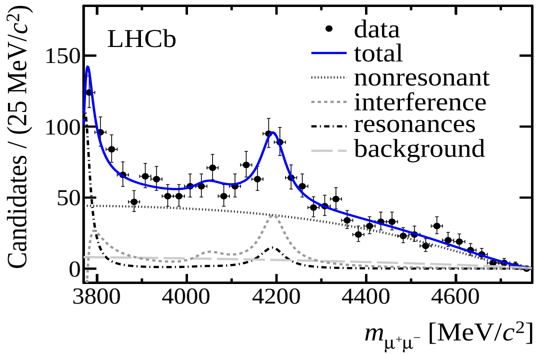
- *Charm loop*: Insertion of 4-quark operators ($\mathcal{O}^c_{1,2}$) or penguin operators ($\mathcal{O}_{3-6}$) are important and influence the extraction of C_9. Perturbative contribution is absorbed in C_9^{eff}. Long distance:
  - Partly cancelled experimentally by removing charmonium resonances.
  - **We followed LCSR computation and prescription** from KMPW to recast the effect inside C_9 as an effective contribution (different for each amplitude). See plots above taken from KMPW.

Result:
  - An increase of charm mass, for instance, from 1.27 to 1.4 GeV shifts C_9^{NP} by $+0.3$ in the third bin, reducing its negative value.
  - On the contrary **we checked explicitly** that this long distance charm contributions obtained by KMPW will tend to slightly **enhance** the negativity of C_9^{NP} $\Rightarrow$ **It increases the anomaly in P_5'**.

Low recoil

- It corresponds to large $\sqrt{q^2} \sim \mathcal{O}(m_b)$ above Ψ' mass, i.e., E_K is around GeV or below.
- Methodology:
  - Operator Product Expansion in $E_K/\sqrt{q^2}$ or $\Lambda_{QCD}/\sqrt{q^2}$
  - Form factors in this region:
    - $\rightarrow$ Extrapolation of LCSR FF above 14 GeV^2 (in my opinion obsolete)
    - $\rightarrow$ Lattice form factors [Bouchard et al,. S. Meinel et al.]
- **Main problem:**

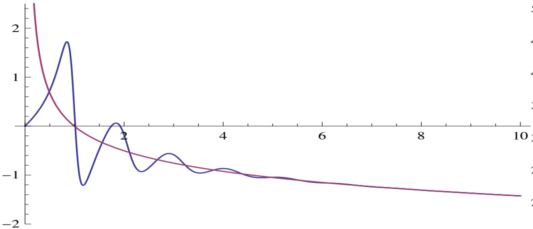
Existence of $c\bar{c}$ **resonances** in this region (clearly seen $\psi(4160)$ in $B^- \to K^-\mu^+\mu^-$), but many more expected to seen from BESS-II data.

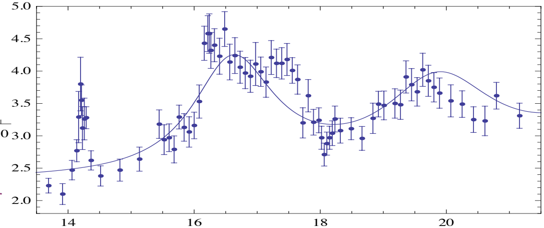


$\Rightarrow$ Quark-hadron duality-violations. How to estimate quark-hadron duality violations:

- First estimate from [Grinstein-Pirjol] combining OPE/HQET gives violation around 5%
- Analysis from [Beylich,Buchalla,Feldmann] estimated to 2%. BUT:
  - Simple toy model estimate based on Shifman's model: OPE+ Δ_2 (oscillating function, exponentially suppressed: duality violating term). Right: simple model based on BES data summing pairs of close resonances.

Assumption: Neglect interactions between $c\bar{c}$ and $B \to K$ system.





Still open questions:

1. Integrate all bin (starting from 15 and not 14.18 GeV^2 to avoid $\psi(3770)$ problem) and then assign a duality-violation: 2%?, 5%?, 10%?
2. Focus near the endpoint where resonances are exponentially suppressed?
3. Results of bin-by-bin at low-recoil are obsolete if no model included.

Interesting experiment by S. Meinel (removed 1st low-recoil bin) in a lattice FF analysis. Result: C_9^{NP} remained negative, while C_9' became compatible with zero in agreement with our pattern.

[JM, N. Serra'14]

Using symmetries (transformation of amplitudes that leaves distribution invariant) we get:

$$P_2 = \frac{1}{2}\left[P'_4 P'_5 + \frac{1}{\beta}\sqrt{(-1 + P_1 + P'^2_4)(-1 - P_1 + \beta^2 P'^2_5)}\right]$$

It allows to predict P_2 using data for $P'_{4,5}$ and P_1.

I. This equation is **not fulfilled** if:
   - Statistical fluctuation on data (**large/low recoil**)
   - There is NP in weak phases inside Wilson coefficients or scalars/tensors (**large/low recoil**).
   - Large strong phases due to resonances (**only at low-recoil**), first low recoil bin large deviation?

II. New constraints on data: $P'^2_5 - 1 \leq P_1 \leq 1 - P'^2_4$

III. Relation between the zero q_0^2 of A_{FB} and P'_5 anomaly: $[P'^2_4 + P'^2_5]|_{q^2=q_0^2} = 1 - \eta(q_0^2)$:

The highest the position of q_0^2 the smallest the value of $P'_5(q_0^2)$

IV. Some of the endpoint symmetries [Zwicky et al] obtained automatically ($P_1 = -1 \to P'_5 = 0 = P_2$).

V. It opens the possibility for a full angular analysis under the hypothesis of real Wilson coefficients and not scalars.

These hypotheses can be tested and partly relaxed.

Conclusions

- The analysis of LHCb data on the 4-body angular distribution of $B \to K^*(\to K\pi)\mu^+\mu^-$ using clean $P_i^{(\prime)}$, A_{FB} + radiative observables gives **the pattern**:

$$\mathbf{C_9^{NP}} \sim [-1.6, -0.9], \quad \mathbf{C_7^{NP}} \sim [-0.05, -0.01], \quad \mathbf{C_9'} \sim \pm\delta \quad \mathbf{C_{10}, C'_{7,10}} \sim \pm\epsilon$$

  where δ is small (at maximum half $|\mathbf{C_9^{NP}}|$) and ϵ is smaller.

- **Large-recoil: 0.1-8.6 GeV^2**: We have shown that the 'usual suspects' does not help in explaining the pattern of deviations in front of a NP explanation:
  - **Charm loops:** The first results from this kind of contributions [Khodjamirian et al.'10] show that they add a positive contribution to C_9, enlarging the size of the discrepancy of data with SM prediction.
  - **Factorizable Power Corrections:** A consistent implementation of correlations between PC + the freedom to choose an appropriate scheme to define soft FF + carefully avoiding inconsistent mixing between q^2 dependence of soft FF shows that PC are substantially **smaller** than previous guestimates.

Naive statement "It is QCD" is in tension with a consistent analysis.

- **Low-recoil: 15-19.22 GeV^2**: The difficulty to establish the size of quark-hadron duality violations in this region (2%?, 5%?, ...) complicates the analysis. Different possibilities: i) model resonances, ii) integrate over the whole q^2 region and assign an error,iii) take only the bin near the endpoint...

Proposal: focus on the well established region 1-7 GeV^2

Back-up slides

[S. Descotes, T. Hurth, JM, J. Virto'13], [J.Matias'12]

- Another possible source of uncertainty is the **S-wave contribution** coming from $B \to K_0^* l^+ l^-$. [Becirevic, Tayduganov '13], [Blake et al.'13]
- We assume that both P and S waves are described by q^2-dependent FF $\times$ a Breit-Wigner function.
- The **distinct** angular dependence of the S-wave terms in **folded** distributions allow to disentangle the signal of the P-wave from the S-wave: $P_i^{(\prime)}$ *can be* **disentangled** *from S-wave pollution* [JM'12].

The modified distribution including the **S-wave**:

$$\frac{1}{\Gamma'_{full}} \frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = Pdf_{K^*}(1 - \mathbf{F_S}) + \frac{1}{\Gamma'_{full}} \mathbf{W_S}$$

$$\frac{\mathbf{W_S}}{\Gamma'_{full}} = \frac{3}{16\pi} \left[\mathbf{F_S} \sin^2\theta_\ell + \mathbf{A_S} \sin^2\theta_\ell \cos\theta_K + \mathbf{A_S^4} \sin\theta_K \sin 2\theta_\ell \cos\phi \right.$$
$$\left. + \mathbf{A_S^5} \sin\theta_K \sin\theta_\ell \cos\phi + \mathbf{A_S^7} \sin\theta_K \sin\theta_\ell \sin\phi + \mathbf{A_S^8} \sin\theta_K \sin 2\theta_\ell \sin\phi \right]$$

$\Gamma'_{full} = \Gamma'_{K^*} + \Gamma'_S$ and the longitudinal polarization fraction associated to Γ'_S is

$$\mathbf{F_S} = \frac{\Gamma'_S}{\Gamma'_{full}} \quad \text{and} \quad 1 - \mathbf{F_S} = \frac{\Gamma'_{K^*}}{\Gamma'_{full}}$$

We can get **bounds** on the size of the S-wave polluting terms from Cauchy-Schwartz

$$\mathbf{A_S} = 2\sqrt{3}\frac{1}{\Gamma'_{full}} \int \mathrm{Re}\left[(A_0'^{L} A_0^{L*} + A_0'^{R} A_0^{R*}) BW_{K_0^*}(m_{K\pi}^2) BW_{K^*}^{\dagger}(m_{K\pi}^2)\right] dm_{K\pi}^2$$

$$|\mathbf{A_S}| \leq 2\sqrt{3}\frac{1}{\Gamma'_{full}} \times \sqrt{[|A_0'^{L}|^2 + |A_0'^{R}|^2][|A_0^{L}|^2 + |A_0^{R}|^2]}\,\mathbf{Z} = 2\sqrt{3}\sqrt{\mathbf{F_S(1-F_S)F_L}}\,\mathbf{Z}/\sqrt{\mathbf{XY}}$$

$\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ collect the Breit-Wigner.

[S.Descotes,T, Hurth, JM,J. Virto 1303.5794]

Coefficient	Large recoil ∞ Range	Low recoil ∞ Range	Large Recoil Finite Range	Low Recoil Finite Range
$\lvert A_S\rvert$	0.33	0.25	0.67	0.49
$\lvert A_S^4\rvert$	0.05	0.10	0.11	0.19
$\lvert A_S^5\rvert$	0.11	0.11	0.22	0.23
$\lvert A_S^7\rvert$	0.11	0.19	0.22	0.38
$\lvert A_S^8\rvert$	0.05	0.06	0.11	0.11

Table : Illustrative values of the size of the bounds for the choices of F_S, F_L, P_1 and $\mathbf{F} = \mathbf{Z}/\sqrt{\mathbf{XY}}$

- **Large-recoil**: $F_S \sim 7\%$ (like $B^0 \to J/\psi K^+\pi^-$), $F_L \sim 0.7$ and $P_1 \sim 0$
- **Low-recoil**: $F_S \sim 7\%$, $F_L \sim 0.38$ and $P_1 \sim -0.48$.

This may help in estimating the **systematics** associated to S-wave.