Precision calculations for the LHC

or

Theoretical errors and their prospects in the next 10 years

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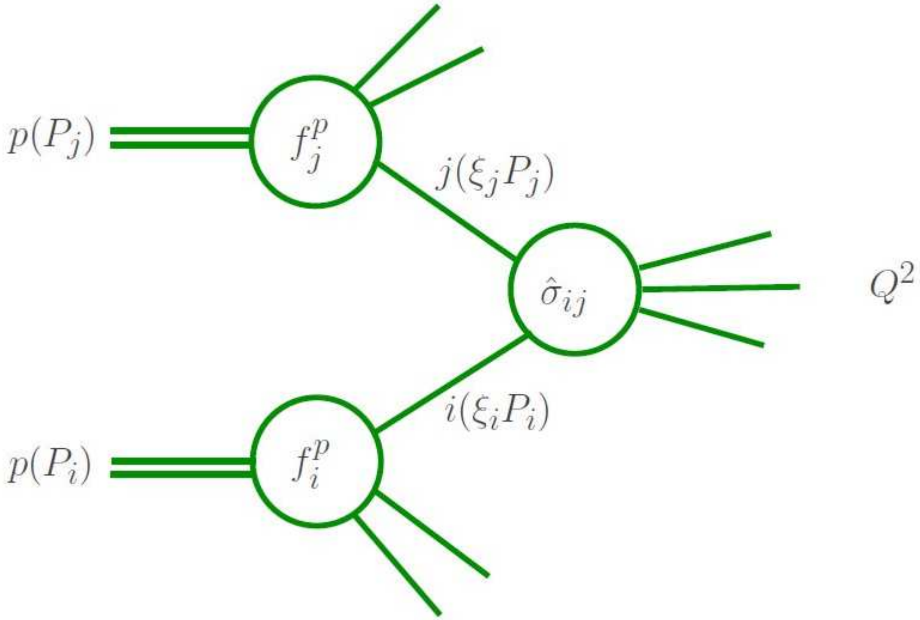
Theoretical Uncertainties

- Missing Higher Order corrections (MHO)
  - truncation of the perturbative series
  - often estimated by scale uncertainties - renormalisation/factorisation
  - ✓ systematically improvable by inclusion of higher orders
- Uncertainties in input parameters
  - parton distributions
  - masses, e.g., m_W, m_h, $[m_t]$
  - couplings, e.g., $\alpha_s(M_Z)$
  - ✓ systematically improvable by better description of benchmark processes
- Uncertainties in parton/hadron transition
  - fragmentation (parton shower)
  - ✓ systematically improvable by matching/merging with higher orders
  - hadronisation (model)
  - underlying event (tunes)

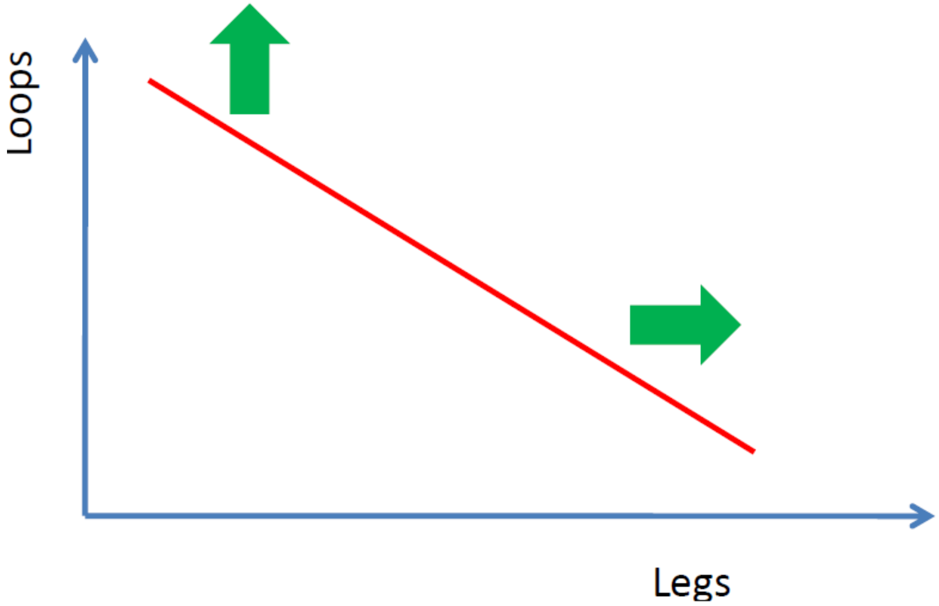
Theoretical Uncertainties

- Missing Higher Order corrections (MHO)
  - ✓ improvements on how to estimate MHO
  - ✓ new higher order calculations
- Uncertainties in input parameters
  - ✓ improved determinations with LHC data
- Uncertainties in parton/hadron transition
  - ✓ improvements in accuracy of event simulation



What is the hold up?

Rough idea of complexity of process $\sim$ #Loops + #Legs (+ #Scales)



- loop integrals are ultraviolet/infrared divergent
- complicated by extra mass/energy scales
- loop integrals often unknown
  - ✓ completely solved at NLO
- real (tree) contributions are infrared divergent
- isolating divergences complicated
  - ✓ completely solved at NLO
- currently far from automation
  - ✓ mostly solved at NLO

Current standard: NLO

1. Estimating MHO

Estimating uncertainties of MHO

Consider a generic observable $\mathcal{O}$ (e.g. σ_H)

$$\mathcal{O}(Q) \quad \sim \quad \mathcal{O}_k(Q,\mu) + \Delta_k(Q,\mu)$$

where

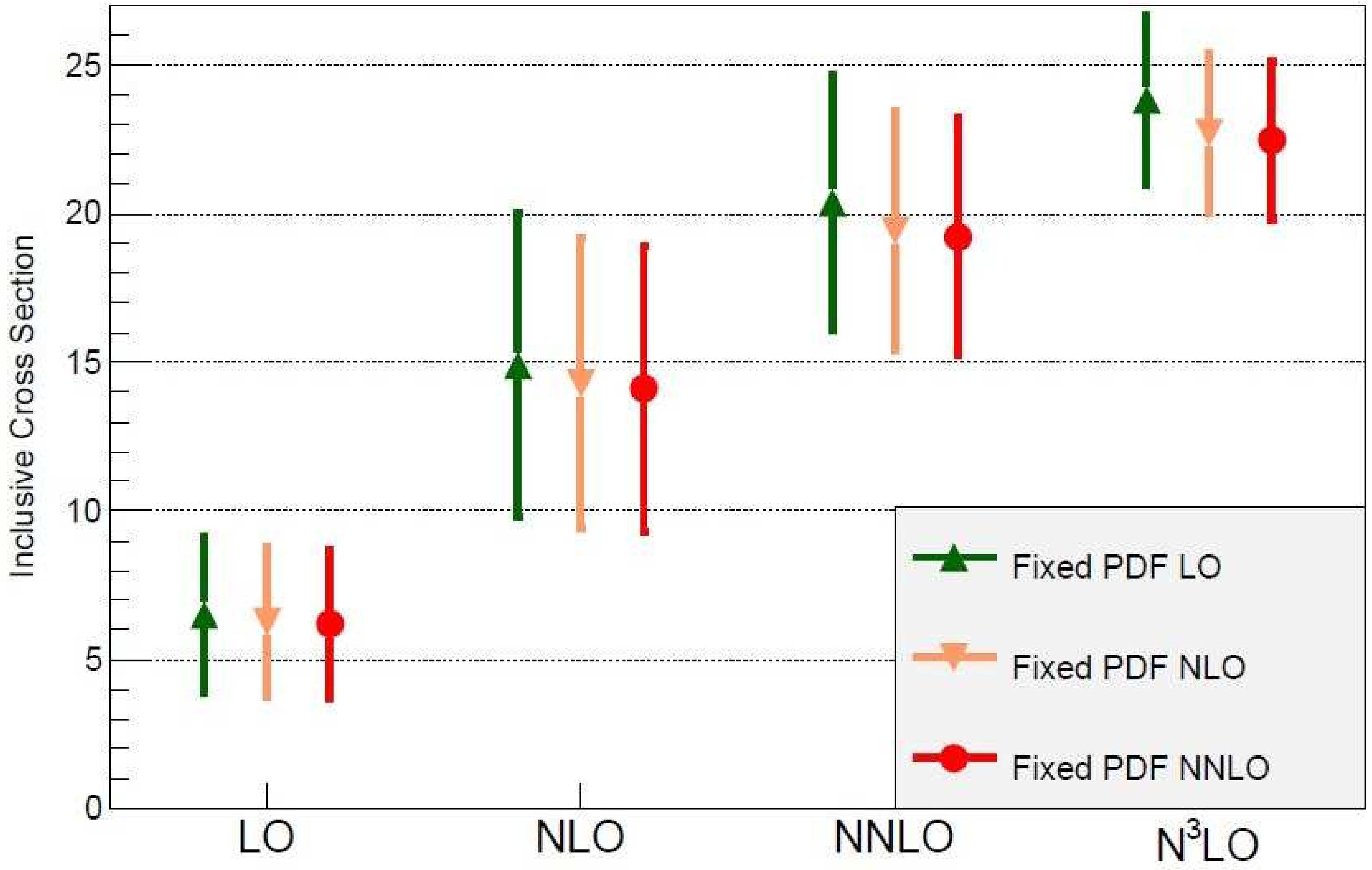
$$\mathcal{O}_k(Q,\mu) \equiv \sum_{n=0}^{k} c_n(Q,\mu)\alpha_s(\mu)^n, \qquad\qquad \Delta_k(Q,\mu) \equiv \sum_{n=k+1}^{\infty} c_n(Q,\mu)\alpha_s(\mu)^n$$

Usual procedure is to use scale variations to estimate Δ_k,

$$\Delta_k(Q,\mu) \quad \sim \quad \max\left[\mathcal{O}_k\left(Q,\frac{\mu}{2}\right),\mathcal{O}_k(Q,2\mu)\right] \sim \alpha_s(\mu)^{k+1}$$

where μ is chosen to be a typical scale of the problem.
Choice of μ and varation of factor 2 is convention

Theoretical error on σ_H



Forte, Isgro, Vita

Scale variation errors may not give an accurate estimate of the cross section!

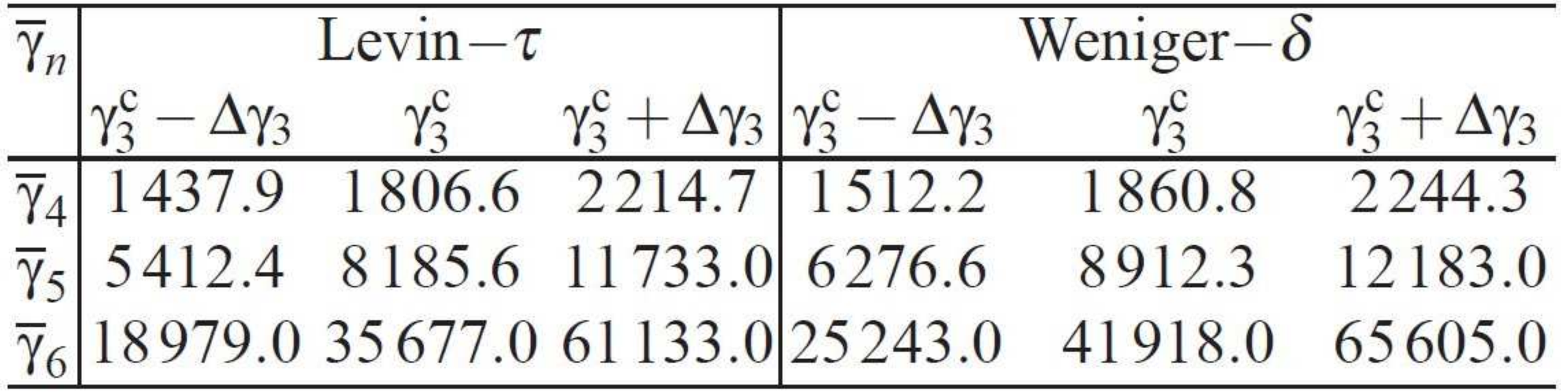
Going beyond scale uncertainties

- ✓ statistical estimate of unknown coefficients — Cacciari, Houdeau

  make the assumption that all the coefficients c_n share a (process dependent) upper bound $\bar{c} > 0$ leading to density functions $f(c_n|\bar{c})$ and $f(\ln\bar{c})$

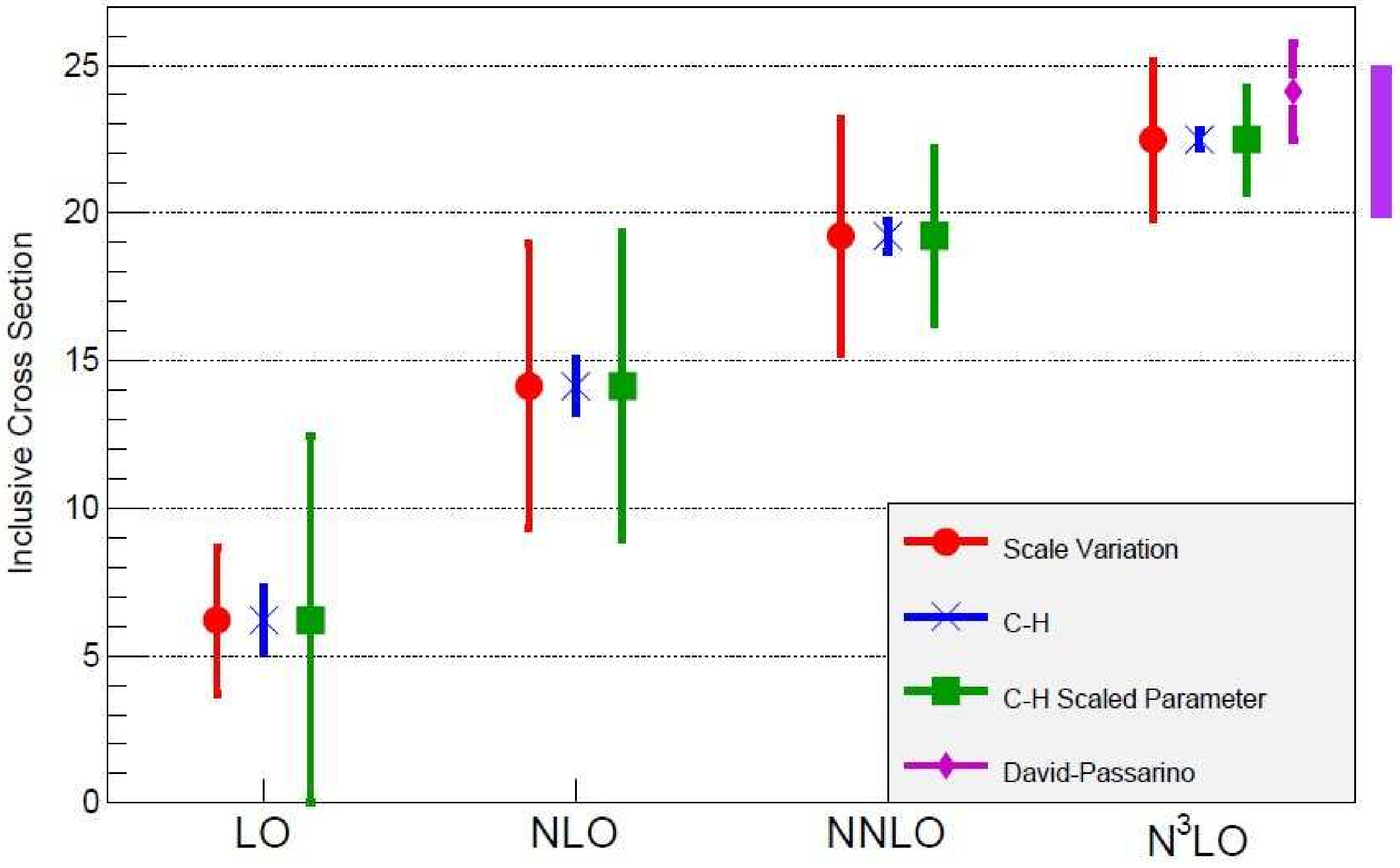
- ✓ series acceleration — David, Passarino

  sequence transformations gives estimates of some of the unknown terms in series



$\overline{\gamma}_n$	Levin$-\tau$			Weniger$-\delta$		
	$\gamma_3^c - \Delta\gamma_3$	γ_3^c	$\gamma_3^c + \Delta\gamma_3$	$\gamma_3^c - \Delta\gamma_3$	γ_3^c	$\gamma_3^c + \Delta\gamma_3$
$\overline{\gamma}_4$	1437.9	1806.6	2214.7	1512.2	1860.8	2244.3
$\overline{\gamma}_5$	5412.4	8185.6	11733.0	6276.6	8912.3	12183.0
$\overline{\gamma}_6$	18979.0	35677.0	61133.0	25243.0	41918.0	65605.0

Theoretical error on σ_H revisited



Forte, Isgro, Vita

Estimating σ_H at N3LO

- ✓ Estimate coefficients using information on the singularity structure of the Mellin space cross section coming from all order resummation Ball et al
  - large N (soft gluon, Sudakov)
  - small N (high energy, BFKL)
- ✓ Accepting that scale variation does not give reliable error estimate, can predict the part of the N^{3}LO cross section coming from scale variations. Knowing the first n terms, then

$$c_{n+1}(Q,\mu) \sim \sum_{\ell=1}^{n} c_{n,\ell} \left(\ln \frac{\mu^2}{Q^2} \right)^{\ell}$$

where $c_{n,\ell}$ is constructed from the known c_n and QCD β function coefficients

$\Rightarrow$ very small scale uncertainty NNLO+

- ✓ pressure building to find better solution

2. Improved precision for signal

Les Houches wishlist for Higgs processes

Process	State of the Art	Desired
H	$\mathrm{d}\sigma$ @ NNLO QCD (expansion in $1/m_t$) full m_t/m_b dependence @ NLO QCD and @ NLO EW NNLO+PS, in the $m_t \to \infty$ limit	$\mathrm{d}\sigma$ @ NNNLO QCD (infinite-m_t limit) full m_t/m_b dependence @ NNLO QCD and @ NNLO QCD+EW NNLO+PS with finite top quark mass effects
$H+j$	$\mathrm{d}\sigma$ @ NNLO QCD (g only) and finite-quark-mass effects @ LO QCD and LO EW	$\mathrm{d}\sigma$ @ NNLO QCD (infinite-m_t limit) and finite-quark-mass effects @ NLO QCD and NLO EW
$H+2j$	σ_{tot}(VBF) @ NNLO(DIS) QCD $\mathrm{d}\sigma$(VBF) @ NLO EW $\mathrm{d}\sigma(gg)$ @ NLO QCD (infinite-m_t limit) and finite-quark-mass effects @ LO QCD	$\mathrm{d}\sigma$(VBF) @ NNLO QCD + NLO EW $\mathrm{d}\sigma(gg)$ @ NNLO QCD (infinite-m_t limit) and finite-quark-mass effects @ NLO QCD and NLO EW
$H+V$	$\mathrm{d}\sigma$ @ NNLO QCD $\mathrm{d}\sigma$ @ NLO EW $\sigma_{\mathrm{tot}}(gg)$ @ NLO QCD (infinite-m_t limit)	with $H \to b\bar{b}$ @ same accuracy $\mathrm{d}\sigma(gg)$ @ NLO QCD with full m_t/m_b dependence
tH and $\bar{t}H$	$\mathrm{d}\sigma$(stable top) @ LO QCD	$\mathrm{d}\sigma$(top decays) @ NLO QCD and NLO EW
$t\bar{t}H$	$\mathrm{d}\sigma$(stable tops) @ NLO QCD	$\mathrm{d}\sigma$(top decays) @ NLO QCD and NLO EW
$gg \to HH$	$\mathrm{d}\sigma$ @ NLO QCD (leading m_t dependence) $\mathrm{d}\sigma$ @ NNLO QCD (infinite-m_t limit)	$\mathrm{d}\sigma$ @ NLO QCD with full m_t/m_b dependence

Improved calculations for Higgs processes

✓ Inclusive Higgs cross section
scales: $\hat{s},\, m_h;\, m_t,\, m_b,\, m_w$

✓ Higgs plus jet
scales: $\hat{s},\, m_h,\, p_T^h,\, E_T^j,\, R;\, m_t,\, m_b,\, m_w$

✓ Higgs plus more jets
scales: $\hat{s},\, m_h,\, p_T^h,\, E_T^{j_1},\, E_T^{j_2},\, E_T^{j_3},\, \ldots,\, R,\, \Delta\eta_{j_1 j_2};\, m_t,\, m_b,\, m_w$

☠ whenever large ratios of scales can be produced, then resummation of the large logarithms may be necessary

- small transverse momentum
- threshold logarithms
- large transverse momentum
- large rapidity separations
- . . .

$$(1+\delta^{NLO}_{QCD}+\delta^{NLO}_{EW}) \qquad \text{or} \qquad (1+\delta^{NLO}_{QCD})\times(1+\delta^{NLO}_{EW})$$

First assault on Higgs production at N3LO $m_t \to \infty$

Aim to reduce the theoretical error for the inclusive Higgs cross section via gluon fusion to $\mathcal{O}(5\%)$

- ✓ Ingredients: Three-loop H+0 parton, Two-loop H+1 parton, One-loop H+2 parton, Tree-level H+3 parton - all known as matrix elements for $m_t \to \infty$
  - key part is to extract the infrared singularities
- ✓ Threshold corrections: Major new result

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

$$\hat{\sigma}_{ij}(m_H^2, \hat{s}) \propto \sum \left(\frac{\alpha_s}{\pi}\right)^k \hat{\eta}_{ij}^k(z)$$

with

$$\hat{\eta}^k = \left[\delta_{ig}\delta_{jg}\hat{\eta}^k(z) + \mathcal{O}(1-z)^0\right]$$

and $\hat{\eta}_{ij}^k(z)$ contains contributions from distributions $1/(1-z)_+$ and $\delta(1-z)$.

- ✓ Opens up the possibility of full N3LO corrections in future.

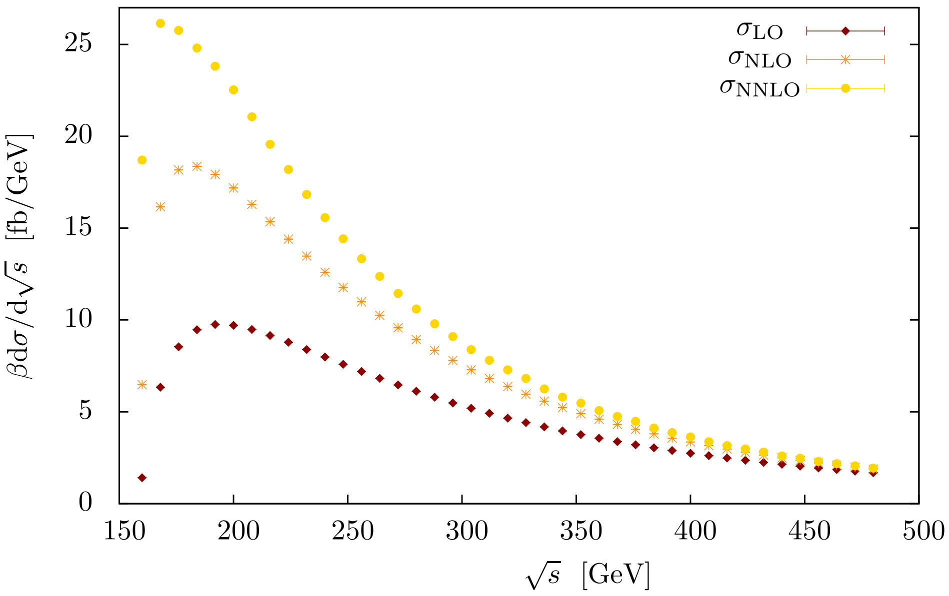
pp $\rightarrow$ H + jet production at NNLO $m_t \rightarrow \infty$

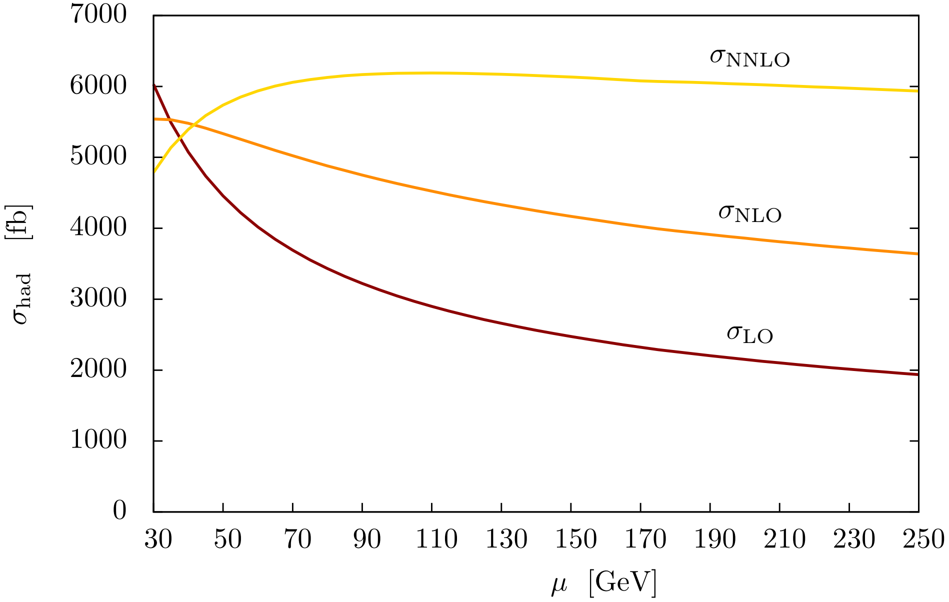
- ✓ Key goal: Establish properties of the Higgs boson!
- ✓ experimental event selection according to number of jets
  - ✓ different backgrounds for different jet multiplicities
  - ✓ H+0 jet known at NNLO

    Anastasiou, Melnikov, Petriello; Catani, Grazzini
  - ✓ H+n jets (n=1,2,3) known at NLO
  - ✓ H+0 jet and H+1 jet samples of similar size
- ✓ NNLO H+1 jet crucial, particularly for WW channel
  - ✓ gluons-only total cross section computed

    Boughezal, Caola, Melnikon, Petriello, Schulze
    - ✓ sector-improved subtraction for real radiation
    - ✓ numerical cancellation of infrared singularities
    - ⚠ distributions in progress

pp $\to$ H + jet (gluons only) at NNLO $m_t \to \infty$





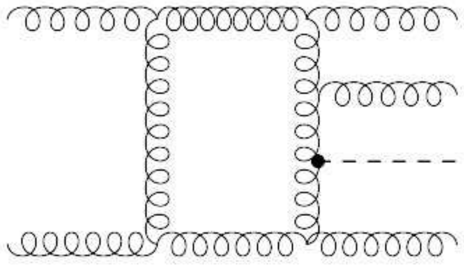
- ✓ large effects near partonic threshold
- ✓ large K-factor

$$\sigma_{NLO}/\sigma_{LO} \sim 1.6$$
$$\sigma_{NNLO}/\sigma_{NLO} \sim 1.3$$

- ✓ significantly reduced scale dependence $\mathcal{O}(4\%)$
- ✓ gg-channel is dominant for phenomenological studies: at NLO gg(70%), qg (30%)
- ⚠ other channels needed at this level of precision - in progress

pp $\rightarrow$ H + 3 jets at NLO $m_t \rightarrow \infty$

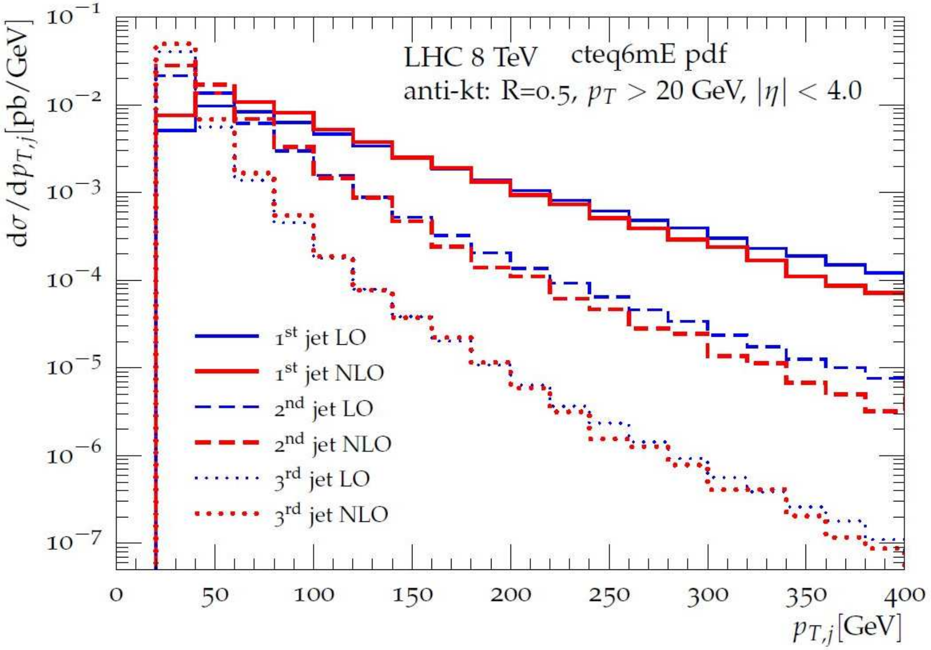
GoSaM: Cullen et al

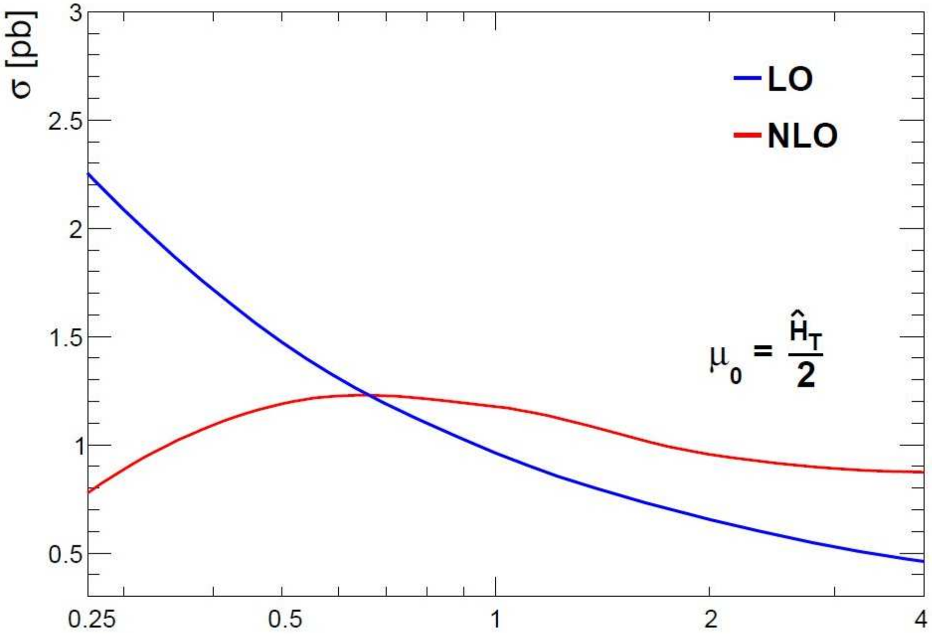


Uses SHERPA and MadDipole/MadEvent for real contribution

Observe

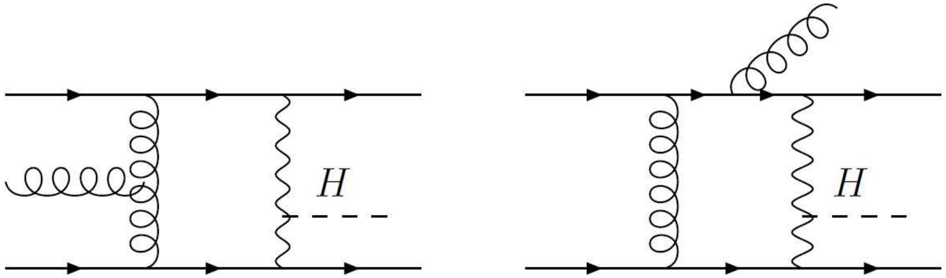
- ✓ strong reduction in scale dependence
- ✓ increased steepness in p_T of Higgs and leading jets





VBF pp → H + 3 jets at NLO

Campanario, Figy, Platzer and Sjodahl



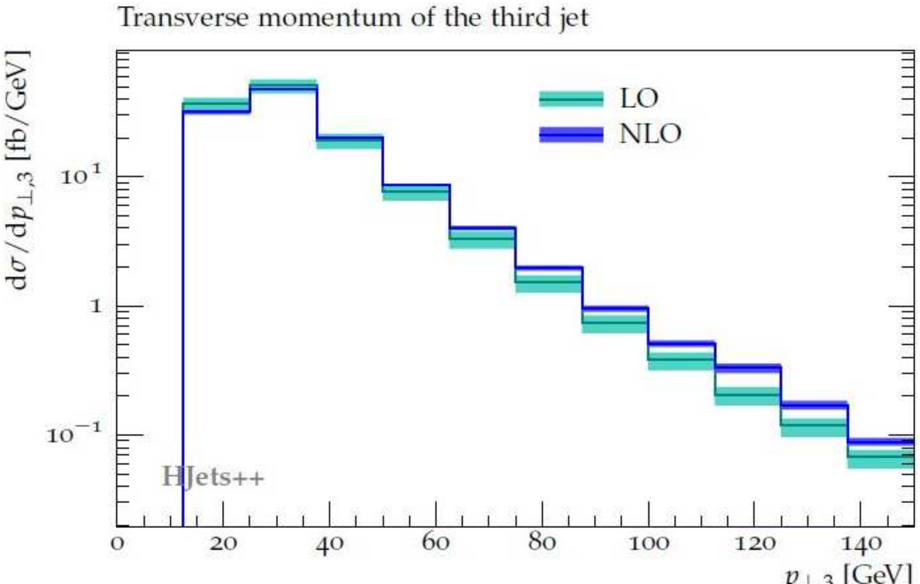
Uses HERWIG++ for real contribution

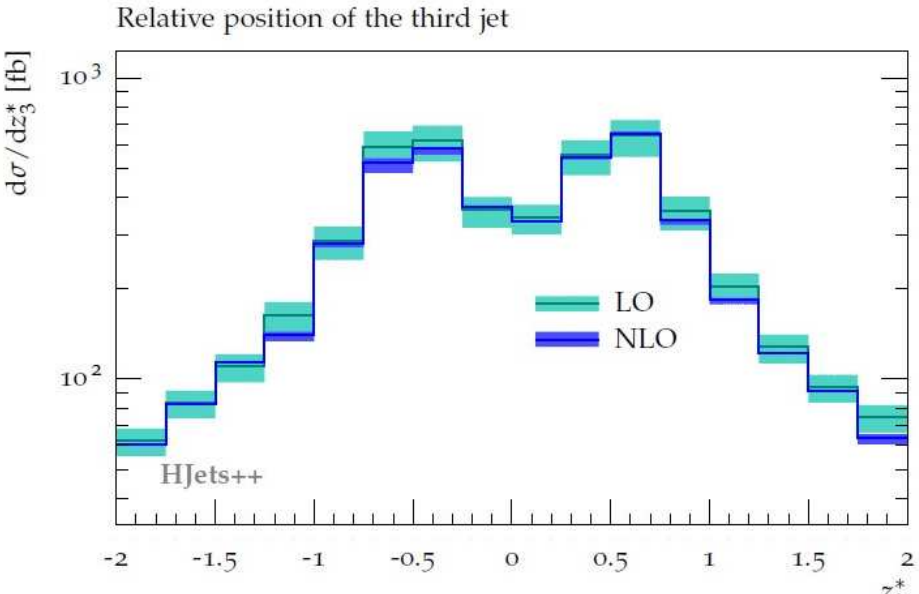
Observe

✓ NLO corrections are moderate for inclusive cuts

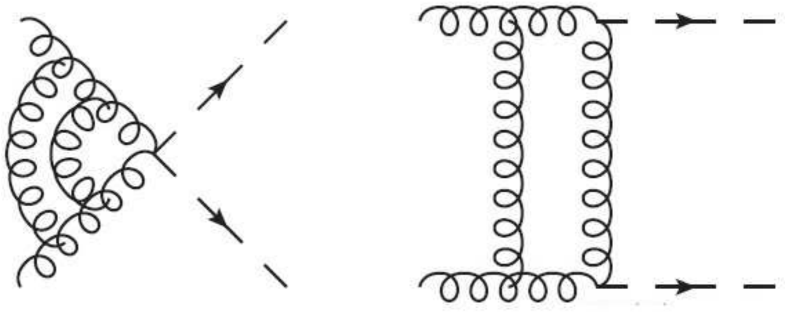
✓ scale uncertainty significantly decreases

✓ third jet tends to accompany the tagging jet


Transverse momentum of the third jet

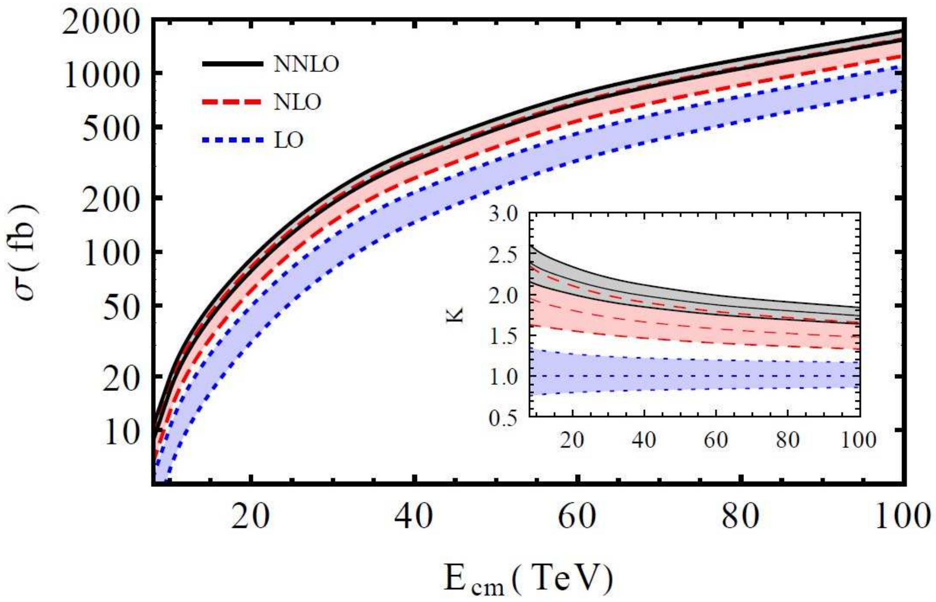

Relative position of the third jet

pp $\to$ HH at NNLO $m_t \to \infty$



$\mathcal{O}(g_S^6)$

de Florian, Mazzitelli

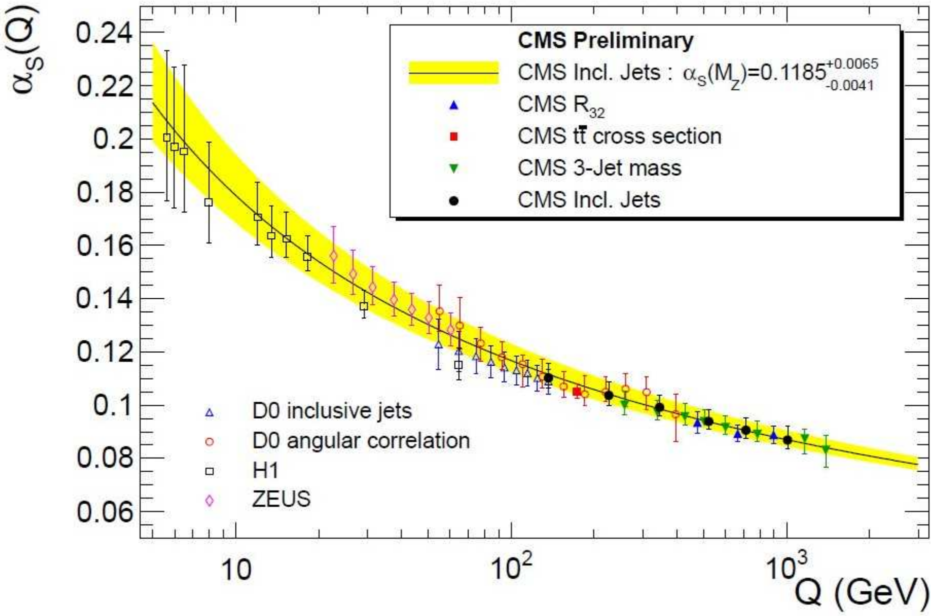


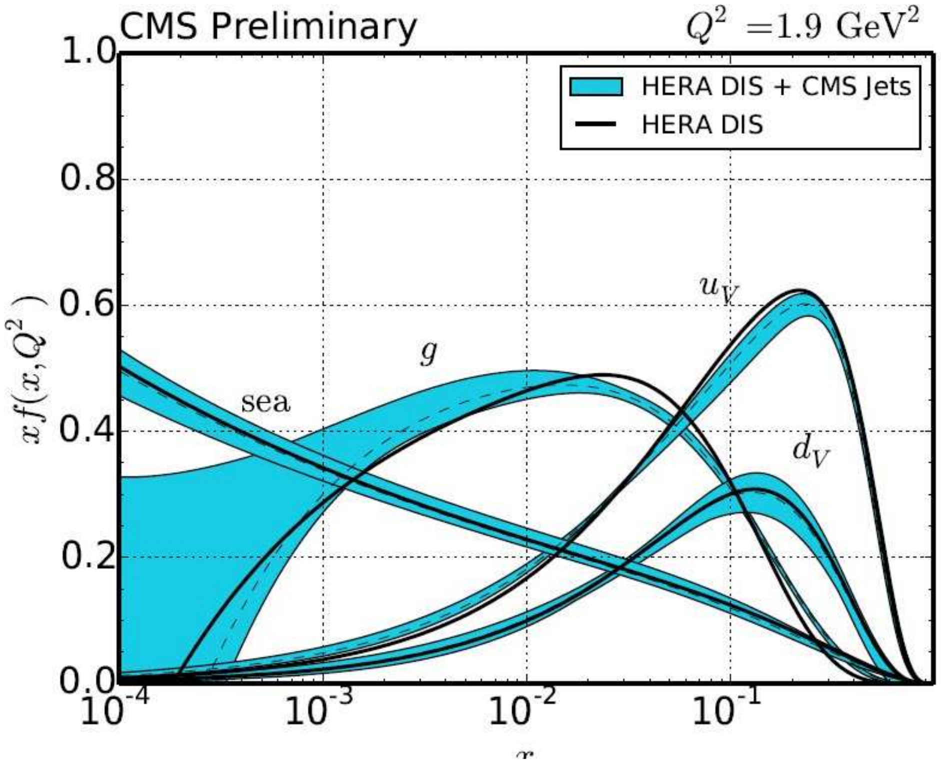
Observe

- ✓ NLO/LO ~ 1.9
- ✓ NNLO/NLO ~ 1.2
- ✓ scale uncertainty significantly decreases

3. Improved precision for input parameters

Potential of LHC data





- ✓ More precise measurements of strong coupling
- ✓ Improved parton distributions

NNLO - for precision measurements

Improvements over NLO

- ✓ reduced scale uncertainty - typically 10%
- ✓ more reliable normalisation and shape of distributions
- ✓ better description of extra radiation
- ✓ more dependence on the jet algorithm

Mandatory for benchmark processes measured to few per cent accuracy

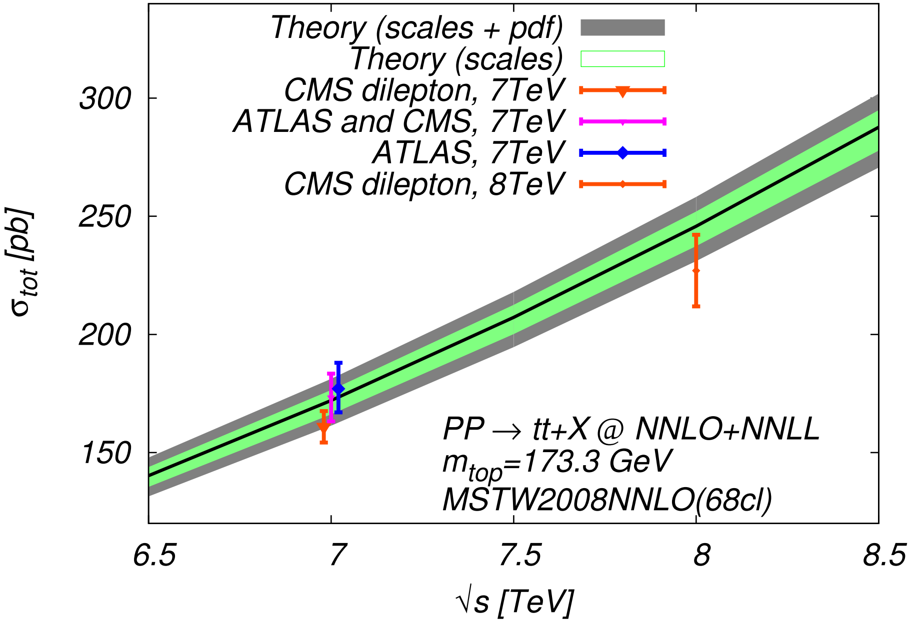
- ✓ jet production
- ✓ vector boson (+ jet) production
- ✓ top quark production

pp $\rightarrow t\bar{t}$ at NNLO

- ✓ Total cross section completed

  Czakon, Fielder, Mitov

  - ✓ STRIPPER for real radiation
  - ✓ purely numerical double virtual
  - ⚠ distributions in progress
- ✓ NNLO theory uncertainty similar to experimental error



Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$+0.259(3.7\%)$ $-0.374(5.3\%)$	$+0.169(2.4\%)$ $-0.121(1.7\%)$
LHC 7 TeV	167.0	$+6.7(4.0\%)$ $-10.7(6.4\%)$	$+4.6(2.8\%)$ $-4.7(2.8\%)$
LHC 8 TeV	239.1	$+9.2(3.9\%)$ $-14.8(6.2\%)$	$+6.1(2.5\%)$ $-6.2(2.6\%)$
LHC 14 TeV	933.0	$+31.8(3.4\%)$ $-51.0(5.5\%)$	$+16.1(1.7\%)$ $-17.6(1.9\%)$

NNLO

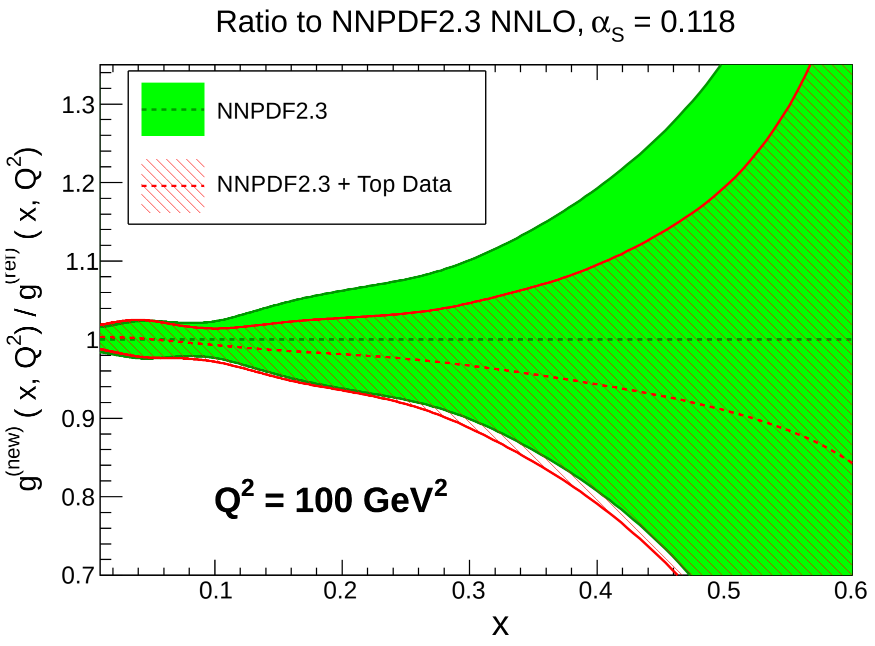
Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$+0.110(1.5\%)$ $-0.200(2.8\%)$	$+0.169(2.4\%)$ $-0.122(1.7\%)$
LHC 7 TeV	172.0	$+4.4(2.6\%)$ $-5.8(3.4\%)$	$+4.7(2.7\%)$ $-4.8(2.8\%)$
LHC 8 TeV	245.8	$+6.2(2.5\%)$ $-8.4(3.4\%)$	$+6.2(2.5\%)$ $-6.4(2.6\%)$
LHC 14 TeV	953.6	$+22.7(2.4\%)$ $-33.9(3.6\%)$	$+16.2(1.7\%)$ $-17.8(1.9\%)$

NNLO+NNLL

Impact on gluon distribution

- ✓ Top production at the LHC dominated by qg and gg channels
- ✓ Total cross section sensitive to gluon PDF
- ✓ Impacts the NNLO global parton distribution fit

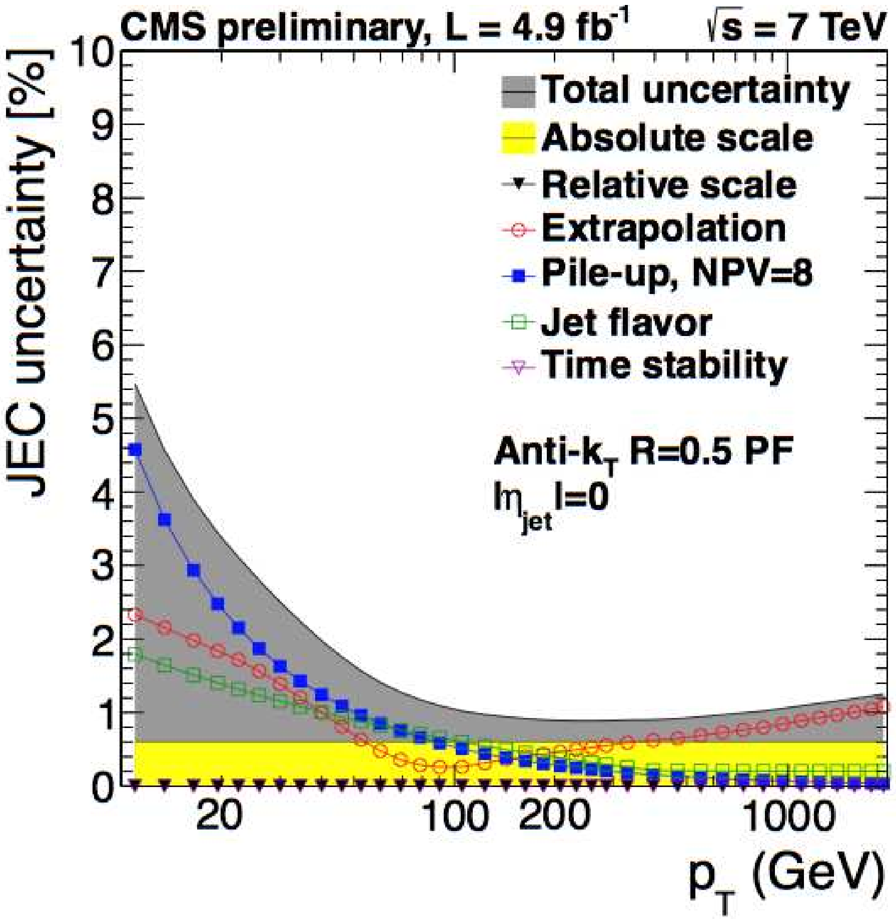
Czakon, Mangano, Mitov, Rojo



- ✓ leads to reduced gluon uncertainty at large x

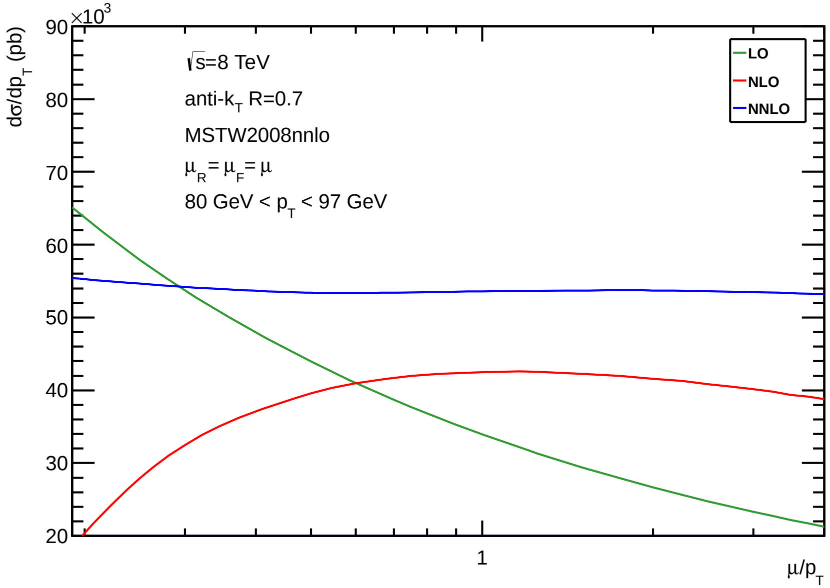
Measuring fundamental quantities with Jets

- ✓ Impressive control over experimental uncertainties
- ✓ With 2011 data CMS Jet Energy Scale Uncertainty below 1% for $p_T = 150 - 600$ GeV in barrel at $|y| < 1.3$.
- ⇒ Experimental uncertainties in Single Jet Inclusive distribution at the 5-10% level
- ⇒ Need for pQCD predictions at NNLO accuracy

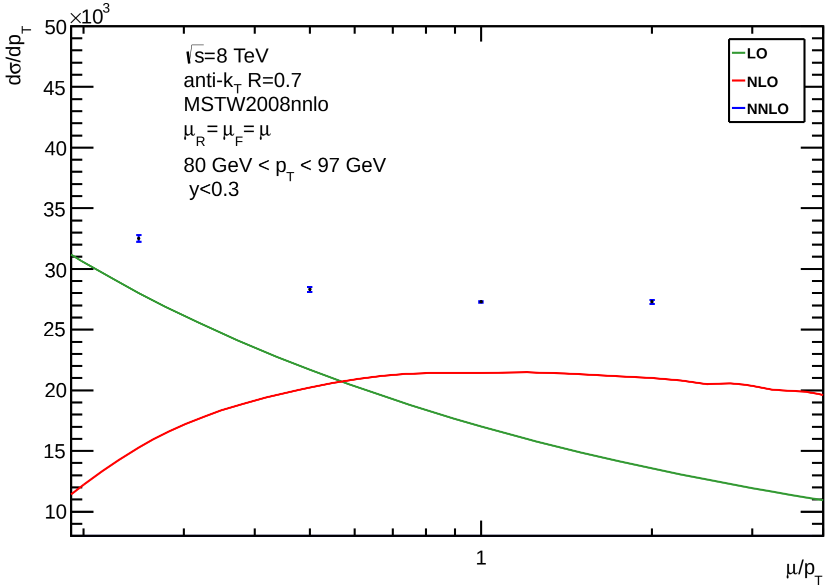


pp → 2 jets (gluons only) at NNLO

Currie, Gehrmann-De Ridder, Gehrmann, Pires, NG



$|y| < 4.4$, 80 GeV $< p_T <$ 97 GeV

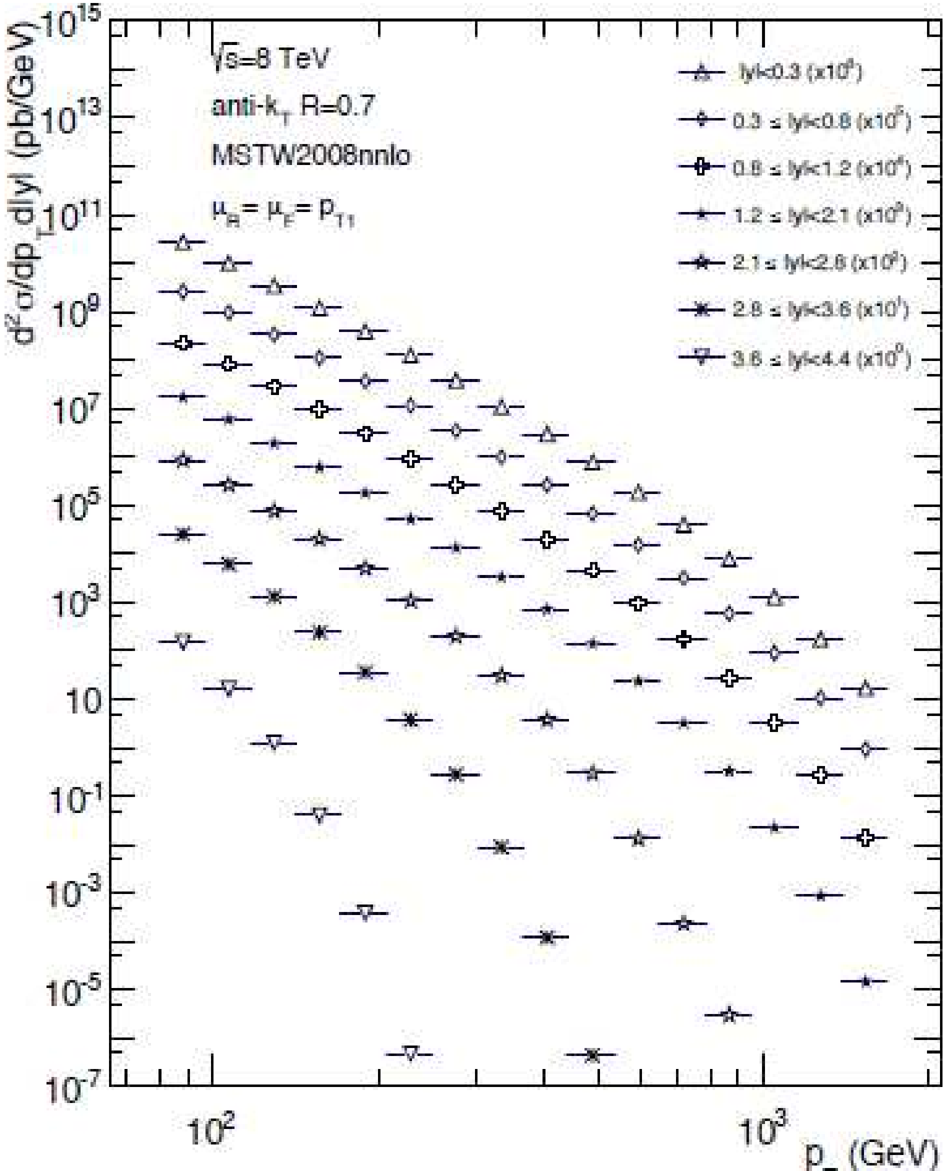


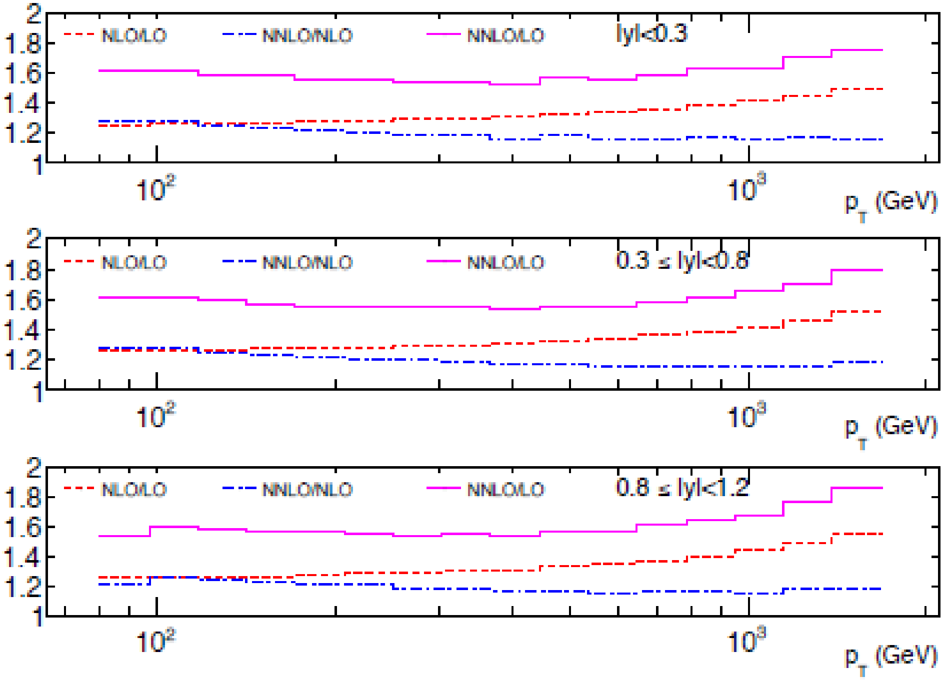
$|y| < 0.3$, 80 GeV $< p_T <$ 97 GeV

- ✓ Scale variation much reduced for $0.5 < \mu/p_T < 2$.
- ✓ . . . but depends on rapidity slice

Jet p_T distribution (gluons only) at NNLO

Gehrmann-De Ridder, Gehrmann, Pires, NG; Currie, Gehrmann-De Ridder, Pires, NG

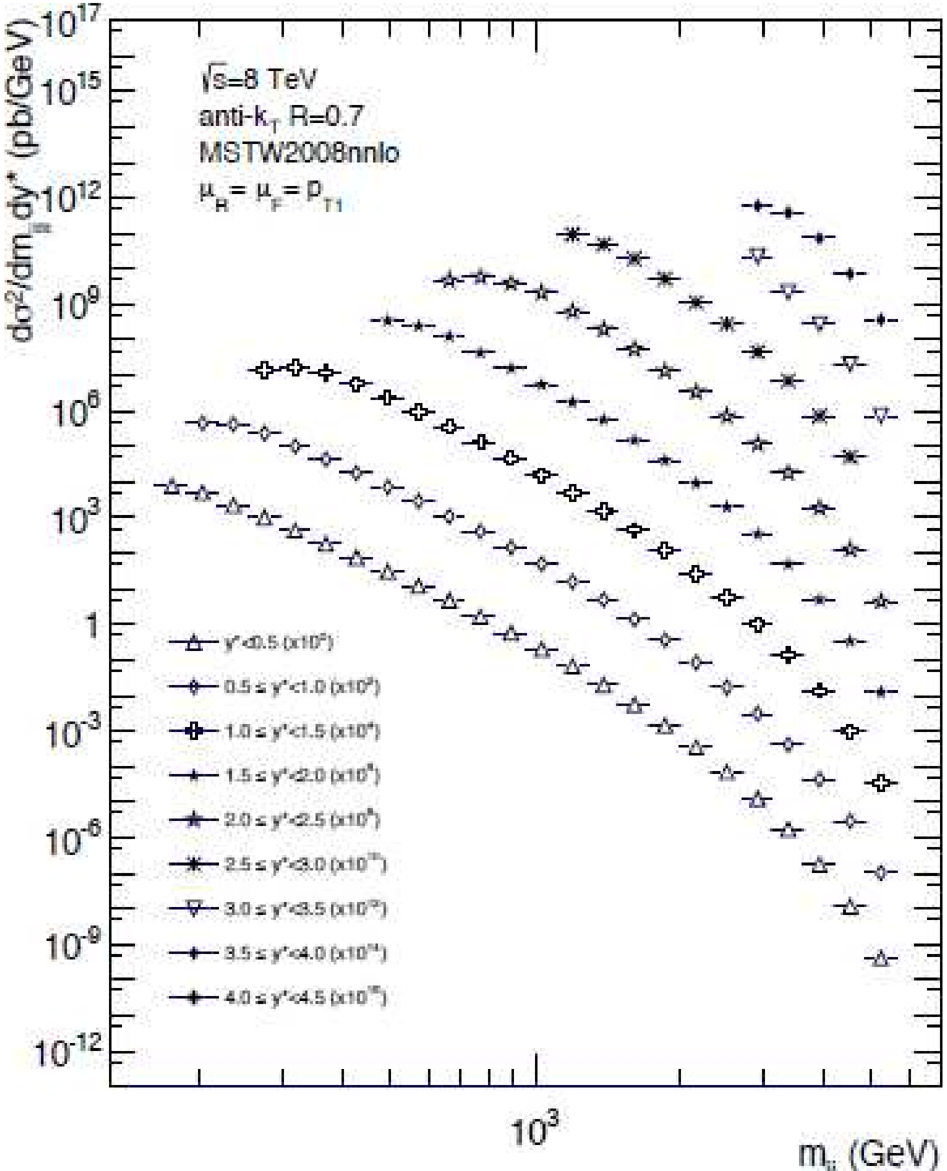


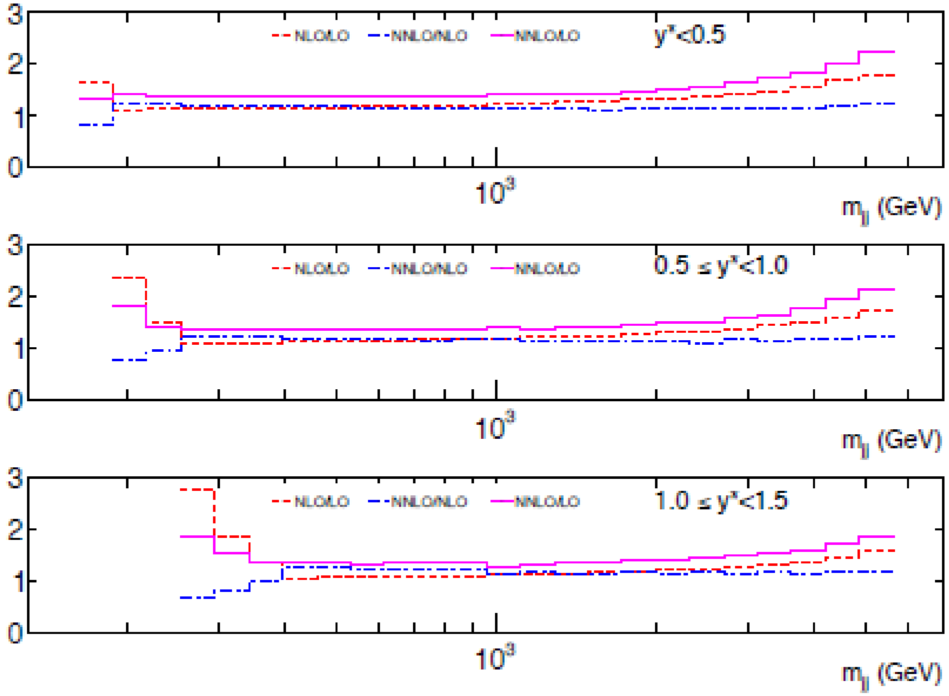


- ✓ NNLO corrections $\sim$25% wrt NLO
- ✓ similar behavior for different rapidity slices

Di-jet mass distribution (gluons only) at NNLO

Gehrmann-De Ridder, Gehrmann, Pires, NG; Currie, Gehrmann-De Ridder, Pires, NG





- ✓ NNLO corrections ∼25% wrt NLO
- ✓ similar behavior for different rapidity slices

Les Houches wishlist for Top/Jet processes

Process	State of the Art	Desired
$t\bar{t}$	$\sigma_{\rm tot}$(stable tops) @ NNLO QCD $\mathrm{d}\sigma$(top decays) @ NLO QCD $\mathrm{d}\sigma$(stable tops) @ NLO EW	$\mathrm{d}\sigma$(top decays) @ NNLO QCD + NLO EW
$t\bar{t}+j(j)$	$\mathrm{d}\sigma$(NWA top decays) @ NLO QCD	$\mathrm{d}\sigma$(NWA top decays) @ NNLO QCD + NLO EW
$t\bar{t}+Z$	$\mathrm{d}\sigma$(stable tops) @ NLO QCD	$\mathrm{d}\sigma$(top decays) @ NLO QCD + NLO EW
single-top	$\mathrm{d}\sigma$(NWA top decays) @ NLO QCD	$\mathrm{d}\sigma$(NWA top decays) @ NNLO QCD + NLO EW
dijet	$\mathrm{d}\sigma$ @ NNLO QCD (g only) $\mathrm{d}\sigma$ @ NLO EW (weak)	$\mathrm{d}\sigma$ @ NNLO QCD + NLO EW
$3j$	$\mathrm{d}\sigma$ @ NLO QCD	$\mathrm{d}\sigma$ @ NNLO QCD + NLO EW
$\gamma+j$	$\mathrm{d}\sigma$ @ NLO QCD $\mathrm{d}\sigma$ @ NLO EW	$\mathrm{d}\sigma$ @ NNLO QCD + NLO EW

Les Houches wishlist for W/Z processes

Process	State of the Art	Desired
V	$\mathrm{d}\sigma$(lept. V decay) @ NNLO QCD $\mathrm{d}\sigma$(lept. V decay) @ NLO EW	$\mathrm{d}\sigma$(lept. V decay) @ NNNLO QCD and @ NNLO QCD+EW NNLO+PS
$V+j(j)$	$\mathrm{d}\sigma$(lept. V decay) @ NLO QCD $\mathrm{d}\sigma$(lept. V decay) @ NLO EW	$\mathrm{d}\sigma$(lept. V decay) @ NNLO QCD + NLO EW
VV'	$\mathrm{d}\sigma$(V decays) @ NLO QCD $\mathrm{d}\sigma$(on-shell V decays) @ NLO EW	$\mathrm{d}\sigma$(decaying off-shell V) @ NNLO QCD + NLO EW
$gg \to VV$	$\mathrm{d}\sigma$(V decays) @ LO QCD	$\mathrm{d}\sigma$(V decays) @ NLO QCD
$V\gamma$	$\mathrm{d}\sigma$(V decay) @ NLO QCD $\mathrm{d}\sigma$(PA, V decay) @ NLO EW	$\mathrm{d}\sigma$(V decay) @ NNLO QCD + NLO EW
$Vb\bar{b}$	$\mathrm{d}\sigma$(lept. V decay) @ NLO QCD massive b	$\mathrm{d}\sigma$(lept. V decay) @ NNLO QCD + NLO EW, massless b
$VV'\gamma$	$\mathrm{d}\sigma$(V decays) @ NLO QCD	$\mathrm{d}\sigma$(V decays) @ NLO QCD + NLO EW
$VV'V''$	$\mathrm{d}\sigma$(V decays) @ NLO QCD	$\mathrm{d}\sigma$(V decays) @ NLO QCD + NLO EW
$VV'+j$	$\mathrm{d}\sigma$(V decays) @ NLO QCD	$\mathrm{d}\sigma$(V decays) @ NLO QCD + NLO EW
$VV'+jj$	$\mathrm{d}\sigma$(V decays) @ NLO QCD	$\mathrm{d}\sigma$(V decays) @ NLO QCD + NLO EW
$\gamma\gamma$	$\mathrm{d}\sigma$ @ NNLO QCD + NLO EW	q_T resummation at NNLL matched to NNLO

4. Improved precision for event simulation

Fixed order versus parton shower

Fixed order calculations

- ✓ Expansion in powers of the coupling constant
- ✓ Correctly describes hard radiation pattern
- ✓ Final states are described by single hard particles
- ✓ NLO: up to two particles in a jet, NNLO: up to three..
- ✓ Soft radiation poorly described

Parton shower

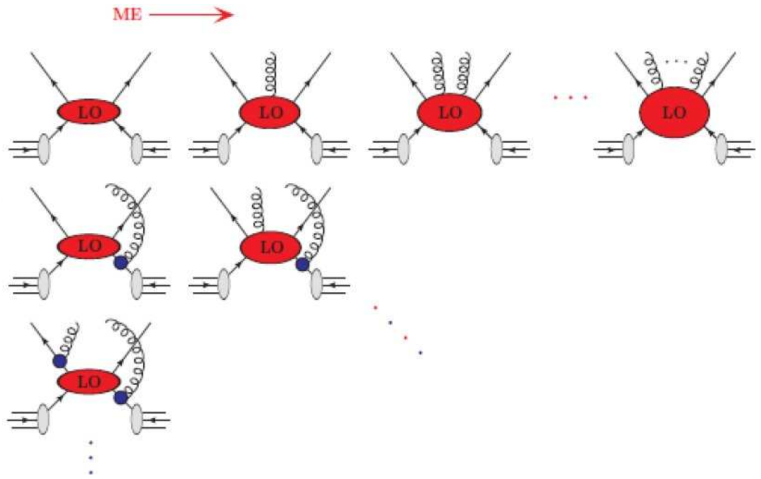
- ✓ Exponentiates multiple soft radiation (leading logarithms)
- ✓ Describes multi-particle dynamics and jet substructure
- ✓ Allows generation of full events (interface to hadronization)
- ✓ Basis of multi-purpose generators (SHERPA, HERWIG, PYTHIA)
- ✓ Fails to account for hard emissions

Ideally: combine virtues of both approaches

Matrix Element improved Parton Shower

matrix elements and parton showers are approximations in different regions of phase space

Merging

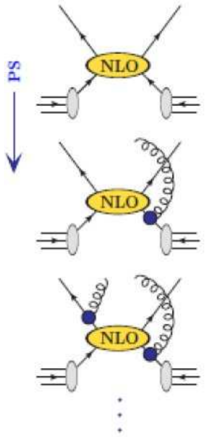


Several fixed order calculations of increasing multiplicity supplemented by PS

CKKW: Catani, Krauss, Kuhn, Webber

MLM: Mangano

Matching



One fixed order calculation supplemented by PS

MC@NLO: Frixione, Webber,

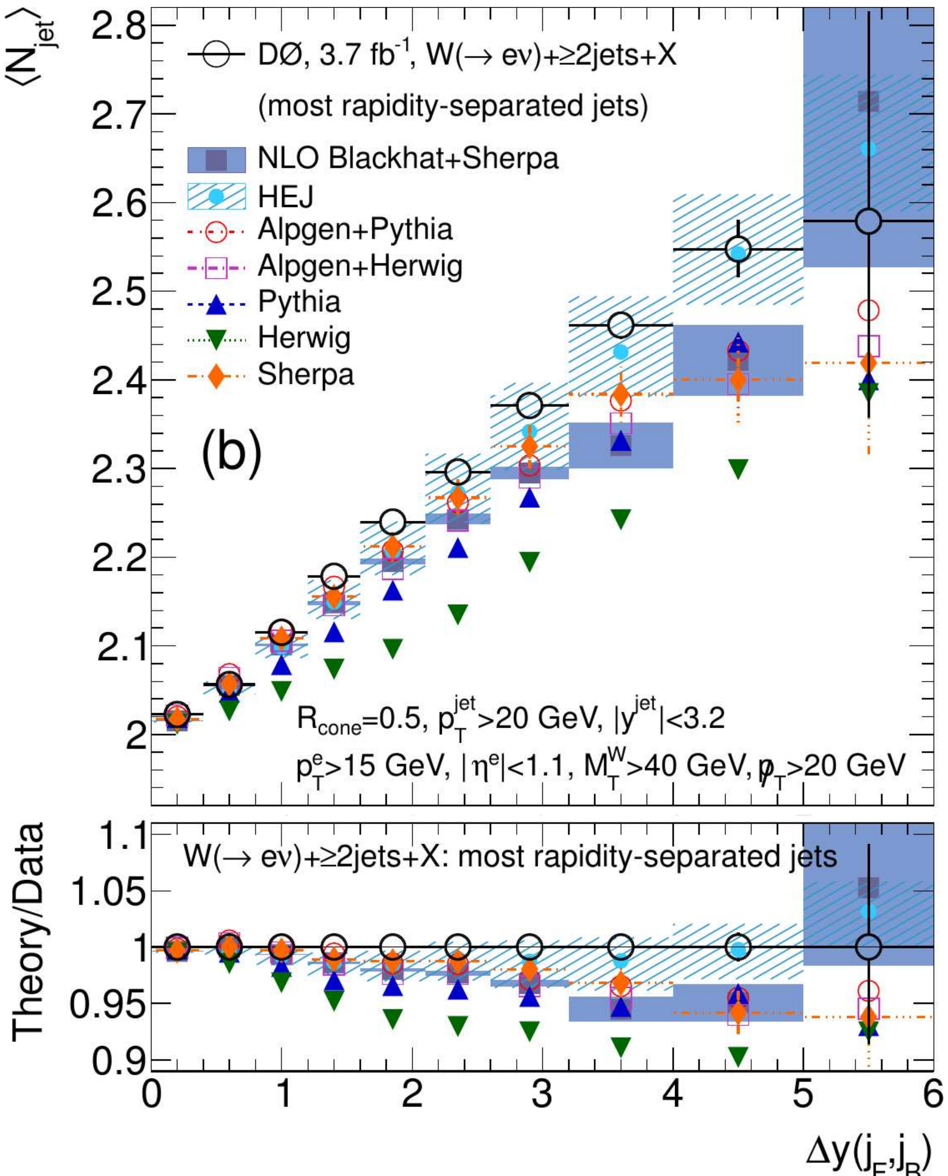
POWHEG: Nason, Oleari

Now benefitting from automation of NLO

aMC@NLO: Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli

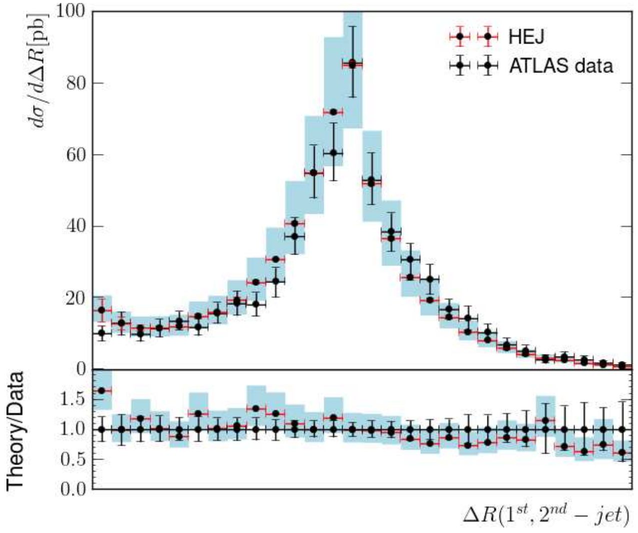
High Energy Jets

Andersen, Smillie

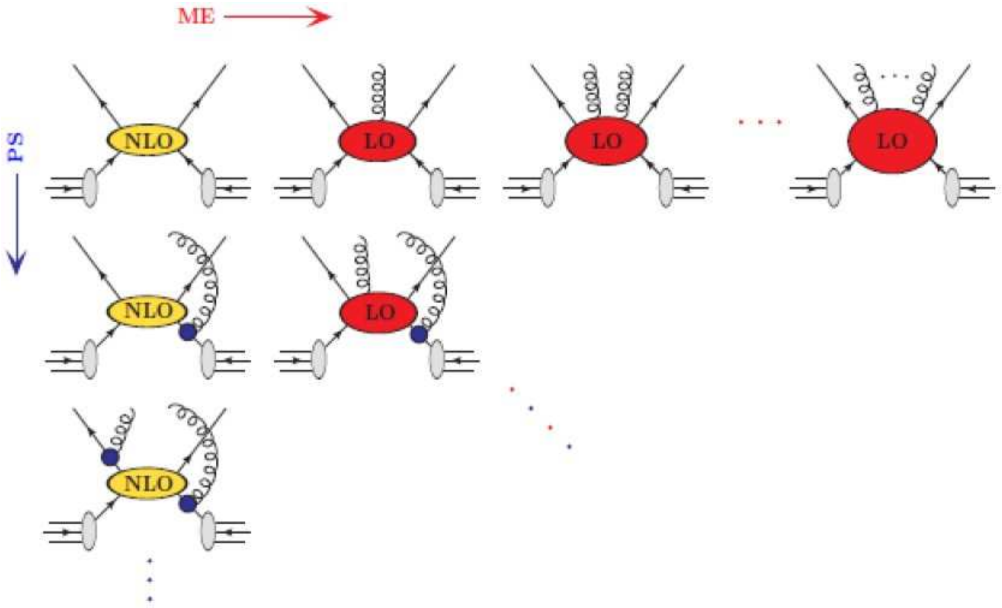


HEJ calculates the real and virtual corrections to the hard scattering ME from wide angle QCD radiation

- ✓ Merges high-multiplicity tree-level ME
- ✓ Describes production of multiple jets of similar E_T
- ✓ Describes dominant corrections at large m_{jj} or large rapidity intervals



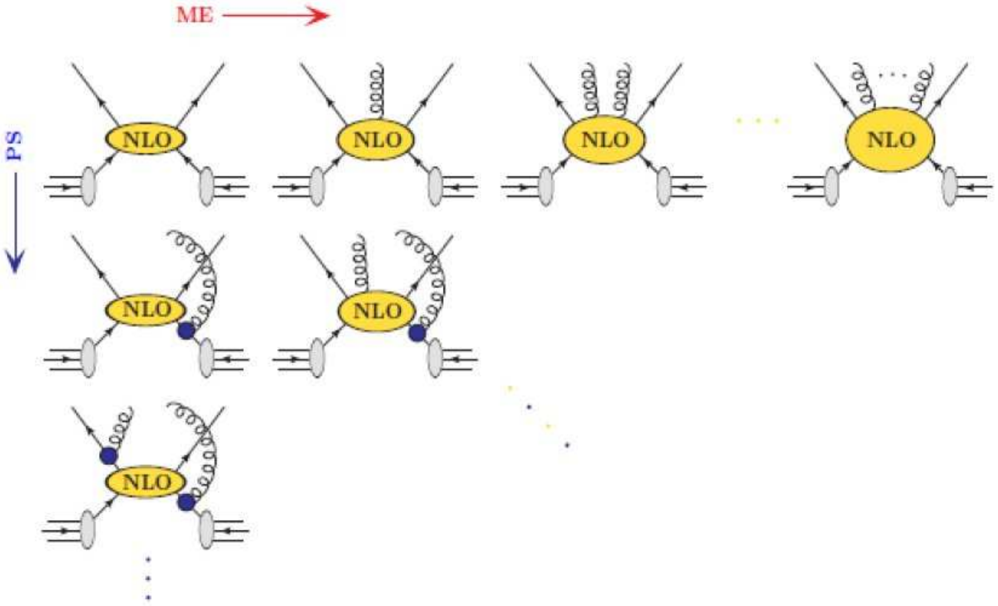
Matrix Element improved Parton Shower



MENLOPS

Supplements core NLOPS with higher multiplicity LOPS

Hamilton, Nason; Hoeche, Krauss, Schonherr, Siegert; Lonnblad, Prestel



MEPS@NLO

Combines multiple NLOPS

Lavesson, Lonnblad; Hoeche, Krauss, Schonherr, Siegert; Frederix, Frixione

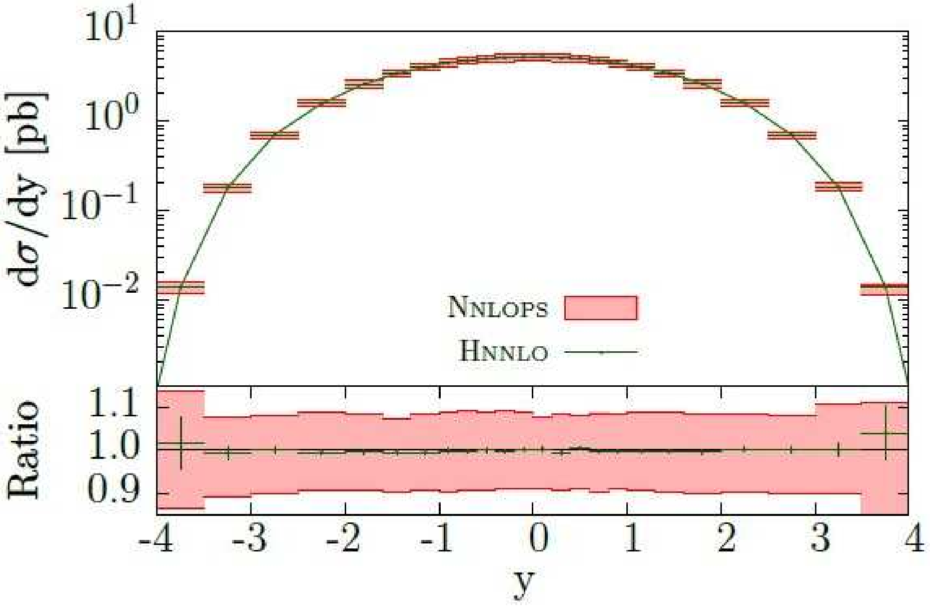
Reaching NNLOPS accuracy

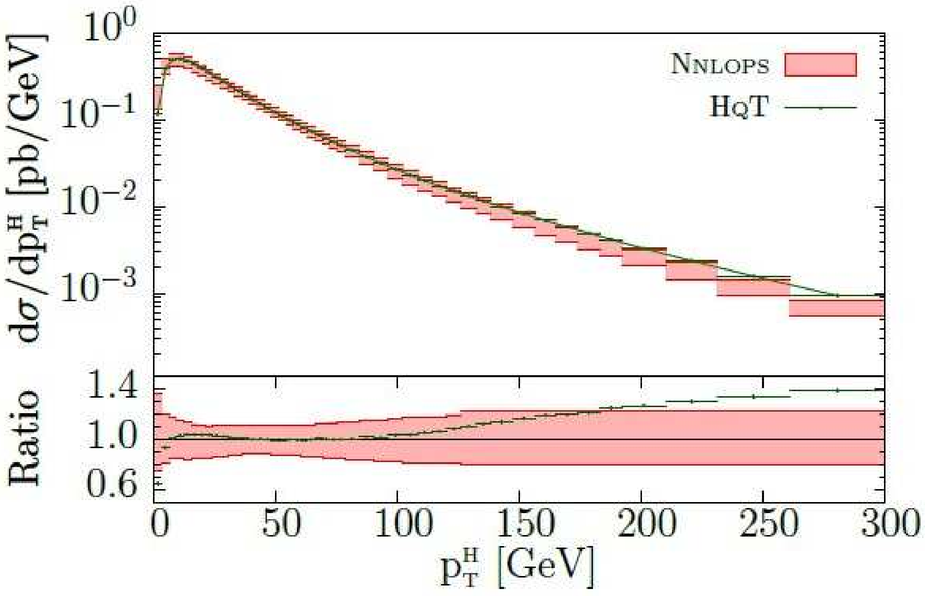
MINLO

Multiscale improved NLO CKKW scale for Born pieces
Sudakov form factors for Born functions in POWHEG

Hamilton, Nason, Zanderighi

Exciting idea! starting from HJ@NLO+PS generate H rapidity distribution at NNLO





Hamilton, Nason, Oleari, Re, Zanderighi

Outlook

Lot of ideas and progress over past few years

- ✓ Incredible conceptual breakthroughs has produced a number of automated NLO solutions for multiparticle processes
- ✓ plus merging with parton showers, etc
- ✓ NLO QCD predictions are established as new standard of theoretical prediction for the LHC
- ✓ NNLO predictions are the new frontier, and results for $2 \rightarrow 2$ processes are in sight

Next few years:

- ✓ Les Houches wishlist to focus theory attention
- ✓ New high precision calculations that will appear such as, e.g. N3LO σ_H, could reduce MHO uncertainty by a factor of two
- ✓ NNLO will emerge as standard for benchmark processes such as dijet production leading to improved pdfs etc. could reduce theory uncertainty due to inputs by a factor of two
- ✓ NNLO calculations will be fully merged with PS, improving parton/hadron transition