(N)(N)LO + parton showers

Stefan Prestel

(DESY)

SM@LHC 2014, Madrid, April 10

Outline

I did not choose the title, and can't deliver what it promises.
However, I can give a heavily biased review of the current status.

- Comments on leading-order ME+PS merging.
- NLO multi-jet merging.
- NNLO outlook.
- Summary.

PYTHIA 8 commercial

- No ep, γp or $\gamma\gamma$ incoming beams.
- Fewer Technicolour and SUSY in-built processes.
+ Evolved MPI model, sophisticated diffractive machinery.
+ τ polarisation in production and decay.
+ Hidden valley showers, R-hadrons.
+ Weak showers.
+ More perturbative physics: Matching and Merging!
+ Simple card files: Should be better match for software frameworks.

PYTHIA 6 development has stopped.

PYTHIA 6 support will stop after the shutdown.

Fixed-order improvements: The ME+PS merging problem

Problem: We want to describe soft/collinear and hard jets in one sample, but we do not know the boundary between "soft" and "hard".
Parton showers are good for describing soft/collinear jets.
Fixed-order matrix elements are good for well-separated jets.

$\Rightarrow$ Should we just "add" several PS and ME samples?

- Just adding ME and PS gives massive double counting.

$\longrightarrow$ Instead, use ME above a cut t_{MS}, and PS below t_{MS}.

- This still has problems: ME overlaps and cut dependence.

$\longrightarrow$ Apply the same weights above and below the cut.

$\longrightarrow$ MEPS merging

CKKW(-L)[1] merging

The CKKW-L prescription is:

- ◇ Calculate the tree-level MEs.

$$\langle \mathcal{O} \rangle = \mathrm{B}_0 \qquad \mathcal{O}(S_{+0j})$$

$$\int \mathrm{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \qquad \mathcal{O}(S_{+1j})$$

[1] JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad) ...

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- Start the PS on the reweighted ME configuration.
  Veto the event if the PS emission if it gives a state in the ME region.
- Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = B_0 \quad \times \Pi_{S_{+0}}(\rho_0, t_{MS}) \quad \mathcal{O}(S_{+0j})$$

$$+ \int B_1 \Theta\left(t\left(S_{+1}\right) - t_{MS}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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CKKW-L results

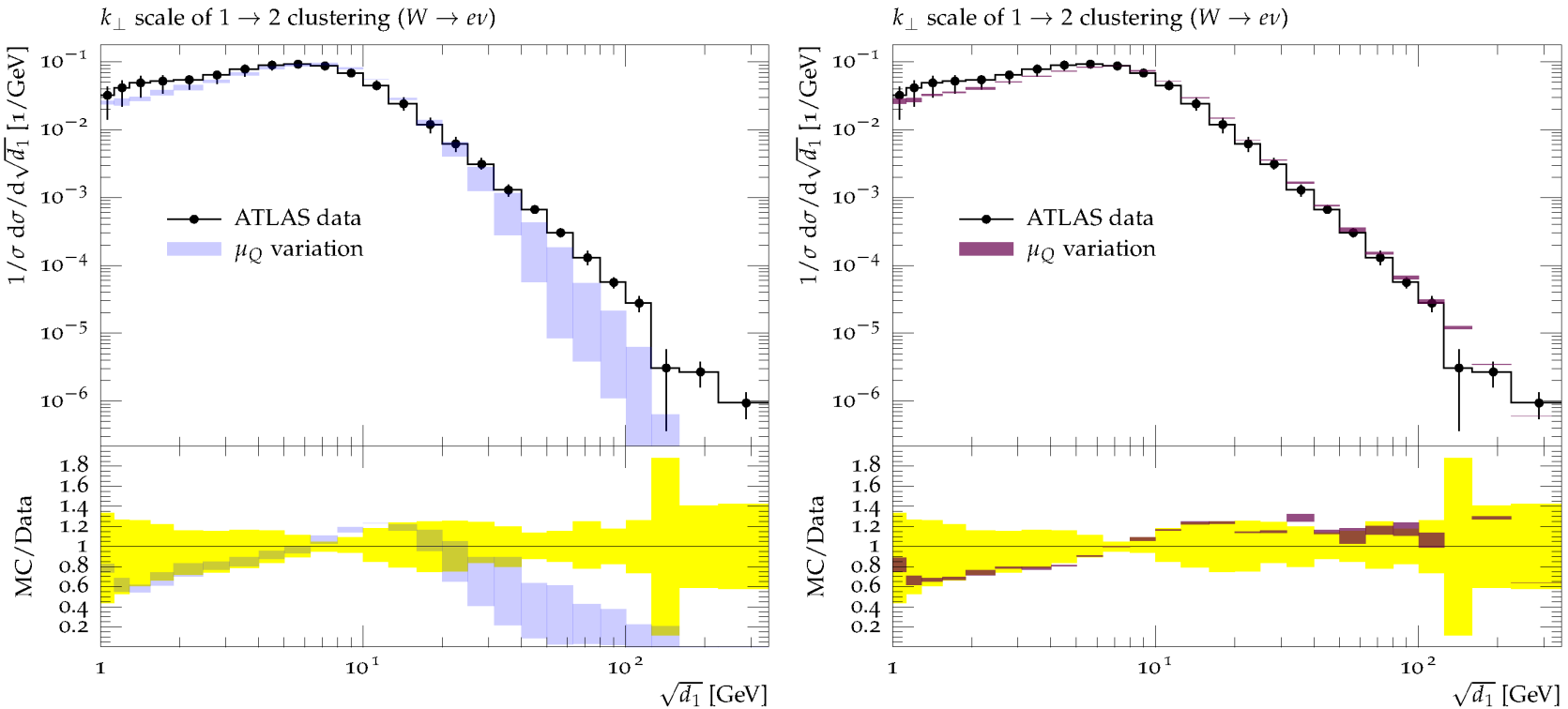

Figure: $k_\perp$-separation between the first and second jet for W+jets, when clustering to exactly two jets. The bands are obtained by varying the PS starting scale in $\mu_Q \in [\frac{1}{2}M_W, 2M_W]$

. . . however

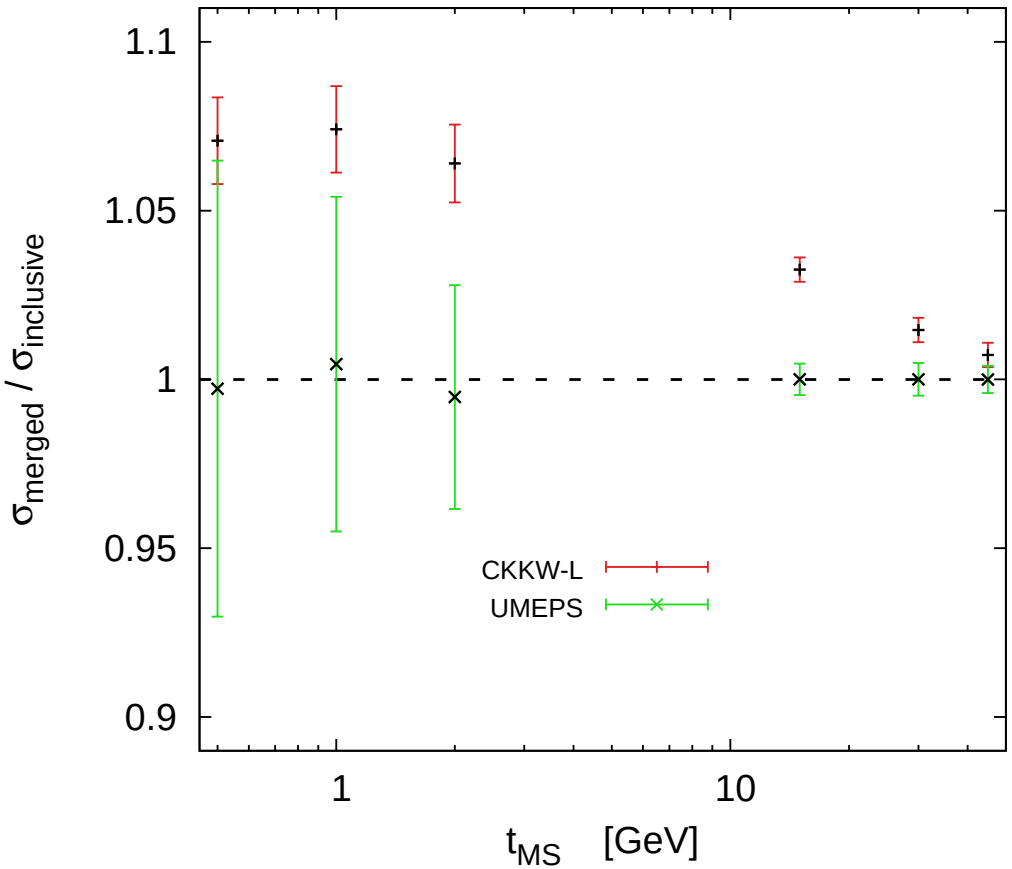

Figure: Ratio of the inclusive cross section after merging, compared to the tree-level inclusive cross section.

Bug vs. Feature in ME+PS

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).
These are the improvements that we need to describe multiple hard jets!

If we simply add samples, the "improvements" will degrade the inclusive cross section: σ_{inc} will contain $\ln(t_{\mathrm{MS}})$ terms.

THE INCLUSIVE CROSS SECTION DOES NOT CONTAIN LOGS RELATED TO CUTS ON HIGHER MULTIPLICITIES.

Traditional approach: Don't use a too small merging scale.
$\rightarrow$ Uncancelled terms numerically not important.

New approach[1]:
Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on t_{MS}.

[1] JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

CKKW(-L)[1] merging

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- Start the PS on the reweighted ME configuration.
  Veto the event if the PS emission if it gives a state in the ME region.
- Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = B_0 \left(1 - \int d\rho w_f^0 w_{\alpha_s}^0 P_0(\rho) \Pi_{S_{+0}}(\rho_0, \rho) \Theta\left(t\left(S_{+1}\right) - t_{MS}\right) \right) \mathcal{O}(S_{+0j})$$
$$+ \int B_1 \Theta\left(t\left(S_{+1}\right) - t_{MS}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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UMEPS merging

The UMEPS prescription is:

- Calculate the tree-level MEs.
- Reweight with Sudakovs, α_s- and PDF-ratios.
- Process 1j-ME again: Reweight, then project onto 0-parton state.
- Start the PS on the reweighted ME configuration.
  Veto the PS emission if it gives a state in the ME region.
- Combine by adding reweighted events, and subtracting reweighted-projected events.

$$\langle \mathcal{O} \rangle = B_0 \left(1 - \int d\rho w_f^0 w_{\alpha_s}^0 \frac{B_1}{B_0} \Pi_{S_{+0}}(\rho_0, \rho) \Theta\left(t\left(S_{+1}\right) - t_{MS}\right) \right) \mathcal{O}(S_{+0j})$$
$$+ \int B_1 \Theta\left(t\left(S_{+1}\right) - t_{MS}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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UMEPS results

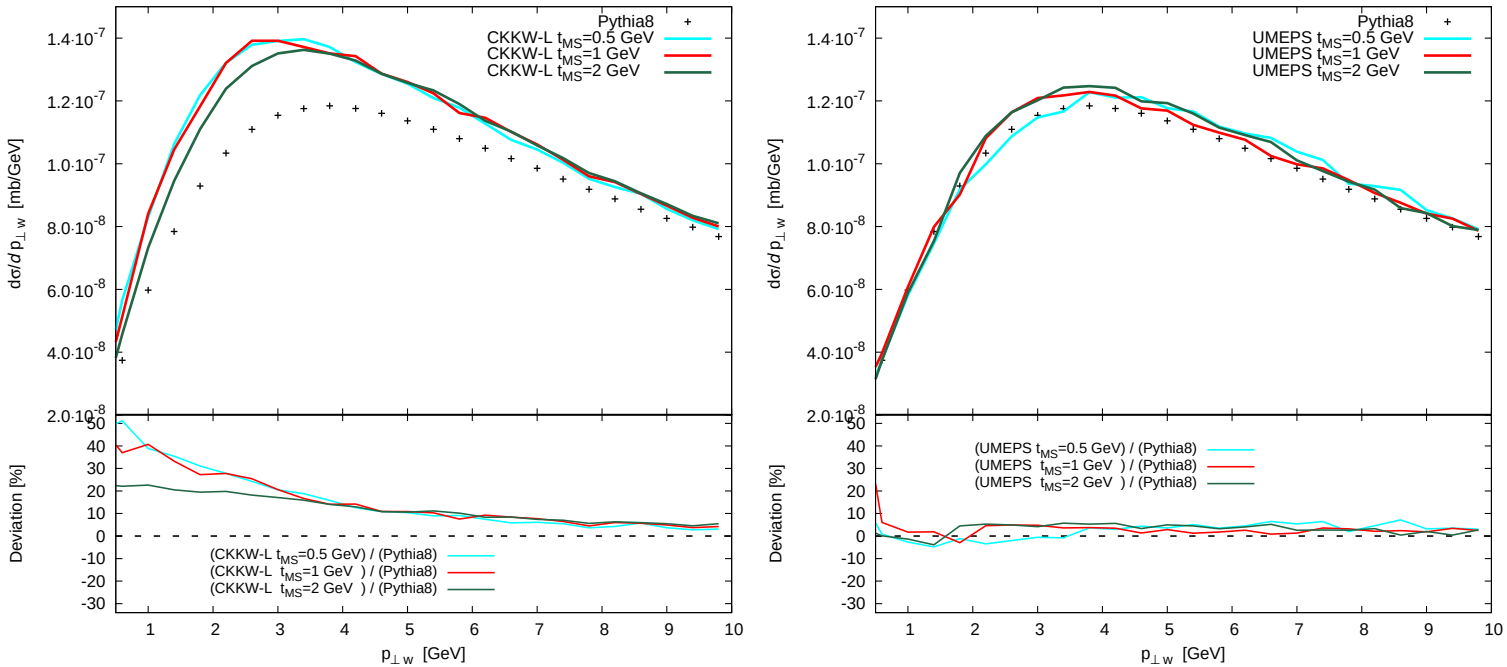


Figure: $p_\perp$ of the W-boson in the Sudakov region (for 2-jet merging, $E_{CM} = 7$ TeV)

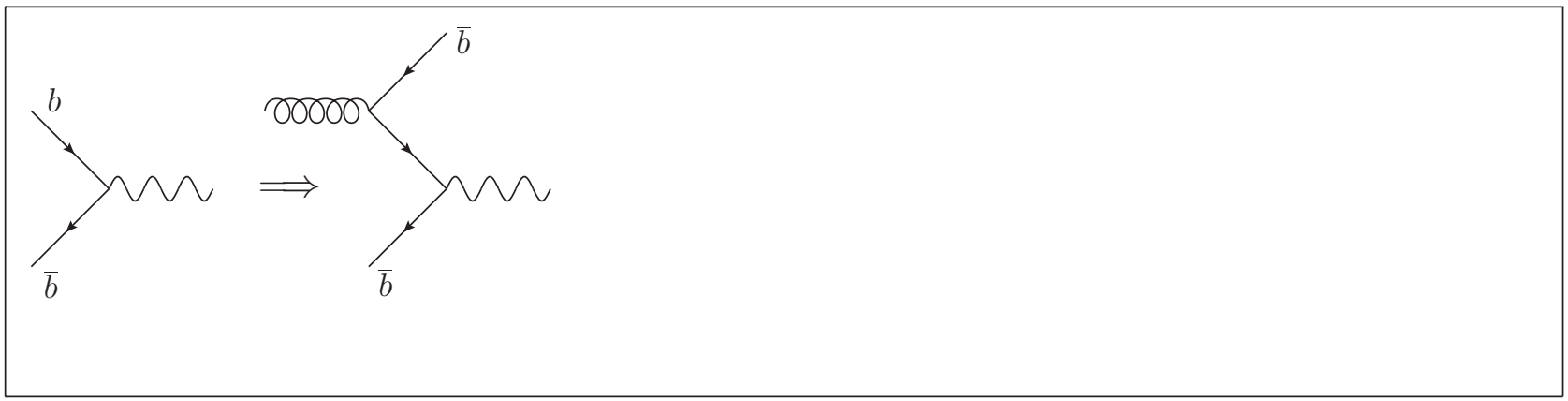
$\Rightarrow$ CKKW-L overshoots for (very) low merging scales.

$\Rightarrow$ UMEPS describes the Sudakov peak nicely.

(For jet observables (high-$p_\perp$ tails etc.) UMEPS does as nicely as CKKW-L.)

Comments on (leading-order) merging

Showers include a treatment of flavour thresholds. These have usually been considered part of the shower.



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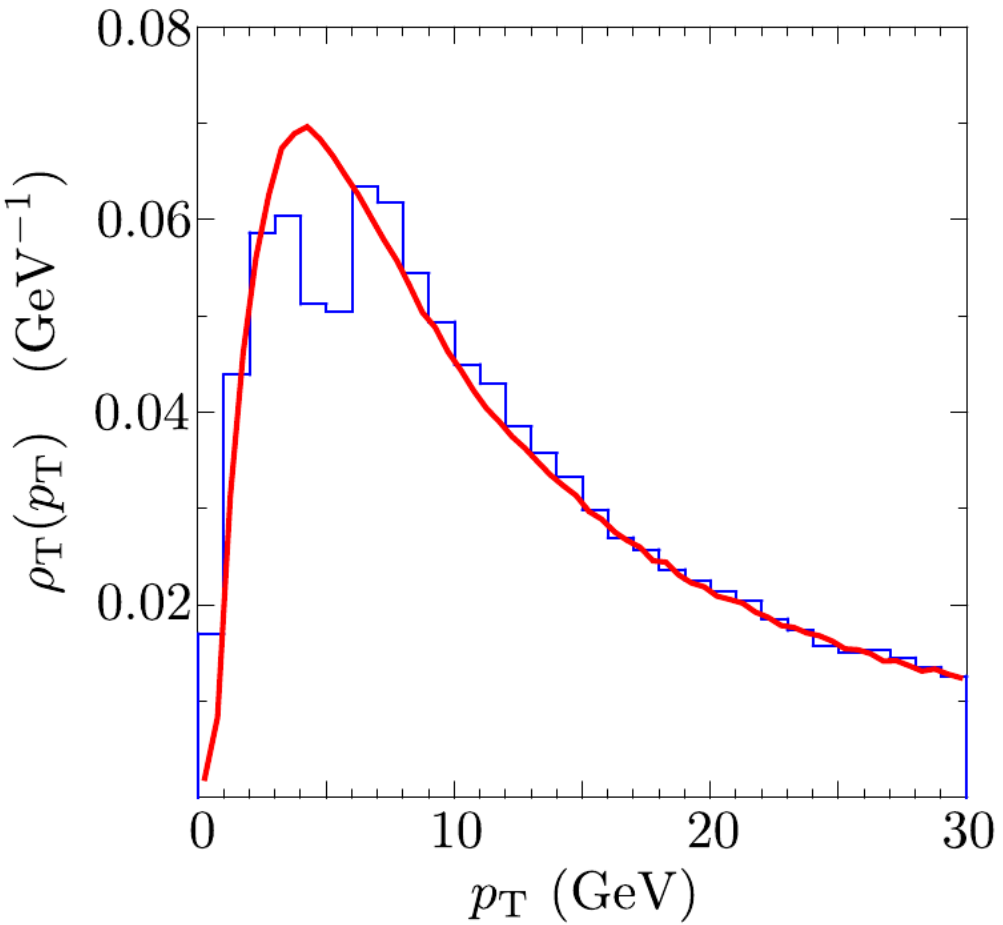

Figure ($p_\perp$ of b-quark in $\mathrm{b\bar{b}} \to Z$, boosted Z) taken from arXiv:1401.6364[hep-ph] by Nagy, Soper

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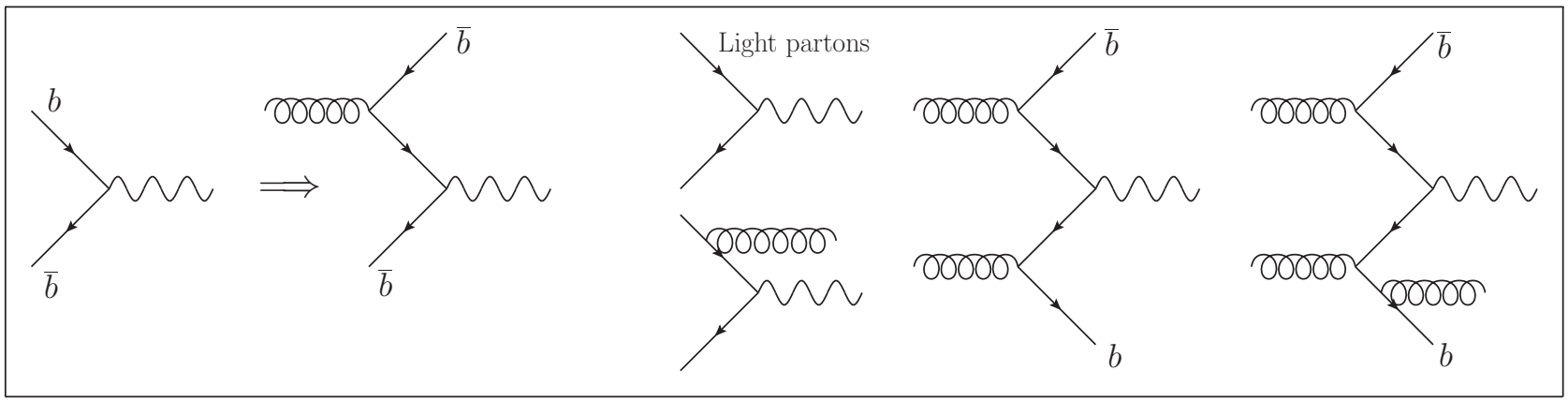

Should we also use matrix elements to describe the flavour thresholds? This only works if the logs are not large[1].

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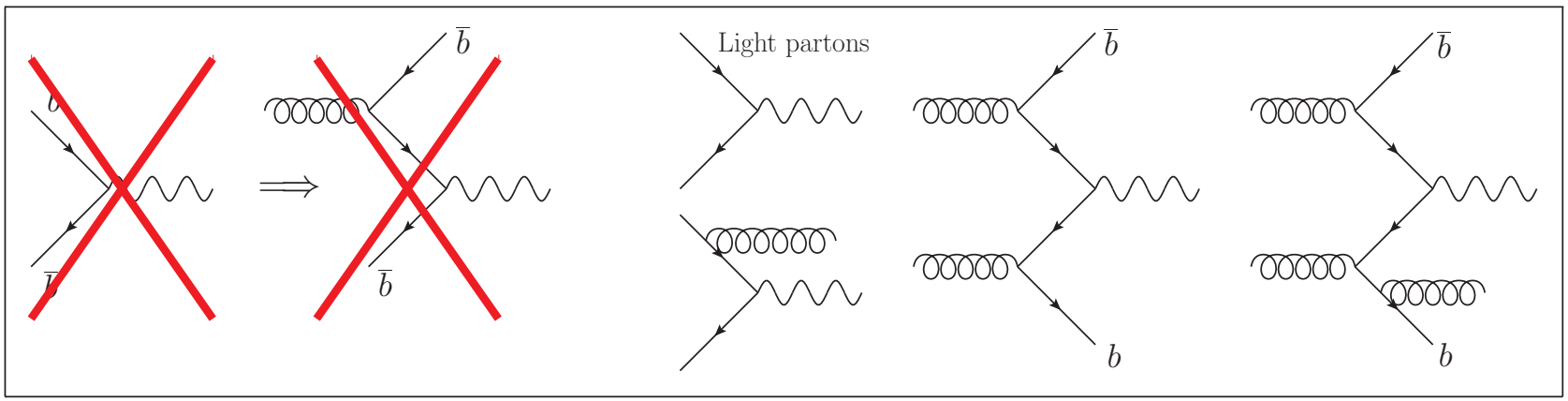
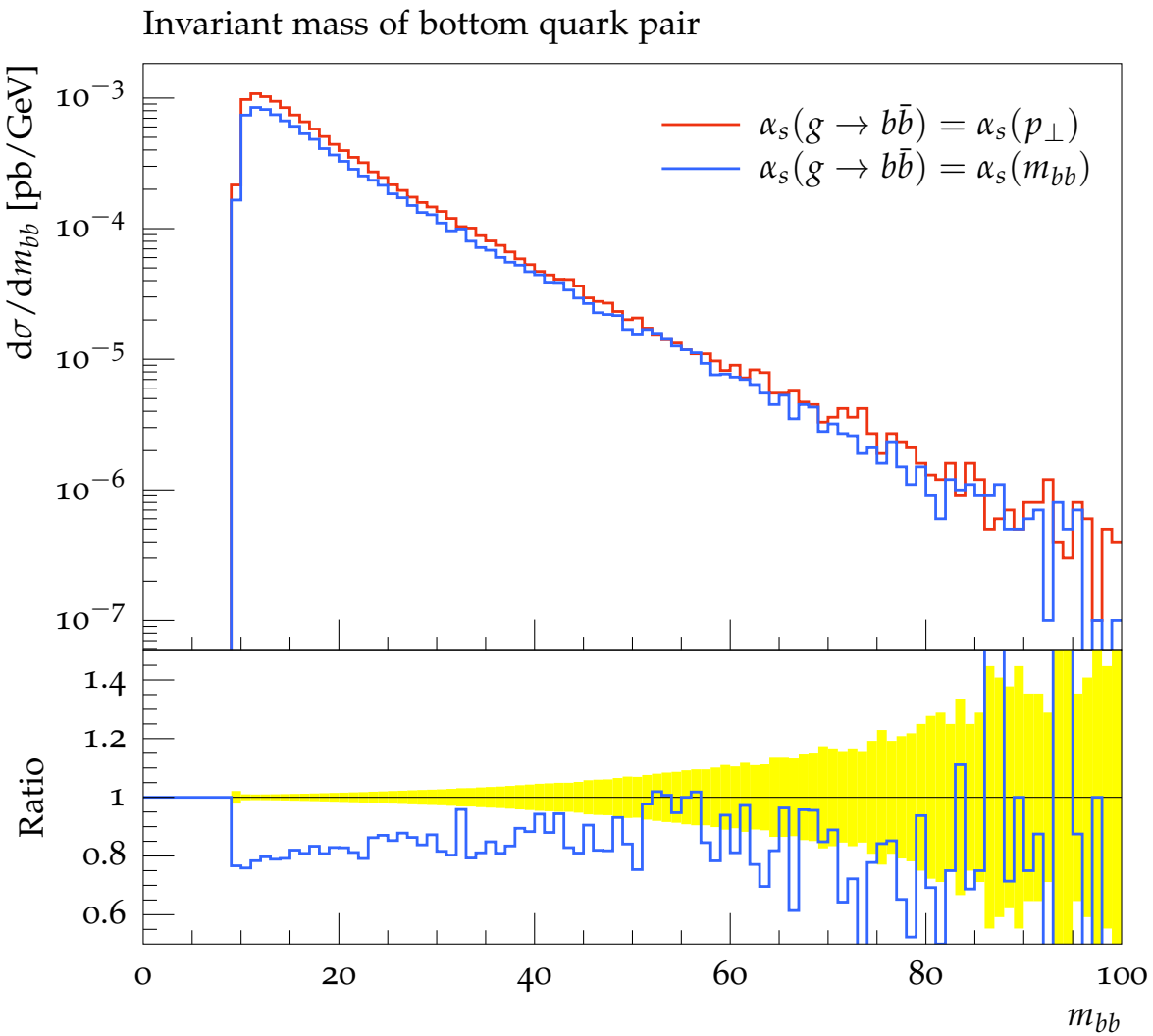


Should we also use matrix elements to describe the flavour thresholds? This only works if the logs are not large[1].

$\Longrightarrow$ To include the flavour thresholds correctly, work in a 4F scheme, i.e. combine multi-light-jet MEs with multi-light/heavy-jet MEs.
Question: How many b's do we need from the ME?
Question: What scale to choose for $\alpha_s(\mathrm{g} \to \mathrm{b\bar{b}})$?

[1] see e.g. JHEP1207(2012)022 by Maltoni, Ridolfi, Ubiali

Comments on (leading-order) merging



Question: What scale to choose for $\alpha_s(\mathrm{g} \to \mathrm{b\bar{b}})$?

Is leading-order enough?

Any leading-order method **X** only ever contains approximate virtual corrections.

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To do NLO multi-jet merging for your preferred LO scheme **X**, do:

- ◇ Subtract approximate **X** $\mathcal{O}(\alpha_s)$-terms, add multiple NLO calculations.
- ◇ Make sure fixed-order calculations do not overlap by cutting, vetoing events and/or vetoing emissions.
- ◇ Adjust higher orders to suit other needs.

$\Rightarrow$ **X**@NLO

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$\Rightarrow$ **X**@NLO

Here, NLO means the NLO cross section excluding phase space regions with "hard" resolved real-emission states, and with divergences present already at Born level regulated with a suitable cut.

UMEPS @NLO

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Basic idea: Do NLO multi-jet merging for UMEPS:

- ◇ Subtract approximate UMEPS $\mathcal{O}(\alpha_s)$-terms, add back full NLO.
- ◇ To preserve the inclusive (NLO) cross section, add approximate NNLO.

$\Rightarrow$ UNLOPS[1].

For UNLOPS merging, we need exclusive NLO inputs:

$$\widetilde{\mathrm{B}}_n = \mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_{n+1|n} + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1|n} \Theta\left(\rho_{\mathrm{MS}} - t\left(S_{+n+1}, \rho\right)\right) - \mathrm{D}_{n+1|n} \right)$$

We can get these e.g. by massaging POWHEG-BOX or MC@NLO output.

The UNLOPS method

Start with UMEPS:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \qquad - \int \widehat{\mathrm{B}}_{1\to 0} \qquad - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$
$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\qquad \widehat{\mathrm{B}}_1 \qquad - \int \widehat{\mathrm{B}}_{2\to 1} \qquad \Bigg) + \iint \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Remove all unwanted $\mathcal{O}(\alpha_s^n)$- and $\mathcal{O}(\alpha_s^{n+1})$-terms:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(- \Big[\int \widehat{\mathrm{B}}_{1\to 0} \Big]_{-1,2} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\Big[\widehat{\mathrm{B}}_1 \Big]_{-1,2} - \Big[\int \widehat{\mathrm{B}}_{2\to 1} \Big]_{-2} \Bigg) + \iint \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Add full NLO results:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \iint \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

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Unitarise:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) + \iint \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

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Iterate for the case of M different NLO calculations, and N tree-level calculations:

$$\begin{aligned}
\langle\mathcal{O}\rangle = \sum_{m=0}^{M-1} & \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \Bigg\{ \widetilde{\mathrm{B}}_m + \left[\widehat{\mathrm{B}}_m \right]_{-m,m+1} + \int_s \mathrm{B}_{m+1\to m} \\
& - \sum_{i=m+1}^{M} \int_s \widetilde{\mathrm{B}}_{i\to m} - \sum_{i=m+1}^{M} \left[\int \widehat{\mathrm{B}}_{i\to m} \right]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s \mathrm{B}^{\uparrow}_{i+1\to m} - \sum_{i=M+1}^{N} \int \widehat{\mathrm{B}}_{i\to m} \Bigg\} \\
& + \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \Bigg\{ \widetilde{\mathrm{B}}_M + \left[\widehat{\mathrm{B}}_M \right]_{-M,M+1} - \left[\int \widehat{\mathrm{B}}_{M+1\to M} \right]_{-M} - \sum_{i=M+1}^{N} \int \widehat{\mathrm{B}}_{i+1\to M} \Bigg\} \\
& + \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int \widehat{\mathrm{B}}_{i\to n} \Bigg\}
\end{aligned}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

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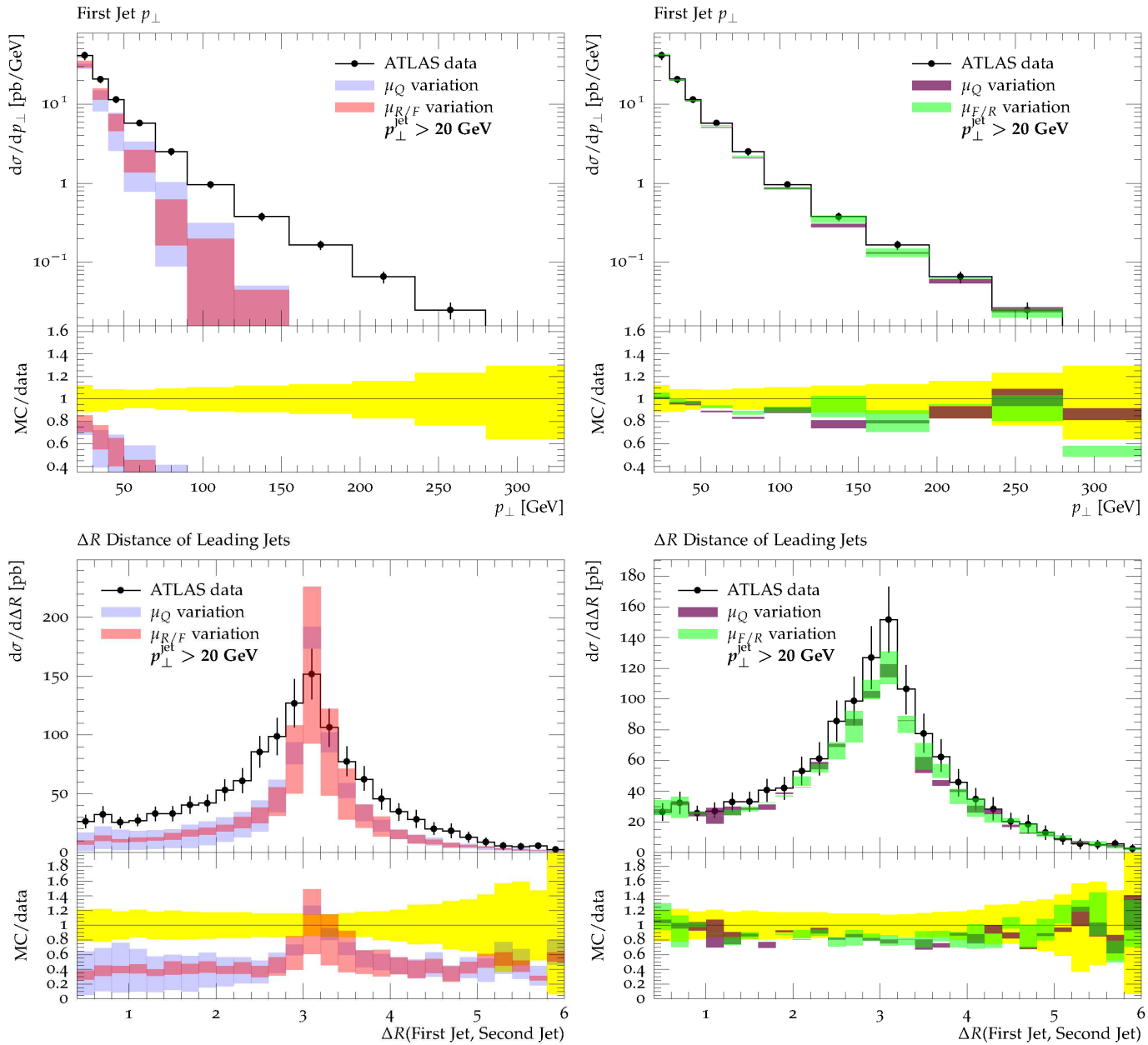
$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int \widehat{\mathrm{B}}_{i\to n} \Bigg\}$$

Inputs (B_n, $\overline{\mathrm{B}}_n$) taken from external tools.

Merging done internally in PYTHIA 8.

UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

NLO merged results (H+jets, the real reason why we unitarise)

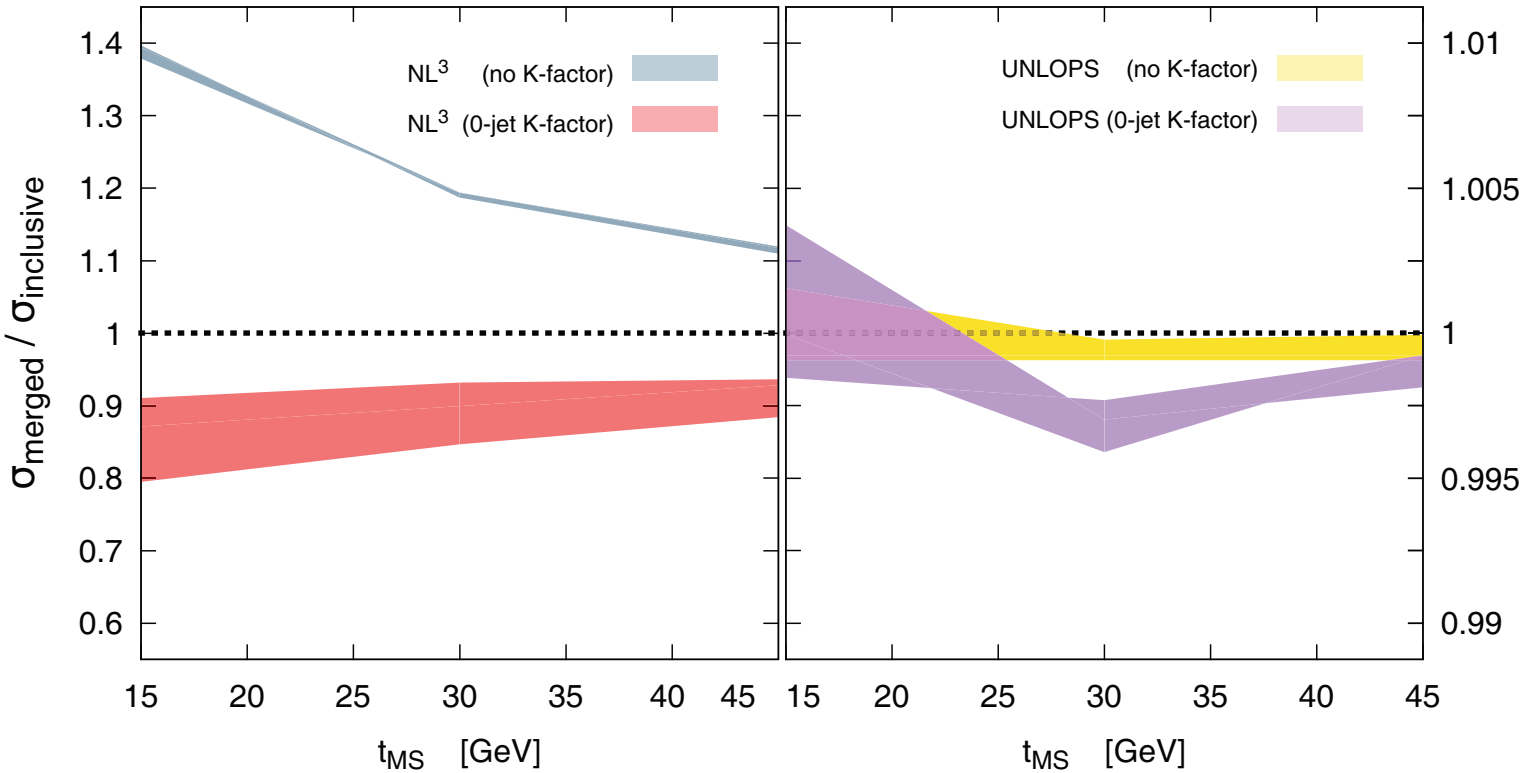


Figure: Ratio of the inclusive cross section for gg→H after merging (H+0)@NLO, (H+1)@NLO and (H+2)@LO, compared to the NLO inclusive cross section.

⇒ NL^3 (=CKKW-L@NLO) has problems for processes with large, loop-driven NLO corrections.

NLO merged results (H+jets)

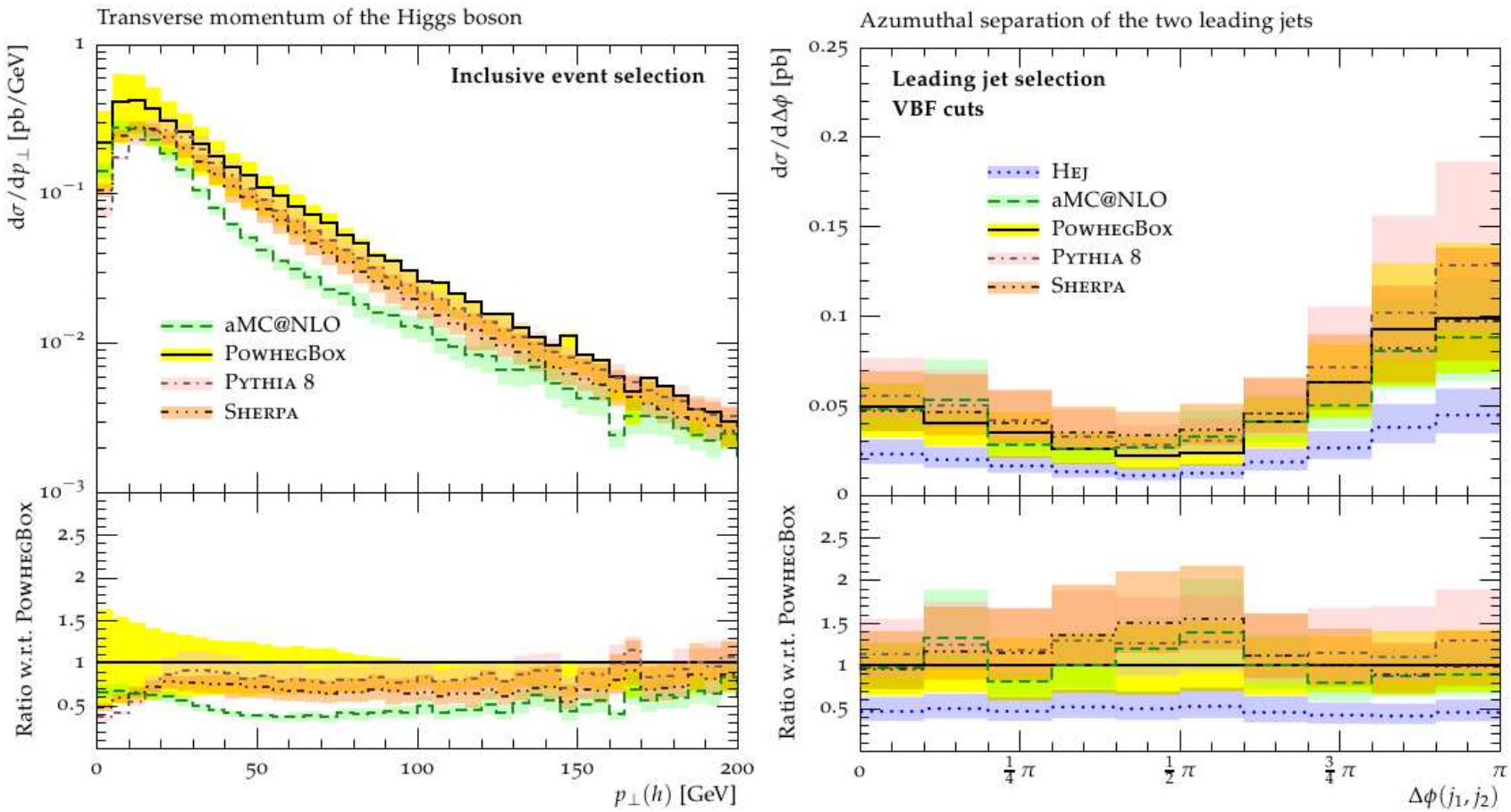


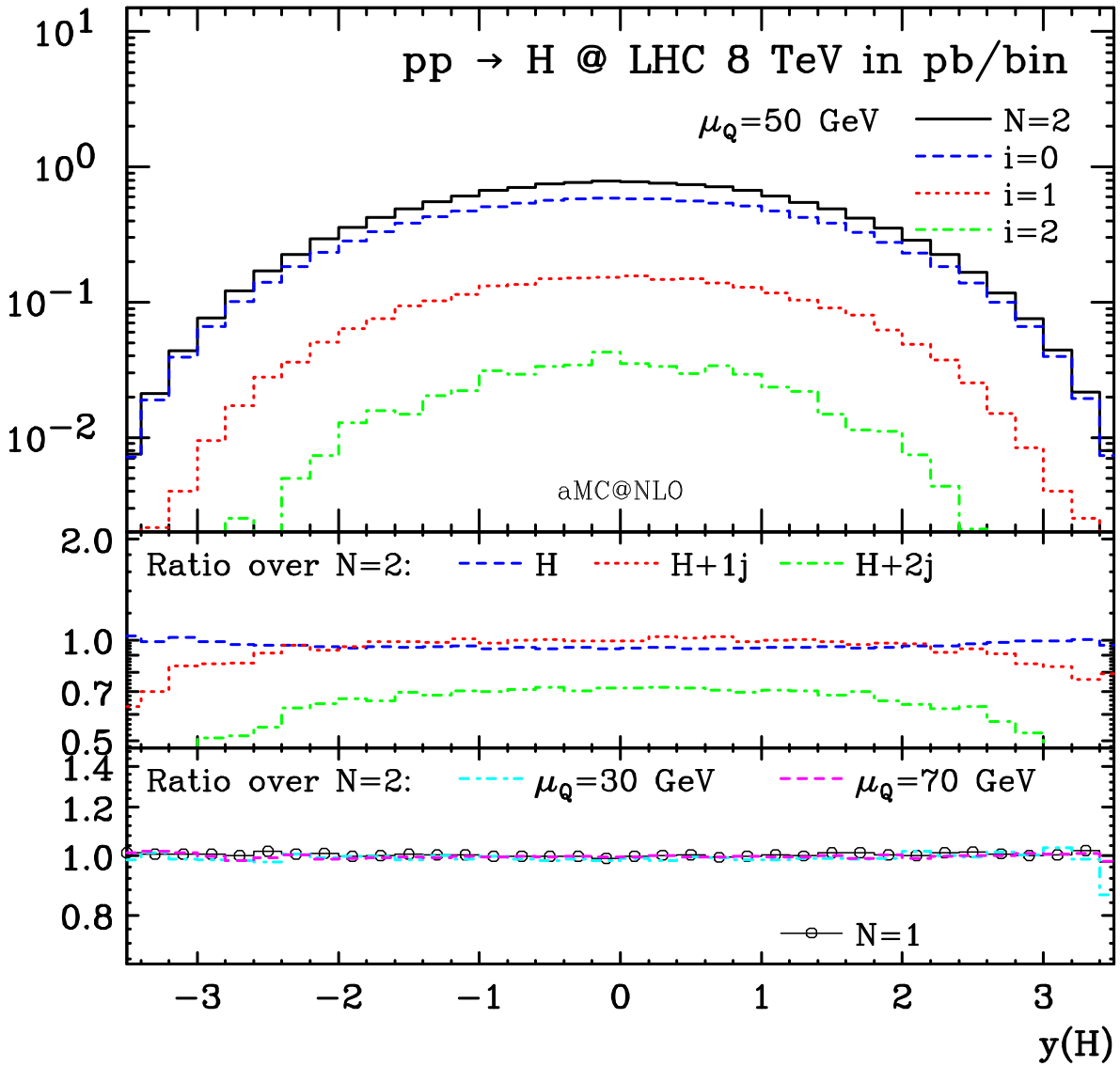
Figure: $p_{\perp,H}$ and $\Delta\phi_{12}$ for gg→H after merging (H+0)@NLO, (H+1)@NLO, (H+2)@NLO, (H+3)@LO, compared to other generators.

⇒ The generators come closer together if enough fixed-order matrix elements are employed. The uncertainties after cuts are still very large.

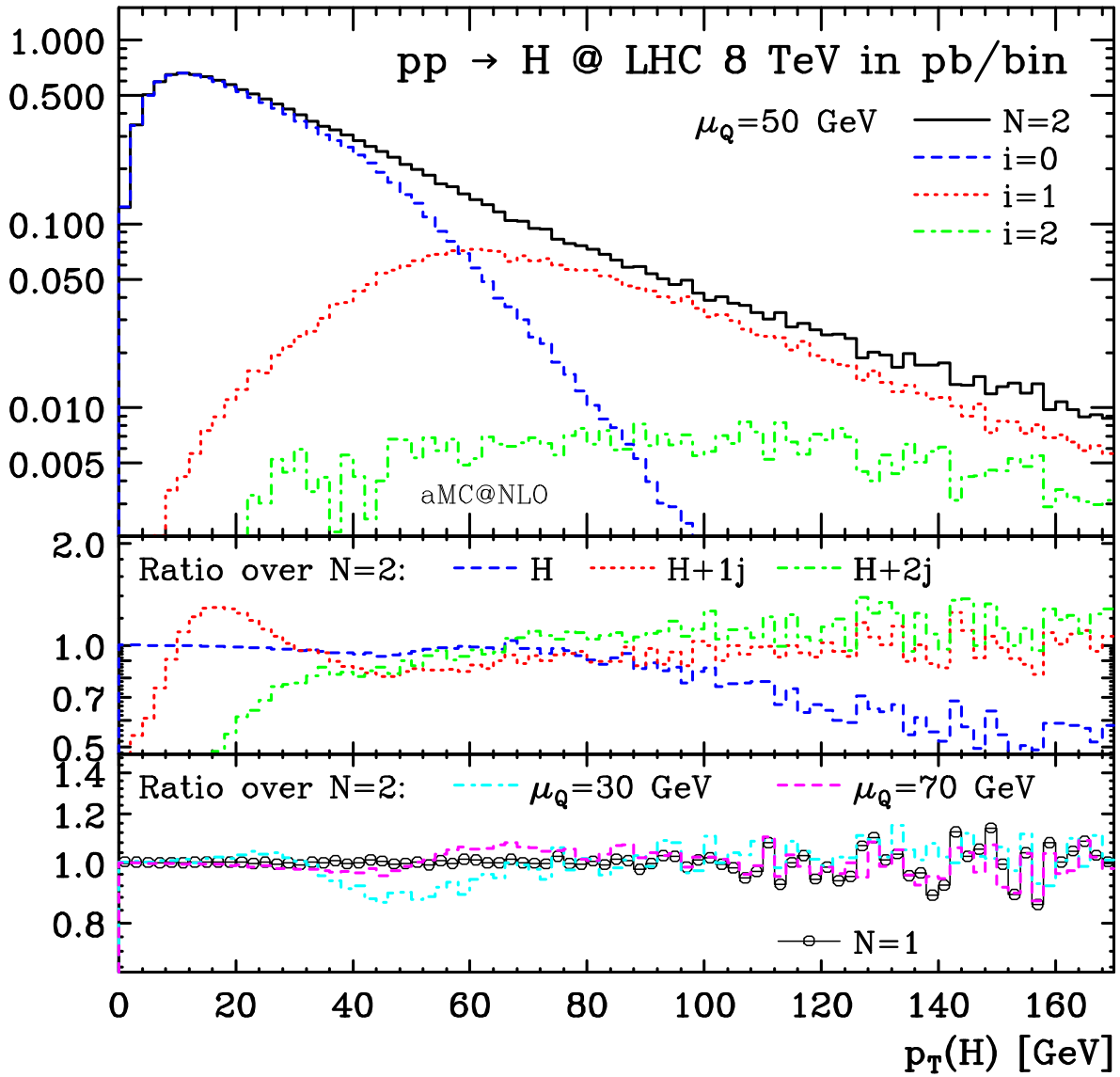
FxFx: Jet matching @ NLO

- Start from MC@NLO calculations.
- Reweight with CKKW-type α_s-running, Sudakov factors (or suppression functions)
- Remove double-counted $\mathcal{O}(\alpha_s^{+1})$-terms
- Match "matrix element jets" to "shower jets" (instead of matching "matrix element partons" to "shower jets")

FxFx plots


pp → H @ LHC 8 TeV in pb/bin
μ_Q=50 GeV
N=2
i=0
i=1
i=2
aMC@NLO
Ratio over N=2:
H
H+1j
H+2j
Ratio over N=2:
μ_Q=30 GeV
μ_Q=70 GeV
N=1
y(H)

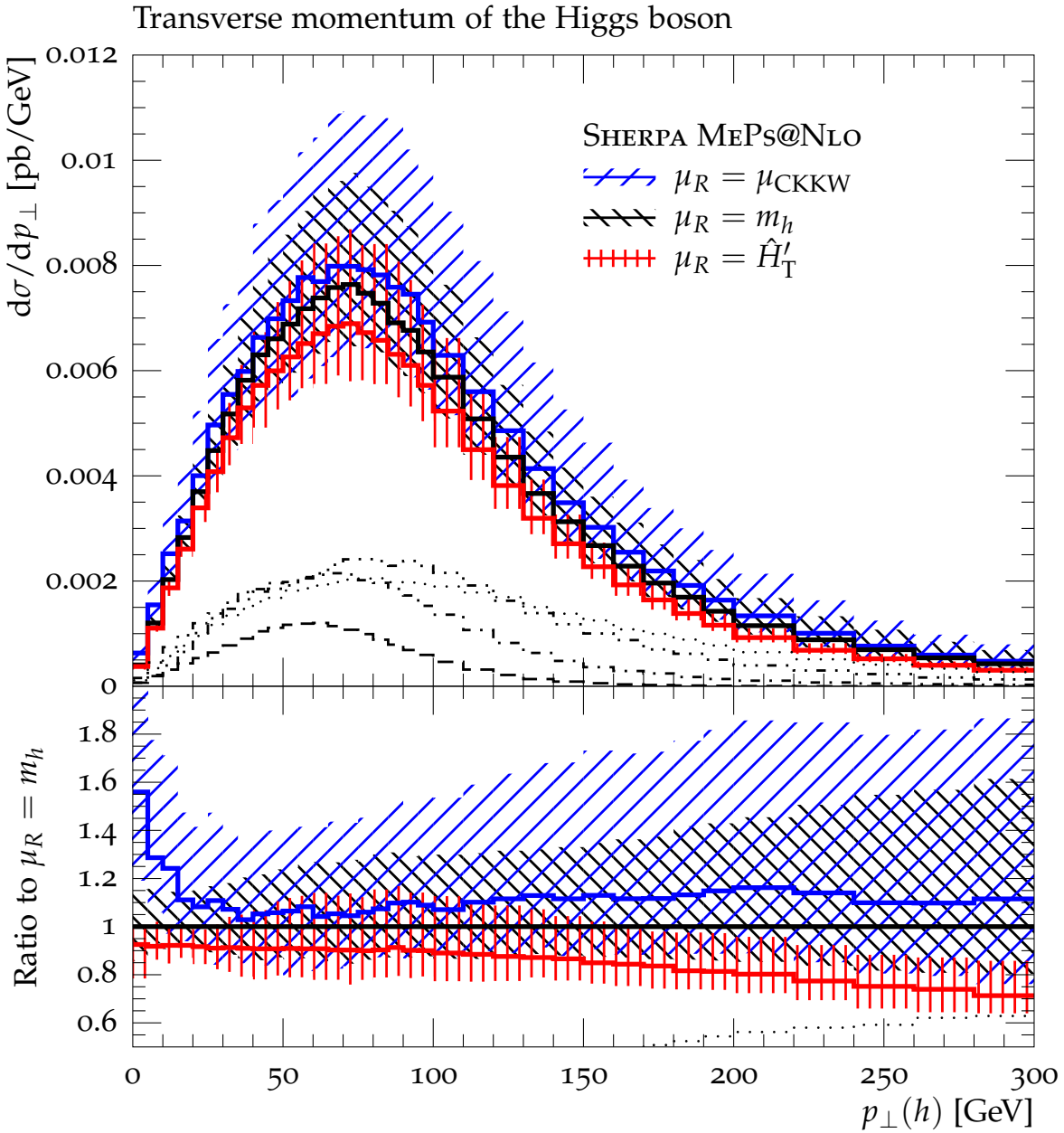
FxFx plots


pp → H @ LHC 8 TeV in pb/bin
μQ=50 GeV
N=2
i=0
i=1
i=2
aMC@NLO
Ratio over N=2:
H
H+1j
H+2j
Ratio over N=2:
μQ=30 GeV
μQ=70 GeV
N=1
pT(H) [GeV]

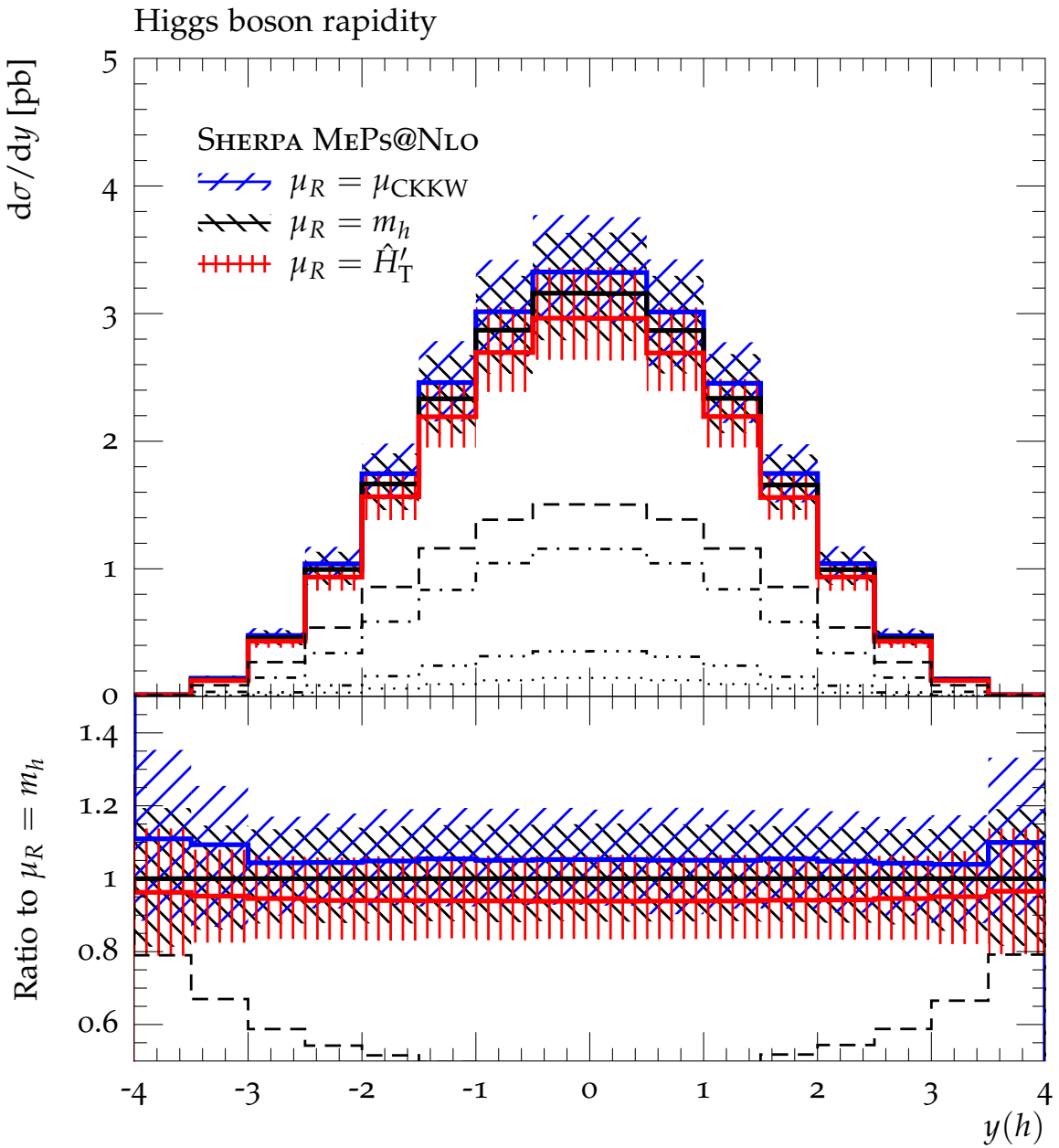
Merging MC@NLO calculations with MEPS@NLO

- Start from S-MC@NLO calculations.
- Disallow real-emission states above t_{MS}.
- Reweight with CKKW-type α_s/PDF-running, carefully preserving NLO accuracy by subtractions
- Reweight with $\mathcal{O}(\alpha_s^{+1})$-subtracted PS Sudakov factors (generated by "forgetful" shower)
- Reweight with $\mathcal{O}(\alpha_s^{+1})$-subtracted MC@NLO Sudakov factors
- When iterating, do not veto hard real emissions for highest multiplicity, and do not subtract the S-MC@NLO Sudakov

MEPS@NLO plots



MEPS@NLO plots



Combining two POWHEG calculations without merging within MiNLO

. . . see below.

Comparison of UNLOPS to the other NLO merging schemes

Other public codes don't go through the trouble of unitarisation. Why?

FxFx: Restricts the range of merging scales. Violation numerically small.
Probably fewest counter events.

MEPS@NLO: Improved, colour-correct Sudakov of MC@NLO for the first emission. Small (non-logarithmic?) violation only comes from (R-D)-type events.
Improved resummation in process-independent way.

MiNLO: applies analytical (N)NLL Sudakov factors, which cancel the necessary terms when merging two multiplicities.
Can be (and was!) moulded into an NNLO matching.

So what's next? NNLO+PS?

NNLO+PS has already arrived: MiNLO-NNLOPS

- Begin with HJ-POWHEG
- Use CKKW-style running α_s, carefully keeping NLO accuracy.
- Reweight with analytic Sudakov factors.
- Choose these Sudakov factors so that HJ-POWHEG $\otimes$ α_s-weight $\otimes$ Sudakovs $= \sigma^{\text{NLO}} + \mathcal{O}(\alpha_s^2)$.
- Reweight with a "differential NNLO K-factor" to achieve NNLO accuracy for inclusive observables (e.g. Higgs rapidity).

Pro : First NNLO+PS result! Analytic Sudakov factors nominally better than PS resummation.

Con : Process-dependent through the MiNLO method. Not currently obvious how to include higher multiplicities.

NNLO+PS (1)

- HJ-MiNLO* differential cross section $(d\sigma/dy)_{\mathrm{HJ-MiNLO}}$ is NLO accurate

$$W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathrm{HJ-MiNLO}}} = \frac{c_2\alpha_{\mathrm{S}}^2 + c_3\alpha_{\mathrm{S}}^3 + c_4\alpha_{\mathrm{S}}^4}{c_2\alpha_{\mathrm{S}}^2 + c_3\alpha_{\mathrm{S}}^3 + d_4\alpha_{\mathrm{S}}^4} \simeq 1 + \frac{c_4 - d_4}{c_2}\alpha_{\mathrm{S}}^2 + \mathcal{O}(\alpha_{\mathrm{S}}^3)$$

- thus, reweighting each event with this factor, we get NNLO+PS
  * obvious for y_H, by construction
  * α_{S}^4 accuracy of HJ-MiNLO* in 1-jet region not spoiled, because $W(y) = 1 + \mathcal{O}(\alpha_{\mathrm{S}}^2)$
  * if we had NLO$^{(0)}$ $+ \mathcal{O}(\alpha_{\mathrm{S}}^{2+3/2})$, 1-jet region spoiled because

$$[\mathrm{NLO}^{(1)}]_{\mathrm{NNLOPS}} = \mathrm{NLO}^{(1)} + \mathcal{O}(\alpha_{\mathrm{S}}^{4.5})$$

---

* Variants for W are possible:

$$W(y, p_T) = h(p_T)\frac{\int d\sigma_A^{\mathrm{NNLO}}\delta(y - y(\Phi))}{\int d\sigma_A^{\mathrm{MiNLO}}\delta(y - y(\Phi))} + (1 - h(p_T))$$

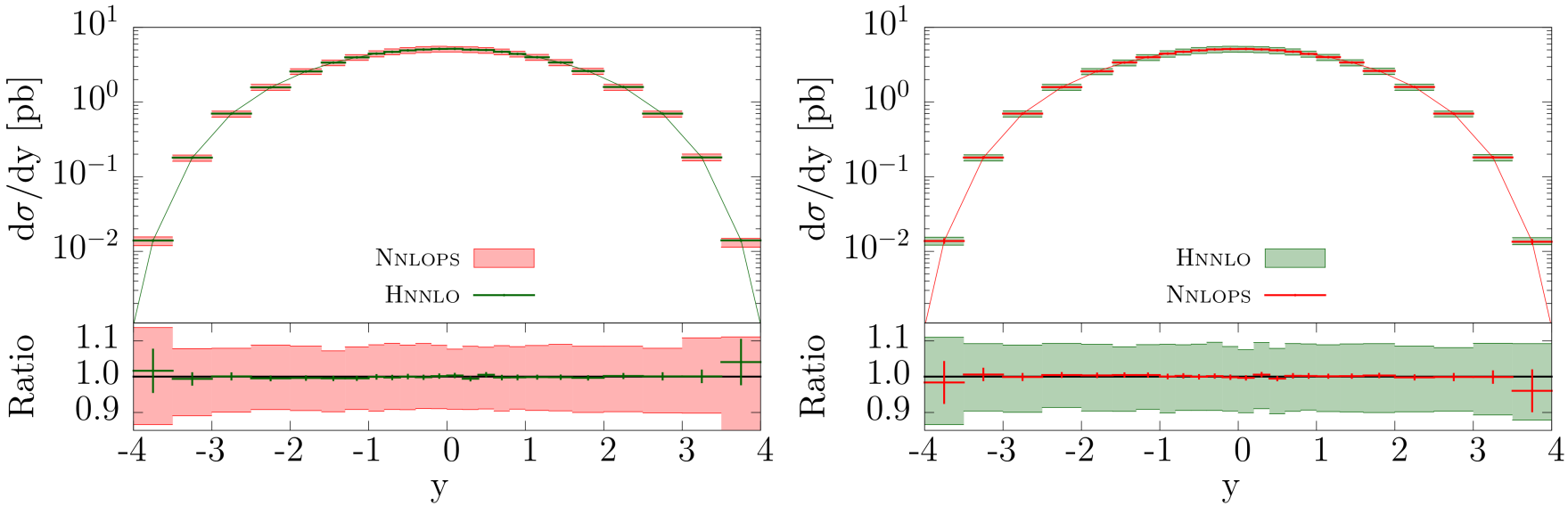
$$d\sigma_A = d\sigma\ h(p_T), \qquad d\sigma_B = d\sigma\ (1 - h(p_T)), \qquad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}$$

* $h(p_T)$ controls where the NNLO/NLO K-factor is spread
* with above W, we get $(d\sigma/dy)_{\mathrm{NNLOPS}} = (d\sigma_A/dy)_{\mathrm{NNLO}} + (d\sigma_B/dy)_{\mathrm{HJ-MiNLO}}$

Slide taken from Emanuele Re, Talk given at ZPW 2014

NNLO+PS (fully incl.)

- NNLO with $\mu = m_H/2$, HJ-MiNLO "core scale" m_H [NNLO from HNNLO, Catani,Grazzini]
- $(7_{\rm Mi} \times 3_{\rm NN})$ pts scale var. in NNLOPS, 7pts in NNLO



☞ Notice: band is 10%

[Until and including $\mathcal{O}(\alpha_{\rm S}^4)$, PS effects don't affect y_H (first 2 emissions controlled properly at $\mathcal{O}(\alpha_{\rm S}^4)$ by MiNLO+POWHEG)]

Process-independent NNLO+PS?

Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of...

$$
\begin{aligned}
\langle \mathcal{O} \rangle \;=\; & \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \left(\widetilde{\mathrm{B}}_0 \;-\; \int_s \widetilde{\mathrm{B}}_{1\to 0} \;+\; \int_s \mathrm{B}_{1\to 0} \;-\; \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} \right. \\
& \qquad \left. -\; \int_s \mathrm{B}^{\uparrow}_{2\to 0} \;-\; \int \widehat{\mathrm{B}}_{2\to 0} \right) \\
& + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} \;-\; \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) \\
& + \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2
\end{aligned}
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\mathcal{O}(\alpha_\mathrm{s}) \qquad & \qquad - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) \\
\mathcal{O}(\alpha_\mathrm{s}^2) \qquad & + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) \\
& + \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2
\end{aligned}
$$

Terms entering due to PS weights are $\mathcal{O}(\alpha_\mathrm{s}^2)_{\mathrm{PS}} \times \mathcal{O}(\alpha_\mathrm{s}^1)_{\mathrm{ME}}$ and $\mathcal{O}(\alpha_\mathrm{s}^0)_{\mathrm{PS}} \times \mathcal{O}(\alpha_\mathrm{s}^2)_{\mathrm{ME}}$

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& \qquad\qquad - \left[\int \widehat{\mathrm{B}}_{2\to 0} \right]_{-2} \Bigg) \\
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\end{aligned}
$$

...now remove these terms

Process-independent NNLO+PS?

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$$\begin{aligned}\langle\mathcal{O}\rangle \;=\; \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\widetilde{\mathrm{B}}}_0 &- \left[\int \widehat{\mathrm{B}}_{1\to 0}\right]_{-1,2} \\ &- \left[\int \widehat{\mathrm{B}}_{2\to 0}\right]_{-2} \Bigg) \\ + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1\right]_{-1,2} &- \left[\int \widehat{\mathrm{B}}_{2\to 1}\right]_{-2} \Bigg) \\ + \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 &\end{aligned}$$

...now remove these terms ...and instead use the exclusive NNLO result[1]

$\widetilde{\widetilde{\mathrm{B}}}_0 \approx \overline{\overline{\mathrm{B}}}_0 - \overline{\overline{\mathrm{B}}}_1$ (more formal defintion, see back-up)

[1] see sketch in JHEP1303(2013)166 (Leif Lönnblad, SP)

The matching formula at NNLO is:

$$\begin{aligned}\langle \mathcal{O} \rangle \;=\; \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \left(\widetilde{\widetilde{\mathrm{B}}}_0 \;-\; \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} \;-\; \left[\int \widehat{\mathrm{B}}_{2\to 0} \right]_{-2} \right) \\ + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} \;-\; \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) \\ + \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \Bigg\}\end{aligned}$$

The inclusive cross section is

$$\int d\phi_0 \mathcal{O}(S_{+0j}) \left(\widetilde{\widetilde{\mathrm{B}}}_0 \;+\; \int_s \mathrm{B}_{2\to 0} \right) \;+\; \int d\phi_0 \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 \;+\; \int_s \mathrm{B}_{2\to 1} \right)$$

which is just the NNLO result, provided that $\widetilde{\widetilde{\mathrm{B}}}_0$ does not contain phase space regions with jet separations above ρ_{MS} (see back-up).

NNLO with UNLOPS

Alternatively, we can use the inclusive cross section for NNLO matching:

$$\begin{aligned}\langle\mathcal{O}\rangle \;=\; \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left(\overline{\overline{\mathrm{B}}}_0 \;-\; \int_s \overline{\mathrm{B}}_{1\to 0} \;-\; \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} \;-\; \int \widehat{\mathrm{B}}_{2\to 0} \right) \\ & + \int \mathcal{O}(S_{+1j}) \left(\overline{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} \;-\; \int \widehat{\mathrm{B}}_{2\to 1} \right) \\ & + \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \Bigg\}\end{aligned}$$

- Relatively clean only because the NLO cross section was not reweighted. Process-independent.
- ρ_{MS}-definition must be infrared safe at NNLO[1].
- Needs NNLO generator to produce $\widetilde{\overline{\mathrm{B}}}_0$ or $\overline{\overline{\mathrm{B}}}_0$. For $\widetilde{\overline{\mathrm{B}}}_0$, the ρ_{MS}-definition has to be known within the NNLO calculation. Then, for simple processes, a MiNLO-NNLOPS reweighting factor is possible. Otherwise, $\overline{\overline{\mathrm{B}}}_0$ should best be fully differential, so that the PS can project onto 0-parton states.

[1] see e.g. discussion in arXiv:1311.0286 (Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi)

Comments on UNLOPS@NNLO

UNLOPS has one important construction principle:
The exclusive $+n-$parton cross section should exactly be NLO + UMEPS higher orders, and nothing else.

For UNLOPS@NNLO, the construction principle says that the exclusive 0-parton cross section should be given by NNLO + UMEPS higher orders.

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Mixed blessings: Separation into jet bins much like in $q_\perp$-subtraction.

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The cons can be improved upon by reordering the formula, and reweighting NLO (or NNLO) parts. This then introduces the same type "smearing/profile ambiguity" as in MiNLO-NNLOPS.

However, the PS resummation is not improved.

NNLO + LL in Geneva

The Geneva collaboration has proposed a common language for NNLO matching[1]. Process:

- Define the accuracies to be matched.
- This defines the merging criterion, and through it, necessary subtractions. The criterion is not unambiguous. Choices arise when deciding how to reweight $\mathcal{O}(\alpha_s^{+1})$-terms, and how to treat subtractions.
- If a merging scale exists, it has to be defined sufficiently "global" (and be IR safe at NNLO).
- Nicely compares Geneva, MiNLO-NNLOPS and UNLOPS.

[1] arXiv:1311.0286 (Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi)

Dreams of MC@NNLO

(Almost) everyone in the MC community wants an MC@NNLO. Loosely

MC@NNLO $\approx$ all NNLO subtractions are generated by the shower

This means controlling un-ordered emissions (and the 4-gluon vertex?).
Leads to a "natural" prescription how to smear inclusive NNLO cross section into real-emission phase space.

This would also mean a significant improvement of the shower.
Which could finally lead to a NLL parton shower.

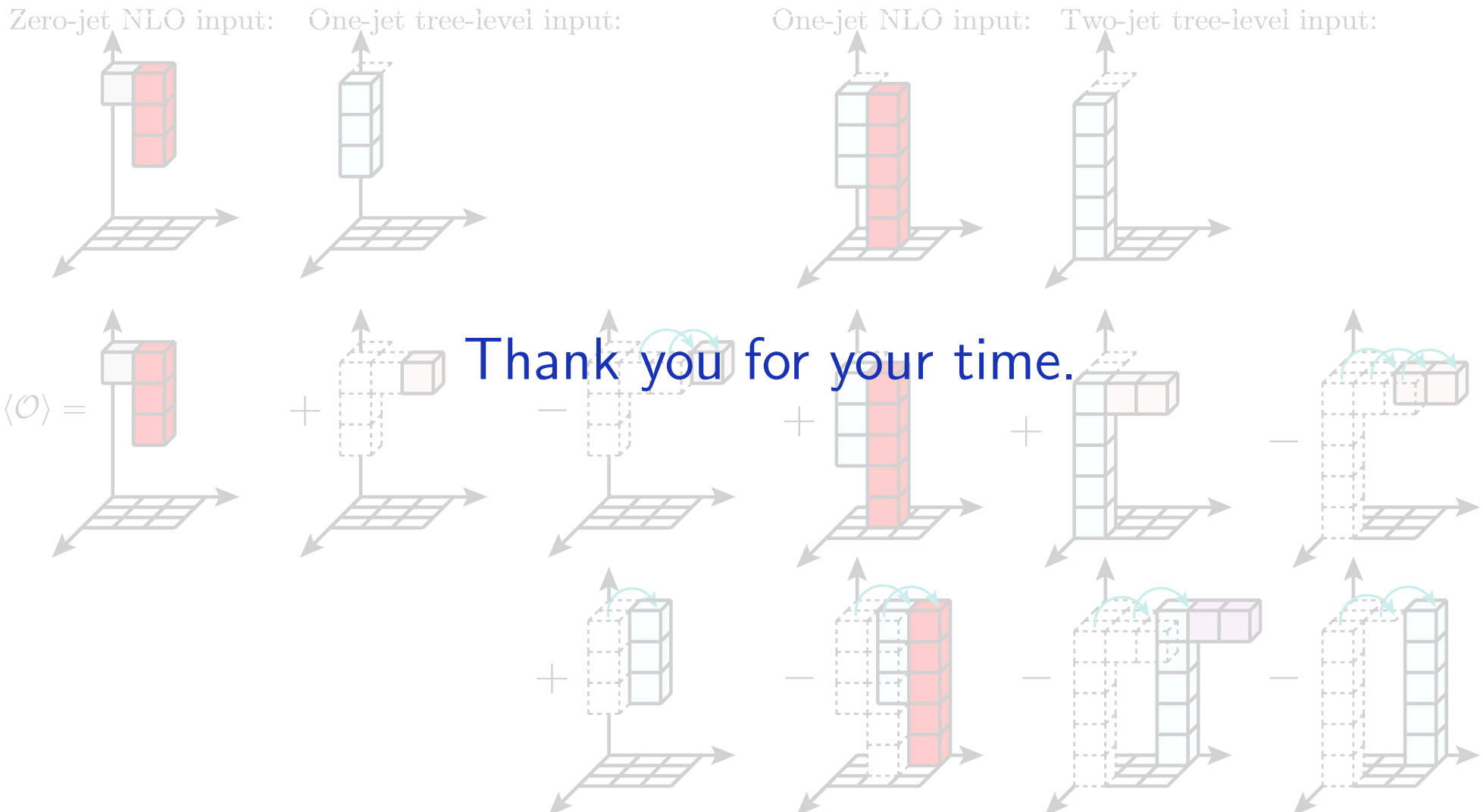
$\Longrightarrow$ Smaller shower uncertainty.

Summary

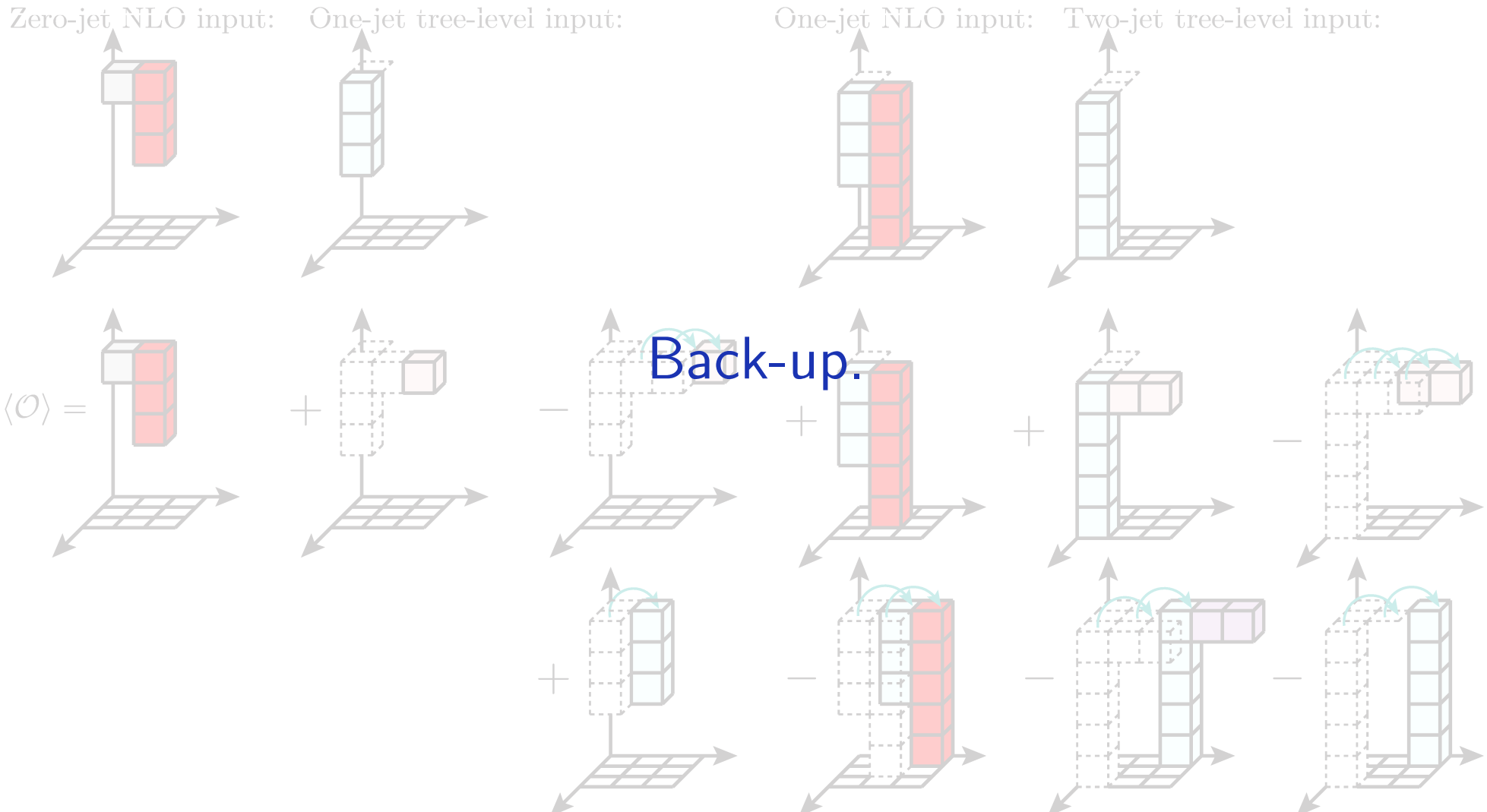
- We need to infuse parton showers with matrix elements.
- Tree-level merging is an old hat
  . . . but still useful to set the scene . . . and still has issues.
  . . . and it's not NLO.
- Multiple NLO merging schemes have appeared, and are being used (e.g. MEPS@NLO-SHERPA, (semi-automatic)
  aMC@NLO-FxFx-HERWIG6, aMC@NLO-FxFx-PYTHIA8
  uMATCHBOX-HERWIG++, NL^3-PYTHIA8, UNLOPS-PYTHIA8).
  All schemes differ in the treatment of sub-leading contributions.
- Ideas for NNLO matching have appeared. So far, they all rely on a NLO-merging scheme that integrates to the NLO cross section.
  MiNLO-NNLOPS publicly available.
  Other implementations are (probably) in the pipe line. These will most likely again differ in sub-leading contributions.

Summary

Summary



Thank you for your time.


Zero-jet NLO input:
One-jet tree-level input:
One-jet NLO input:
Two-jet tree-level input:
⟨O⟩ =

Back-up.

Illustration of leading-order merging

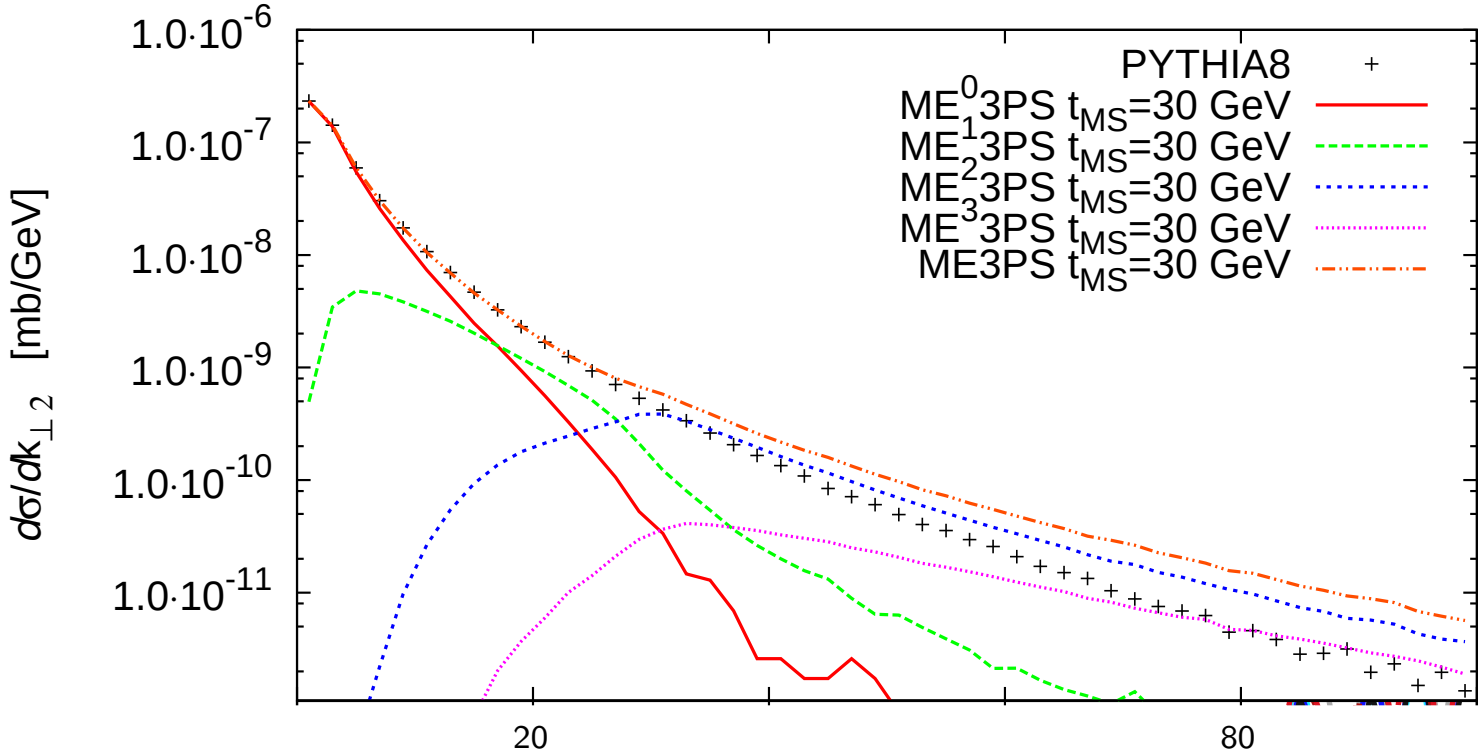


Figure: Separation between the first and second jet for W+jets, when clustering to exactly two jets. The coloured lines show the different reweighted multi-parton MEs.

UMEPS definitions

$$
\begin{aligned}
w_n =& \frac{x_n^+ f_n^+(x_n^+,\rho_n)}{x_n^+ f_n^+(x_n^+,\mu_F)} \frac{x_n^- f_n^-(x_n^-,\rho_n)}{x_n^- f_n^-(x_n^-,\mu_F)} \\
& \times \prod_{i=1}^{n} \left[\frac{\alpha_s(\rho_i)}{\alpha_s(\mu_R)} \frac{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+,\rho_{i-1})}{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+,\rho_i)} \frac{x_{i-1}^- f_{i-1}^-(x_{i-1}^-,\rho_{i-1})}{x_{i-1}^- f_{i-1}^-(x_{i-1}^-,\rho_i)} \Pi_{S_{+i-1}}(x_{i-1},\rho_{i-1},\rho_i) \right] \\
\widehat{\mathrm{B}}_n =& \mathrm{B}_n w_n
\end{aligned}
$$

$$
\int \widehat{\mathrm{B}}_{n\to m} = \left[\prod_{a=m+1}^{n-1} \int d\rho_a dz_a d\varphi_a \Theta(\rho_{\mathrm{MS}} - \rho_a) \right] \int d\rho_n dz_n d\varphi_n \mathrm{B}_n w_n
$$

$$
\langle \mathcal{O} \rangle = \sum_{n=0}^{N} \int d\phi_0 \int \ldots \int \mathcal{O}(S_{+nj}) \left\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int \widehat{\mathrm{B}}_{i\to n} \right\} .
$$

In CKKW-L, w_n contains an additonal factor $\Pi_{S_{+n}}(x_n,\rho_n,\rho_{\mathrm{MS}})$. UMEPS induces this through $\int \widehat{\mathrm{B}}_{n+1\to n}$ instead.

Comments on UMEPS

This sketch directly extends to the case of (α_s- or Sudakov-) weighted $+n$-jet states ($\widehat{\mathrm{B}}_n$), e.g. two-jet UMEPS merging:

$$\begin{aligned}\langle\mathcal{O}\rangle \;=\; & \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 - \int \widehat{\mathrm{B}}_{1\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \right] \\ & + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 - \int \widehat{\mathrm{B}}_{2\to 1} \right] \\ & + \int\cdots\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \Bigg\}\end{aligned}$$

The merging scale is finally a *technical* parameter, much in the same way that the shower cut-off (or `ptminsq` in the POWHEG-BOX) is.

Integrated version of the real-emission matrix elements are available anyway in MEPS since we need them to perform the Sudakov weighting.

Unitarisation should be helpful for inclusive observables, heavy flavour (5F vs. 4F, 4F vs. 3F) treatments, MC tuning in the presence of merging,...

Further comments on UMEPS

UMEPS merging for up to two additional jets:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 - \int \widehat{\mathrm{B}}_{1\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \right] + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 - \int \widehat{\mathrm{B}}_{2\to 1} \right] + \int \cdots \int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \Bigg\}$$

The "subtract what you add" prescription means that this will produce *counter-events* with negative weight $\rightarrow$ Statistics problematic?

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The "subtract what you add" prescription means that this will produce *counter-events* with negative weight $\rightarrow$ Statistics problematic?

To remove a lot of cancellations, we can directly rewrite this as

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 \Pi_{S_{+0}}(\rho_0, \rho_{\mathrm{MS}}) - \int w_1 \Pi_{S_{+0}}(\rho_0, \rho) \left[\mathrm{B}_1 - \mathrm{B}_0 P \right]_{1\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \right] + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 \Pi_{S_{+1}}(\rho_1, \rho) - \int w_2 \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \left[\mathrm{B}_2 - \mathrm{B}_1 P \right]_{2\to 1} \right] + \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \Bigg\} .$$

The "unitarisation" is just given by Sudakov-reweighted, projected MC@NLO H-events. Such an implementation should be quite stable.

UMEPS $'$

Here, we illustrate how to include the full kinematical information also below the merging scale.

$$d\sigma^u_{+0} \equiv \widehat{\mathrm{B}}_{+0} - \int \widehat{\mathrm{B}}_{0+1\to 0} \qquad\qquad d\sigma^u_{+1} \qquad\qquad \equiv \widehat{\mathrm{B}}_{+1}$$

So far, we have

$$\begin{aligned}
\langle\mathcal{O}\rangle = {} & d\sigma^u_{+0} \times [\Delta(\rho_0,\rho_c)\mathcal{O}_0(S_{+0}) \;+\; \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times (\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\ldots)] \\
& + d\sigma^u_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\
& \qquad \times \Theta(\rho_1-\rho_{\mathrm{MS}})
\end{aligned}$$

But we could write

$$\begin{aligned}
\langle\mathcal{O}\rangle = {} & d\sigma^u_{+0} \times [\Delta(\rho_0,\rho_c)\mathcal{O}_0(S_{+0}) \;+\; \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times (\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \qquad)] \\
& + d\sigma^u_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\
& \qquad \times \Theta(\rho_1-\rho_{\mathrm{MS}}) \\
& + d\sigma^d_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\
& \qquad \times \Theta(\rho_{\mathrm{MS}}-\rho_1) \\
& - d\sigma^u_{+0} \times \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times \Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \\
& - \int \Big\{ d\sigma^d_{+1} \times [\Delta(\rho_1,\rho_c) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c) \;+\ldots)] \times \Theta(\rho_{\mathrm{MS}}-\rho_1) \\
& - d\sigma^u_{+0} \times \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times \Delta(\rho_1,\rho_c) \Big\} \mathcal{O}_0(S_{+0})
\end{aligned}$$

UMEPS $'$ = UMEPS

This is just

$$\begin{aligned}\langle\mathcal{O}\rangle = {} & d\sigma^u_{+0}\mathcal{O}_0(S_{+0}) - \int d\sigma^d_{+1} \times \Theta(\rho_{\mathrm{MS}} - \rho_1)\mathcal{O}_0(S_{+0}) \\ & + d\sigma^u_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \; + \; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \; + \ldots)] \\ & \qquad \times \Theta(\rho_1 - \rho_{\mathrm{MS}}) \\ & + d\sigma^d_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \; + \; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \; + \ldots)] \\ & \qquad \times \Theta(\rho_{\mathrm{MS}} - \rho_1)\end{aligned}$$

which for $d\sigma^d_{+1} = d\sigma^u_{+1}$ becomes

$$\begin{aligned}\langle\mathcal{O}\rangle = {} & d\sigma^u_{+0}\mathcal{O}_0(S_{+0}) \\ & + d\sigma^u_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \; + \; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \; + \ldots)] \\ & \qquad \times \Theta(\rho_1 - \rho_c)\end{aligned}$$

i.e. simply the UMEPS result for $\rho_{\mathrm{MS}} = \rho_c$.
Note that this does not depend on the form of $d\sigma^u_{+0}$ or $d\sigma^u_{+1}$, and can thus also be applied for UNLOPS to add real-emission kinematics with $\rho(S_{+n+1}) < \rho_{\mathrm{MS}}$.

Extreme merging scale values

In unitarised merging, $t_{\rm MS}$ can be varied between the PS cut-off ρ_c and ∞. The $t_{\rm MS}$ dependence is almost exactly cancelled (caveat: jet definition)

For $t_{\rm MS} \to \infty$, the spectrum of the first add. emission is given by the PS. The below-$t_{\rm MS}$ behaviour can be fixed[1], leading e.g. to the following form:

$$\langle \mathcal{O} \rangle = \left[d\sigma^u_{+0} - \int d\sigma^d_{+1} \times \Theta(\rho_{\rm MS} - \rho_1) \right] \mathcal{O}_0(S_{+0}) + \mathbf{PS}\left[d\sigma^u_{+1} \Theta(\rho_1 - \rho_{\rm MS}) \right] + \mathbf{PS}\left[d\sigma^d_{+1} \Theta(\rho_{\rm MS} - \rho_1) \right]$$

which for $d\sigma^d_{+1} = d\sigma^u_{+1}$ becomes

$$\langle \mathcal{O} \rangle = d\sigma^u_{+0} \mathcal{O}_0(S_{+0}) + \mathbf{PS}\left[d\sigma^u_{+1} \Theta(\rho_1 - \rho_c) \right]$$

i.e. simply the UMEPS result for $\rho_{\rm MS} = \rho_c$.

The merging scale is a *technical* parameter, much in the same way that the shower cut-off (or `ptminsq`/`hfact` in the POWHEG-BOX) is.

[1] See also arXiv:1211.5467 (Simon Plätzer)

How do we get $\widetilde{\mathrm{B}}_n$?

Like everyone else, we need exclusive NLO cross sections as input:

$$\widetilde{\mathrm{B}}_n \quad = \quad \mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_{n+1|n} + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1|n} \Theta \left(\rho_{\mathrm{MS}} - t \left(S_{+n+1}, \rho \right) \right) - \mathrm{D}_{n+1|n} \right)$$

First, observe that

$$\begin{aligned} \widetilde{\mathrm{B}}_n &= \mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_{n+1|n} + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1|n} - \mathrm{D}_{n+1|n} \right) - \int d\Phi_{\mathrm{rad}} \mathrm{B}_{n+1|n} \Theta \left(t \left(S_{+n+1}, \rho \right) - \rho_{\mathrm{MS}} \right) \\ &= \overline{\mathrm{B}}_n - \int_{\rho_{\mathrm{MS}}} d\Phi_{\mathrm{rad}} \mathrm{B}_{n+1|n} \end{aligned}$$

and remember that the POWHEG-BOX produces

$$\overline{B}_n \Delta(p_{\perp min}) + \overline{B}_n \frac{B_{n+1}}{B_n} \Delta(p_\perp) = \overline{B}_n \left[1 - \int_{p_\perp min} \frac{B_{n+1}}{B_n} \Delta(p_\perp) \right] + \overline{B}_n \frac{B_{n+1}}{B_n} \Delta(p_\perp)$$

Thus, if we project onto an underlying Born for radiative events, we get $\overline{B}_n$.
By having a subtraction sample $\int_{\rho_{\mathrm{MS}}} d\Phi_{\mathrm{rad}} \mathrm{B}_{n+1|n}$, we get $\widetilde{\mathrm{B}}_n$.

What is $\widetilde{\overline{\mathrm{B}}}_0$?

For UNLOPS@NNLO, $\widetilde{\overline{\mathrm{B}}}_0$ is given by

$$\widetilde{\overline{\mathrm{B}}}_0 \mathcal{O}_{+0} = \left[\overline{\overline{\mathrm{B}}}_0 - \overline{\mathrm{B}}_1 \Theta\left(\rho\left(S_{+1}, \rho\right) - \rho_{\mathrm{MS}}\right)\right] \mathcal{O}_{+0}$$

where we need to project

$$\begin{aligned}
\overline{\overline{\mathrm{B}}}_0 \;=\; & \left[\mathrm{B}_0 + \mathrm{V}_0 + \int \mathrm{D}_{1|0}\right] \mathcal{O}_{+0} + \int \left[\mathrm{B}_1 \mathcal{O}_{+1} - \mathrm{D}_{1|0} \mathcal{O}_{+0}\right] \\
+ \; & \left[\mathrm{W}_0 + \iint \mathrm{G}_{2|0} + \iint (1-h) \mathrm{H}_{2|0}\right] \mathcal{O}_{+0} \\
+ \; & \int \left[\mathrm{V}_1 \mathcal{O}_{+1} + \int \mathrm{D}_{2|1} \mathcal{O}_{+1} - \int \mathrm{E}_{2|0} \mathcal{O}_{+0} + \int \mathrm{F}_{2|0} \mathcal{O}_{+0} + \int h \mathrm{H}_{2|0} \mathcal{O}_{+0}\right] \\
+ \; & \iint \left[\mathrm{B}_2 \mathcal{O}_{+2} - \mathrm{D}_{2|1} \mathcal{O}_{+1} + \mathrm{E}_{2|0} \mathcal{O}_{+0} - \mathrm{F}_{2|0} \mathcal{O}_{+0} - \mathrm{G}_{2|0} \mathcal{O}_{+0} - \mathrm{H}_{2|0} \mathcal{O}_{+0}\right]
\end{aligned}$$

onto 0-parton states. (Regularisation scheme irrelevant, only an example!)
$\mathrm{E}_{2|0}$ cancels spurious div's from integrating $\mathrm{D}_{2|1}$ over the full 2-parton PS;
$\mathrm{F}_{2|0}$ cancels double-unresolved div's in $\iint \mathrm{B}_2$ that would otherwise cancel against $\int \mathrm{V}_1$;
$\mathrm{G}_{2|0}$ cancels double-unresolved div's in $\iint \mathrm{B}_2$ that would cancel against W_0;
$\mathrm{H}_{2|0}$ cancels div's that are genuinely shared between $\iint \mathrm{B}_2$, $\int \mathrm{V}_1$ and W_0 (if present), or other spurious div's (if existing).

What is $\widetilde{\widetilde{\mathrm{B}}}_0$?

With $\Theta_n(>) = \Theta(t(S_{+n},\rho) - \rho_{\mathrm{MS}})$ and $\Theta_n(<) = \Theta(\rho_{\mathrm{MS}} - t(S_{+n},\rho))$,

$$
\begin{aligned}
\widetilde{\widetilde{\mathrm{B}}}_0 \;=\;& \left[\mathrm{B}_0 + \mathrm{V}_0 + \int \mathrm{D}_{1|0}\right]\mathcal{O}_{+0} \\
&+ \int\left[\mathrm{B}_1\Theta_1(<)\,\mathcal{O}_{+1} - \mathrm{D}_{1|0}\mathcal{O}_{+0}\right] + \left[\mathrm{W}_0 + \iint \mathrm{G}_{2|0} + \iint(1-h)\mathrm{H}_{2|0}\right]\mathcal{O}_{+0} \\
&+ \int\left[\mathrm{V}_1\Theta_1(<)\,\mathcal{O}_{+1} + \int \mathrm{D}_{2|1}\Theta_1(<)\,\mathcal{O}_{+1} - \int \mathrm{E}_{2|0}\mathcal{O}_{+0} + \int \mathrm{F}_{2|0}\mathcal{O}_{+0} + \int h\mathrm{H}_{2|0}\mathcal{O}\right. \\
&+ \iint\left[\mathrm{B}_2\Theta_1(<)\,\Theta_2(>)\,\mathcal{O}_{+2} + \mathrm{B}_2\Theta_1(<)\,\Theta_2(<)\,\mathcal{O}_{+2} - \mathrm{D}_{2|1}\Theta_1(<)\,\mathcal{O}_{+1}\right. \\
&\qquad\left.\left. + \mathrm{E}_{2|0}\mathcal{O}_{+0} - \mathrm{F}_{2|0}\mathcal{O}_{+0} - \mathrm{G}_{2|0}\mathcal{O}_{+0} - \mathrm{H}_{2|0}\mathcal{O}_{+0}\right]\right.
\end{aligned}
$$

Note the term $\mathrm{B}_2\Theta_1(<)\,\Theta_2(>)\,\mathcal{O}_{+2}$, which contains a state S_{+2} with two resolved partons, but no state with only one resolved parton. Alternatively, we could have defined $\widetilde{\widetilde{\mathrm{B}}}_0$ without this term. Then, it would be, in an NNLO-matched calculation, be included e.g. through $\int \widehat{\mathrm{B}}_{2\to 0}$. This is how we defined $\widetilde{\widetilde{\mathrm{B}}}_0$ in the main text. Above, we also defined the sum $\overline{\overline{\mathrm{B}}}_0 - \overline{\mathrm{B}}_1$ to not contain such terms.