QCD RESUMMATION

Simone Marzani
Institute for Particle Physics Phenomenology
Durham University

Standard Model @ LHC 2014
8th-11th April 2014

OUTLINE

- Aim of this talk: a brief (and possibly biased) review of last year's results in QCD resummation

- Technical points (SCET vs dQCD), phenomenology and some curiosity

- Because of time constraints my and ability to give a coherent talk I'll be concentrating on "final-state resummation" (i.e. event shapes and jets)

PERTURBATIVE QCD CALCULATIONS

- Precise theoretical predictions needed for the LHC
  - NLO calculations in QCD are now standard

    long list of public (automated) codes

  - NNLO exists for an increasing number of processes

    e.g. Mitov, Czakon (2013)
    Boughezal, Caola, Melnikov, Petriello, Schulze (2013)
    Currie, De Ridder, Glover, Pires (2013)

  - NNNLO has also appeared for Higgs

    Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger (2014)

$$\sigma(x,Q^2) = \sigma_{\rm LO}(x,Q^2) + \alpha_s \sigma_{\rm NLO}(x,Q^2) + \alpha_s^2 \sigma_{\rm NNLO}(x,Q^2) + \alpha_s^3 \sigma_{\rm NNNLO}(x,Q^2) + \dots$$

PERTURBATIVE QCD CALCULATIONS

- Precise theoretical predictions needed for the LHC
  - NLO calculations in QCD are now standard

    long list of public (automated) codes

  - NNLO exists for an increasing number of processes

    e.g. Mitov, Czakon (2013)
    Boughezal, Caola, Melnikov, Petriello, Schulze (2013)
    Currie, De Ridder, Glover, Pires (2013)

  - NNNLO has also appeared for Higgs

    Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger (2014)

$$\sigma(x,Q^2) = \sigma_{\mathrm{LO}}(x,Q^2) + \alpha_s \sigma_{\mathrm{NLO}}(x,Q^2) + \alpha_s^2 \sigma_{\mathrm{NNLO}}(x,Q^2) + \alpha_s^3 \sigma_{\mathrm{NNNLO}}(x,Q^2) + \ldots$$

- Many observables at LHC characterised by multiple scales Q_i
- Multi-scale problems are affected by perturbative logarithmic corrections $\alpha_s^n \log^m(Q_i/Q_j)$
- When $\alpha_s^n \log^m(Q_i/Q_j) \sim 1$ fixed order PT is no longer justified

WHERE DO LOGARITHMS COME FROM ?

- Real emissions diagrams are singular for soft/collinear emissions
- These singularities are cancelled by virtual counterparts
- Finite logarithmic pieces are left over, e.g.

$$-\alpha_s \int_0^{Q_0} \frac{dE}{E}\Big|_{\rm real} + \alpha_s \int_0^{Q} \frac{dE}{E}\Big|_{\rm virtual} = \alpha_s \int_{Q_0}^{Q} \frac{dE}{E}\Big|_{\rm virtual} = \alpha_s \ln \frac{Q}{Q_0}$$

WHERE DO LOGARITHMS COME FROM ?

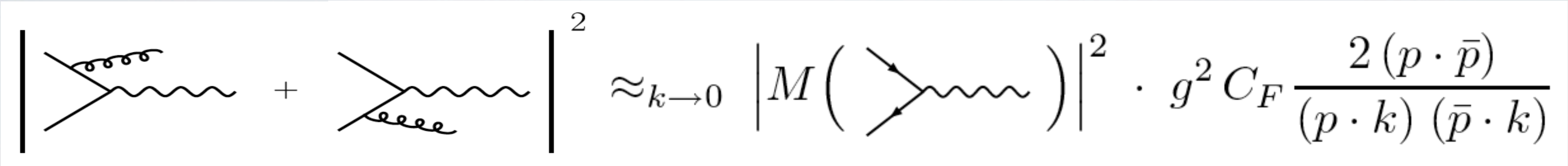
- Real emissions diagrams are singular for soft/collinear emissions
- These singularities are cancelled by virtual counterparts
- Finite logarithmic pieces are left over, e.g.

$$-\alpha_s \int_0^{Q_0} \frac{dE}{E}\Big|_{\mathrm{real}} + \alpha_s \int_0^{Q} \frac{dE}{E}\Big|_{\mathrm{virtual}} = \alpha_s \int_{Q_0}^{Q} \frac{dE}{E}\Big|_{\mathrm{virtual}} = \alpha_s \ln \frac{Q}{Q_0}$$

- This corrections are important for observables that insist on only small deviations from lowest order kinematics (V~0)
- Real radiation is constrained to a small corner of phase space and the logarithms are large
  - event (jet) shapes, e.g. thrust (jet mass): $V=1-T$ ($V= m_{jet}/p_T$)
  - production at threshold: $V=1-M^2/s$
  - transverse momentum: $V= p_T/M$ ...

RESUMMATION: A SKETCH

- All-order calculations are based on factorisation
  - Matrix element factorisation in soft/collinear limit



Born dipole

  - this can be generalised to the multi-gluon case
  - phase space factorisation usually in a conjugate space, e.g.

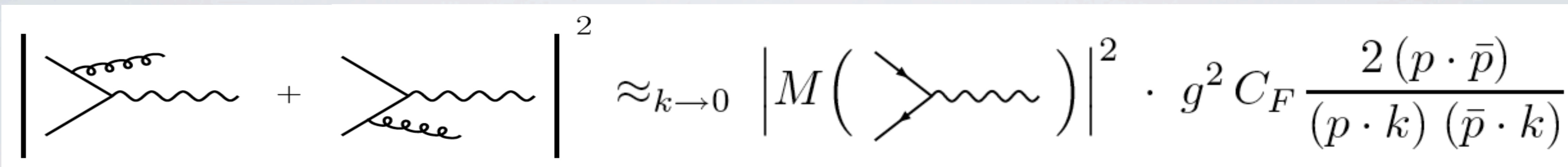
$$\delta^{(2)}\left(\sum_{i=1}^{n} \underline{k}_{Ti} + \underline{Q}_T\right) = \frac{1}{(2\pi)^2}\int d^2\underline{b}e^{i\underline{b}\cdot\underline{Q}_T}\prod_{i=1}^{n} e^{i\underline{b}\cdot\underline{k}_{Ti}}$$

- factorisation then leads to exponentiation

$$\sigma_{res} = g_0 \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots]$$

RESUMMATION: A SKETCH

- All-order calculations are based on factorisation
  - Matrix element factorisation in soft/collinear limit



Born

dipole

- this can be g
- phase sp ce, e.g.

resummation is a systematic re-arrangement of perturbation theory

$\delta^{(2)}$ $\prod e^{ib\cdot k_{Ti}}$

LL

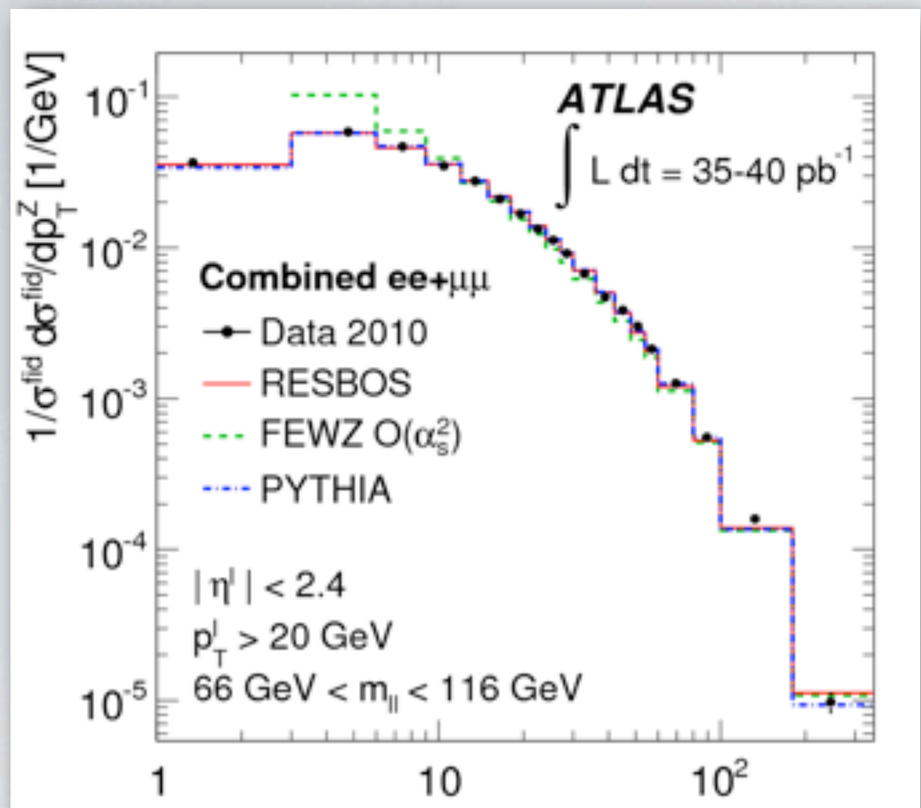
NLL

NNLL

- factorisation then leads to exponentiation

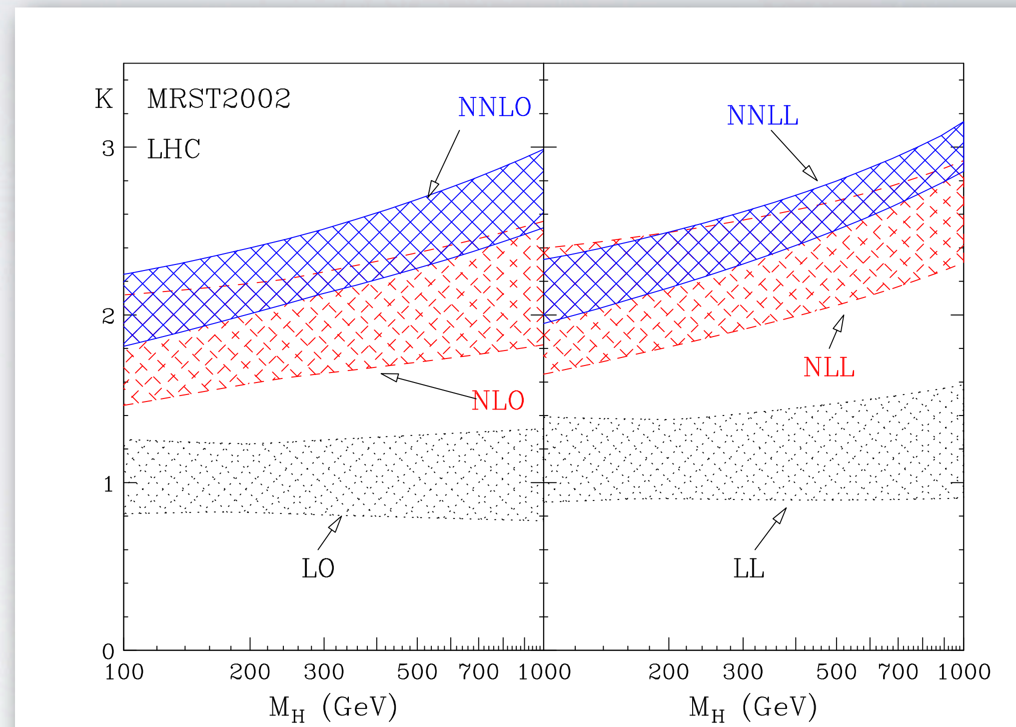
$\sigma_{res} = g_0 \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots]$

RESUMMATION IN ACTION



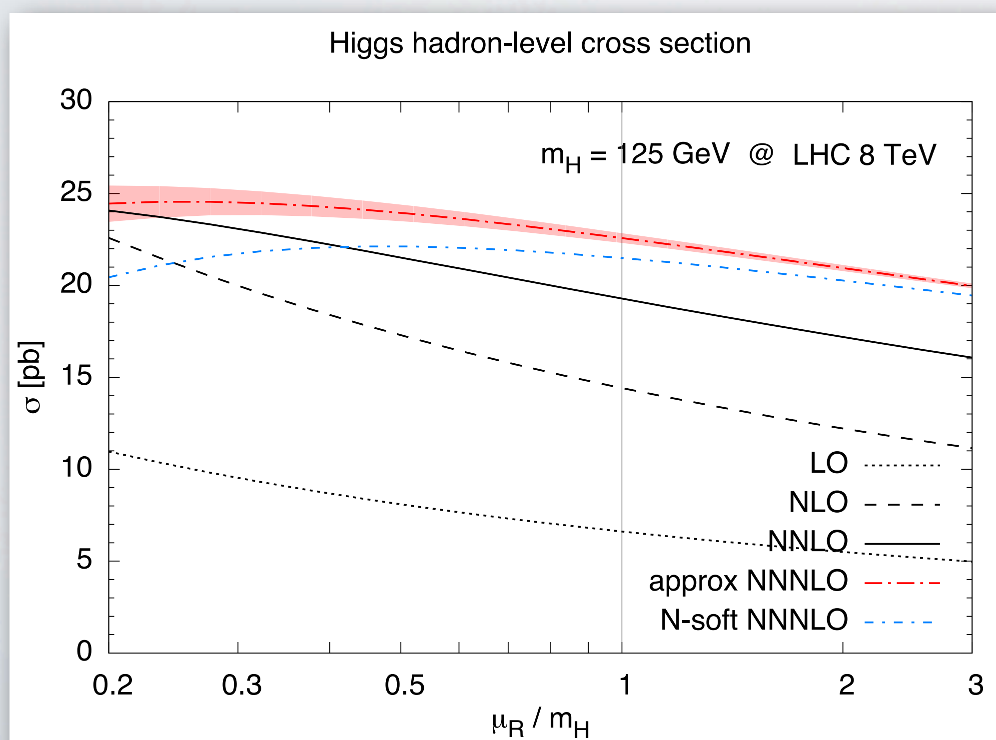
Phys.Lett. B705 (2011) 415-434

1) it's necessary for describing data in particular kinematic limits



Catani, de Florian, Grazzini, Nason (2003)

2) it reduces theoretical uncertainty

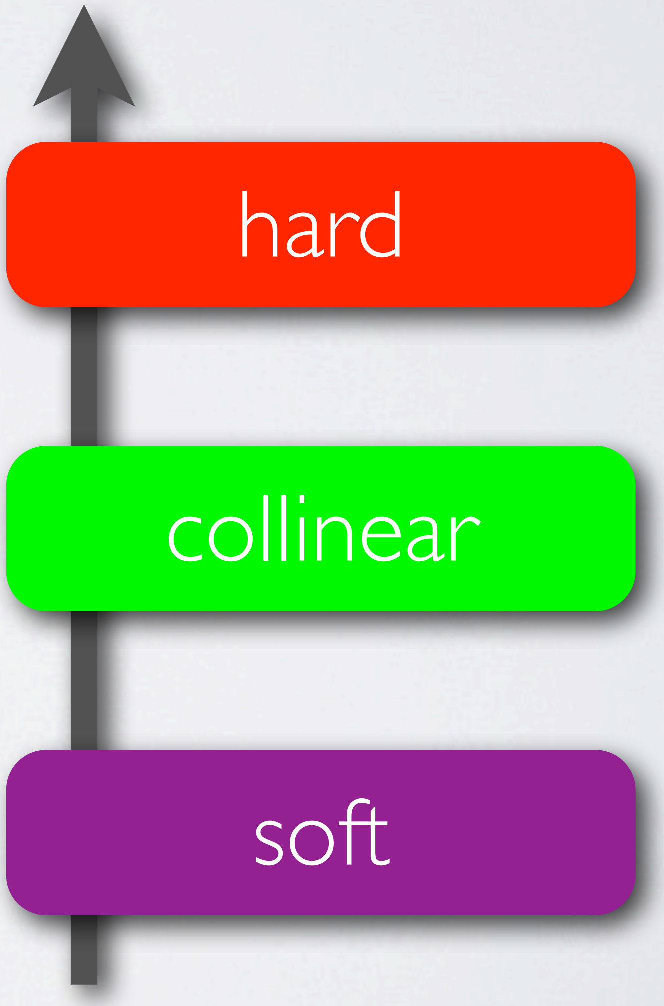


Ball, Bonvini, Forte, SM, Ridolfi (2013)

3) it can be used to approximate higher orders

SOFT COLLINEAR EFFECTIVE THEORY

- Our discussion so far based on factorisation of QCD matrix elements and phase space in the soft/collinear limit (dQCD)
- An alternative framework for resumming large logs is SCET
- In SCET
  - hard modes are integrated out
  - effective Lagrangian for soft & collinear fields
  - separation of scales leads to factorisation
  - resummation is achieved by RG evolution



COMPARING RESUMMATION TECHNIQUES

- dQCD and SCET provide frameworks to approximate full QCD in particular kinematic limits

To all logarithmic orders the answer better be the same, but do they agree to a given log accuracy ?

COMPARING RESUMMATION TECHNIQUES

- dQCD and SCET provide frameworks to approximate full QCD in particular kinematic limits

To all logarithmic orders the answer better be the same, but do they agree to a given log accuracy ?

- Not trivial to establish
- Answers are often given in different forms (moment vs momentum space)
- Log counting often differs between the two communities (and between groups of the same community)
- This resulted into many "lively" discussions

RECENT STUDIES

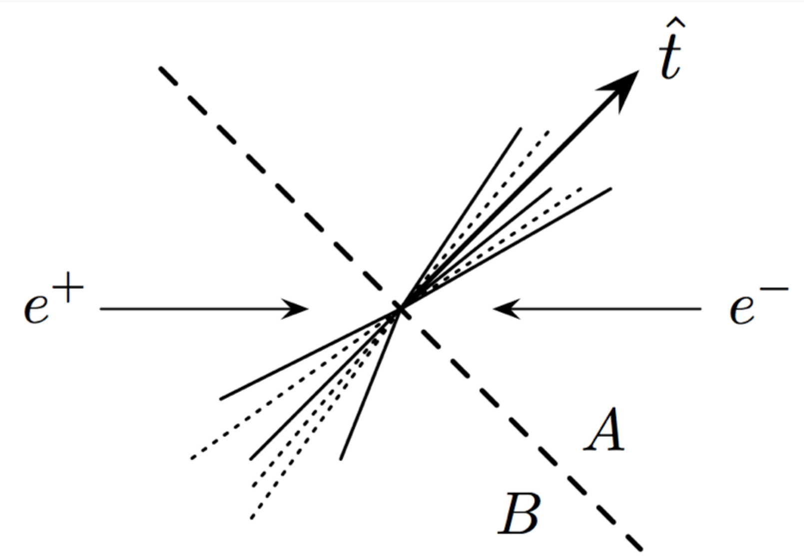
- Most recent example: events shapes in e^+e^-

Almeida, Ellis, Lee, Sterman, Sung, Walsh (2014)

- Work also done in the context of threshold resummation

Bonvini, Forte, Ghezzi, Ridolfi (2012)

Sterman, Zeng (2013)



$$1 - \tau = T = \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$

- Thrust measures the distance from a 2-jet like event

- We are going to consider the cumulative distribution, i.e.

$$1 - \text{thrust} < \tau$$

A WORKED EXAMPLE

cumulative distribution in dQCD

resummed exponent

$$R(\tau_a) = \mathcal{N}(Q)\,\exp(\bar{E})\,\widehat{T}(\bar{E}')\,\frac{1}{\Gamma(1-\bar{E}')}\,.$$

prefactor: no logs

$\mathcal{N}(Q) = 1 + \alpha_s C_1 + \dots$

multiple-emission effects (differential operator)

Almeida, Ellis, Lee, Sterman, Sung, Walsh (2014)

A WORKED EXAMPLE

cumulative distribution in dQCD

resummed exponent

$$R(\tau_a) = \mathcal{N}(Q)\, \exp(\bar{E})\, \widehat{T}(\bar{E}')\, \frac{1}{\Gamma(1-\bar{E}')}\,.$$

prefactor: no logs

$\mathcal{N}(Q) = 1 + \alpha_s C_1 + \dots$

multiple-emission effects (differential operator)

cumulative distribution in SCET

evolution

hard function

$$R(\tau_a) = \exp(K_H + 2K_J + K_S)\Big(\frac{\mu_H}{Q}\Big)^{\omega_H}\Big(\frac{\mu_J}{Q\tau_a^{1/j_J}}\Big)^{2j_J\omega_J}\Big(\frac{\mu_S}{Q\tau_a}\Big)^{\omega_S} H_2(Q^2,\mu_H)$$

$$\times\, \widetilde{J}\Big(\partial_\Omega + \ln\frac{\mu_J^{j_J}}{Q^{j_J}\tau_a},\mu_J\Big)^2\, \widetilde{S}\Big(\partial_\Omega + \ln\frac{\mu_S}{Q\tau_a},\mu_S\Big)\frac{\exp(\gamma_E\Omega)}{\Gamma(1-\Omega)}\,.$$

jet function

soft function

$$\Omega = 2\omega_J + \omega_S$$

$$\omega_i = -\kappa_i \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\text{cusp}}(\alpha_s(\mu'))$$

Almeida, Ellis, Lee, Sterman, Sung, Walsh (2014)

EQUIVALENCE OF THE RESULTS

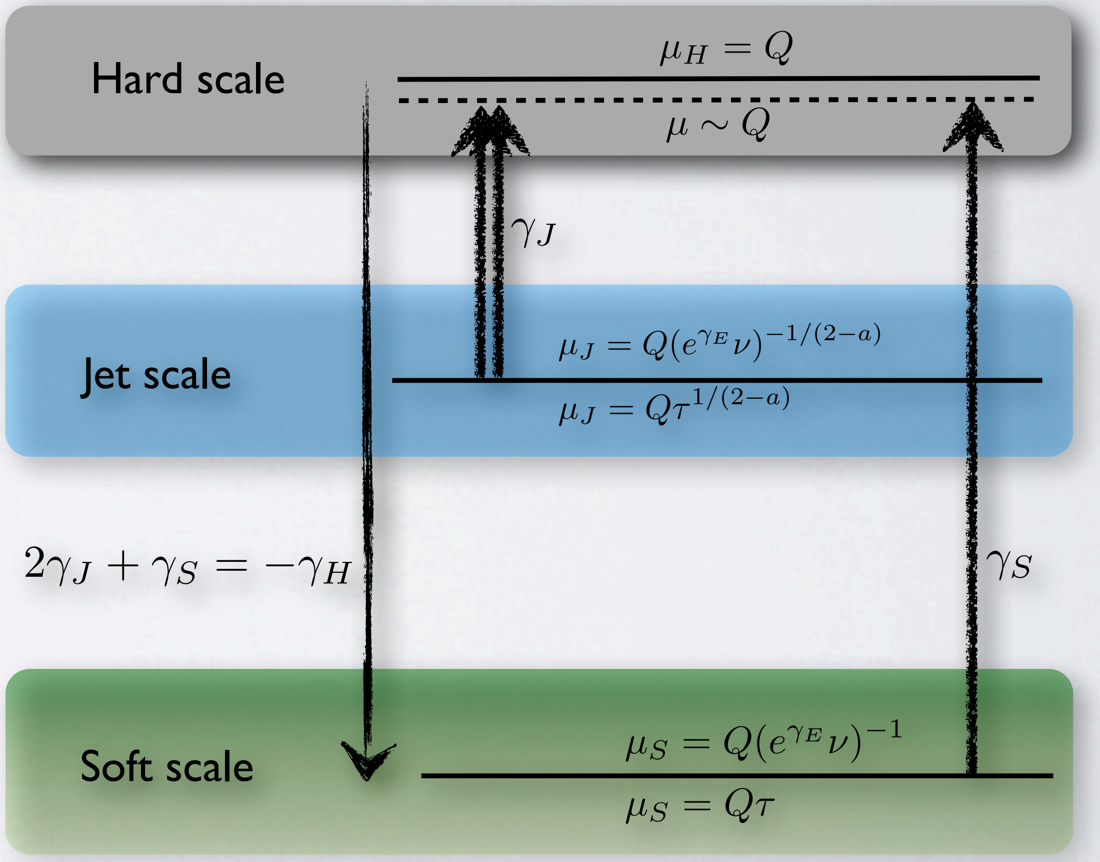
1. The pre-factor in dQCD contains no logs, while the SCET expression does. The difference is beyond the working accuracy, but one can exponentiate all the logs.

2. Scale choice: dQCD expression depends on one scale **μ~Q**, while SCET one on **μ**H, **μ**J and **μ**S.

with the choice

$$\mu_H = Q\,, \quad \mu_J = \bar{\mu}_J \equiv Q\bar{\tau}_a^{1/j_J}\,,$$

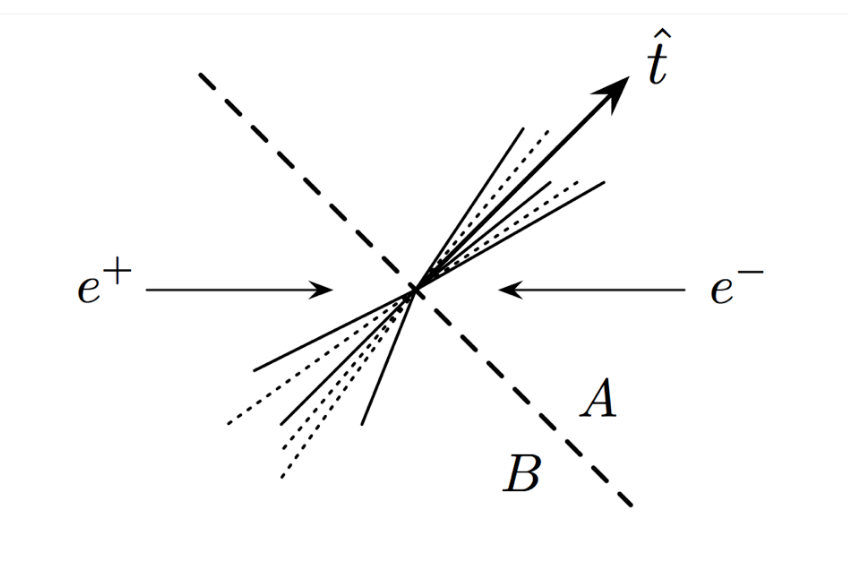
$$\mu_S = \bar{\mu}_S \equiv Q\bar{\tau}_a$$



dQCD and SCET results are completely equivalent !

Almeida, Ellis, Lee, Sterman, Sung, Walsh (2014)

BACK TO PHENOMENOLOGY

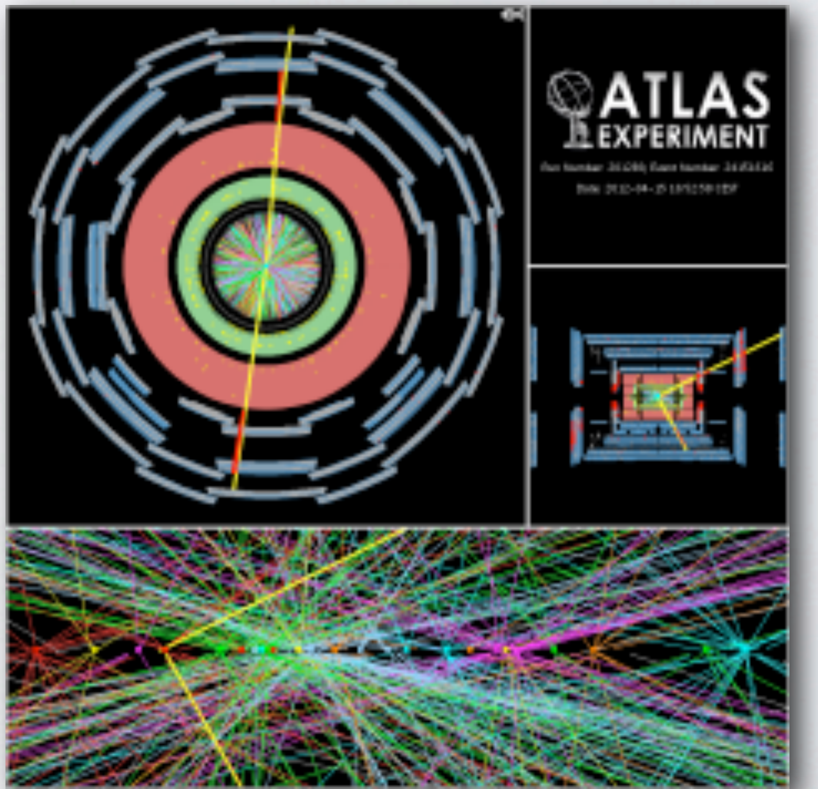


$$1 - \tau = T = \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$

- Event shapes are a powerful tool to study QCD radiation
- We know how to compute them
- They are computed using all particles
- Can we define track-based observables in a meaningful way ?

Advantages in using tracks

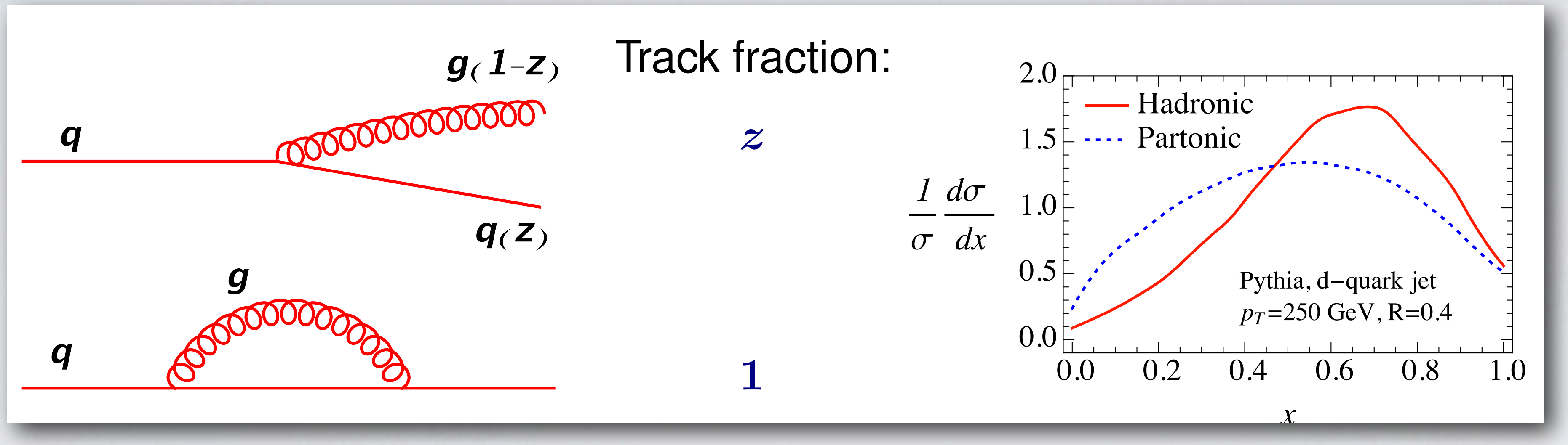
- better vertex reconstruction (for pile-up)
- better angular resolution (for jet substructure)



TRACK-BASED OBSERVABLES

Chang, Procura, Thaler, Waalewijn (2013)

- What should be worry about ? IRC unsafety !



- We have to define track functions (similar to PDFs and FFs)
- $T_i(x,\mu)$: distribution of energy fraction x of parton i converted to tracks

IRC safe

$$\frac{d\sigma}{de} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \delta[e - \hat{e}(\{p_i^\mu\})]$$

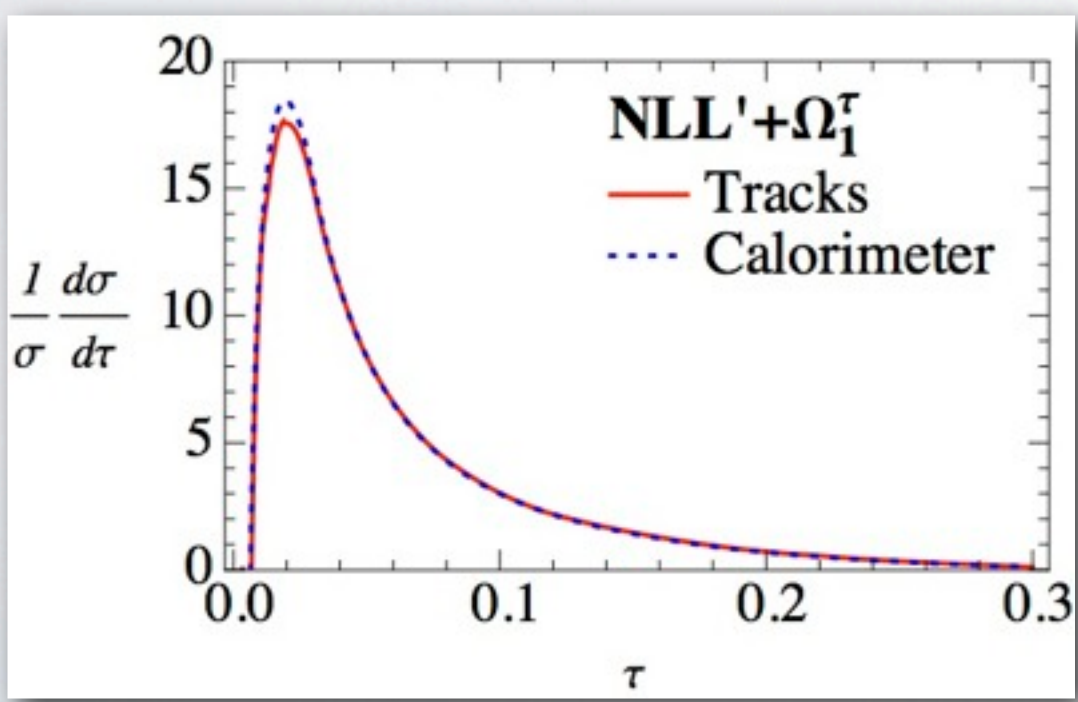
tracks only

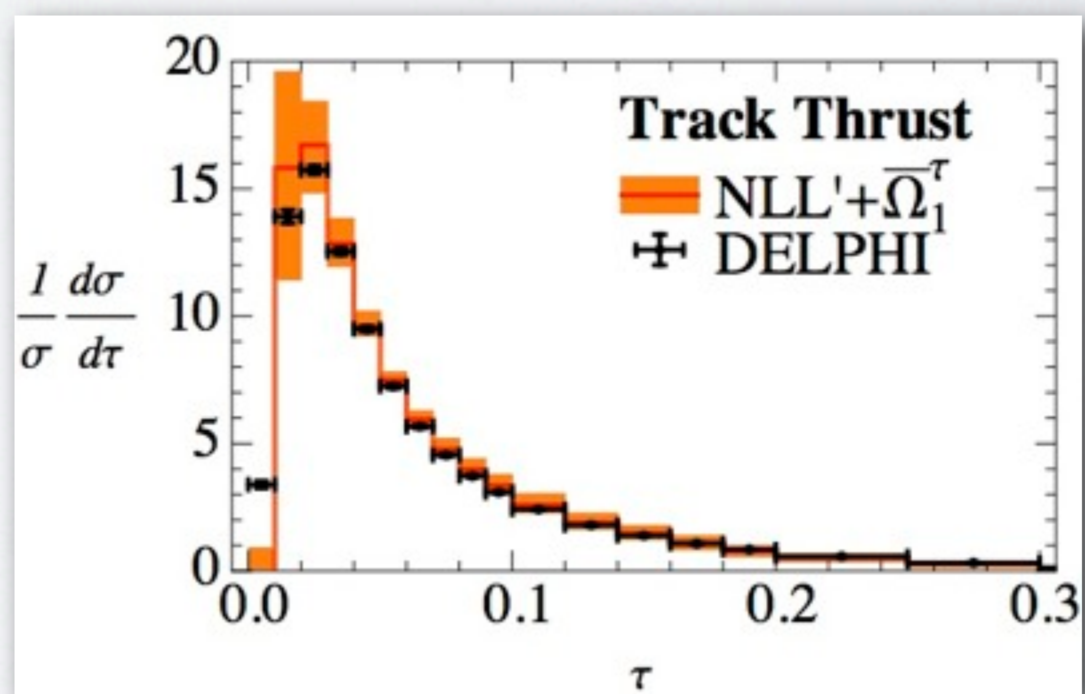
$$\frac{d\sigma}{d\bar{e}} = \sum_N \int d\Pi_N \frac{d\bar{\sigma}_N}{d\Pi_N} \int \prod_{i=1}^{N} dx_i\, T_i(x_i)\; \delta[\bar{e} - \hat{e}(\{x_i p_i^\mu\})]$$

USING TRACK FUNCTIONS

Chang, Procura, Thaler, Waalewijn (2013)

- Similarly to PDFs, track functions can be extracted from data
- In first study they were obtained from MC using dσ/dx (LO and NLO)
- Evolution equation more complicated than DGLAP (multiple convolutions of track functions)
- Resummed calculation for track-trust: very similar to normal thrust (cancellation)
- Good description of DELPHI data





JETS AND THEIR PROPERTIES

- Jets occupy a central role in LHC phenomenology
- The study of their substructure is a rapidly growing field
- Important for searches and QCD measurements
- Resummation: we can re-use a lot of the tools developed for event shapes, with important differences
- Easiest example is the jet mass
- (N)NLL resummation in dQCD & SCET

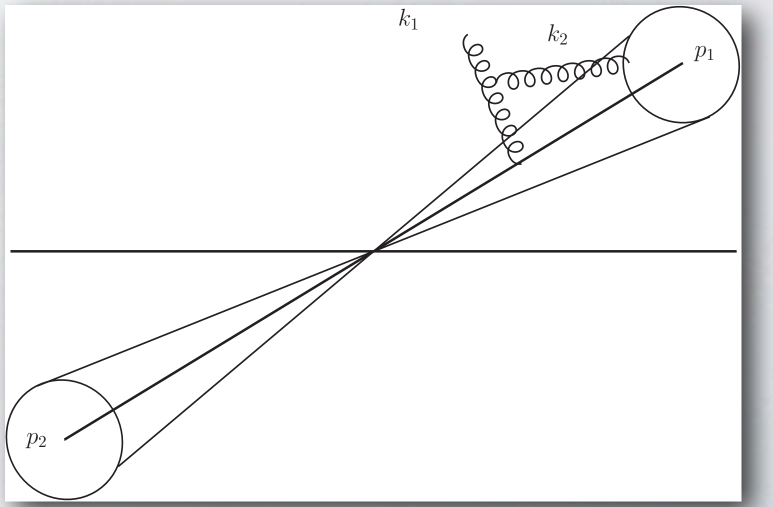
Dasgupta, Khelifa-Kerfa, SM, Spannowsky (2012)
Chien, Kelley, Schwartz and Zhu (2012)
Jouttenus, Stewart, Tackmann and Waalewijn (2013)

constant pre-factor — independent emissions — multiple emissions — correlated emissions

$$\Sigma(\rho) \equiv \frac{1}{\sigma}\int^{\rho} d\,\rho' \frac{d\sigma}{d\rho'} = \mathcal{N}(\alpha_s(p_T))e^{-D(\rho)} \cdot \frac{e^{-\gamma_E D'(\rho)}}{\Gamma(1+D'(\rho))} \cdot \mathcal{S}(\rho)$$

$$\rho = \frac{m_j^2}{p_t^2 R^2}$$

dependence on the jet algorithm & non-global logs: difficult to resum

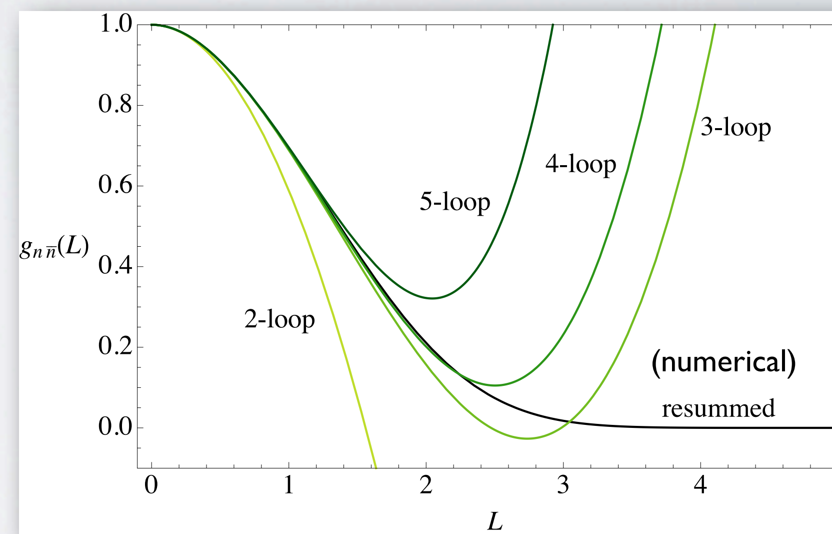


NEW INSIGHTS INTO OLD PROBLEMS

- Numerical resummation of non-global logs at large N_c :
  - Monte Carlo implementation — Dasgupta, Salam (2001)
  - non-linear evolution equation (BMS) — Banfi, Marchesini, Smye (2002)

$$\partial_L g_{\bar{n}b}(L) = \frac{1}{4\pi}\int_0^1 d\cos\theta_j \int_0^{2\pi} d\phi_j \frac{(\bar{n}b)}{(\bar{n}j)(jb)}\left[e^{L(r_{\bar{n}b}-r_{\bar{n}j}-r_{jb})} g_{\bar{n}j}(L) g_{jb}(L) - g_{\bar{n}b}(L)\right]$$

- Iterative solution can be obtained analytically
  - internal symmetries of the equation
  - use of GPLs and symbols to perform polar integrals — Schwartz, Zhu (2014)



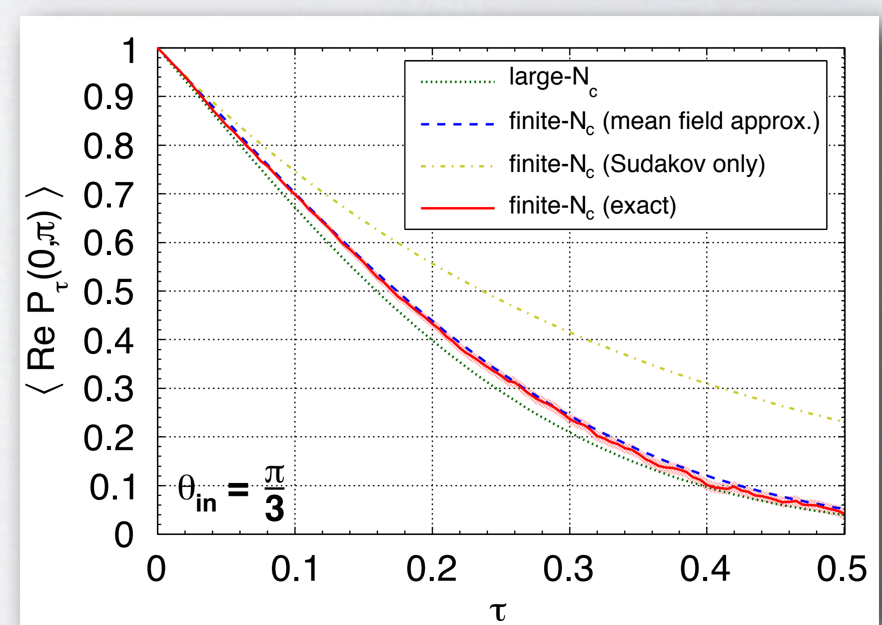
- and finite N_c !

	NGLs		small-x
large N_c	BMS	⟷	BK
	?		
finite N_c	Weigert	⟷	JIMWLK

numerical solution of associate JIMWLK eq.

Weigert (2004)

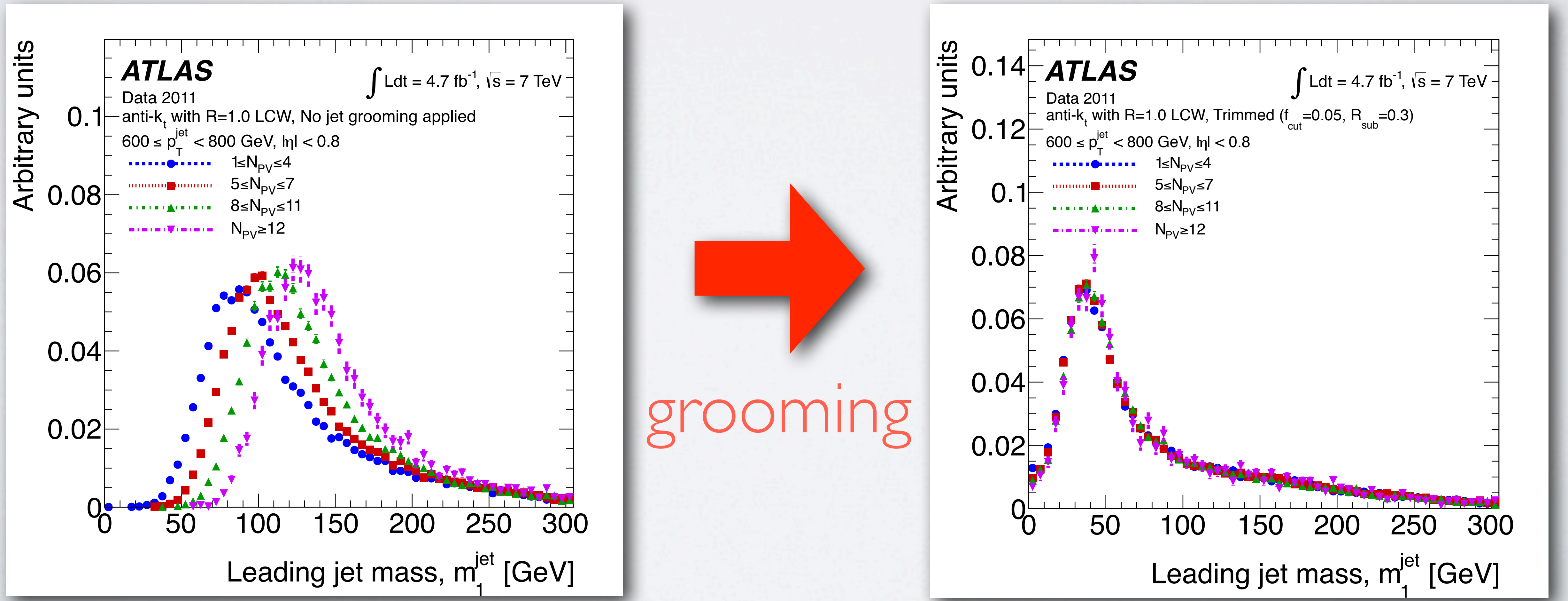
Hatta, Ueda (2013)



GROOMING AND TAGGING

- LHC energy (10^4 GeV) $\gg$ electro-weak scale (10^2 GeV)
- Hadronic decays of boosted particles reconstructed in fat jets
- Exploit jet substructure to distinguish signal form bkgd jets

- Grooming and Tagging:
  1. clean the jets up by removing soft junk
  2. identify the features of hard decays and cut on them
- Grooming provides a handle on UE and pile-up



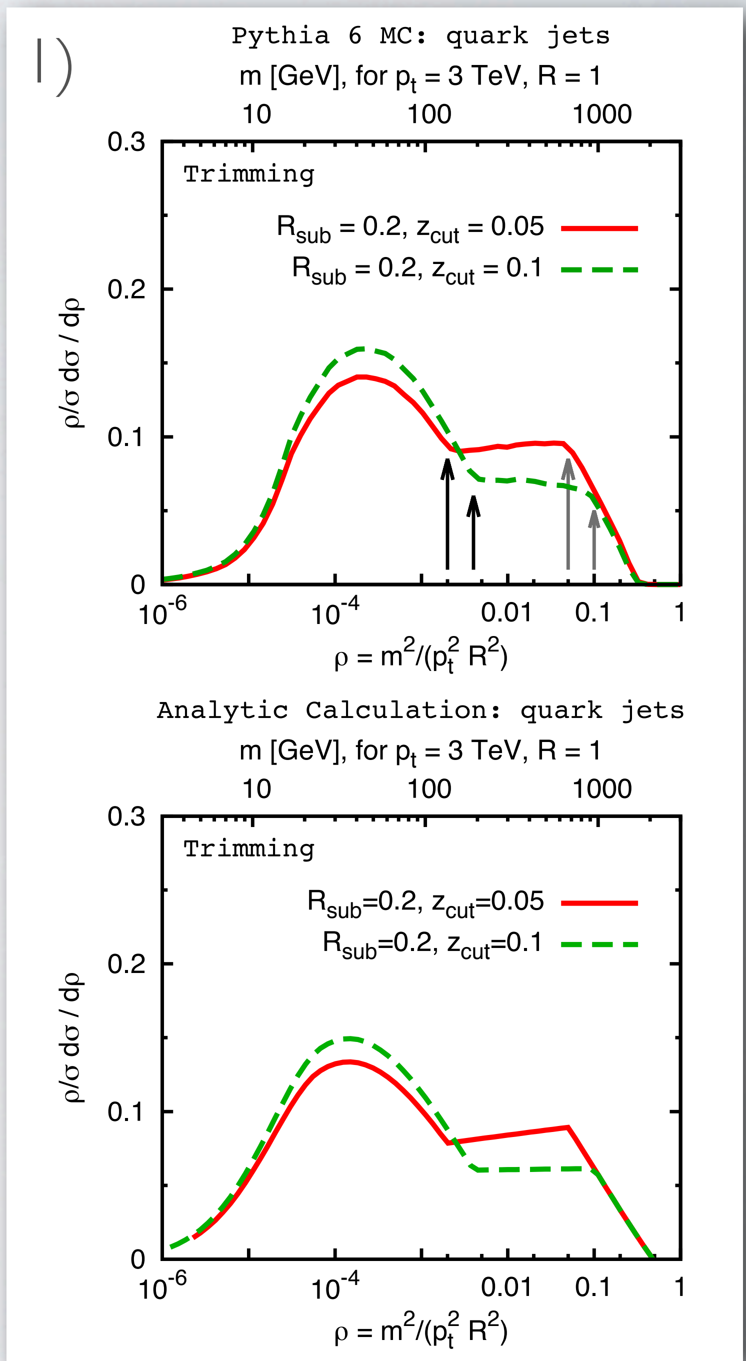
ATLAS, JHEP 1309 (2013) 076

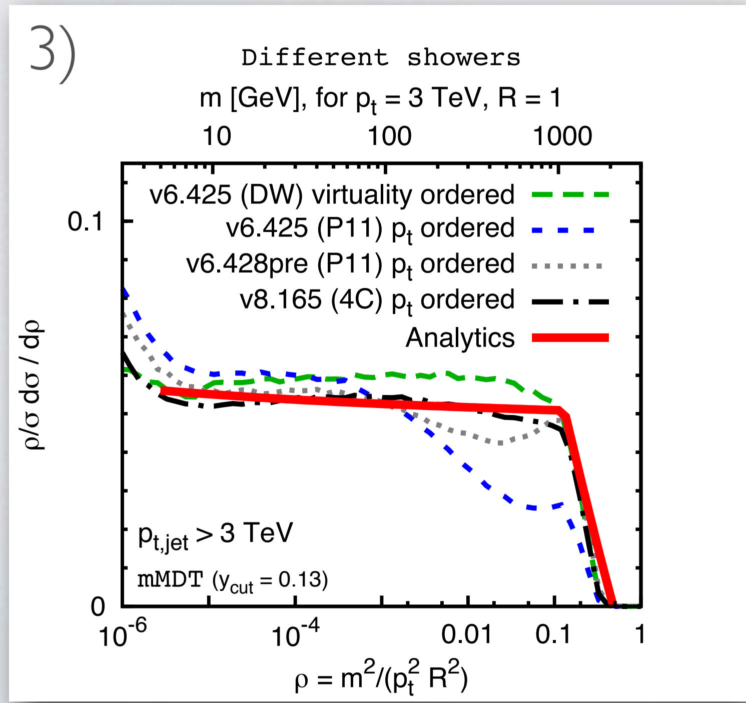
ANALYTIC UNDERSTANDING

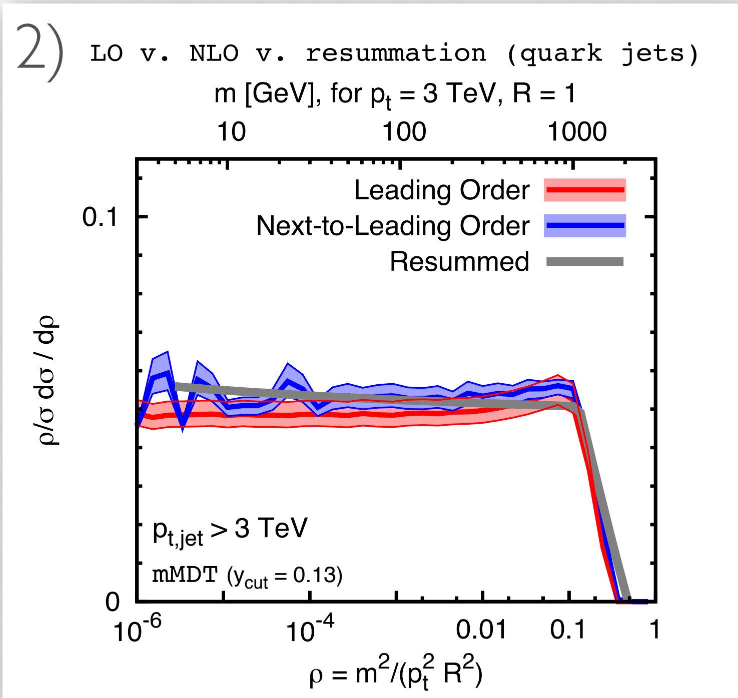
- Grooming / tagging algorithm are fairly complex
- Studies until recently purely based on MCs
- First analytic understanding of groomed jet masses (based on resummation)

1. explanation of features and properties
2. development of better tools
3. checks on MC parton showers

Dasgupta, Fregoso, SM, Salam, (Powling) (2013)





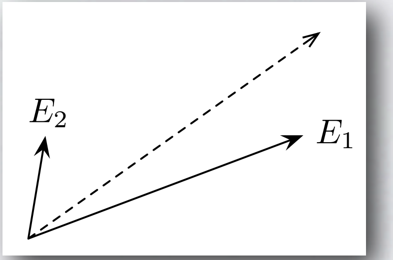


PROPERTIES OF JETS

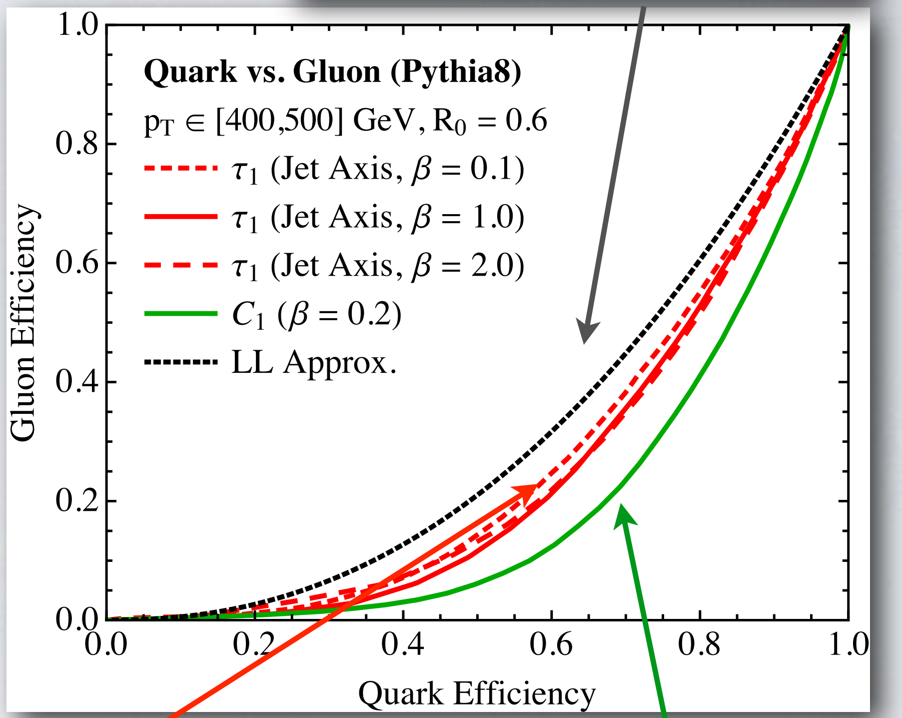
- Angularities & energy correlation functions

$$\tau_1^{(\beta)} = \frac{E_2 E_1^\beta}{(E_1+E_2)^\beta}(\theta_{12})^\beta + \frac{E_1 E_2^\beta}{(E_1+E_2)^\beta}(\theta_{12})^\beta$$

$$C_1^{(\beta)} = \frac{\mathrm{ECF}(2,\beta)}{\mathrm{ECF}^2(1,\beta)} = \frac{E_2(\theta_{12})^\beta}{E_1}$$



LL: all the same $x^{CA/CF}$



recoil sensitive

recoil free: better

- NLL analysis
- Different sensitivity to recoil for β<1
- ECFs more sensitive to collinear splittings, hence better q/g discriminants

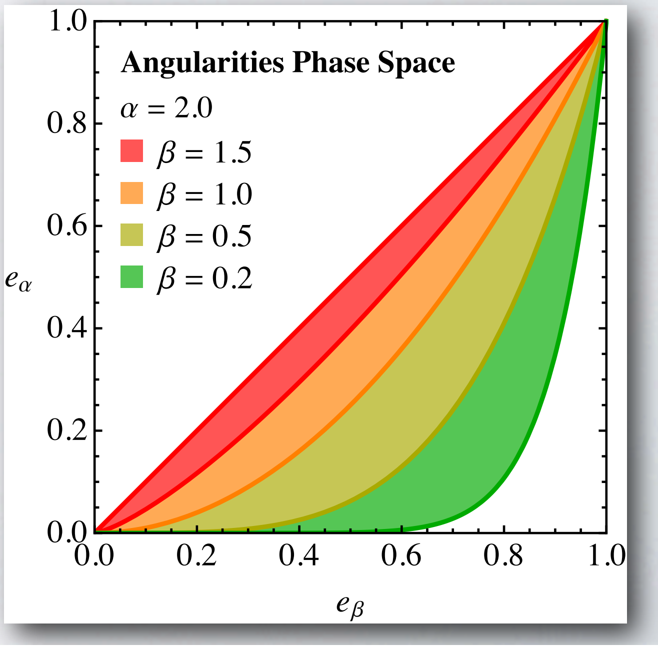
Larkoski, Salam, Thaler (2013)

- Study of jet shapes and angularities with different axis to minimise sensitivity to recoil

Larkoski, Neil, Thaler (2013)

- Analysis of the factorisation properties of double differential distributions

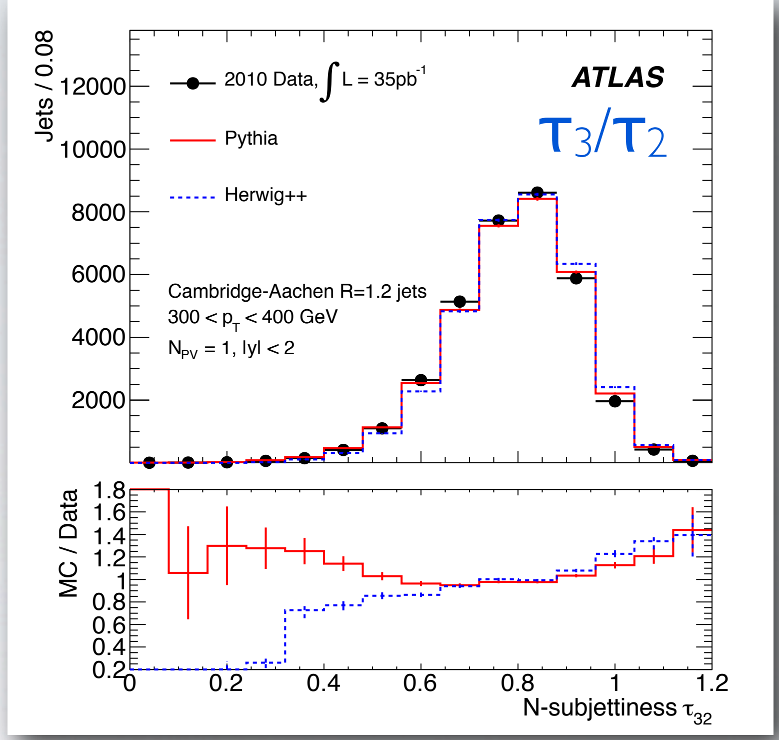
Larkoski, Moult, Neil (2013)



RATIO OBSERVABLES: UNSAFE ...

- *N*-subjettiness aims to identify the number of subjets in a jet
- The ratio $\tau_{23} = \tau_3/\tau_2$ is used as a top tagger
- τ_3 and τ_2 are IRC safe but τ_{23} is not !

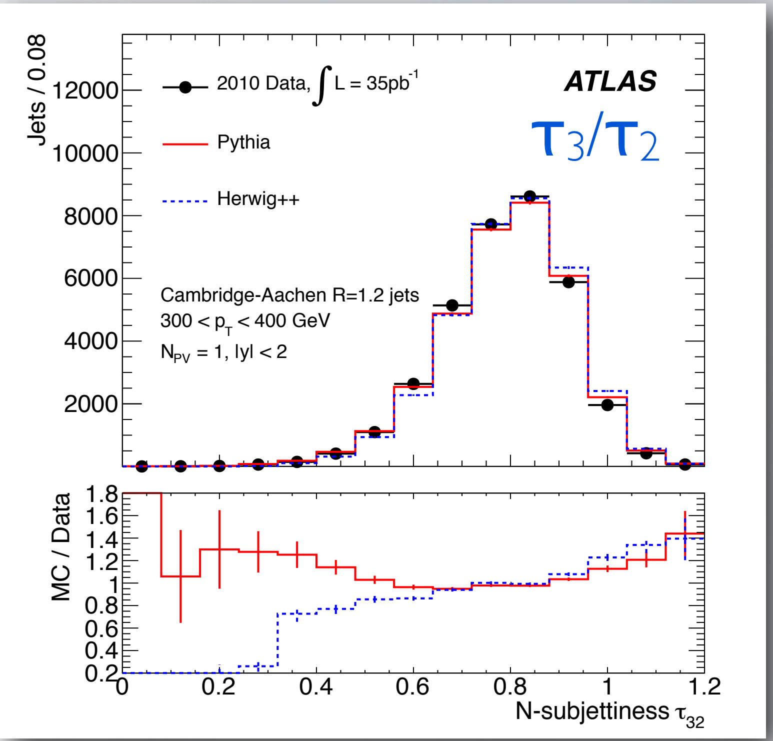
Soyez, Salam, Kim, Dutta, Cacciari (2013)



- τ_{23} is defined order by order in PT only with a cut $\tau_2 > \tau_{cut}$

RATIO OBSERVABLES: UNSAFE ... BUT ...

- *N*-subjettiness aims to identify the number of subjets in a jet
- The ratio $\tau_{23} = \tau_3/\tau_2$ is used as a top tagger
- τ_3 and τ_2 are IRC safe but τ_{23} is not !



Soyez, Salam, Kim, Dutta, Cacciari (2013)

- τ_{23} is defined order by order in PT only with a cut $\tau_2 > \tau_{cut}$
- Let's consider two generic jet angularities

at fixed order

$$\frac{d\sigma^{\rm LO}}{dr} = \int_0^1 de_\beta \int_0^1 de_\alpha \, \frac{d^2\sigma^{\rm LO}}{de_\alpha \, de_\beta} \, \delta\left(r - \frac{e_\alpha}{e_\beta}\right)$$

$$= \int_0^{r^{\frac{\beta}{\alpha-\beta}}} de_\beta \, e_\beta \, \frac{d^2\sigma^{\rm LO}}{de_\alpha \, de_\beta}\bigg|_{e_\alpha = r e_\beta} .$$

singular when

$e_\beta \to 0$

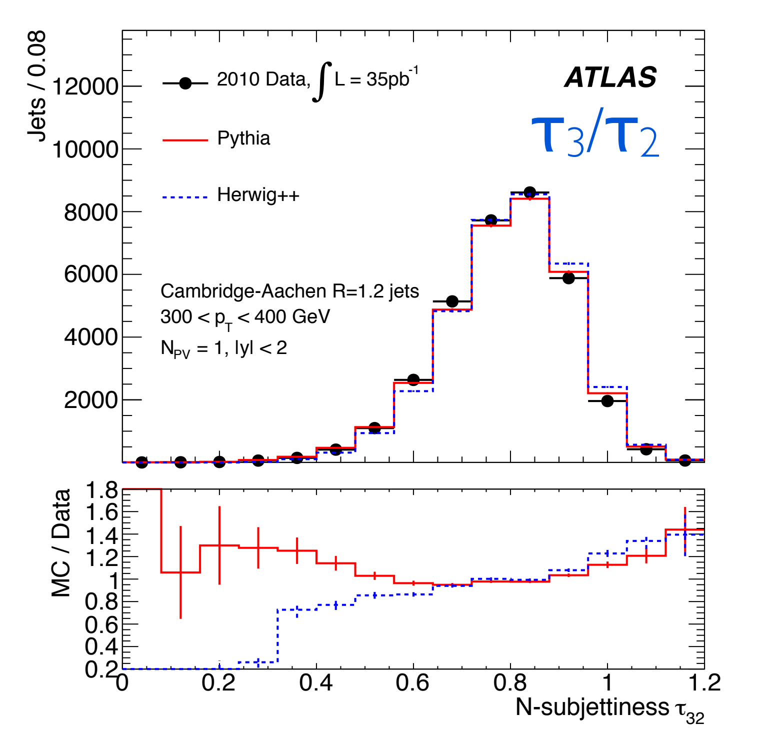
IRC unsafe

Larkoski, Thaler (2013)

RATIO OBSERVABLES: UNSAFE ... BUT ... CALCULABLE

- *N*-subjettiness aims to identify the number of subjets in a jet
- The ratio $\tau_{23} = \tau_3/\tau_2$ is used as a top tagger
- τ_3 and τ_2 are IRC safe but τ_{23} is not !

Soyez, Salam, Kim, Dutta, Cacciari (2013)



- τ_{23} is defined order by order in PT only with a cut $\tau_2 > \tau_{\text{cut}}$
- Let's consider two generic jet angularities

to all orders

$$\frac{d\sigma^{\cancel{\text{LO}}\ \text{LL}}}{dr} = \int_0^1 de_\beta \int_0^1 de_\alpha \frac{d^2\sigma^{\cancel{\text{LO}}\ \text{LL}}}{de_\alpha\, de_\beta}\, \delta\left(r - \frac{e_\alpha}{e_\beta}\right)$$

$$= \int_0^{r^{\frac{\beta}{\alpha-\beta}}} de_\beta\, e_\beta \left.\frac{d^2\sigma^{\cancel{\text{LO}}\ \text{LL}}}{de_\alpha\, de_\beta}\right|_{e_\alpha = r e_\beta} .$$

finite when

$$e_\beta \longrightarrow 0$$

IRC unsafe

but Sudakov safe

- Sudakov suppression acts as a cut-off
- Expansion in fractional powers of α_s

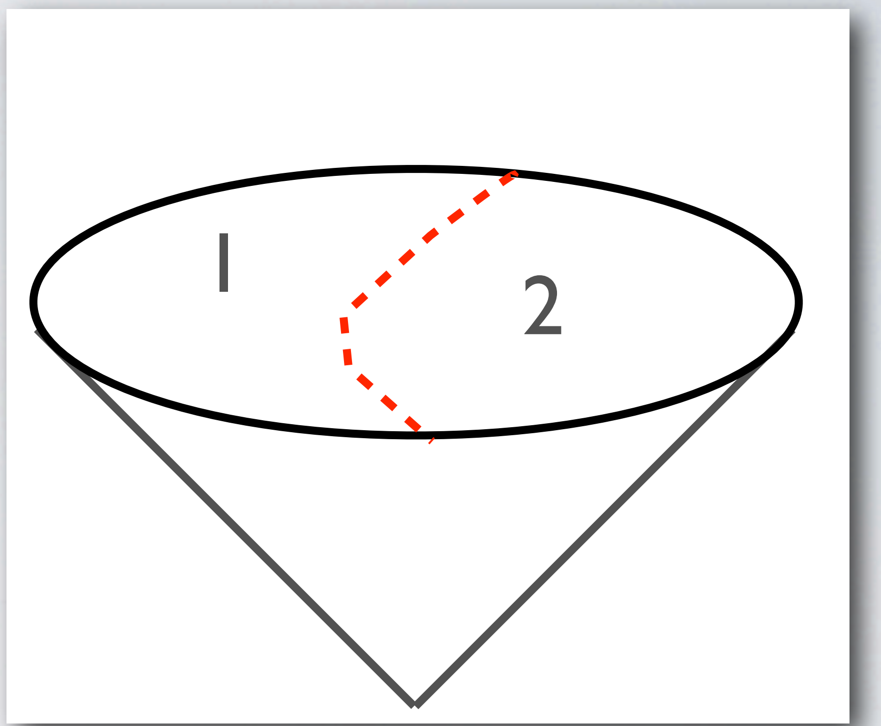
Larkoski, Thaler (2013)

SUDAKOV SAFETY: ANOTHER EXAMPLE

Larkoski, SM, Soyez Thaler (2014)

- Soft Drop: recursive de-clustering of a jet that checks

$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\rm cut} \left(\frac{\Delta R_{12}}{R_0}\right)^{\beta}$$



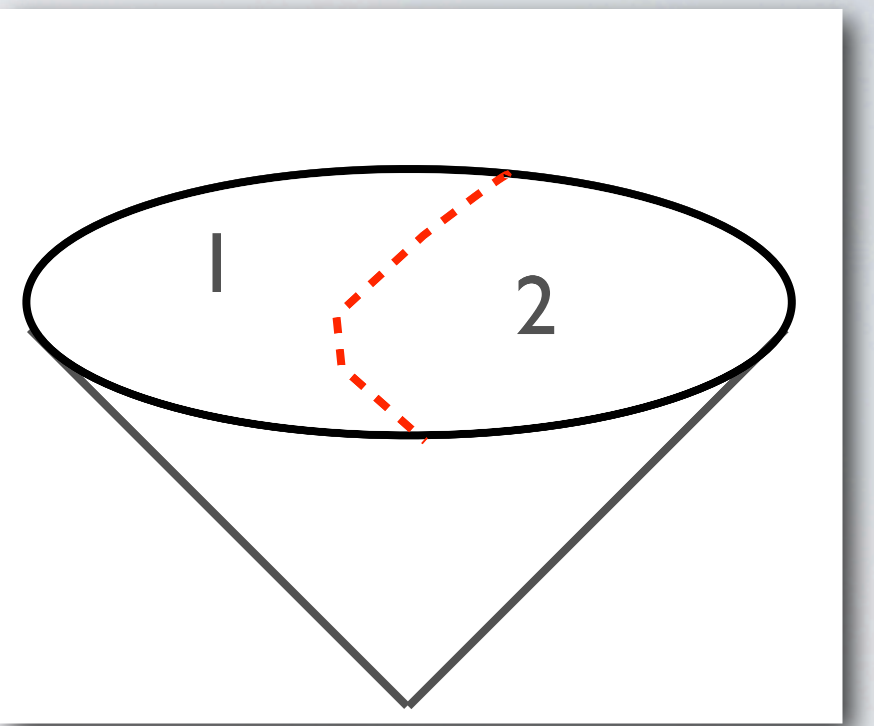
- What is the amount of energy which has been groomed away ?

SUDAKOV SAFETY: ANOTHER EXAMPLE

Larkoski, SM, Soyez Thaler (2014)

- Soft Drop: recursive de-clustering of a jet that checks

$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\rm cut} \left(\frac{\Delta R_{12}}{R_0} \right)^{\beta}$$



- What is the amount of energy which has been groomed away ?
- Not IRC safe for β=0

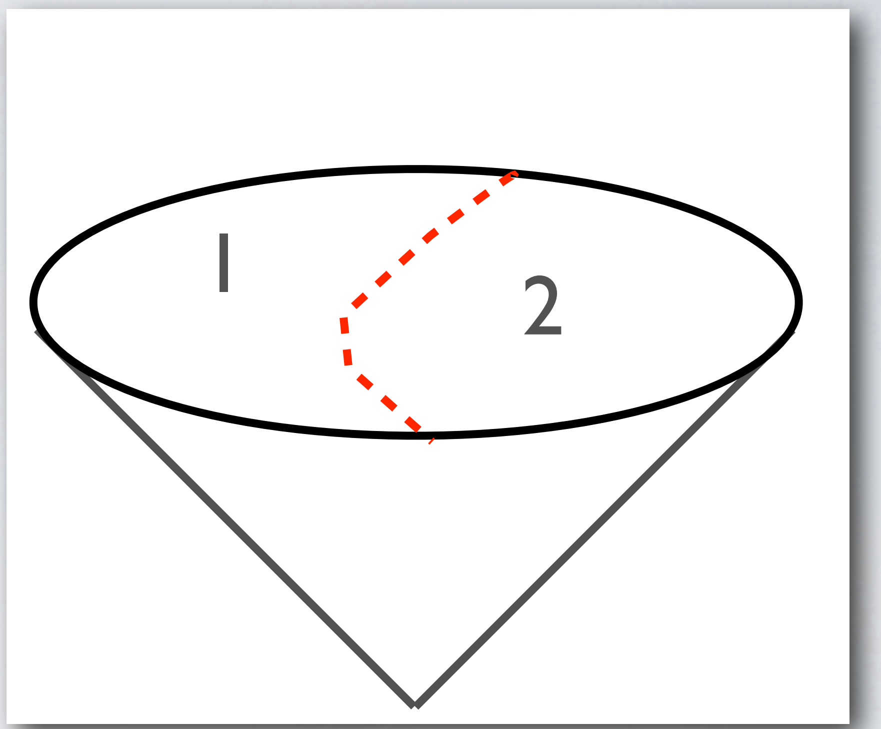
$$\Sigma^{\text{energy-drop}}(\Delta_E) = 1 - \frac{\alpha_s}{\pi} \frac{C_i}{\beta} \log^2 \frac{z_{\rm cut}}{\Delta_E} + \mathcal{O}\left(\left(\frac{\alpha_s}{\beta} \right)^2 \right)$$

SUDAKOV SAFETY: ANOTHER EXAMPLE

Larkoski, SM, Soyez Thaler (2014)

- Soft Drop: recursive de-clustering of a jet that checks

$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\rm cut} \left(\frac{\Delta R_{12}}{R_0}\right)^{\beta}$$

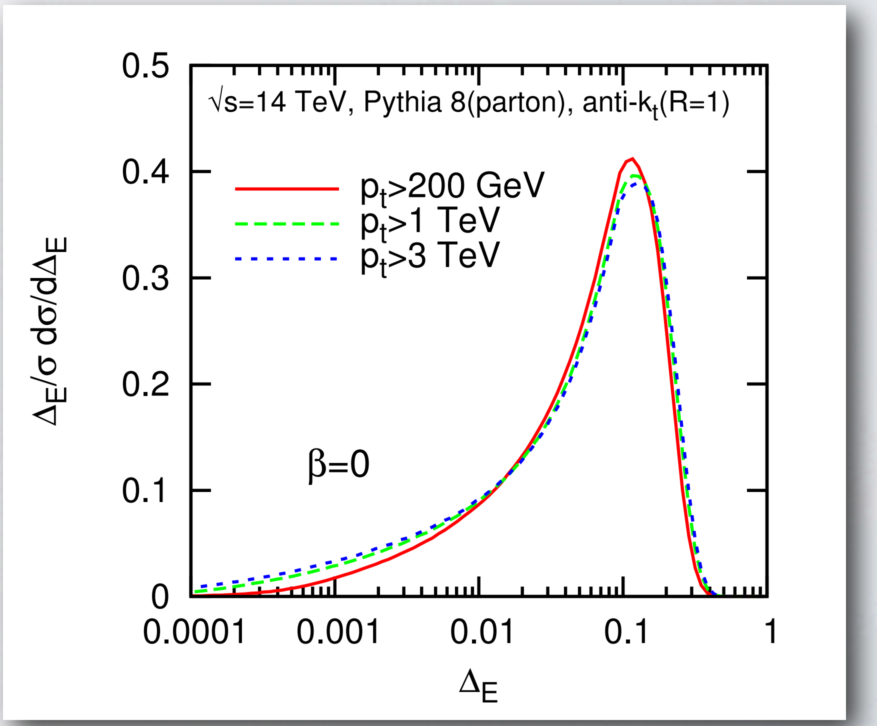


- What is the amount of energy which has been groomed away ?
- Not IRC safe for β=0

$$\Sigma^{\rm energy\text{-}drop}(\Delta_E) = 1 - \frac{\alpha_s}{\pi}\frac{C_i}{\beta}\log^2\frac{z_{\rm cut}}{\Delta_E} + \mathcal{O}\left(\left(\frac{\alpha_s}{\beta}\right)^2\right)$$

- Compute to all orders and then take β=0

$$\Sigma^{\rm energy\text{-}drop}(\Delta_E)_{\beta=0} = \frac{\log z_{\rm cut} - B_i}{\log \Delta_E - B_i}$$



finite result which does not depend on α_s
(at fixed coupling)

THINGS I LEFT OUT
(but covered in this conference)

- Higgs p_T and jet p_T (jet veto) resummation

  see talks by S. Forte and F. Tackmann

- (N)NLO + parton shower

  see talk by S. Prestel

- Resummation effects in Drell-Yan p_T and related variables

  see (exp.) talks by L. Perrozzi and M. Lisovyi

THINGS I LEFT OUT
(not covered in this conference)

- Threshold resummation for heavy particles (tops, stops, *etc.*)

  e,g. Ferroglia, (SM), Pecjak, Yang (2013)
  Broggio, Ferroglia, Neubert, Vernazza, Yang (2013)

- Progress in understanding transverse momentum parton density

  e.g. Gehrmann, Luebbert, Yang (2014)

- Top-pair p_T resummation

  e.g. Zhu, Li, Li, Shao, Yang (2013)

- Forward physics, BFKL, Mueller-Navalet jets

  e.g. Ducloué, Szymanowski, Wallon (2013, 2014)
  Jung, Hautmann et *al.* (Cascade)
  Lipatov, Zotov et *al.* (2013)

and many other interesting papers !

THANK YOU !