Detecting Particles near an AdS Black Hole

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Hawking Radiation

- Black holes emit thermal radiation
- Black holes have a “temperature”
- Hartle-Hawking vacuum contains particles
Hawking Radiation

- Black holes emit thermal radiation
- Black holes have a “temperature”
- Hartle-Hawking vacuum contains particles
- How can we measure the temperature of a black hole?
Measuring Temperature

- Idea: put a two-level system above a black hole
- Model Hawking radiation as a scalar
- Excitation = Detection
- Transition rate should obey thermality condition

\[ \dot{F}(E) = e^{-E/T_{\text{loc}}} \dot{F}(-E) \]
Which Black Hole?

- Schwarzschild
Which Black Hole?

- Schwarzschild

- Done by Hodgkinson, Louko, Ottewill (Phys. Rev. D 89, 104002 (2014))
Schwarzschild Anti-de Sitter Space

Timelike infinity
Schwarzschild Anti-de Sitter Space

- Schwarzschild (uncharged spinless) black hole
- Asymptotically AdS
- Timelike infinity
- Reflective boundary conditions

\[ ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_2^2 \]

\[ f(r) = \frac{r^2}{L^2} + 1 - \frac{r_0}{r}, \quad f(r_+) = 0 \]

\[ T_H = \frac{3r_+^2 + 1}{4\pi r_+} \]
Why SAdS?

- Well-studied system
- Physical importance: AdS/CFT duality
- Conformal coupling similar to Schwarzschild case
- Detector response not done before
The Unruh-Dewitt Detector

\[ H_{int}(\tau) = c\chi(\tau)\mu(\tau)\phi(x(\tau)) \]

- A monopole detector with small coupling constant \( c \), switching function \( \chi(\tau) \)
- Simplest detector has two energy states with gap \( E \)
- First-order transition probability over trajectory:

\[ P(E) = c^2 \left| \langle 0_d | \mu(0) | E_d \rangle \right|^2 F(E) \]
The Response Function $F(E)$

- $F(E)$ is independent of physical details of the detector besides energy gap
- For calculations, can be written in terms of the Wightman function $W(x, x') = \langle \Psi | \phi^\dagger(x) \phi(x') | \Psi \rangle$

\[
F(E) = 2 \lim_{\epsilon \to 0} \text{Re} \int_{-\infty}^{\infty} du \chi(u) \int_{0}^{\infty} ds \chi(u - s)e^{-iEs}W_\epsilon(u, u - s)
\]

\[
W_\epsilon(\tau', \tau'') = W_\epsilon(x(\tau'), x(\tau''))
\]
Special Case: Stationary Trajectory

• In this case, we can integrate over infinite time and take the average, and the regulator can just be taken to zero pointwise.

• Since $W_\varepsilon(u,u-s)=W_\varepsilon(s)$ depends only on $s$, we end up with

$$F(E) = \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} ds \, e^{-iE s} W_\varepsilon(s)$$

$$\dot{F}(E) = \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} ds \, e^{-iE s} W_\varepsilon(s)$$

• Works for static detectors and circular geodesics
Which Vacuum?

- Hartle-Hawking vacuum: black hole in thermal equilibrium with environment, radiation present
- Boulware vacuum: static observers see no particles, no radiation present
Conformal Radial Klein-Gordon Equation

\[ r^* = -\int_r^\infty \frac{dr'}{f(r')} \]

\[ w_{\omega lm} = (4\pi\omega)^{-1/2} e^{-i\omega t} Y_{lm}(\theta, \phi) r^{-1} \Phi_{\omega lm}(r) \]

\[ [\partial_{r^*}^2 + \omega^2 - \tilde{V}(r^*)] \Phi = 0 \]

\[ \tilde{V}(r) = f(r)V(r) \]

\[ V(r) = \frac{l(l + 1)}{r^2} + \frac{r_0}{r^3} \]

\[ R_{\omega lm} = \Phi_{\omega lm}/r \]
Effective Potential

\[ \tilde{V}_l \quad r_+/L = 0.1 \]

\[ \tilde{V}_l(\infty) = l(l + 1) \]

\[ E^2 = T_H^2 \]

\[ l = 1 \]
\[ l = 0 \]
\[ r/r_+ \]
Static Transition Rate

- Dependence on one energy for each angular momentum

\[
\dot{F}_H(E) = \frac{1}{2E} \frac{1}{eE/T_{loc} - 1} \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} R_{\tilde{\omega}l}(r)
\]

\[
\dot{F}_B(E) = \Theta(-E) \frac{1}{2|E|} \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} R_{\tilde{\omega}l}(r)
\]

\[
\tilde{\omega} = \sqrt{f(r)E}, \quad T_{loc} = T_H / \sqrt{f(r)}
\]
Hartle-Hawking vs. Boulware

\[ \frac{r_+}{L} = 0.1, \frac{r}{r_+} = 10 \]
HH Angular Momentum Decomposition

\[ r_+/L = 0.1 \]
Features of Static Transition Rate

- Angular momentum contributions go to zero at certain points: mode at detector is zero
- Peaks: Quasinormal modes ("trapped modes")
- Higher angular momentum contributions are nontrivial at higher detector energies
- Peaks get sharper with higher angular momentum, due to reflective boundary
Effective Potential

\[ \tilde{V}_l \]

\[ r_+/L = 0.1 \]

\[ \tilde{V}_l(\infty) = l(l + 1) \]

\[ \begin{align*}
l &= 1 \\
l &= 0 \\
r/r_+ &
\end{align*} \]
Circular Geodesic Detector

- Solving equations of motion for circular orbit yields

\[ \frac{dt}{d\tau} = \sqrt{\frac{2r}{2r - 3r_0}} \]

\[ \frac{d\phi}{d\tau} = \sqrt{\frac{r_0 + 2r^3}{r^2(2r - 3r_0)}} \]
Hartle-Hawking vs. Boulware

\[ r_+/L = 0.1 \]
Boulware Angular Momentum Decomposition

\[ \frac{r_+}{L} = 0.1 \]
“Steps”

- Contribution nonzero for positive $\omega_- = (mb - E)/a$
- Dropoff is dependent on $l \geq |m|$
- Circular geodesic motion creates excitation regardless of field state
Hartle-Hawking Angular Momentum Decomposition

\[ r_+/L = 0.1 \]
HH Angular Momentum Decomposition

$$r_+/L = 0.01$$
HH Angular Momentum Decomposition

\[ r_+/L = 0.1 \]
Small Black Hole Limit

• When the black hole is very small compared to the AdS length, the spacetime is “almost AdS”
• Higher angular momentum modes contribute less off-peak at energies shown
• Peaks become sharper, and approach AdS normal frequencies

$$\omega L = 2 + l + 2n$$
First Peak Frequency

\[ \omega_R = 2 + l + 2n, \quad l = n = 0 \]
HH Angular Momentum Decomposition

\[ r_+/L = 1 \]
Large(r) Black Hole Limit

- No transition between small and large black holes, since effective potential vanishes at infinity.
- Peaks subsumed by greater exponential decay; not very interesting.
- Boundary conditions mean this is not Schwarzschild-like.
Conclusions

- Radiation is thermal, but not featureless
  - Peaks due to quasinormal resonances
  - Troughs due to zeroes of modes
- Small black holes converge to AdS sharp peaks
- Large black holes have no peaks
- Circular geodesic detectors are excited in either vacuum
Next Steps

- More general trajectories, e.g. radial infall
- More general spacetimes, e.g. SAdS geon
- Multiple detector scenarios
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- Multiple detector scenarios
- Firewalls?
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Thank You!